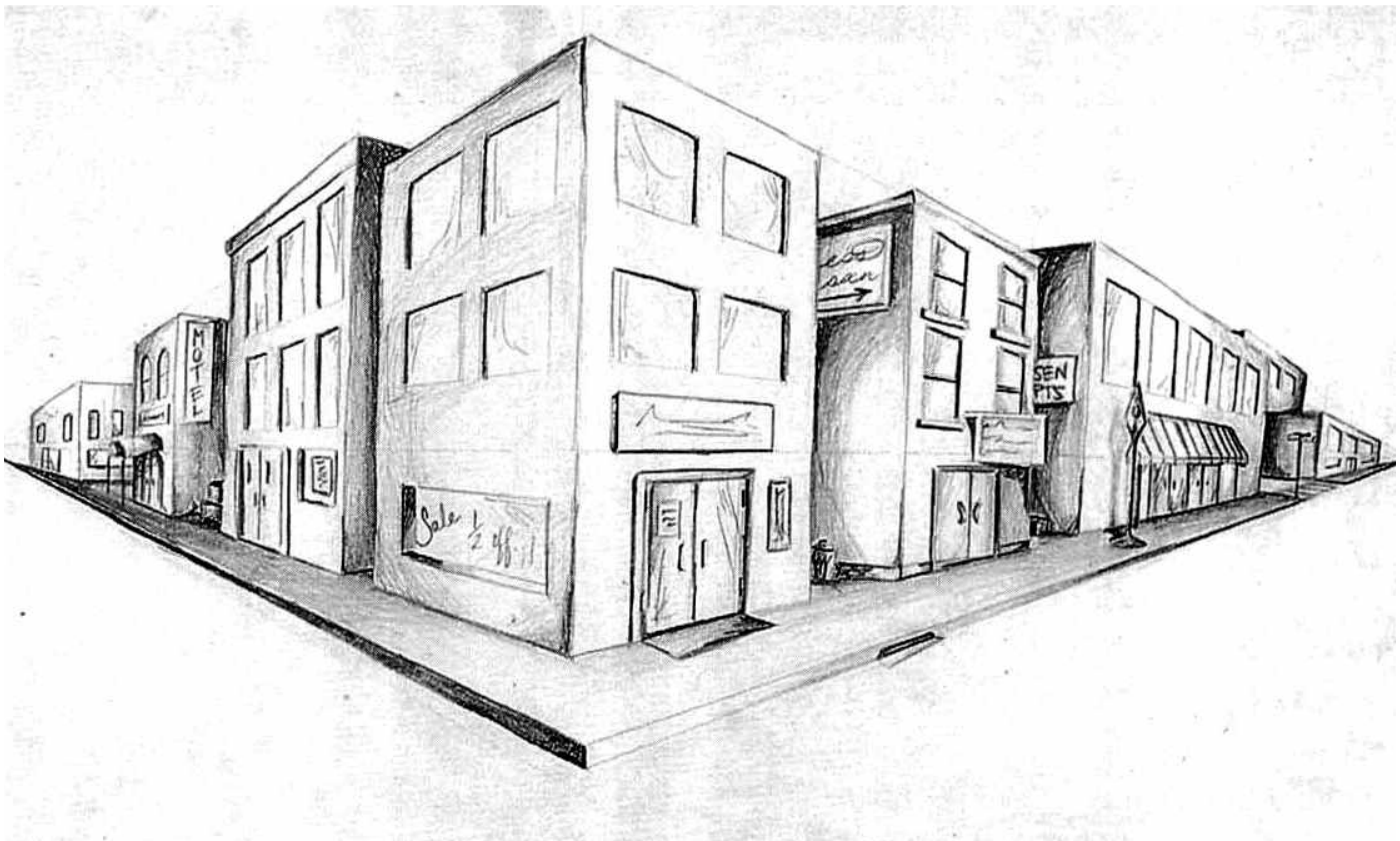
# Detecting corners




16-385 Computer Vision
Fall 2020, Lecture 5

http://16385.cs.cmu.edu/

# Overview of today’s lecture

- Why detect corners?
- Visualizing quadratics.
- Harris corner detector.
- Multi-scale detection.
- Multi-scale blob detection.

# Slide credits

Most of these slides were adapted from:

- Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

- Fredo Durand (MIT).
- James Hays (Georgia Tech).

# Why detect corners?

# Why detect corners?

Image alignment (homography, fundamental matrix)

3D reconstruction

Motion tracking

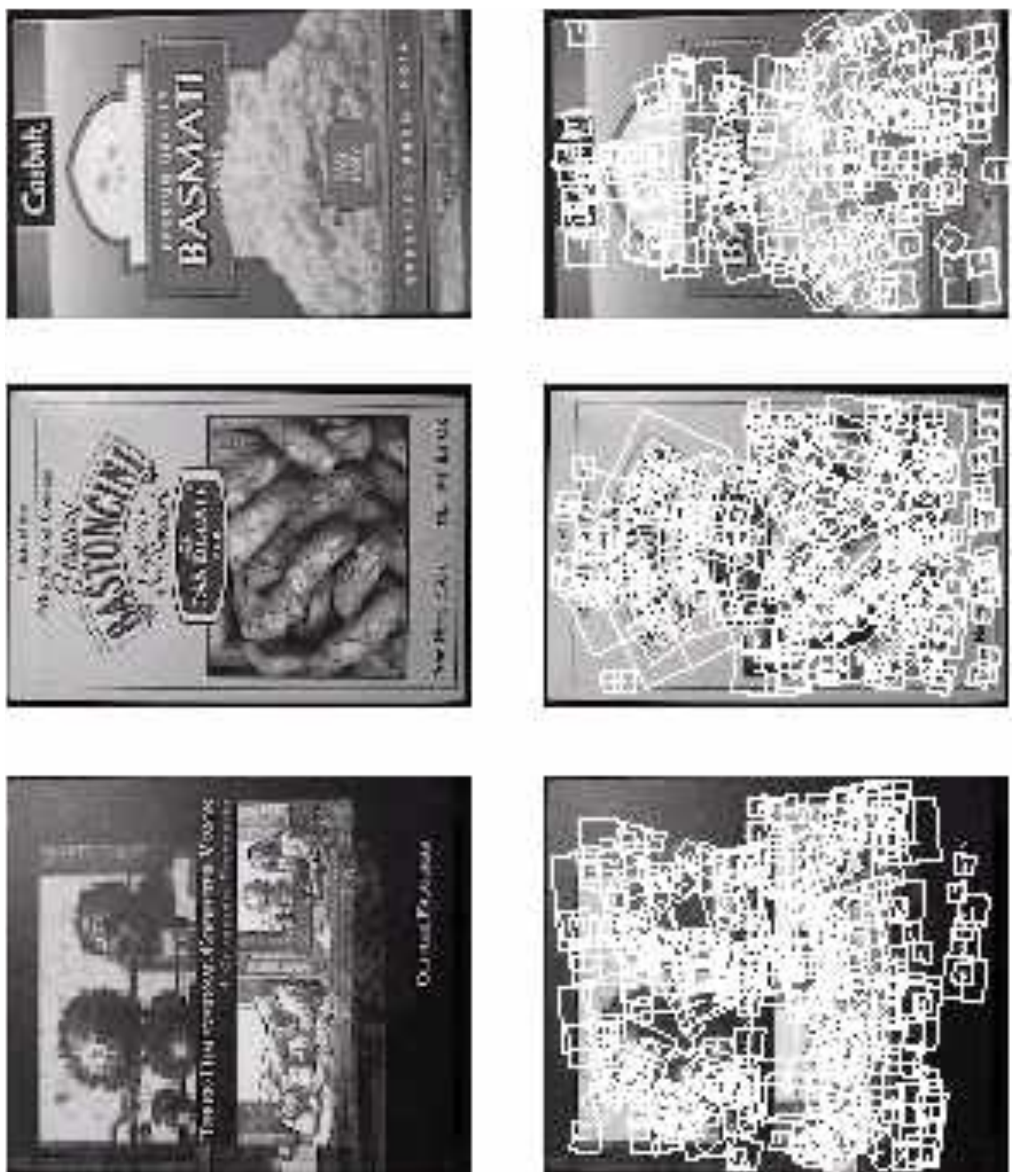
Object recognition

Indexing and database retrieval

Robot navigation

# Planar object instance recognition

Database of planar objects

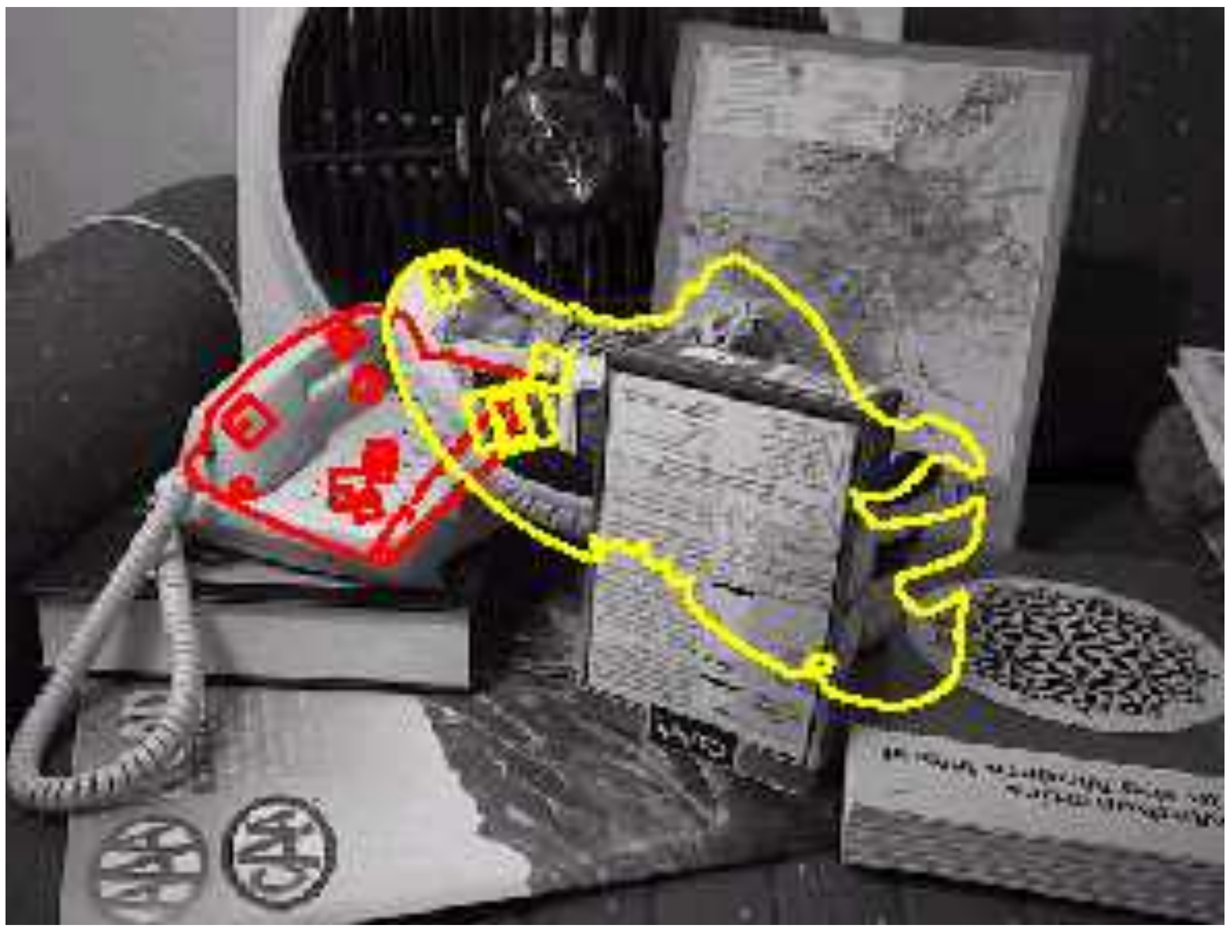
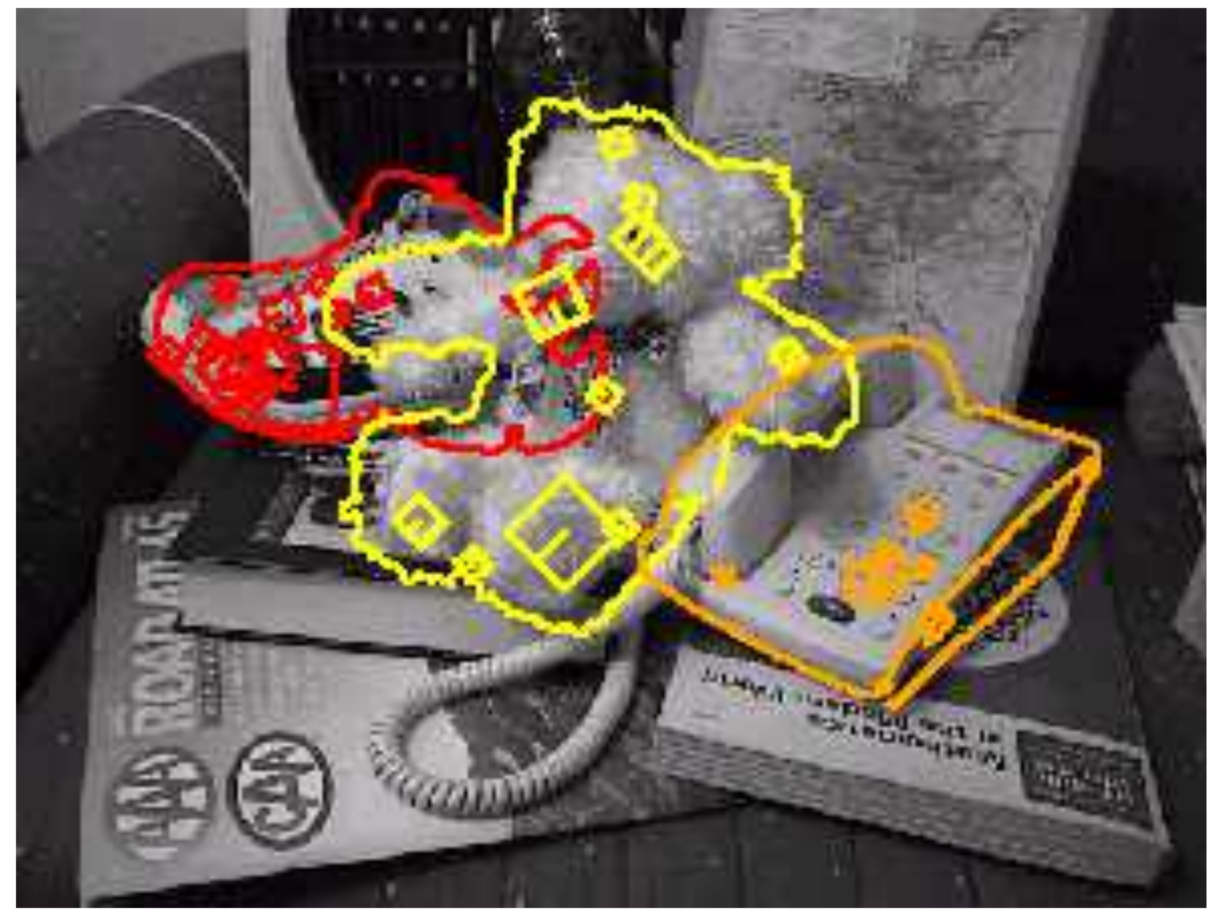


Instance recognition

# 3D object recognition

Database of 3D objects

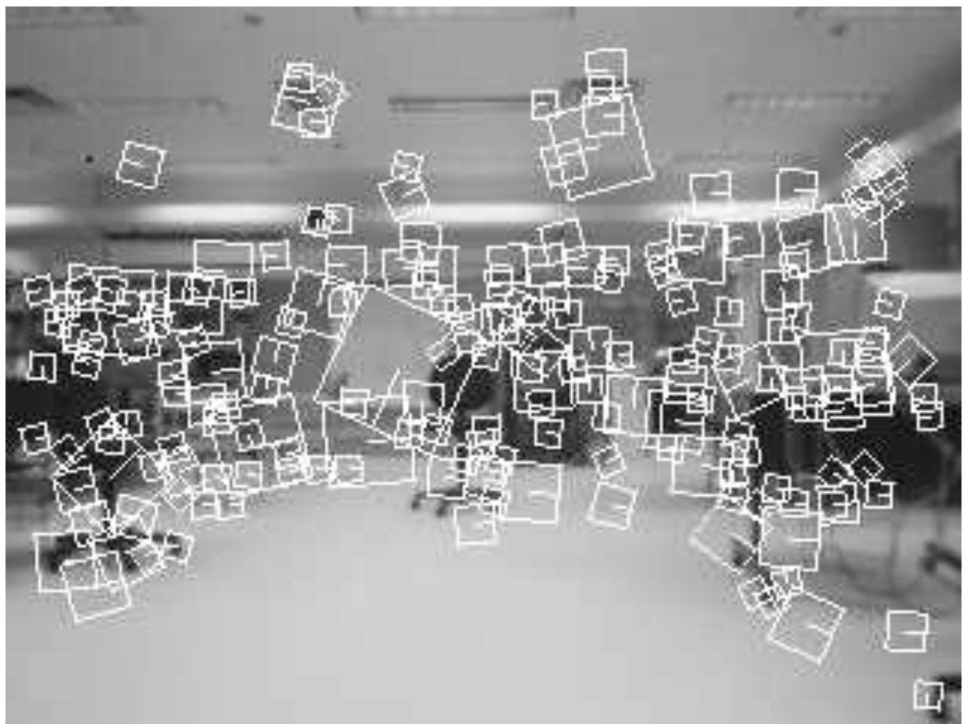
3D objects recognition

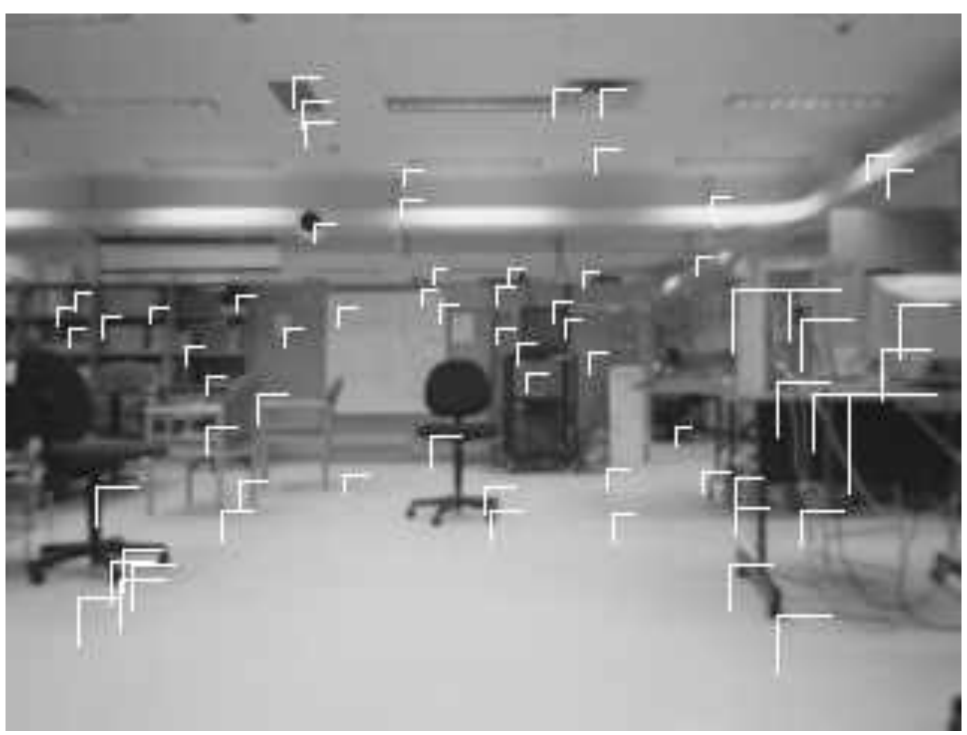
 

Recognition under occlusion

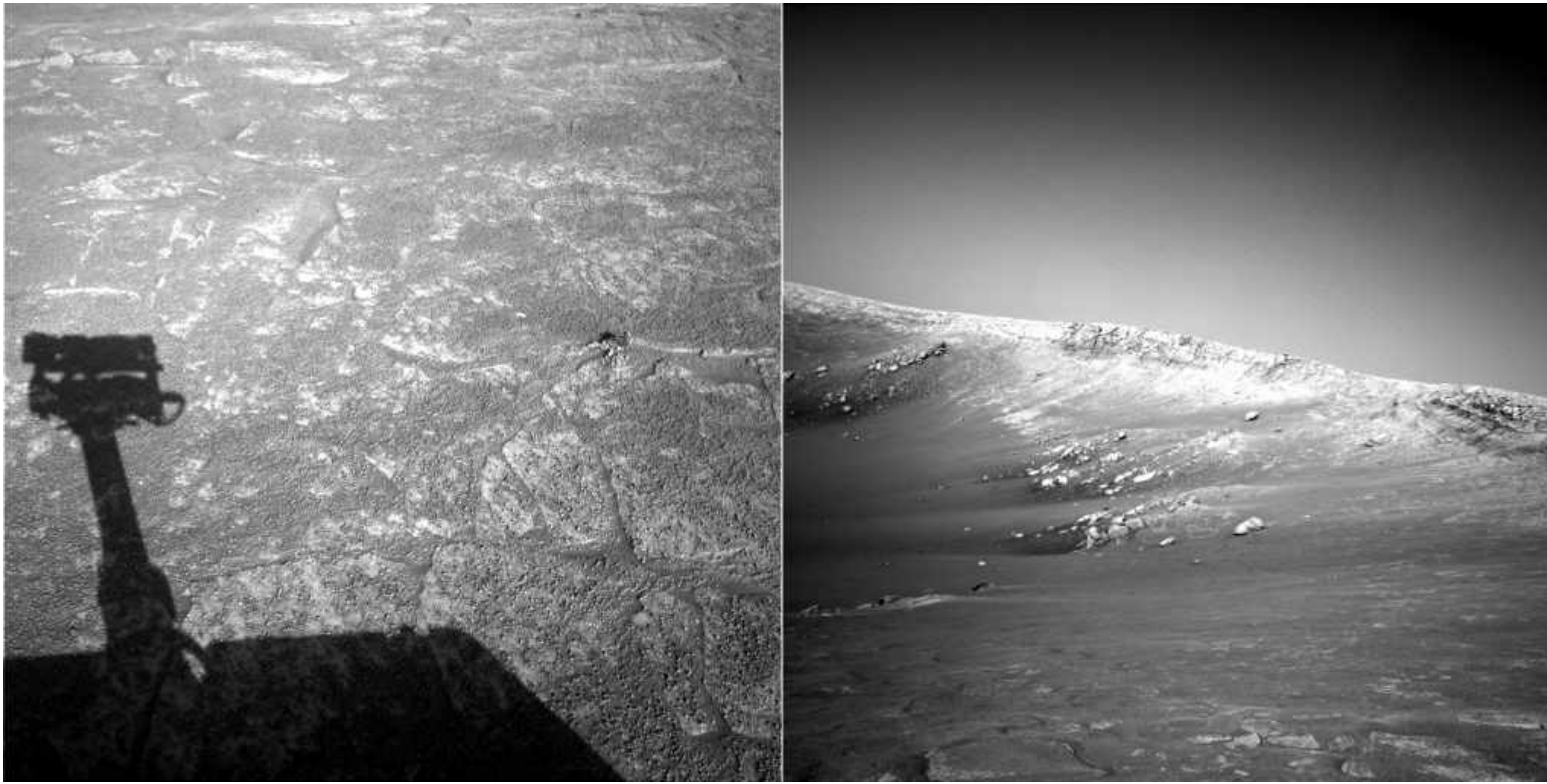
# Location Recognition

# Robot Localization



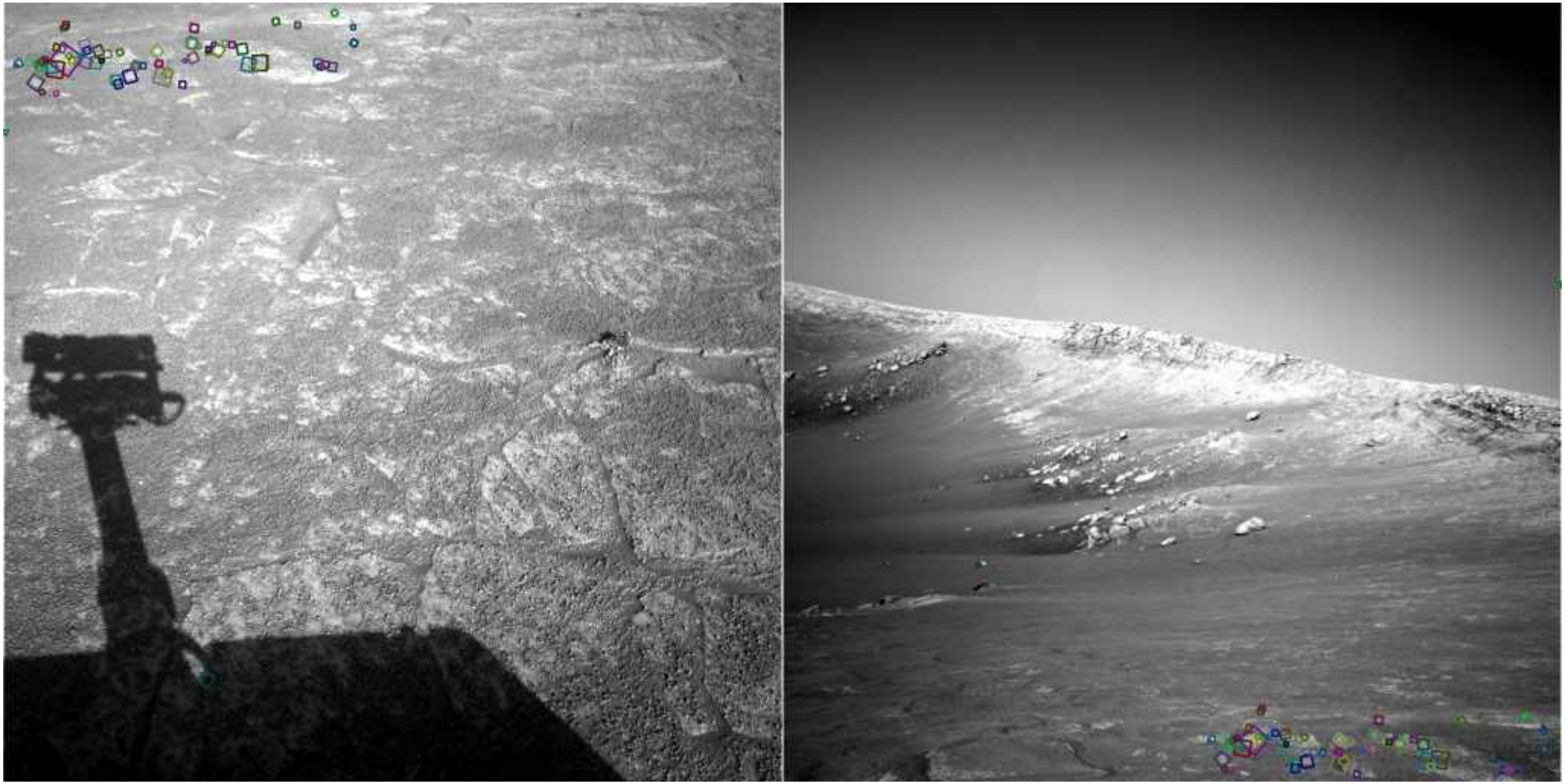


# Image matching

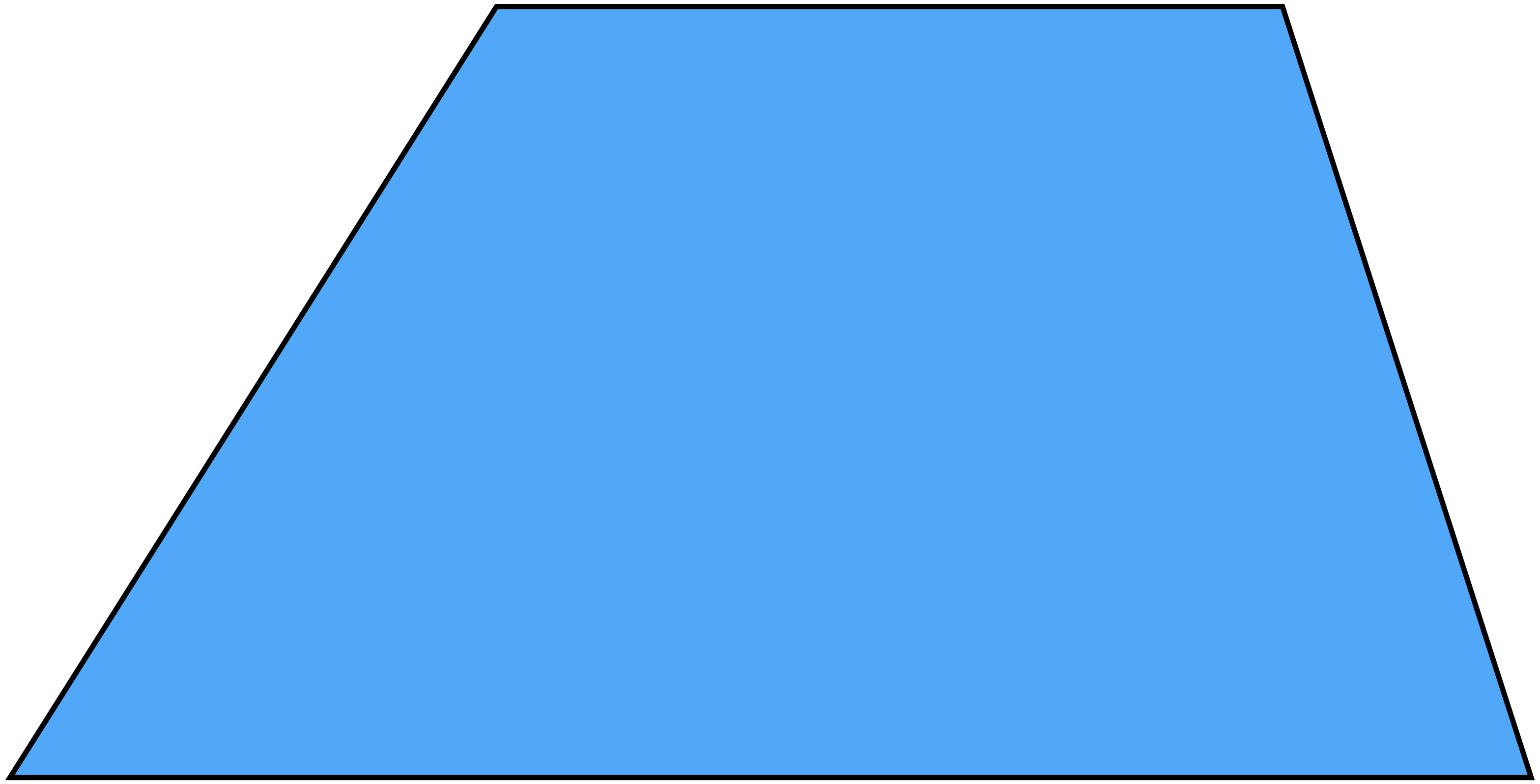


NASA Mars Rover images

*Where are the corresponding points?*

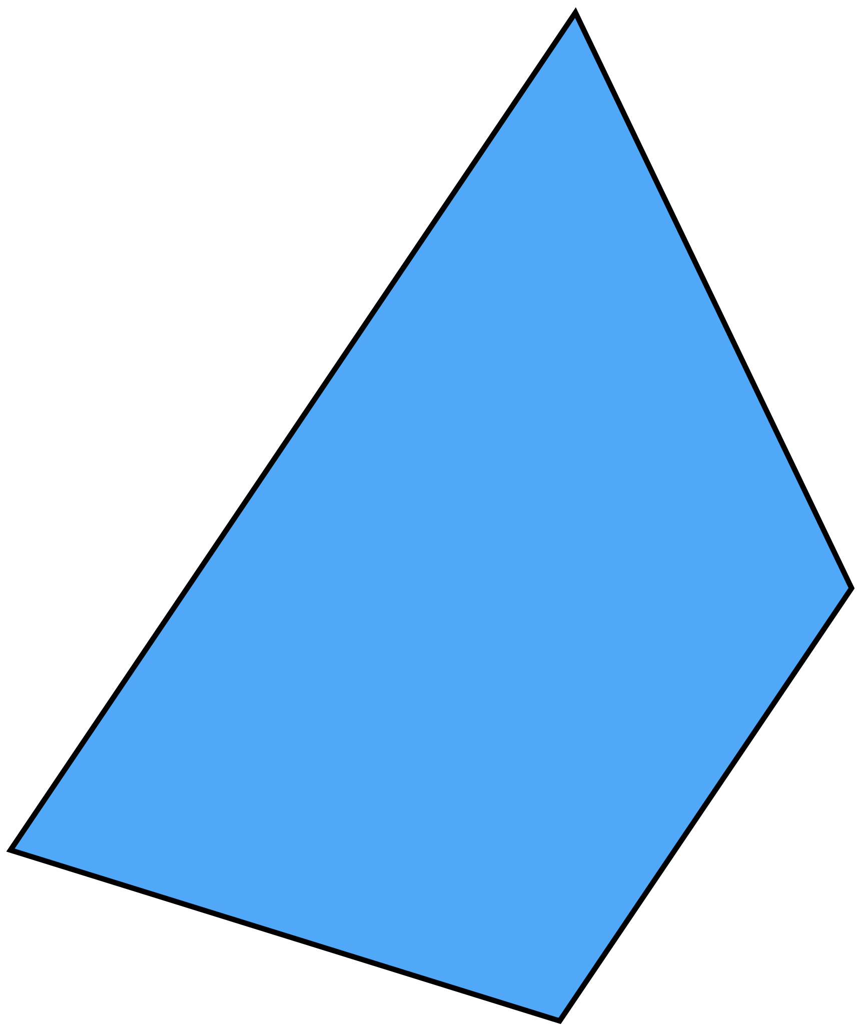


NASA Mars Rover images



Pick a point in the image.
Find it again in the next image.

*What type of feature would you select?*



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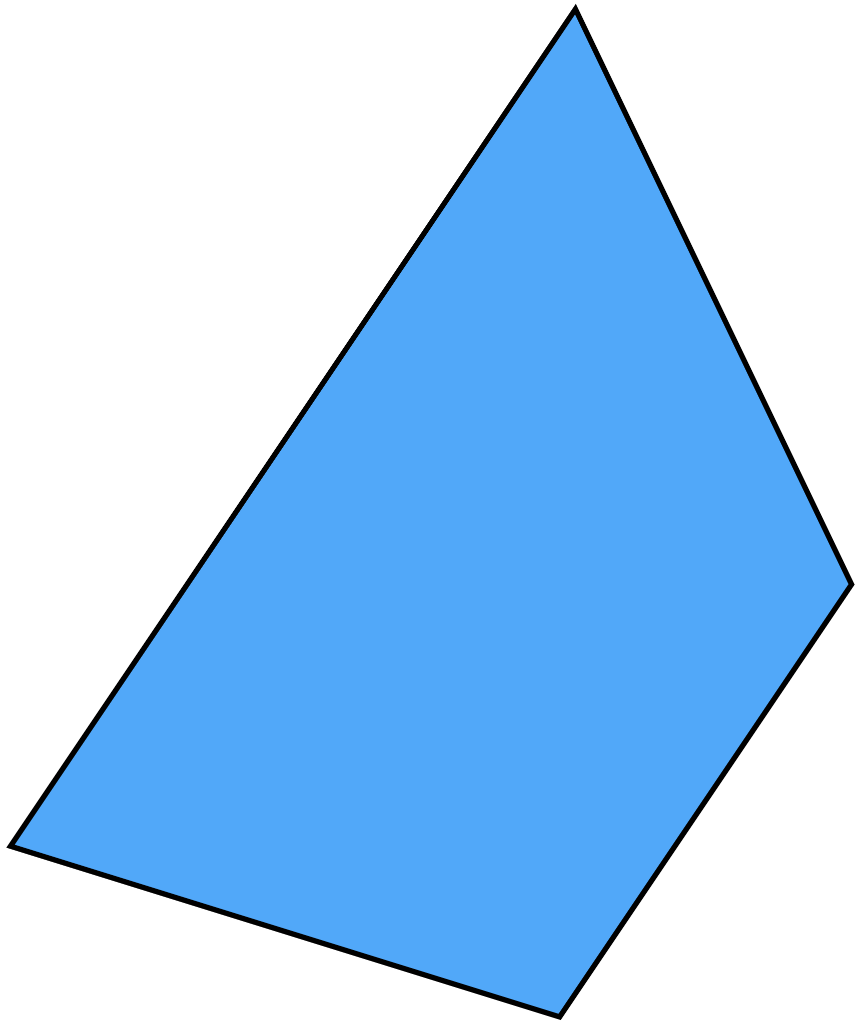
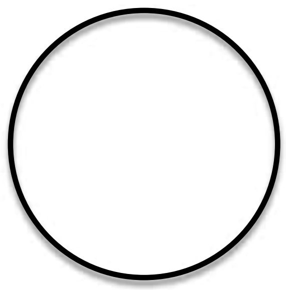


Pick a point in the image.
Find it again in the next image.

*What type of feature would you select?*

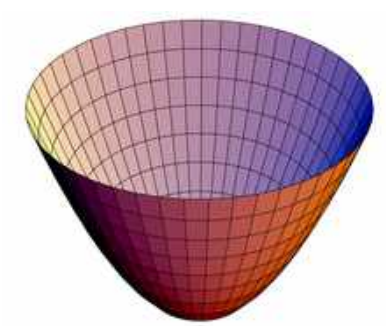
a corner

# Visualizing quadratics



Equation of a circle

$$1 = x^2 + y^2$$



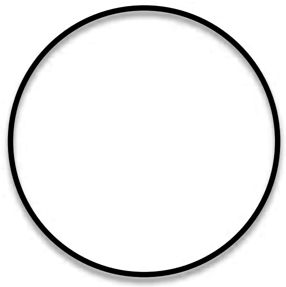
Equation of a 'bowl' (paraboloid)

$$f(x, y) = x^2 + y^2$$

*If you slice the bowl at*

$$f(x, y) = 1$$

*what do you get?*



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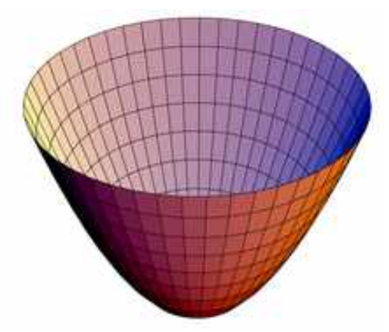

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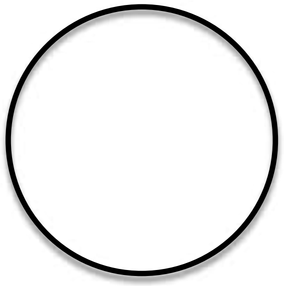

$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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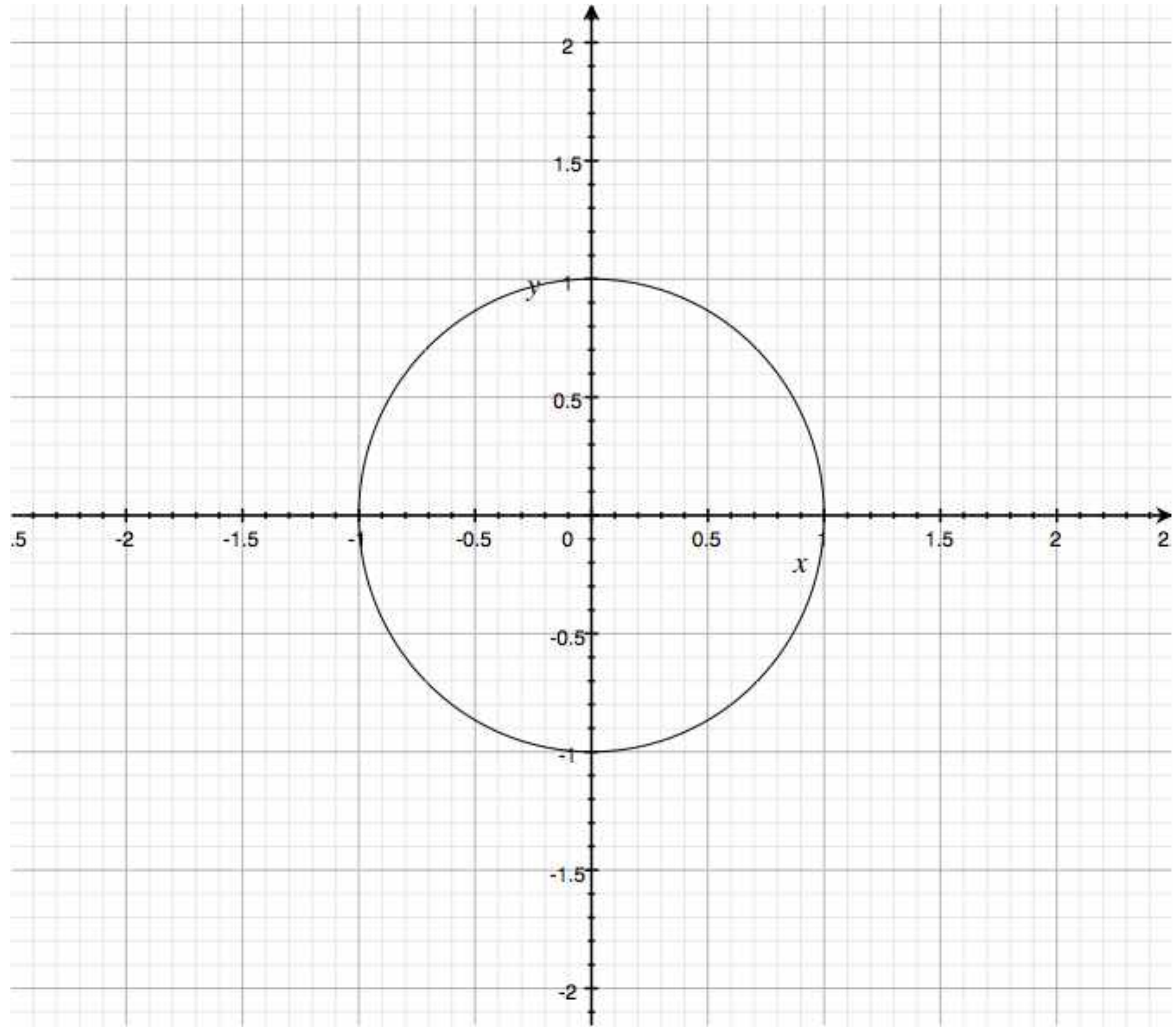
'sliced at 1'



*What happens if you **increase** coefficient on **x**?*

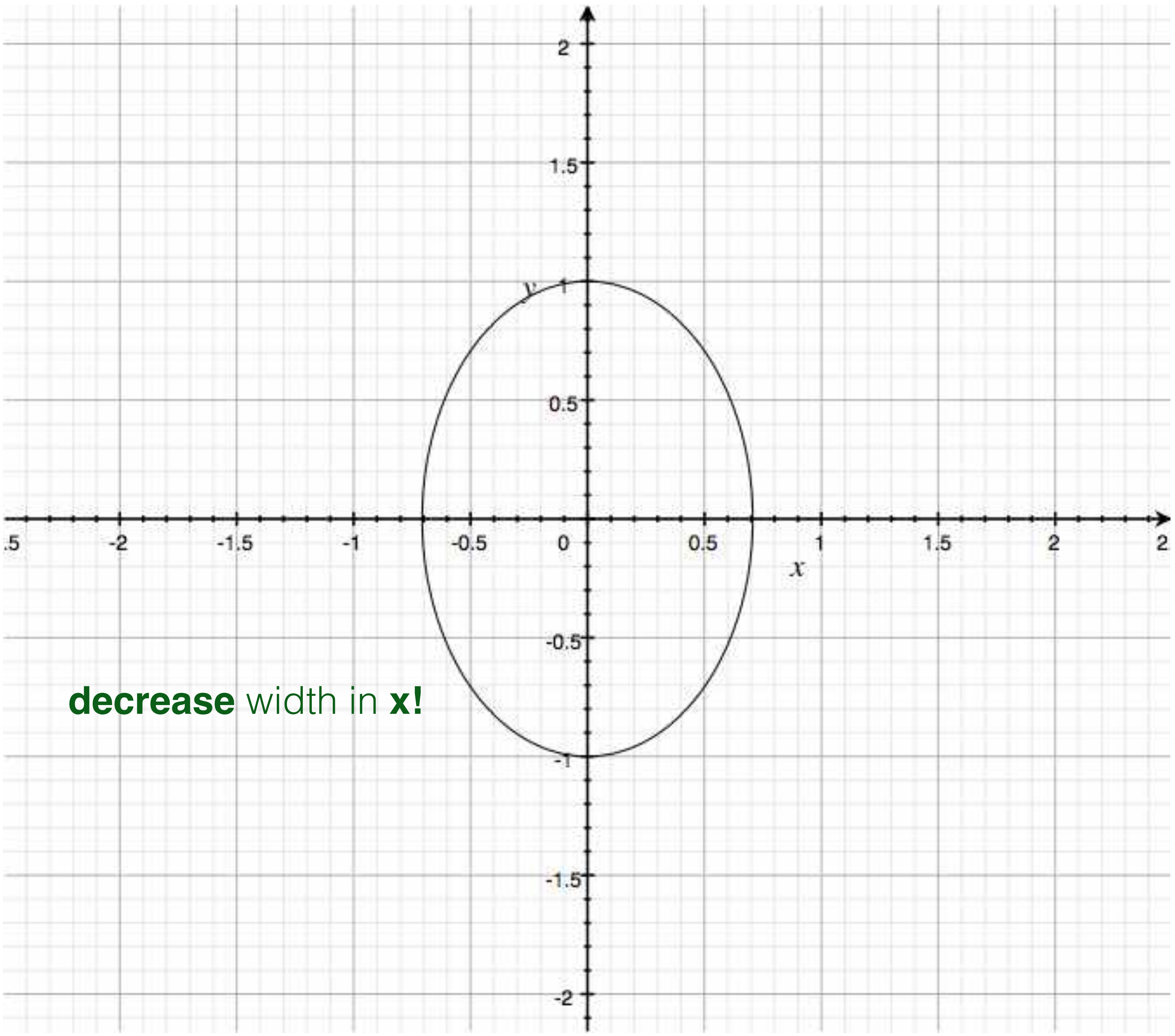
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on **x**?*

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



*What happens if you **increase** coefficient on **y**?*

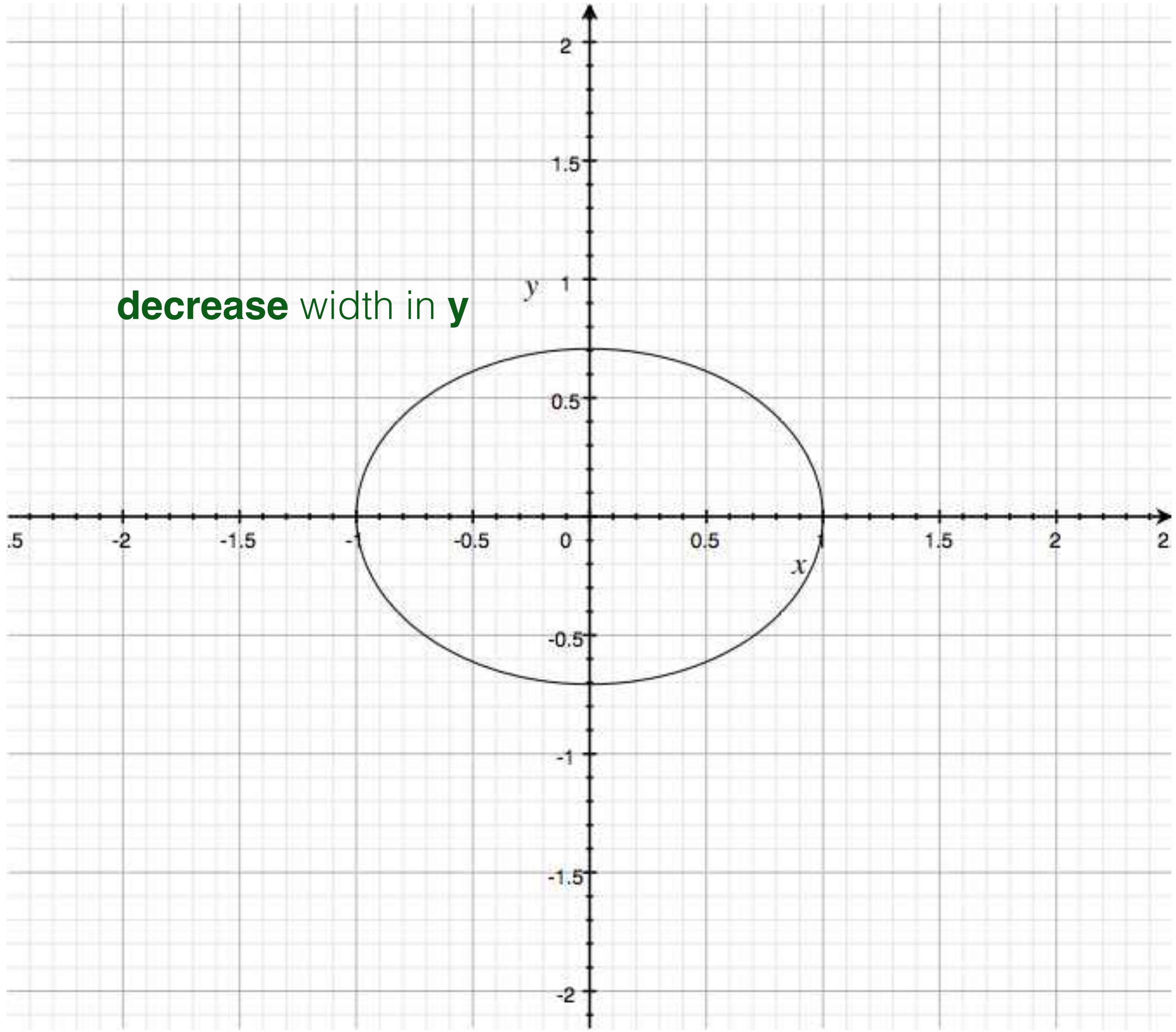
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

*What happens if you **increase** coefficient on **y**?*

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1



$$f(x, y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

*What's the shape?*
*What are the eigenvectors?*
*What are the eigenvalues?*

$$f(x, y) = x^2 + y^2$$

can be written in matrix form like this…

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

**Result of Singular Value Decomposition (SVD)**

eigenvectors

eigenvalues along diagonal

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{\top}$$

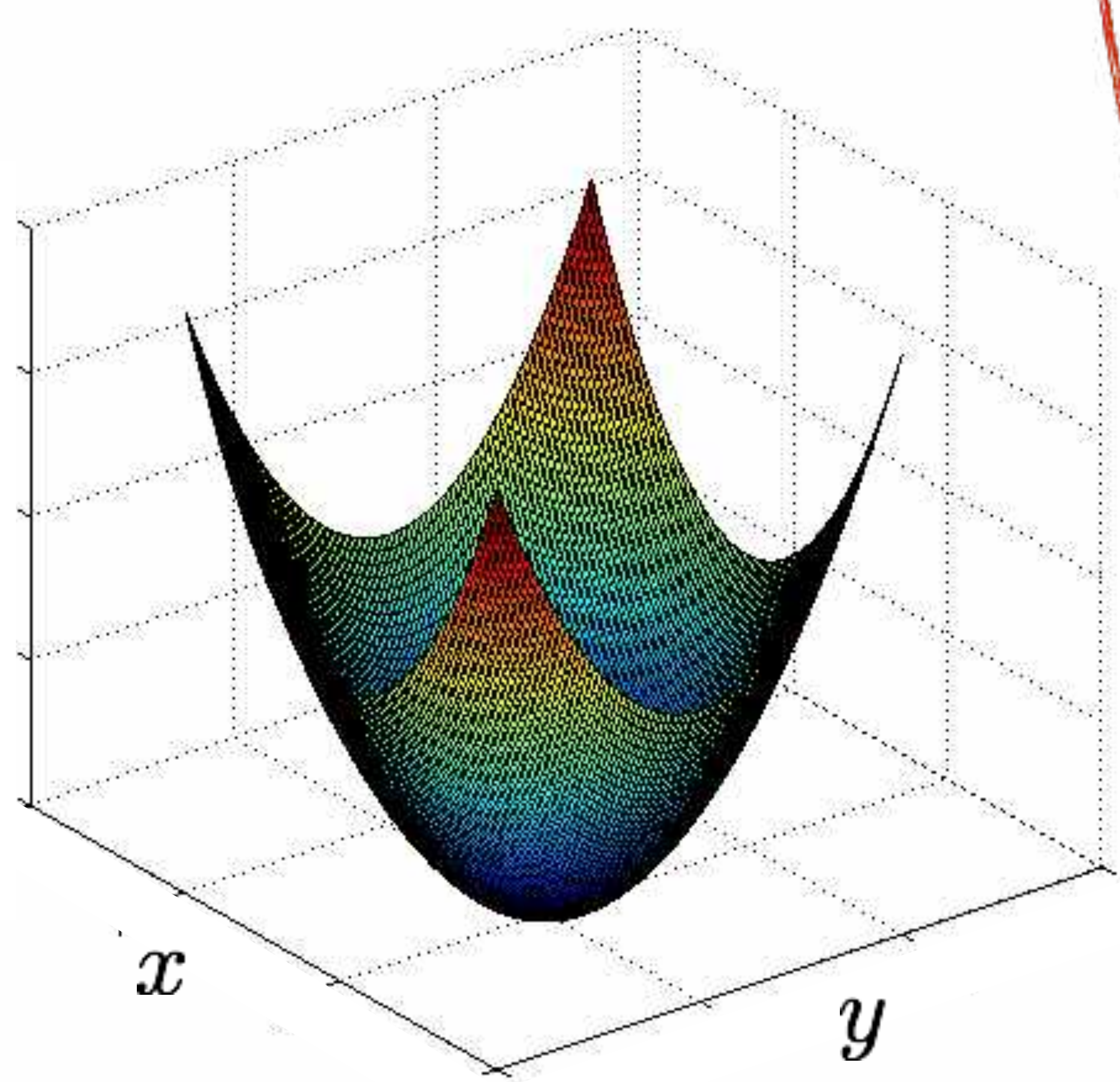
axis of the 'ellipse slice'

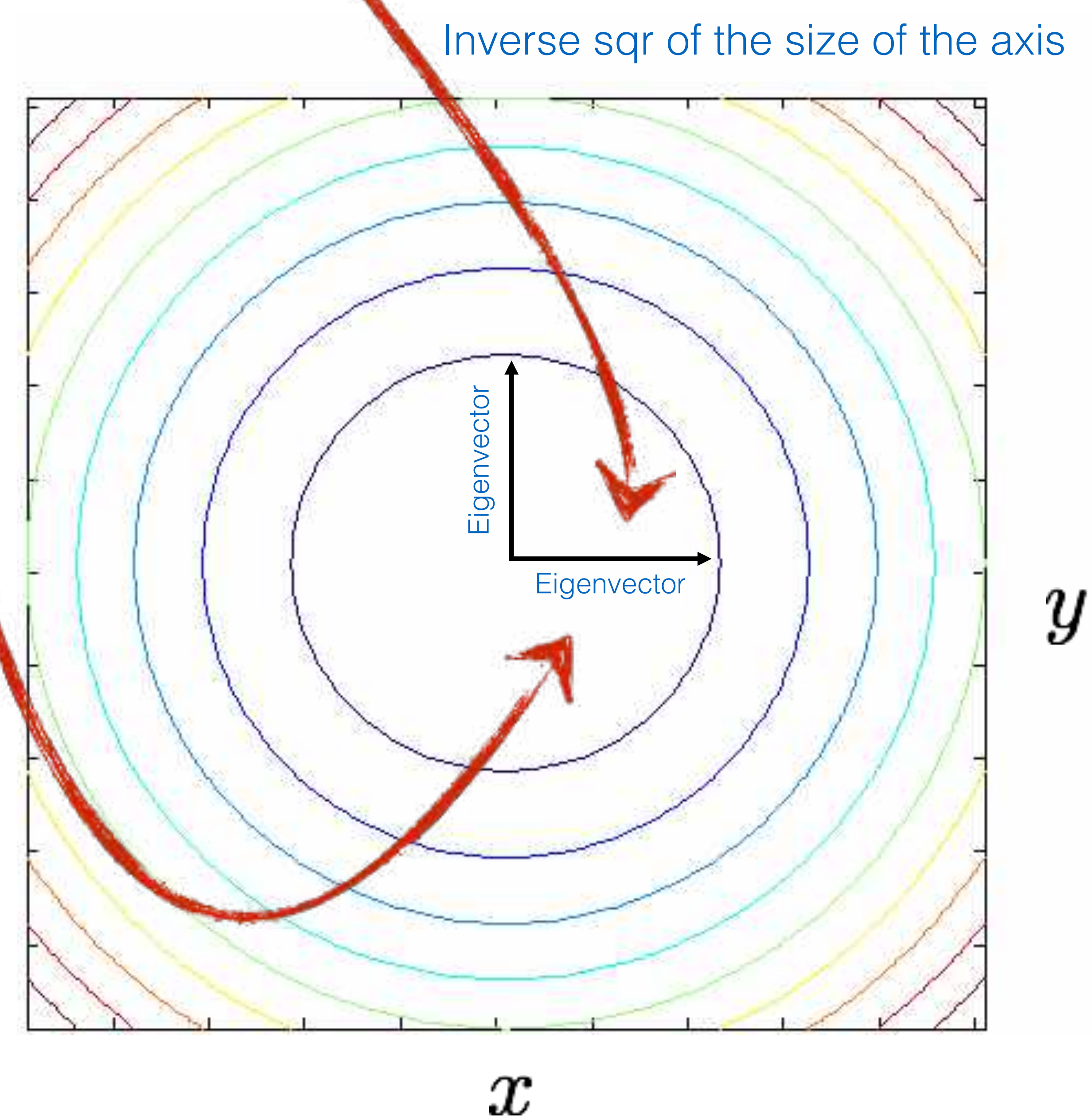
Inverse sqr of length of the quadratic along the axis

Eigenvectors Eigenvalues

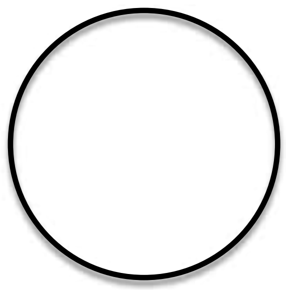
$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

Eigenvectors



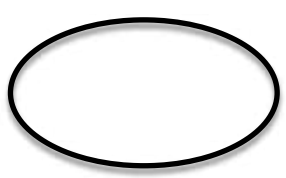



Recall:



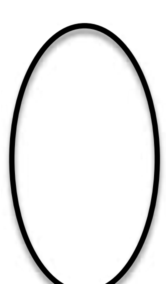
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **y** direction



$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the **x** direction



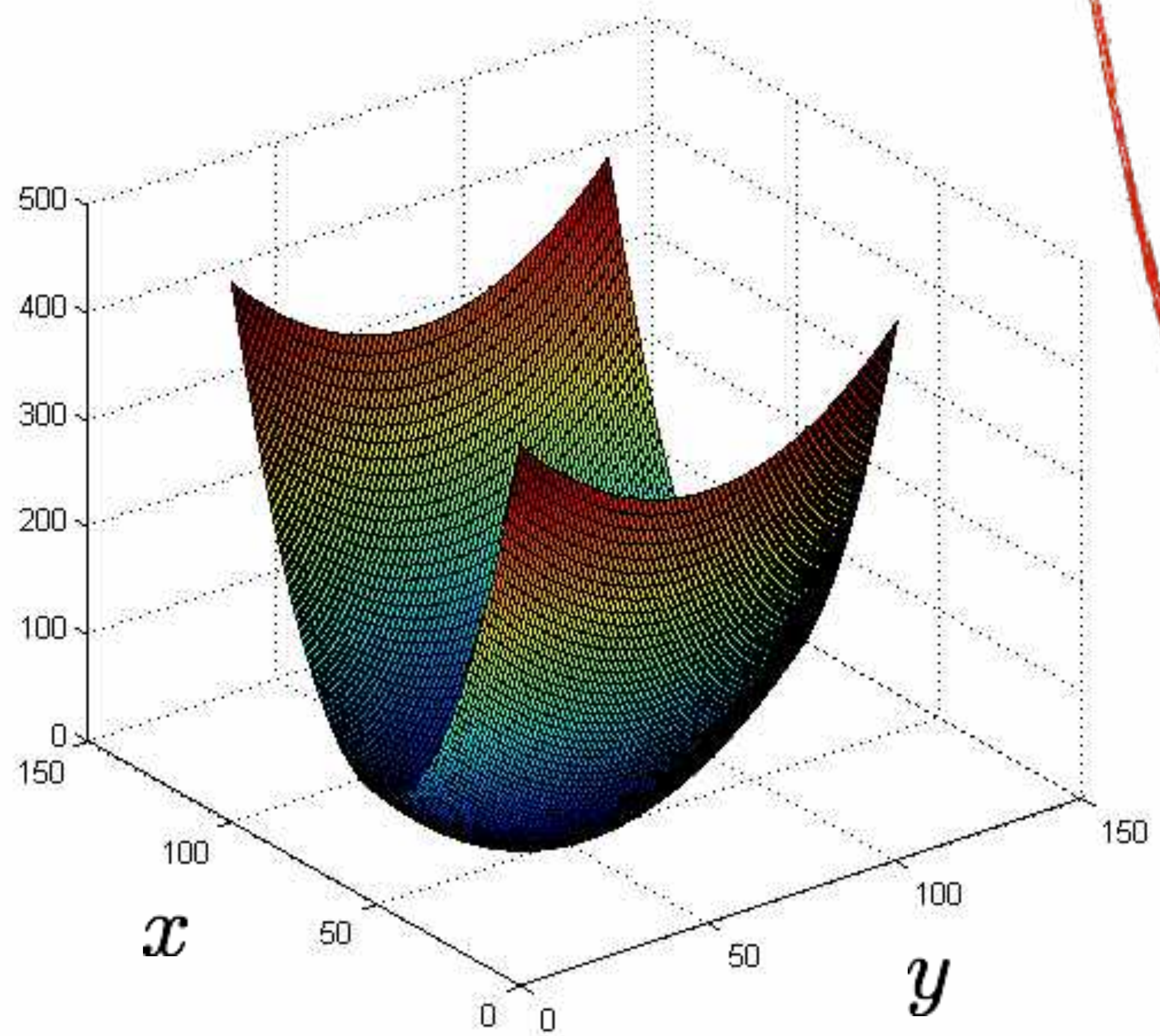
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

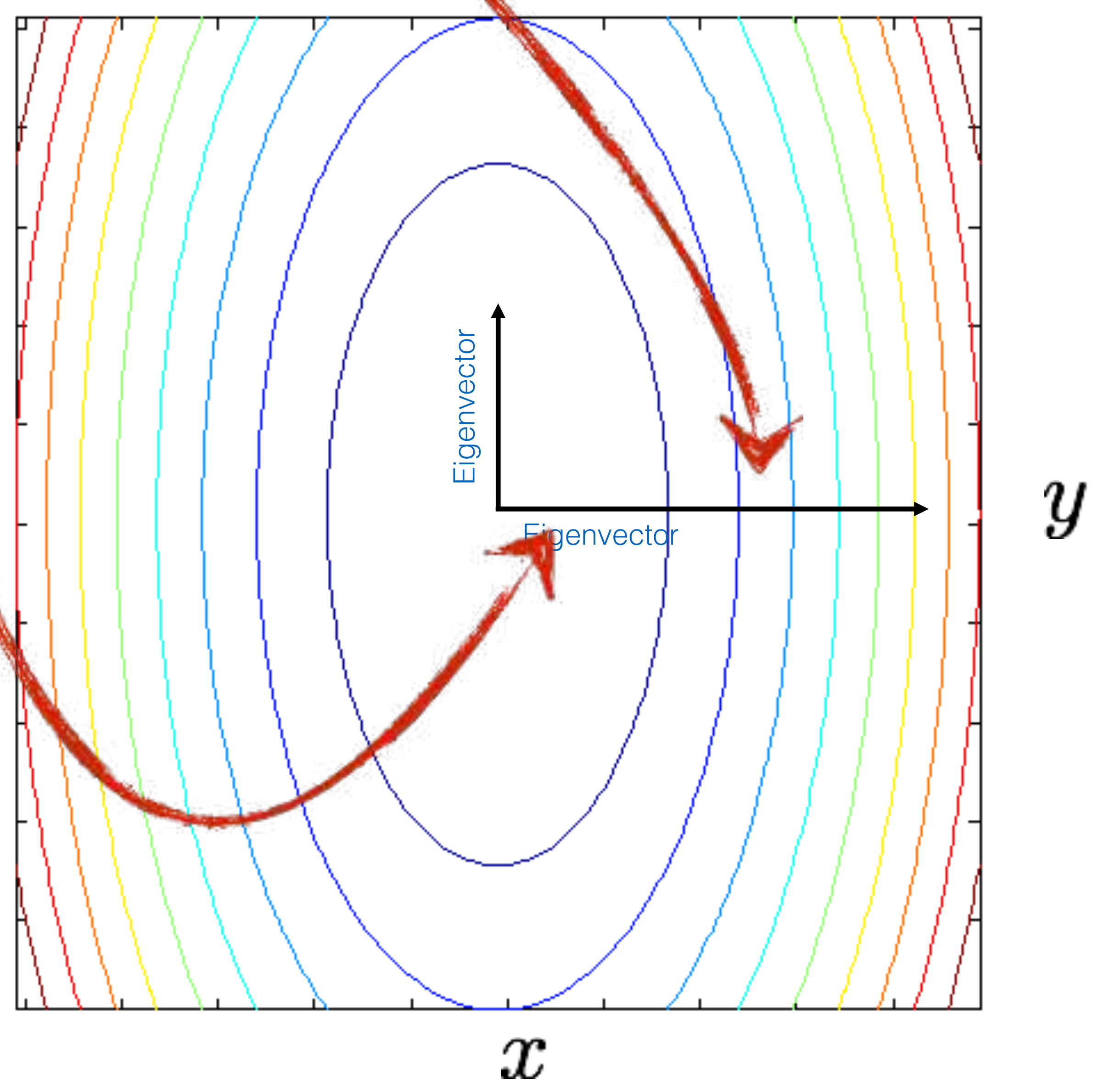
Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

Eigenvectors Eigenvectors

Inverse sqr of length of axis

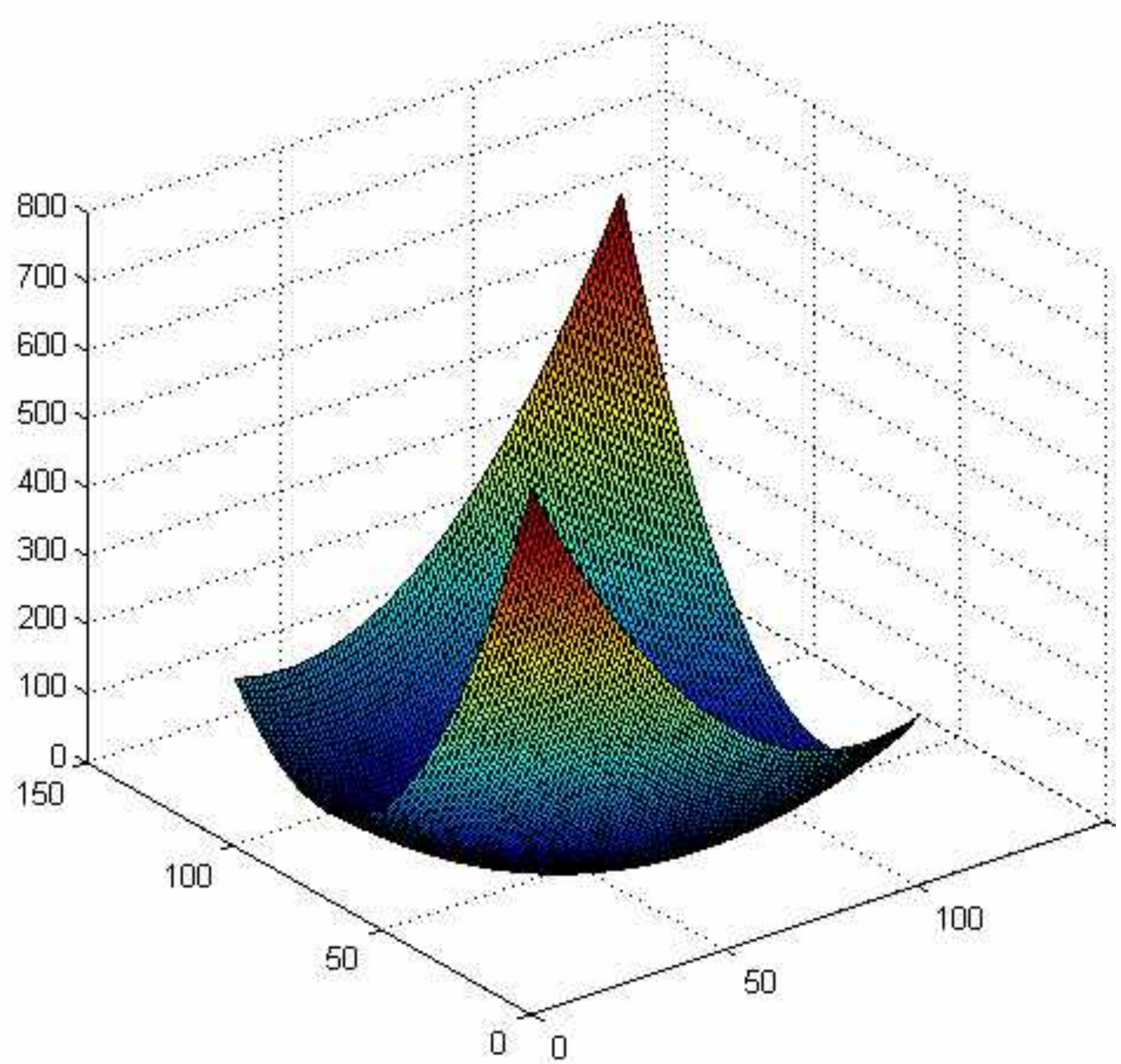


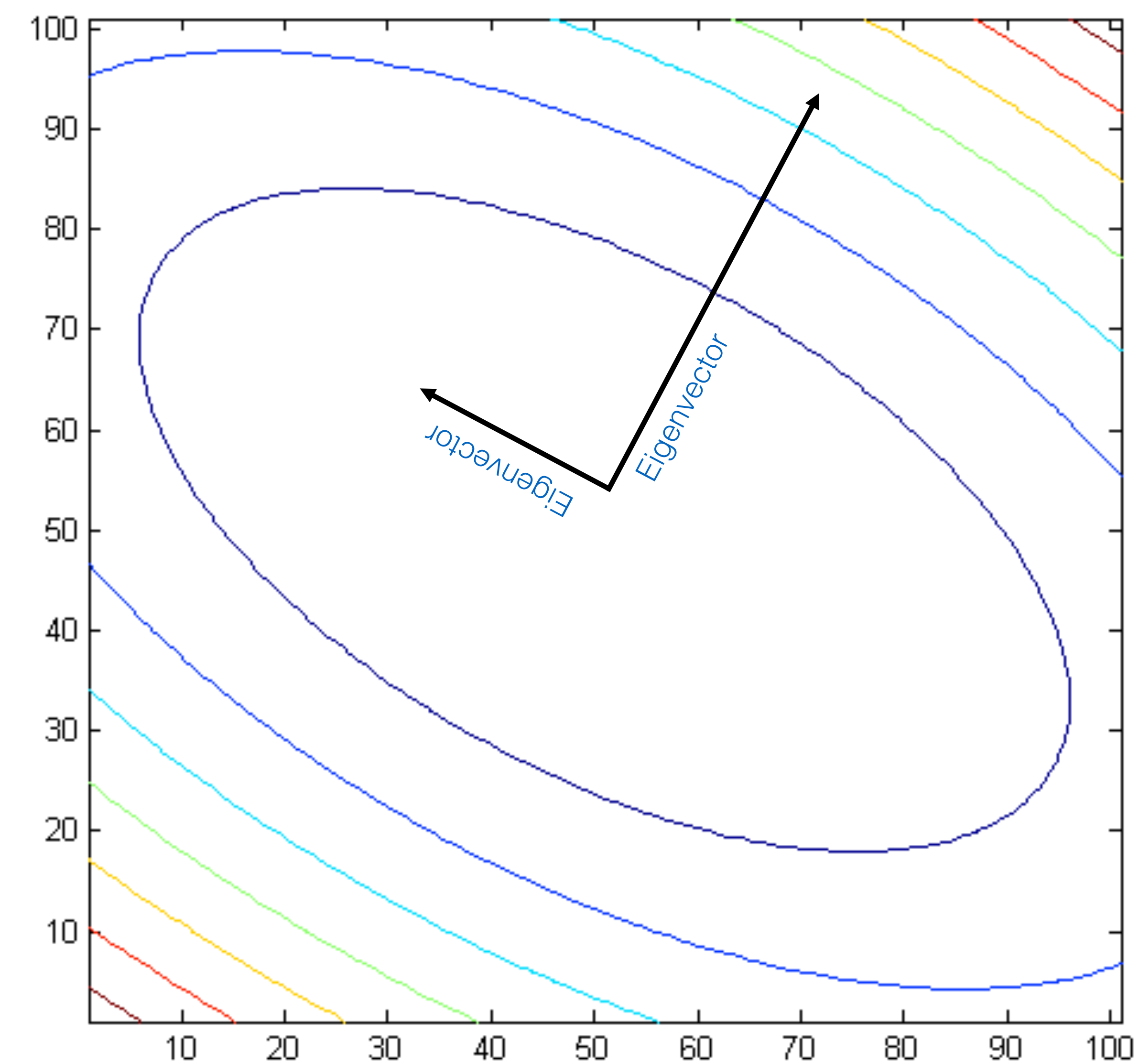



Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$

Eigenvectors Eigenvectors

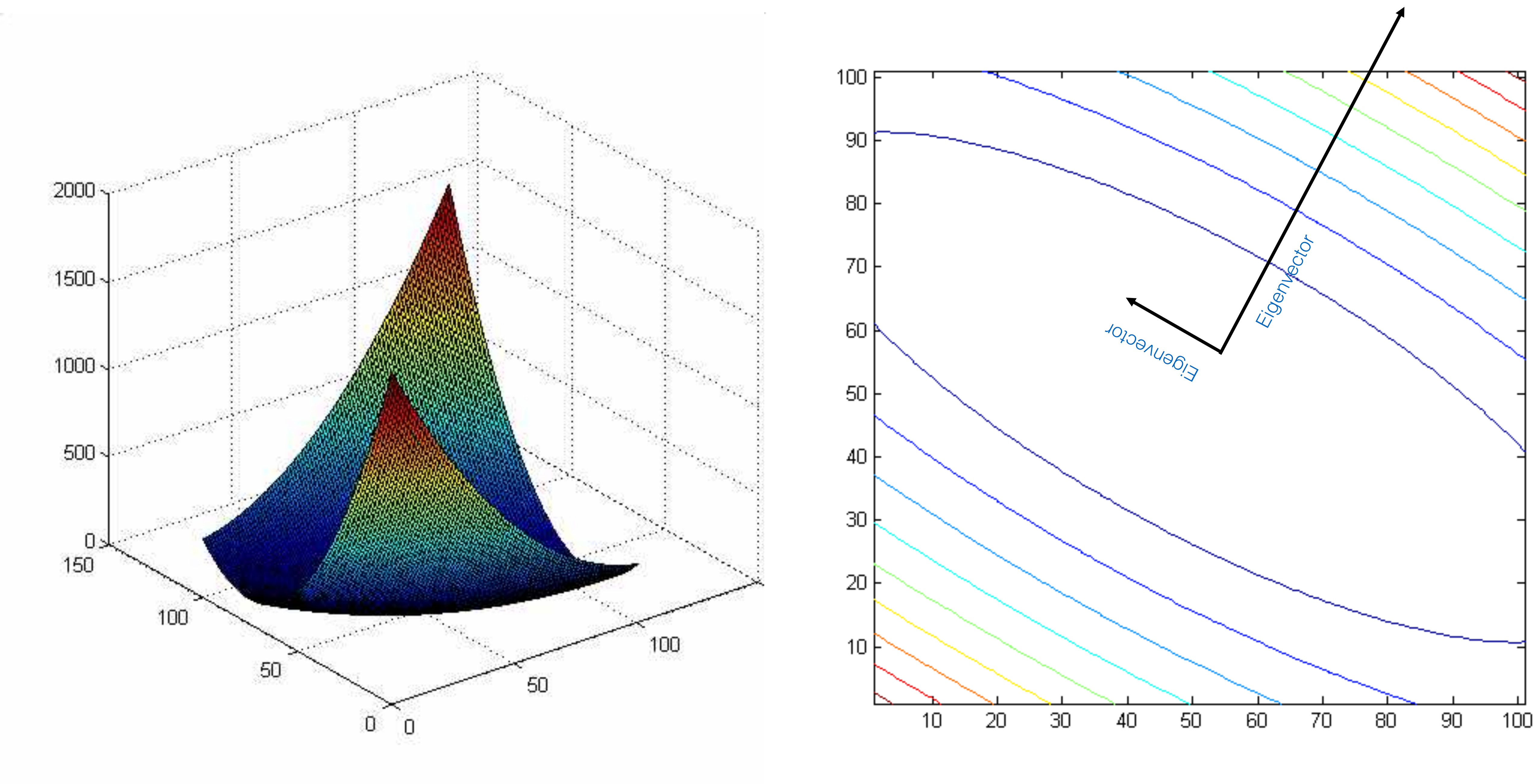


Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$

Eigenvectors Eigenvectors



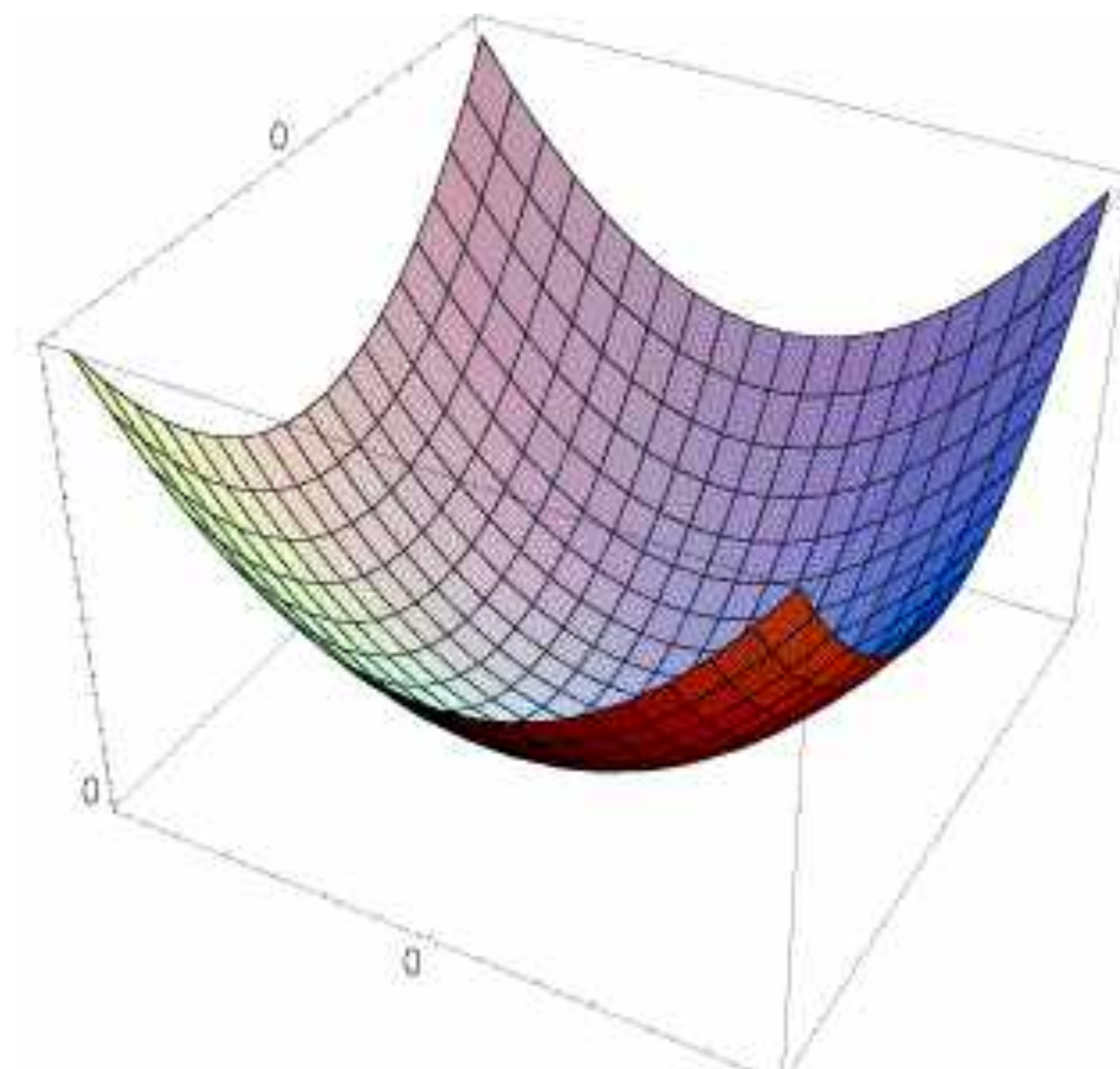
We will need this to understand the…

# Error function for Harris Corners

The surface *E*(*u*,*v*) is locally approximated by a quadratic form

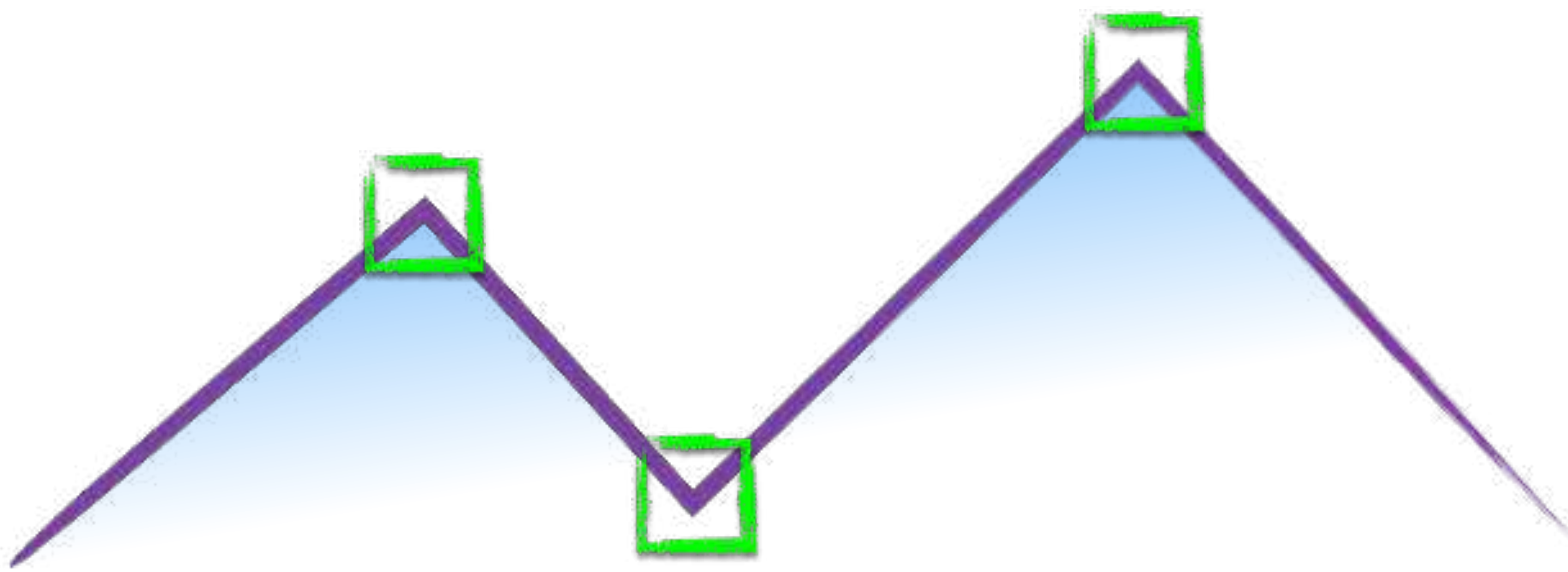
$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



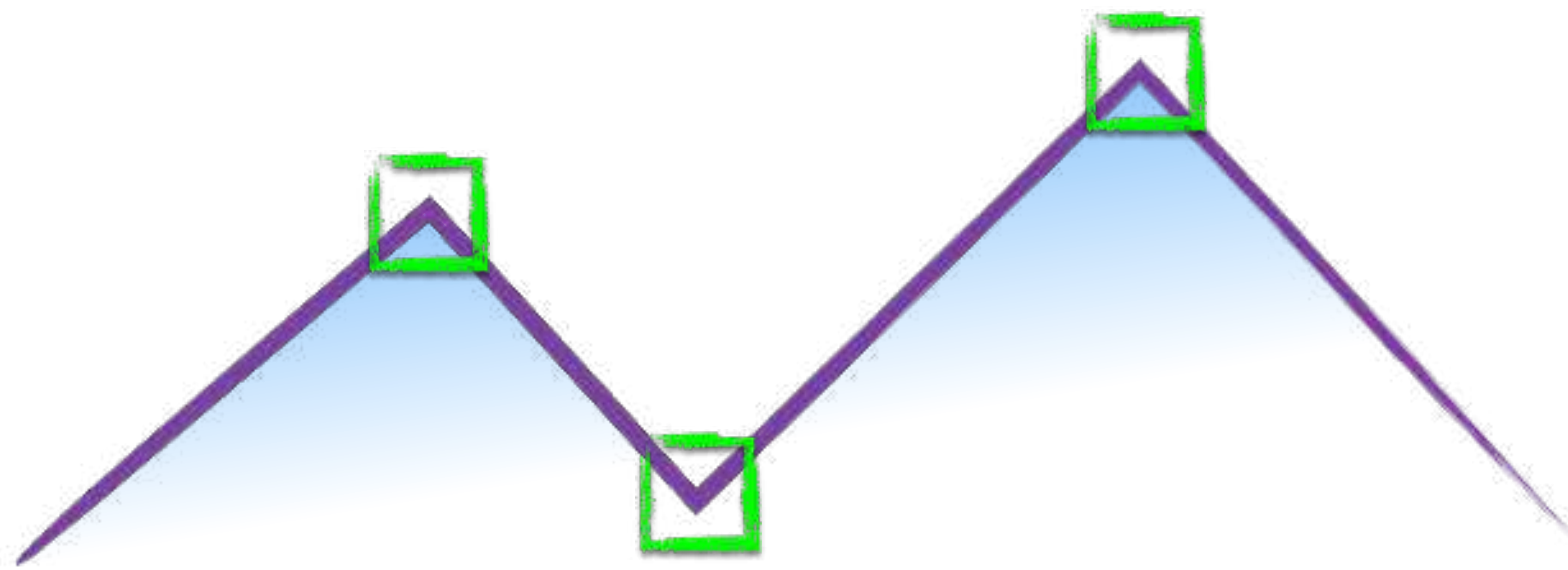
# Harris corner detector

# How do you find a corner?



# How do you find a corner?

[Moravec 1980]



Easily recognized by looking through a small window

Shifting the window should give large change in intensity

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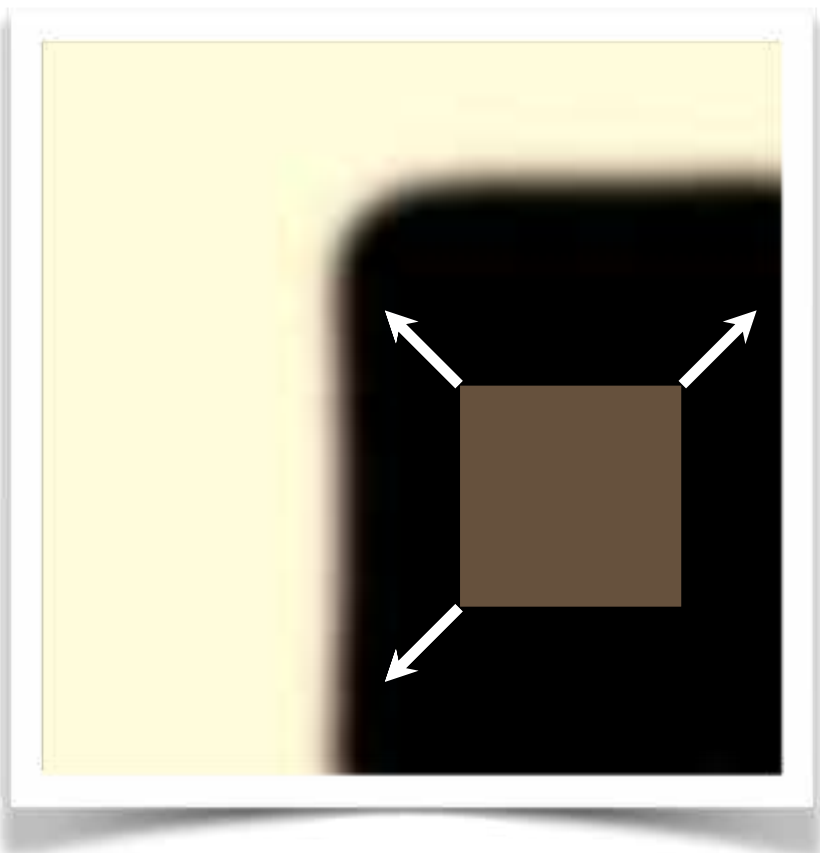
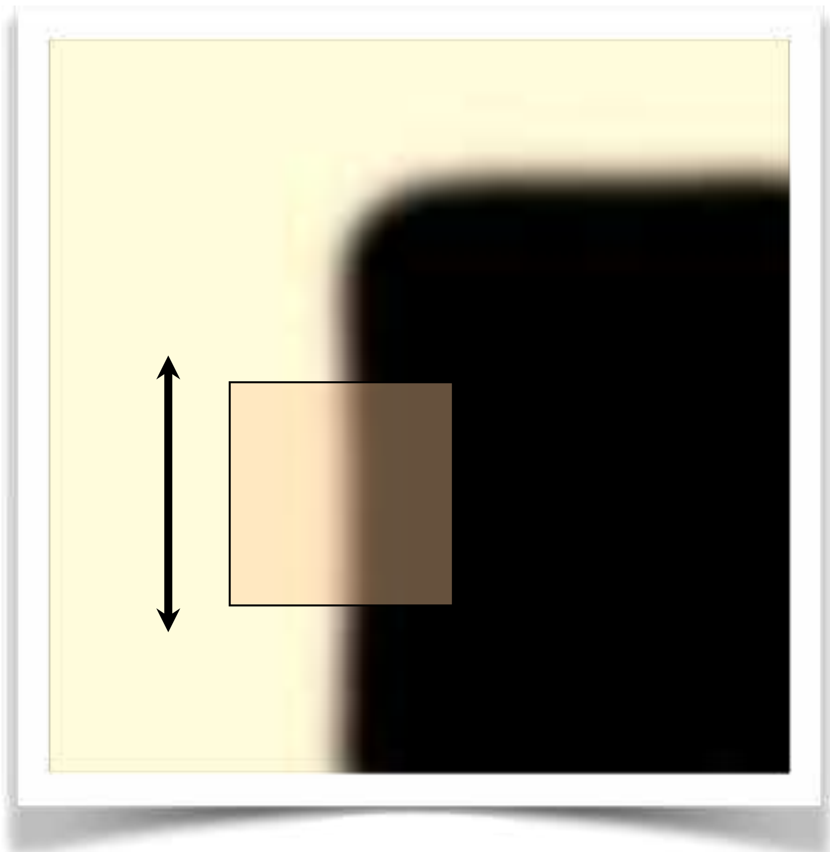
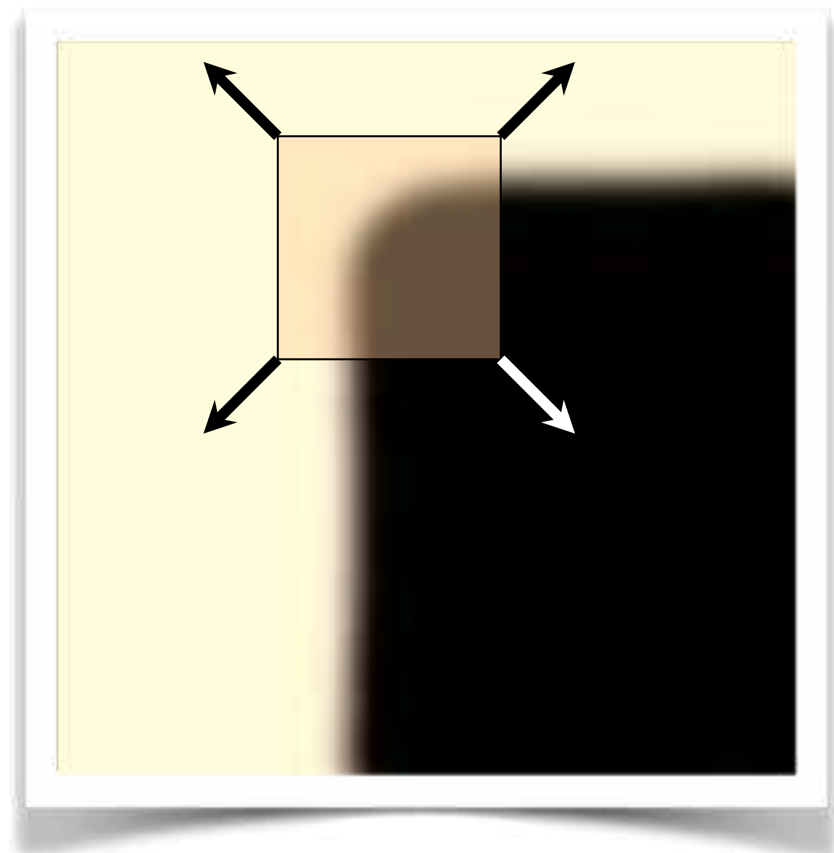


"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

[Moravec 1980]

Design a program to detect corners

(hint: use image gradients)

# Finding corners
## (a.k.a. PCA)

$$I_x = \frac{\partial I}{\partial x}$$

$$I_y = \frac{\partial I}{\partial y}$$

1. Compute image gradients over small region

2. Subtract mean from each image gradient

3. Compute the covariance matrix

$$\begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

4. Compute eigenvectors and eigenvalues

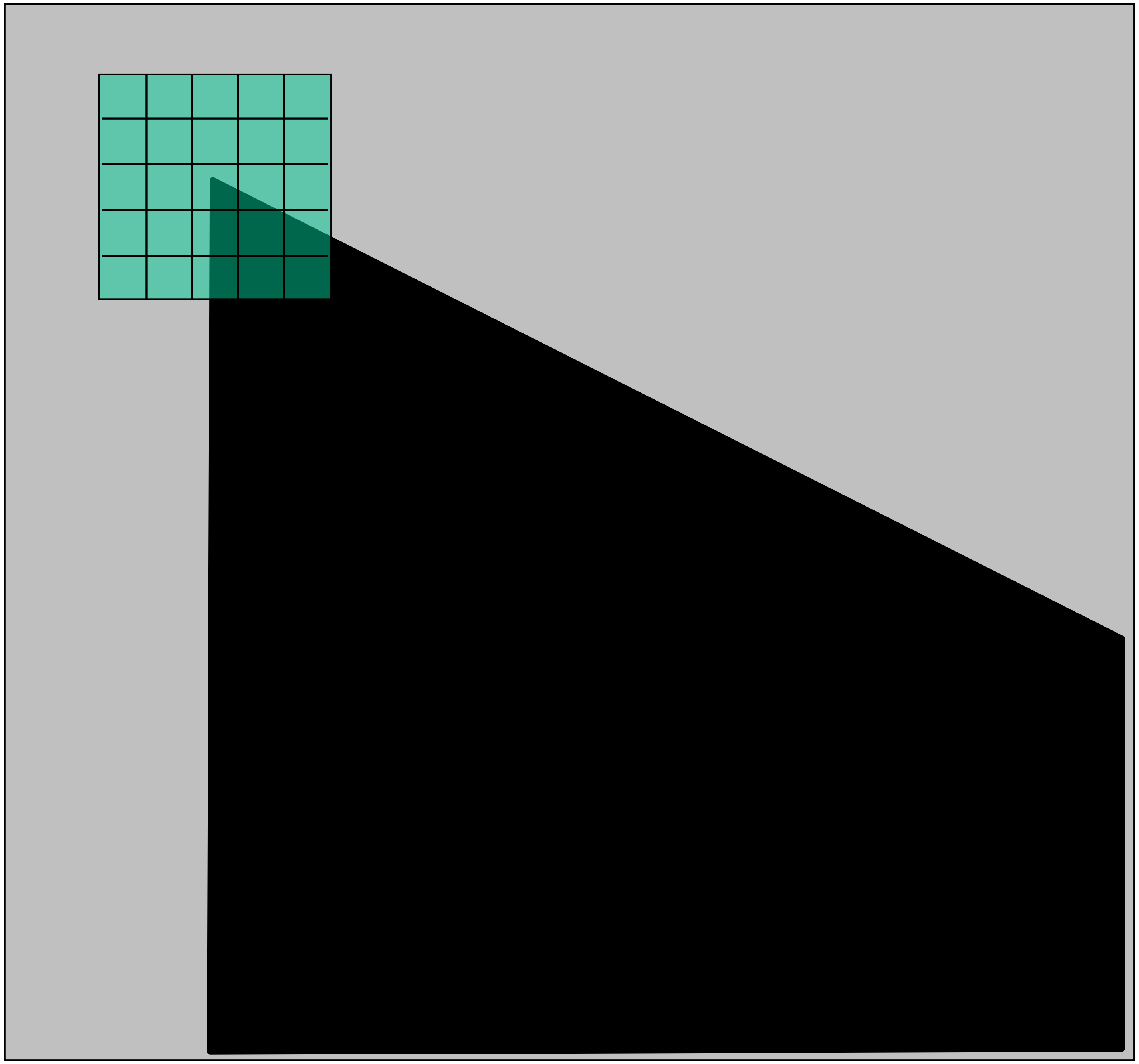
5. Use threshold on eigenvalues to detect corners

1. Compute image gradients over a small region

(not just a single pixel)

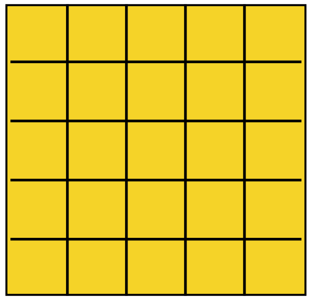
# 1. Compute image gradients over a small region

(not just a single pixel)



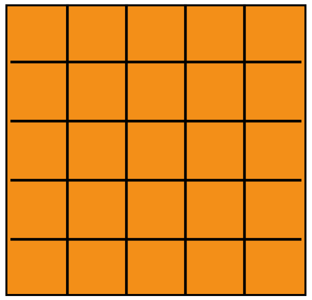
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$

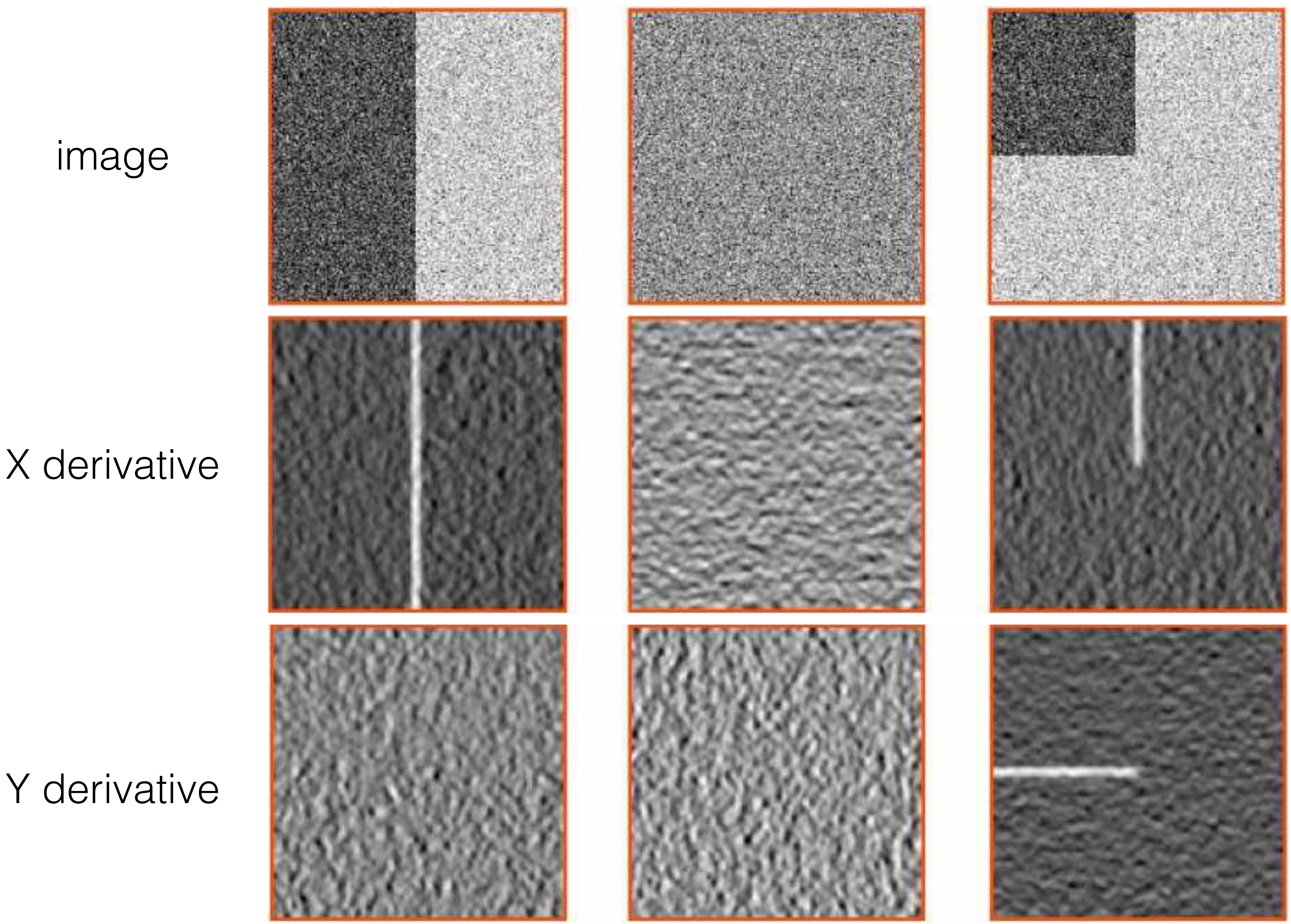


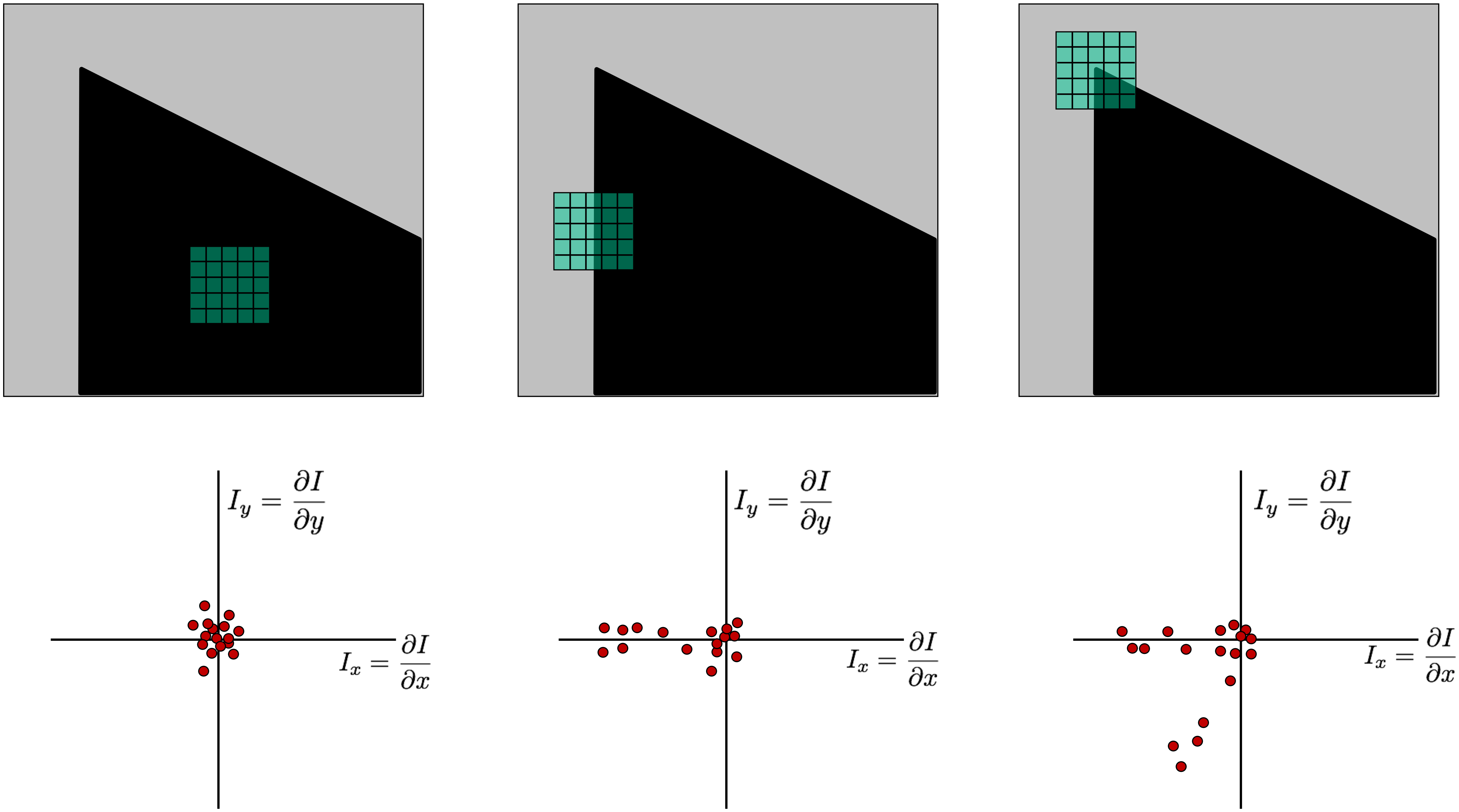
array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$

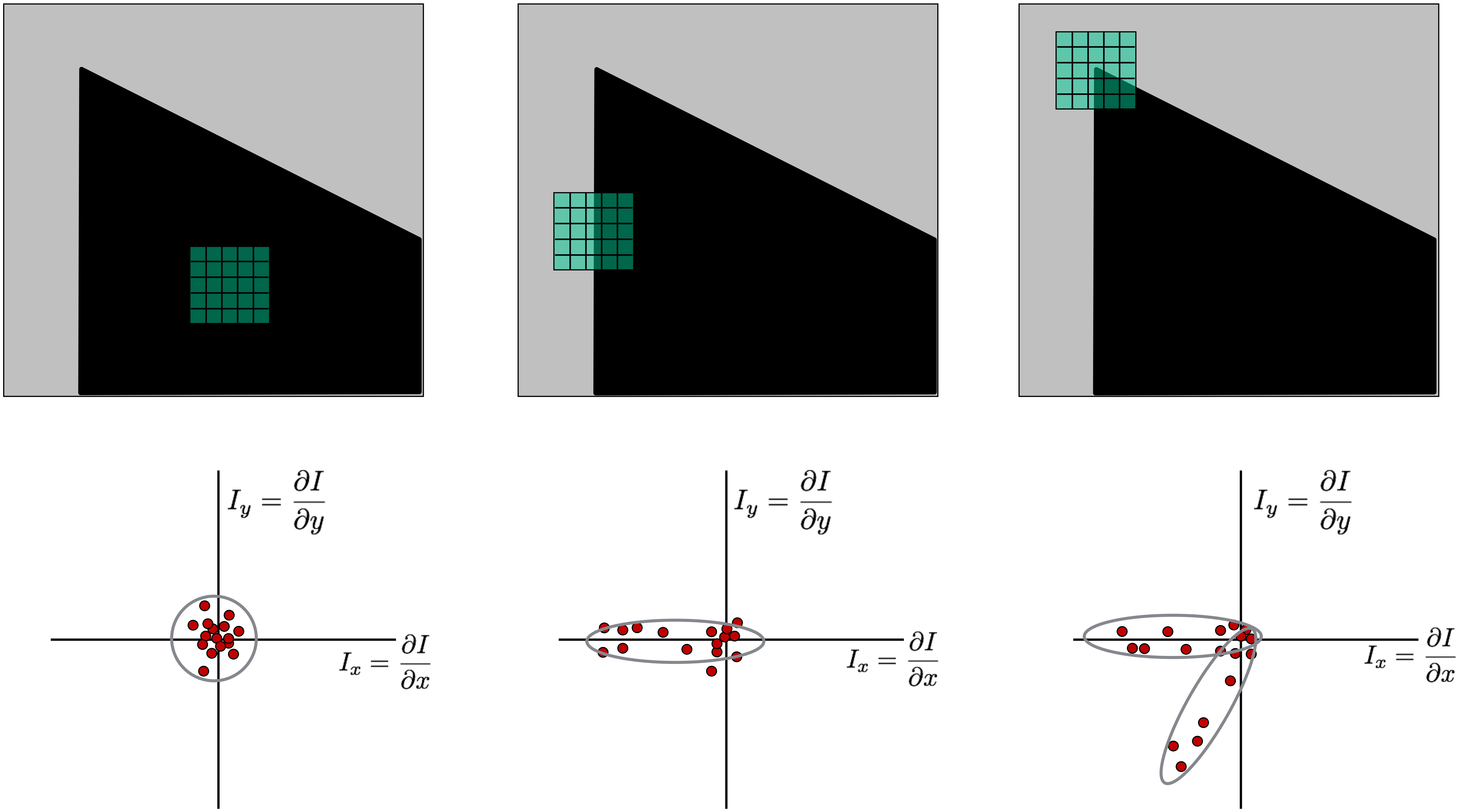


# visualization of gradients

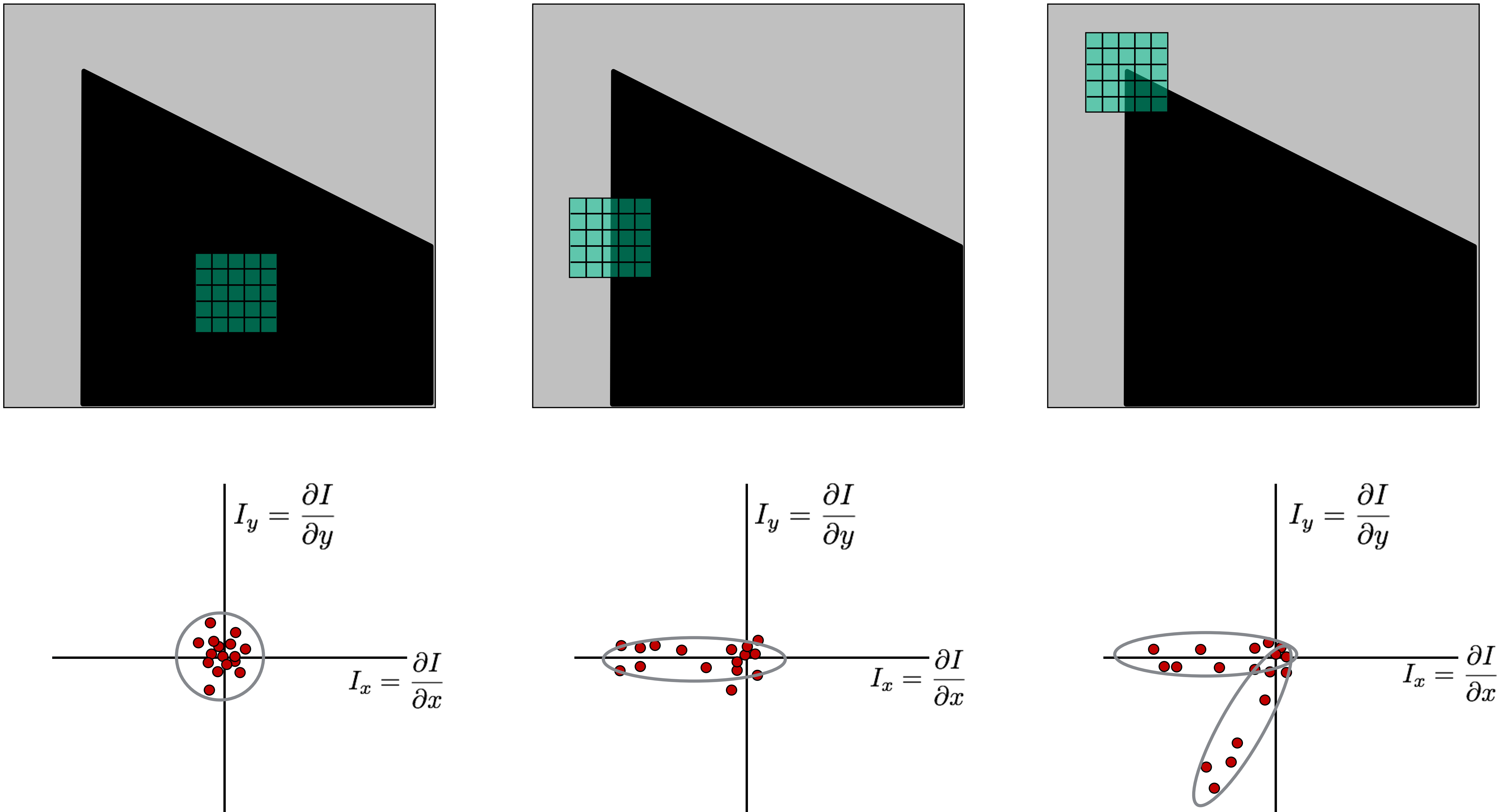





*What does the distribution tell you about the region?*



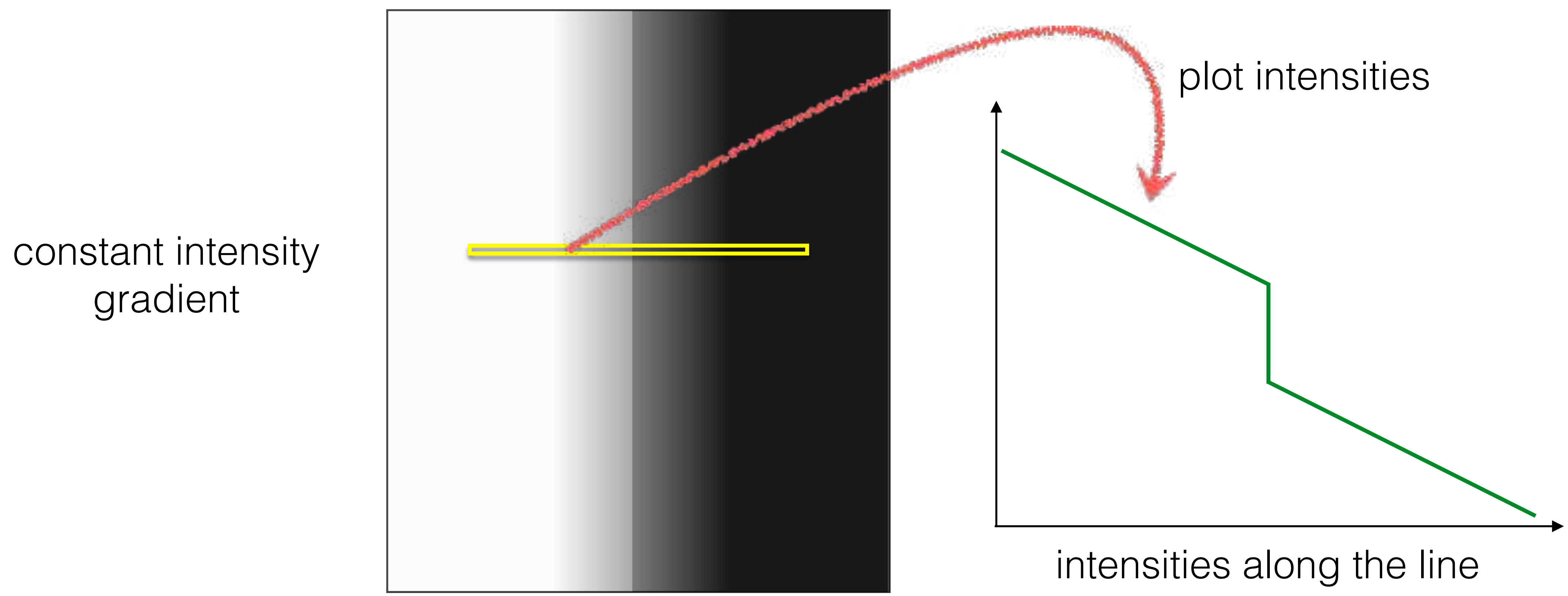

distribution reveals edge orientation and magnitude




*How do you quantify orientation and magnitude?*

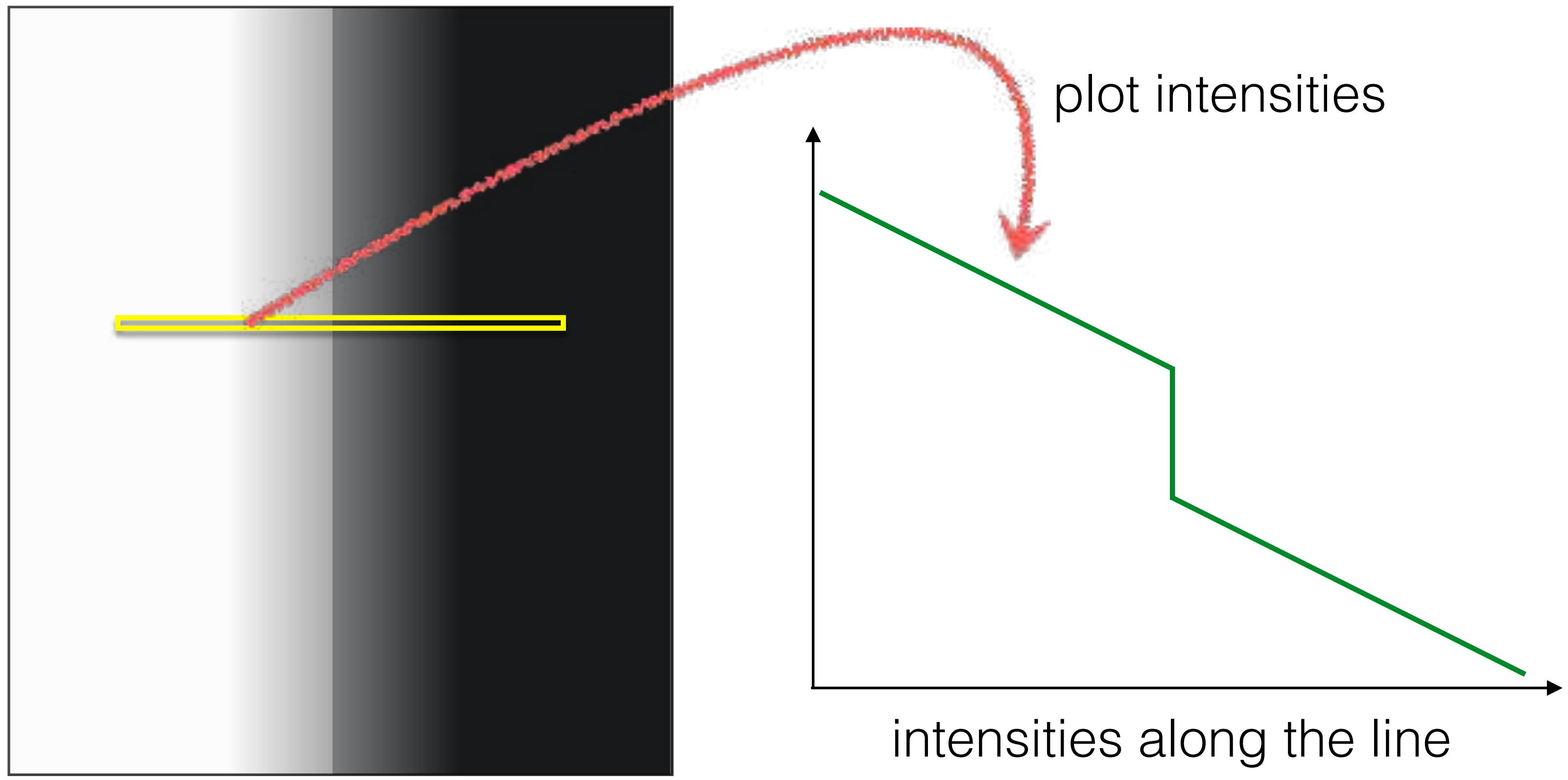
2. Subtract the mean from each image gradient

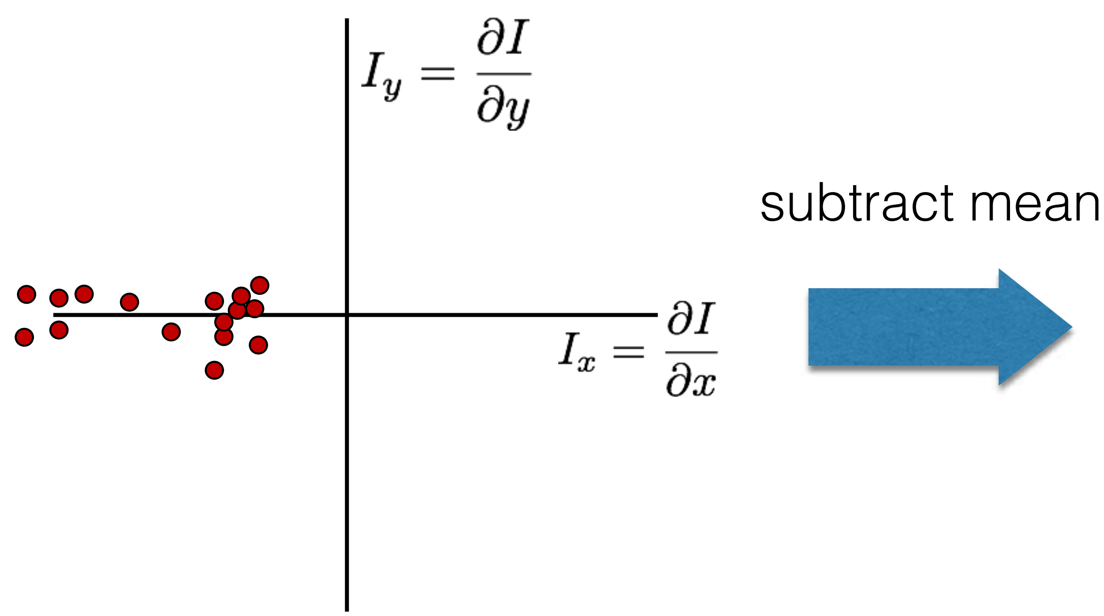
# 2. Subtract the mean from each image gradient



# 2. Subtract the mean from each image gradient

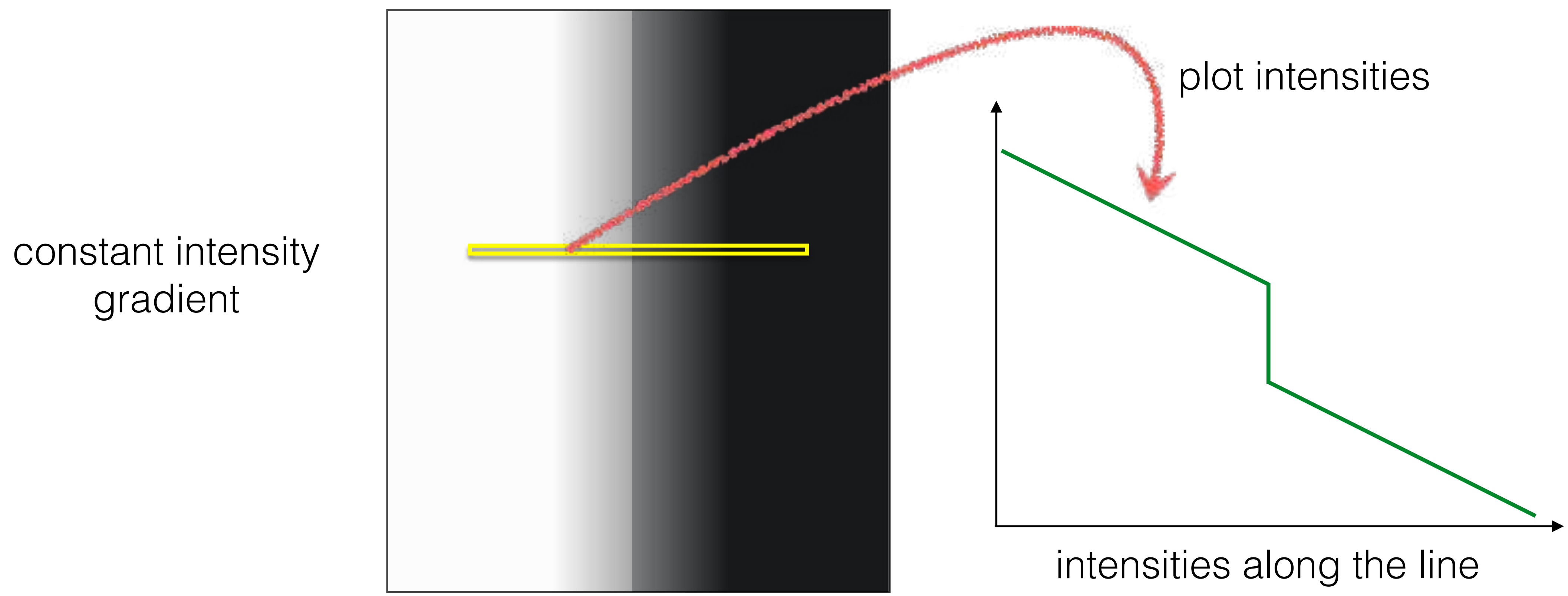
constant intensity gradient

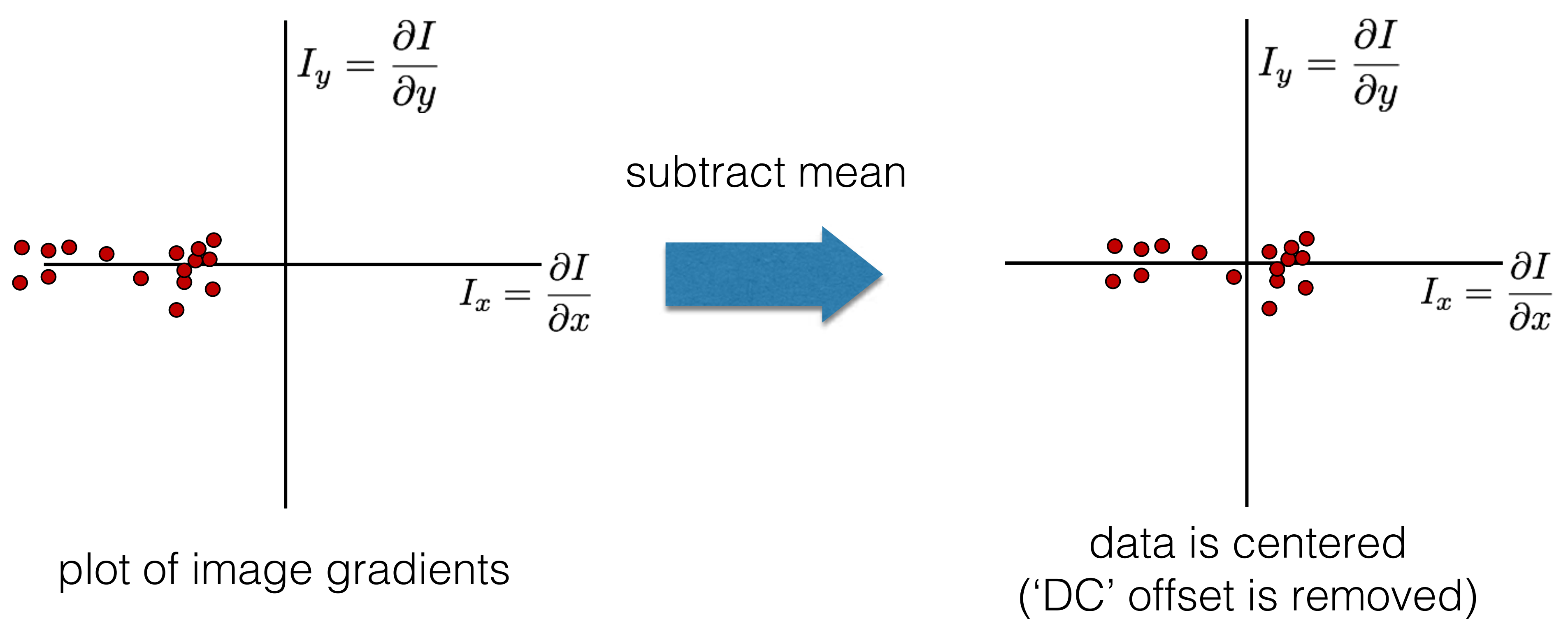



plot of image gradients

# 2. Subtract the mean from each image gradient



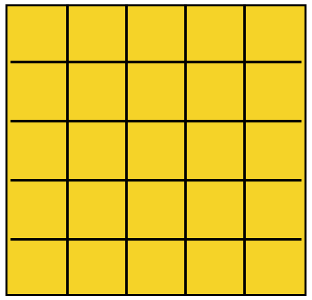
3. Compute the covariance matrix

# 3. Compute the covariance matrix

$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

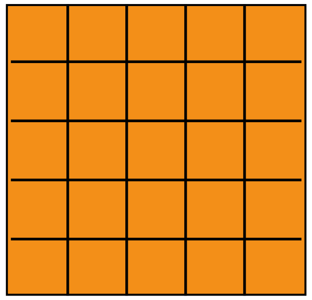
$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$

$\sum_{p \in P} I_x I_y$ =sum(



array of x gradients

*



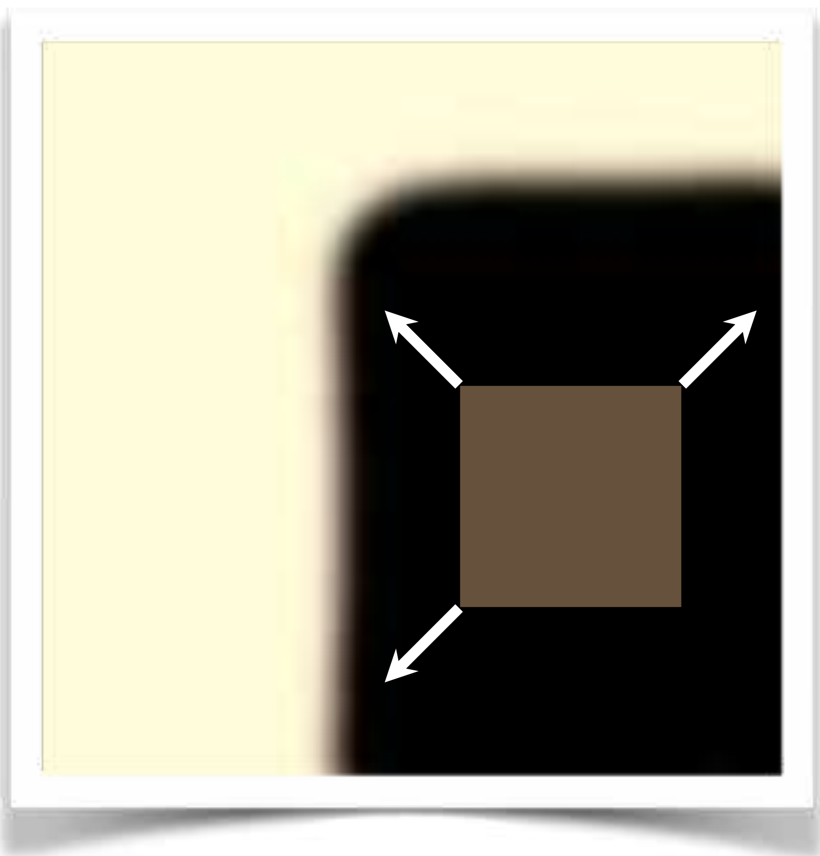
array of y gradients

)

*Where does this covariance matrix come from?*

Easily recognized by looking through a small window

Shifting the window should give large change in intensity



“flat” region:
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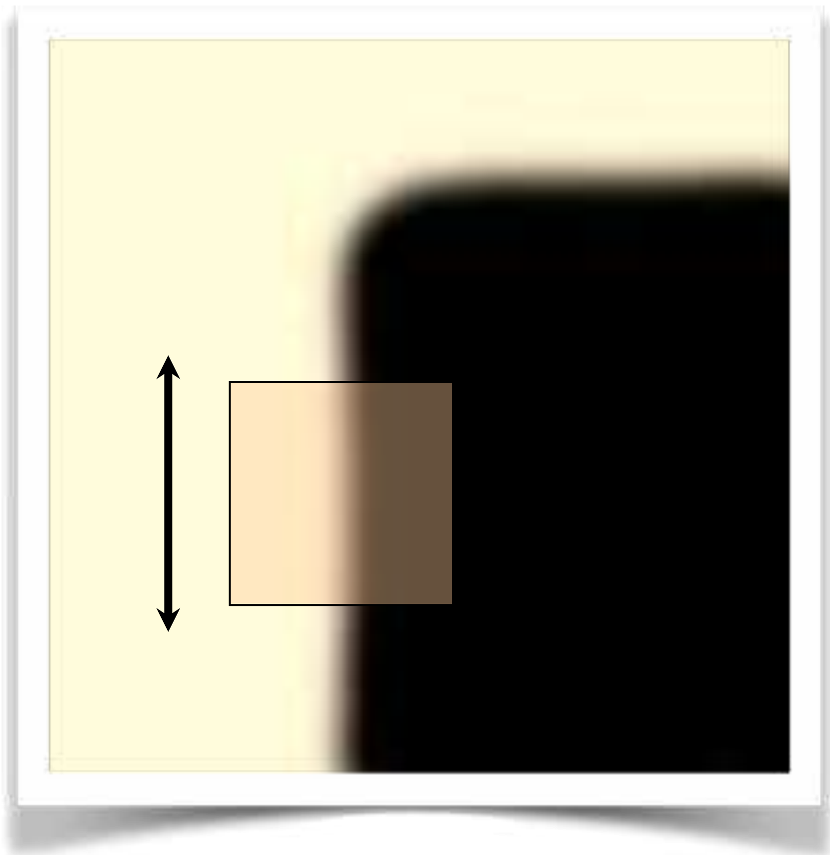
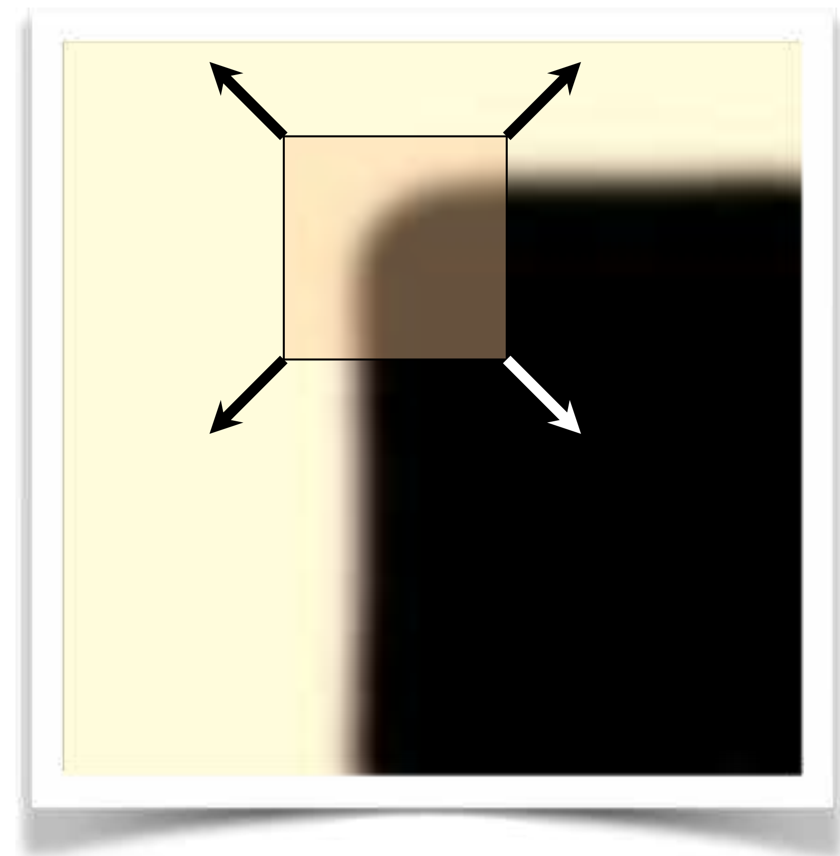


“edge”:
no change along the edge direction



“corner”:
significant change in all directions

[Moravec 1980]

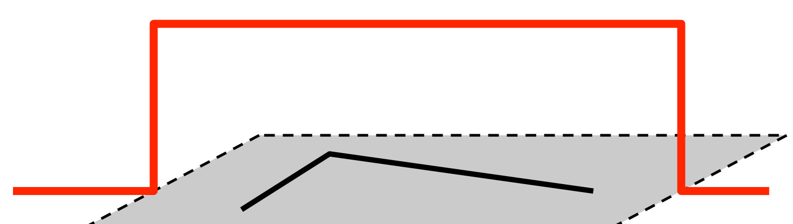
# Error function

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

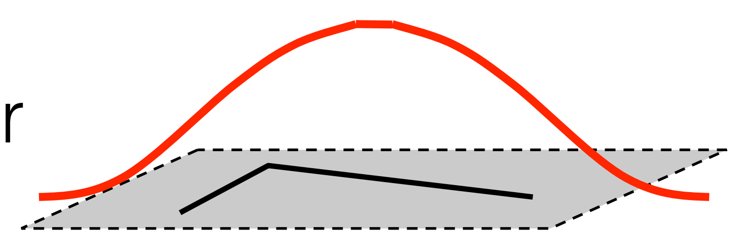
Error function — Window function — Shifted intensity — Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

# Error function approximation

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

First-order Taylor expansion of I(x,y) about (0,0)
(bilinear approximation for small shifts)

# Bilinear approximation

For small shifts $[u, v]$ we have a 'bilinear approximation':

Change in appearance for a shift [u,v]

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

'second moment' matrix
'structure tensor'

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

By computing the gradient covariance matrix…

$$\begin{bmatrix} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{bmatrix}$$

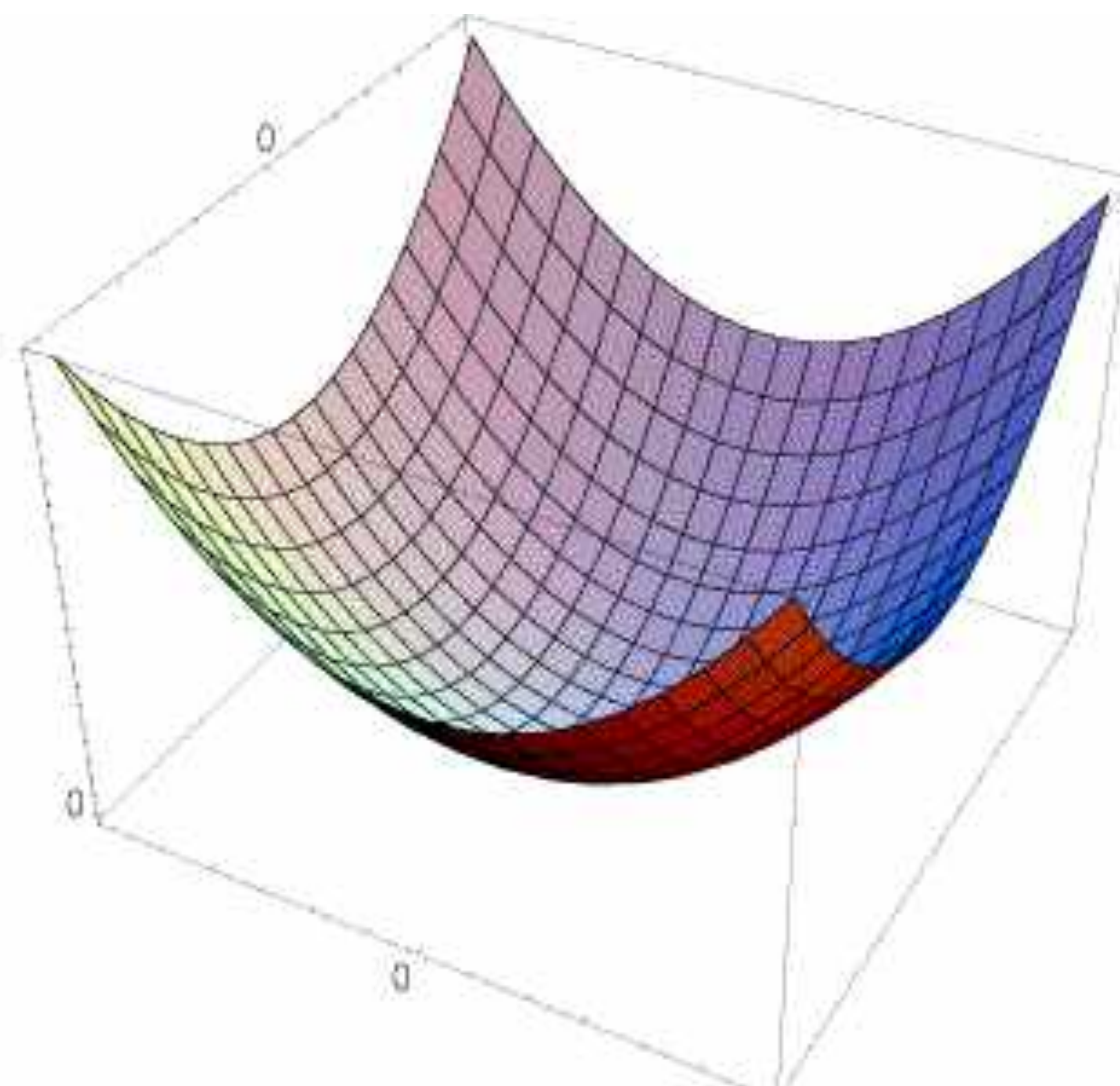
we are fitting a quadratic to the gradients over a small image region

# Visualization of a quadratic

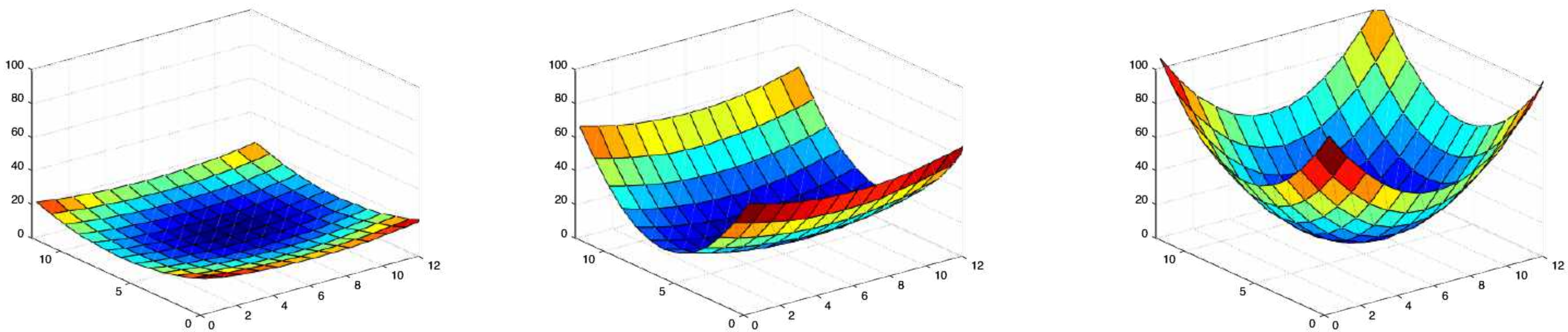
The surface *E*(*u*,*v*) is locally approximated by a quadratic form

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

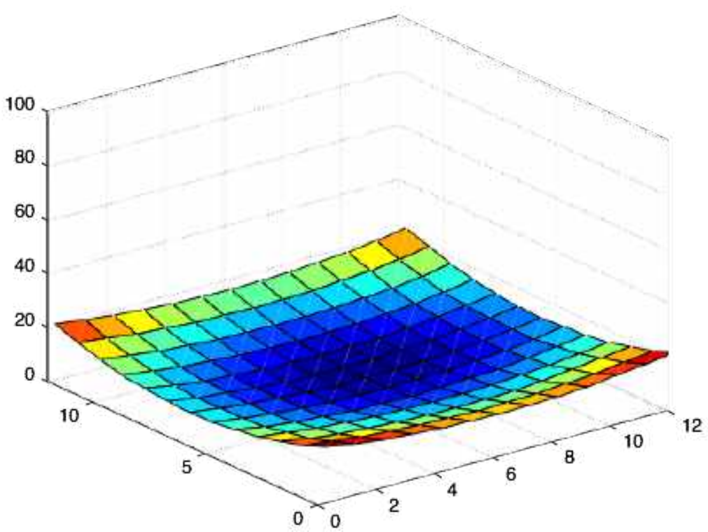


*Which error surface indicates a good image feature?*

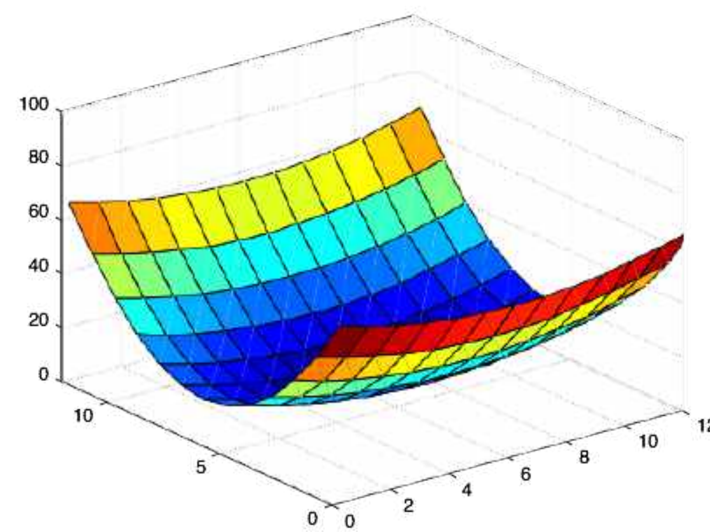


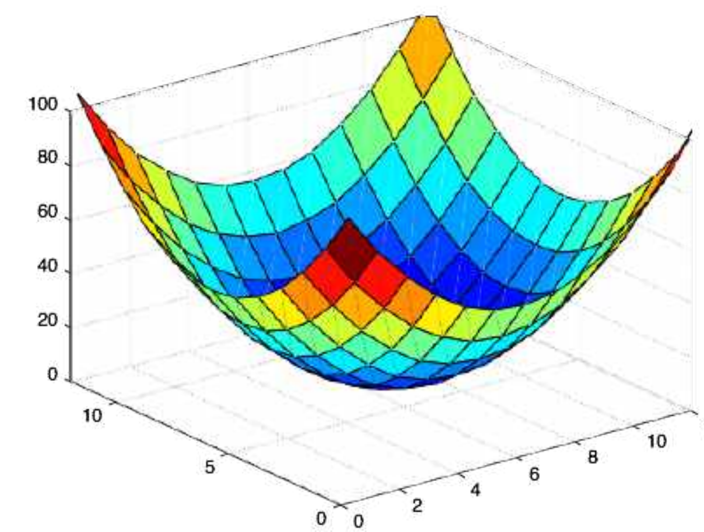

*What kind of image patch do these surfaces represent?*

## *Which error surface indicates a good image feature?*

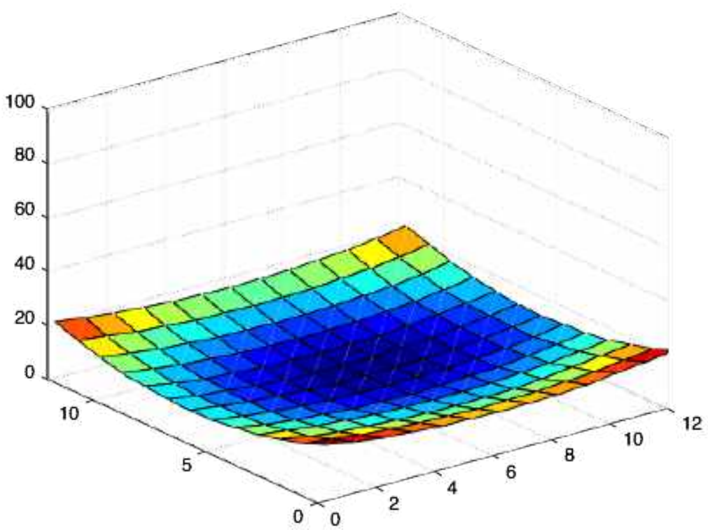



flat

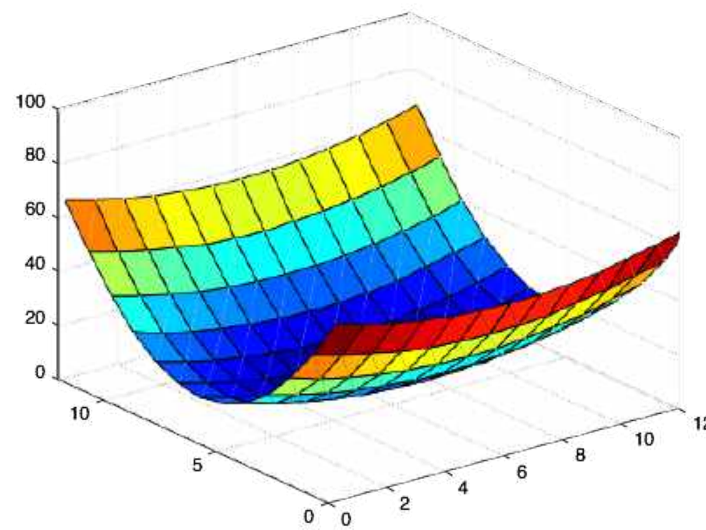


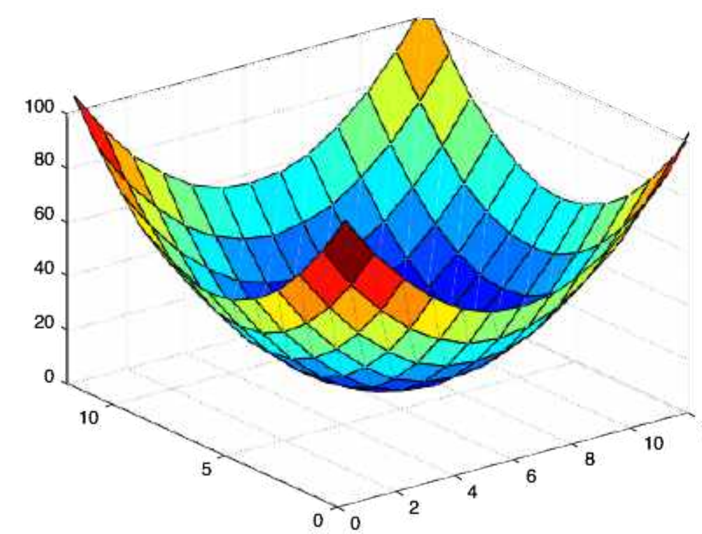
*Which error surface indicates a good image feature?*



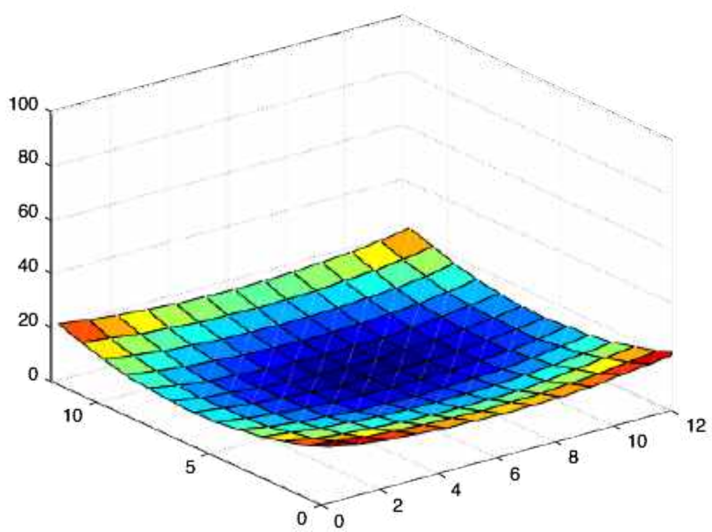

flat



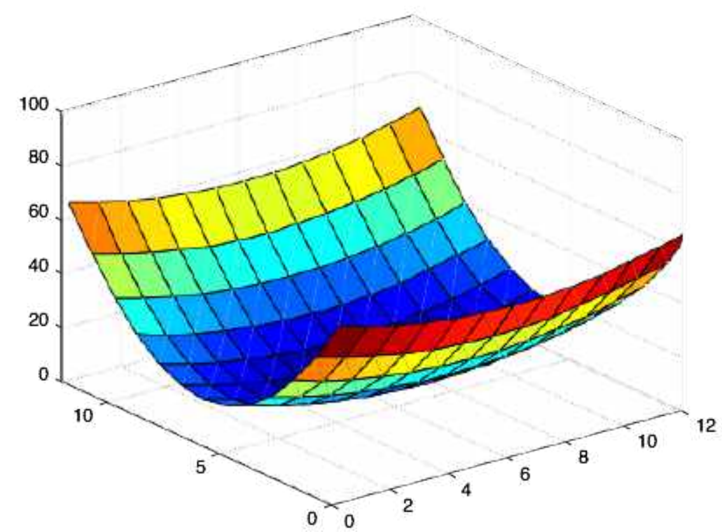

edge
'line'



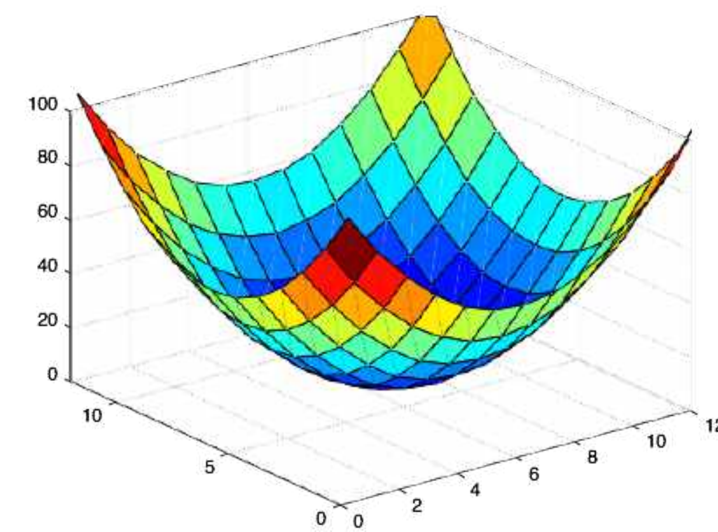
*Which error surface indicates a good image feature?*




flat




edge
‘line’




corner
‘dot’

4. Compute eigenvalues and eigenvectors

# 4. Compute eigenvalues and eigenvectors

eigenvalue

$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

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eigenvalue

$$Me = \lambda e$$

eigenvector

$$(M - \lambda I)e = 0$$

1. Compute the determinant of $M - \lambda I$

(returns a polynomial)

# 4. Compute eigenvalues and eigenvectors

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$$M\boldsymbol{e} = \lambda \boldsymbol{e}$$

eigenvector

$$(M - \lambda I)\boldsymbol{e} = 0$$

1. Compute the determinant of $M - \lambda I$
   (returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$
   (returns eigenvalues)

# 4. Compute eigenvalues and eigenvectors

eigenvalue

$$Me = \lambda e$$

eigenvector

$$(M - \lambda I)e = 0$$

1. Compute the determinant of $M - \lambda I$
(returns a polynomial)

2. Find the roots of polynomial $\det(M - \lambda I) = 0$
(returns eigenvalues)

3. For each eigenvalue, solve $(M - \lambda I)e = 0$
(returns eigenvectors)

```
eig(M)
```

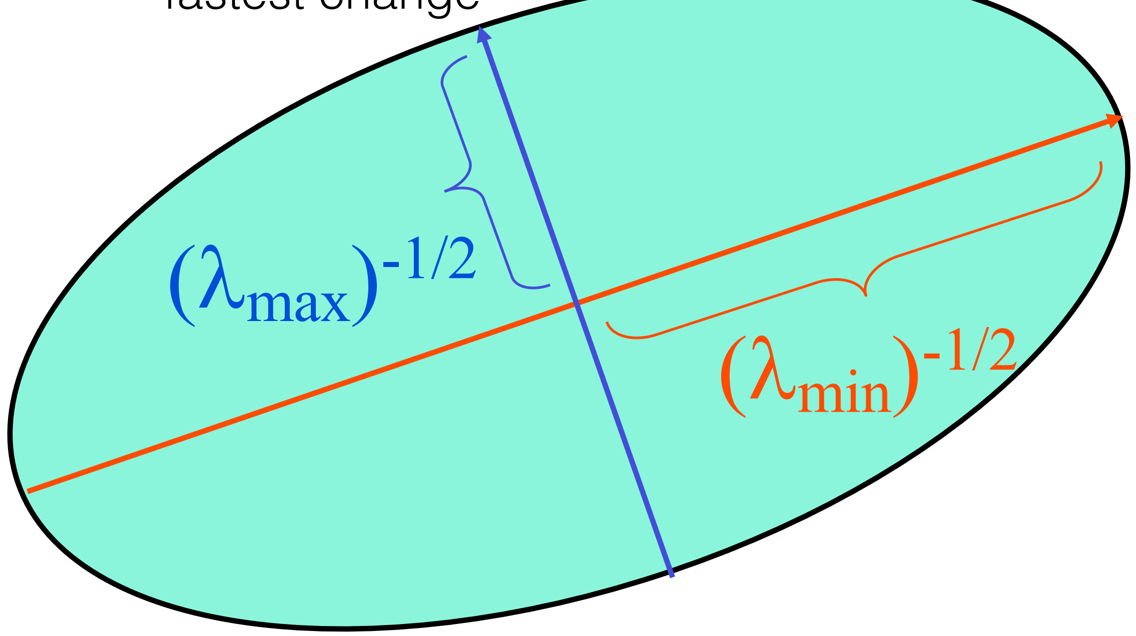
# Visualization as an ellipse

Since M is symmetric, we have $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

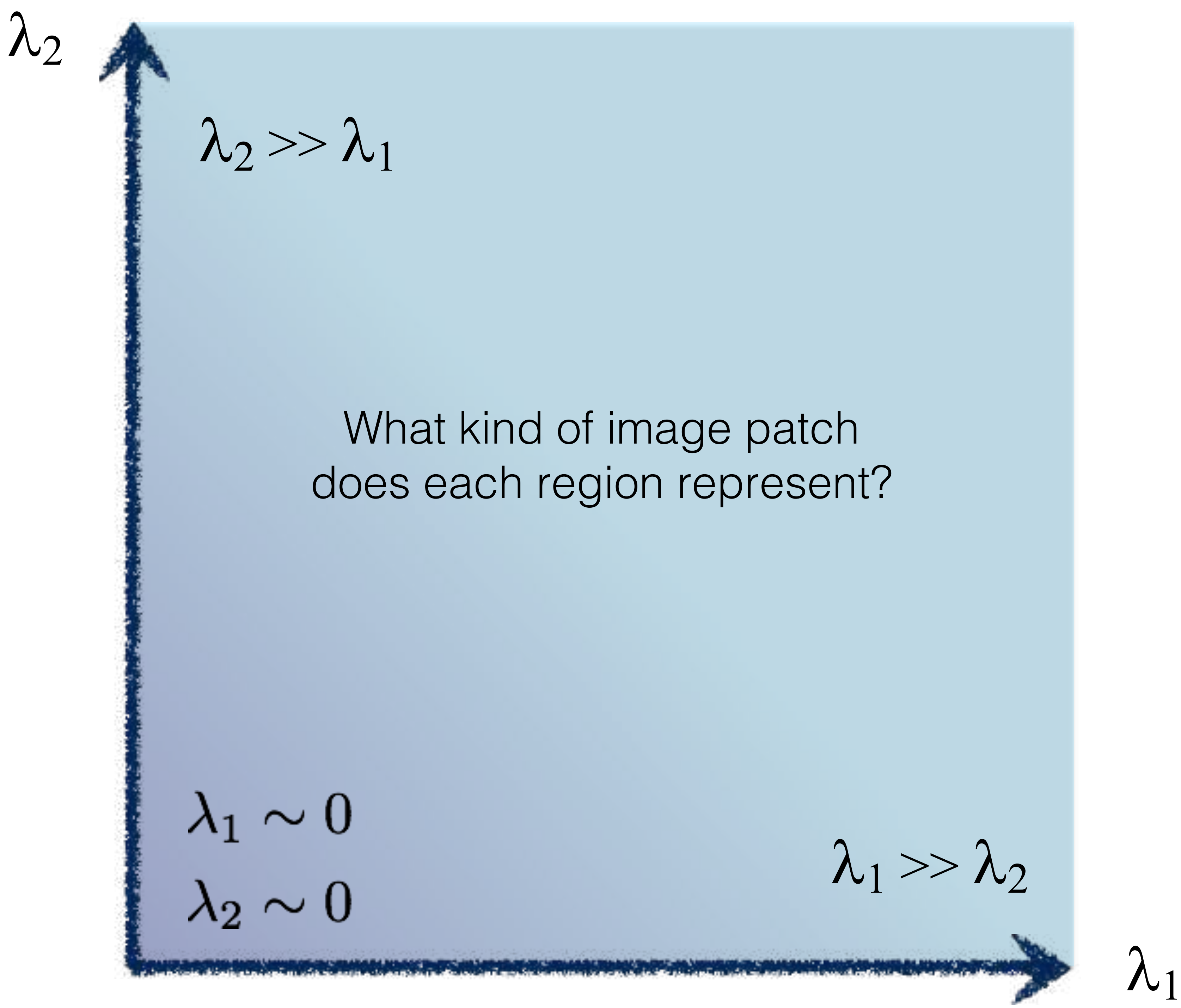
We can visualize ***M*** as an ellipse with axis lengths determined by the eigenvalues and orientation determined by ***R***

Ellipse equation:

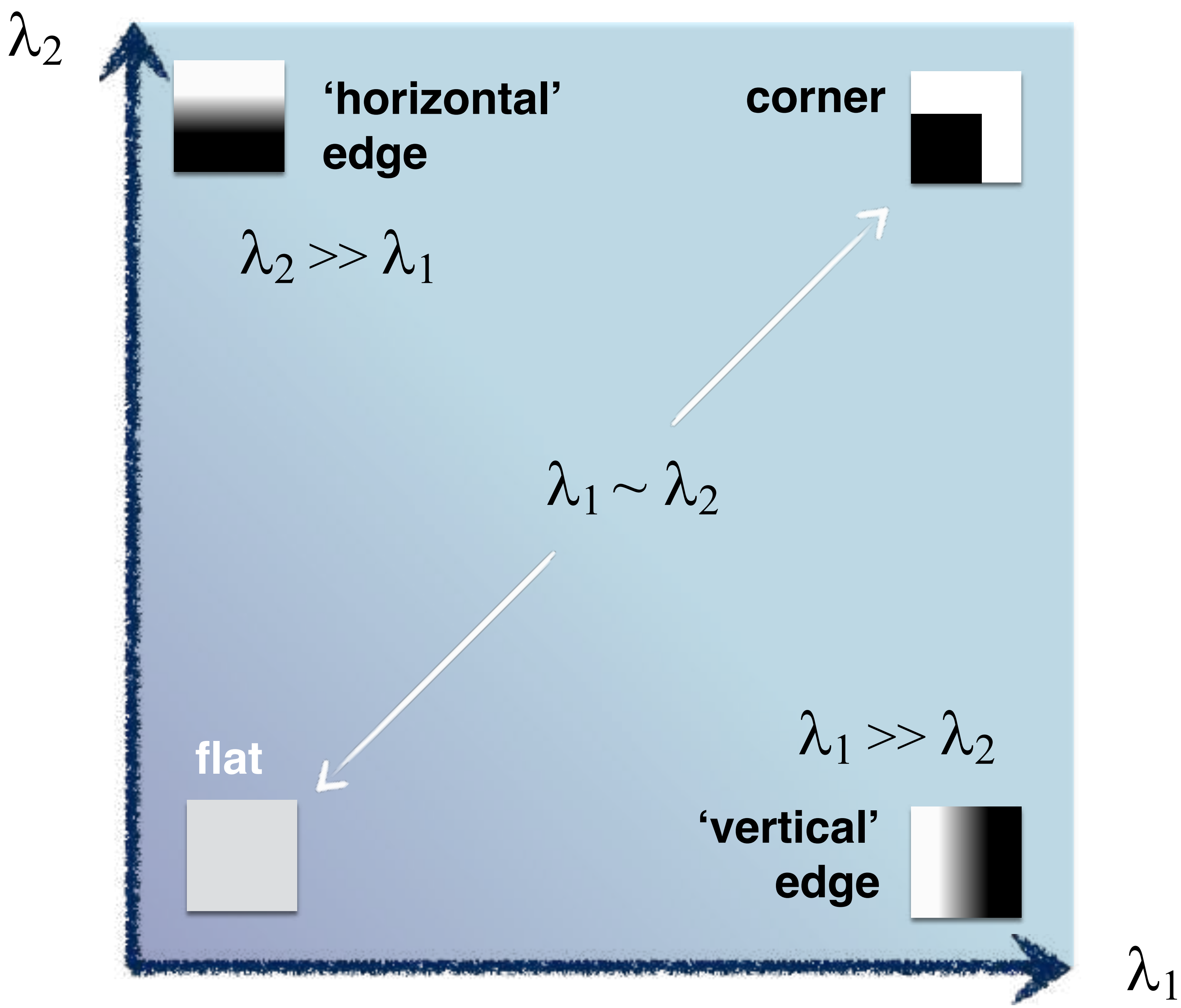
$$[u \;\; v] \;\; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



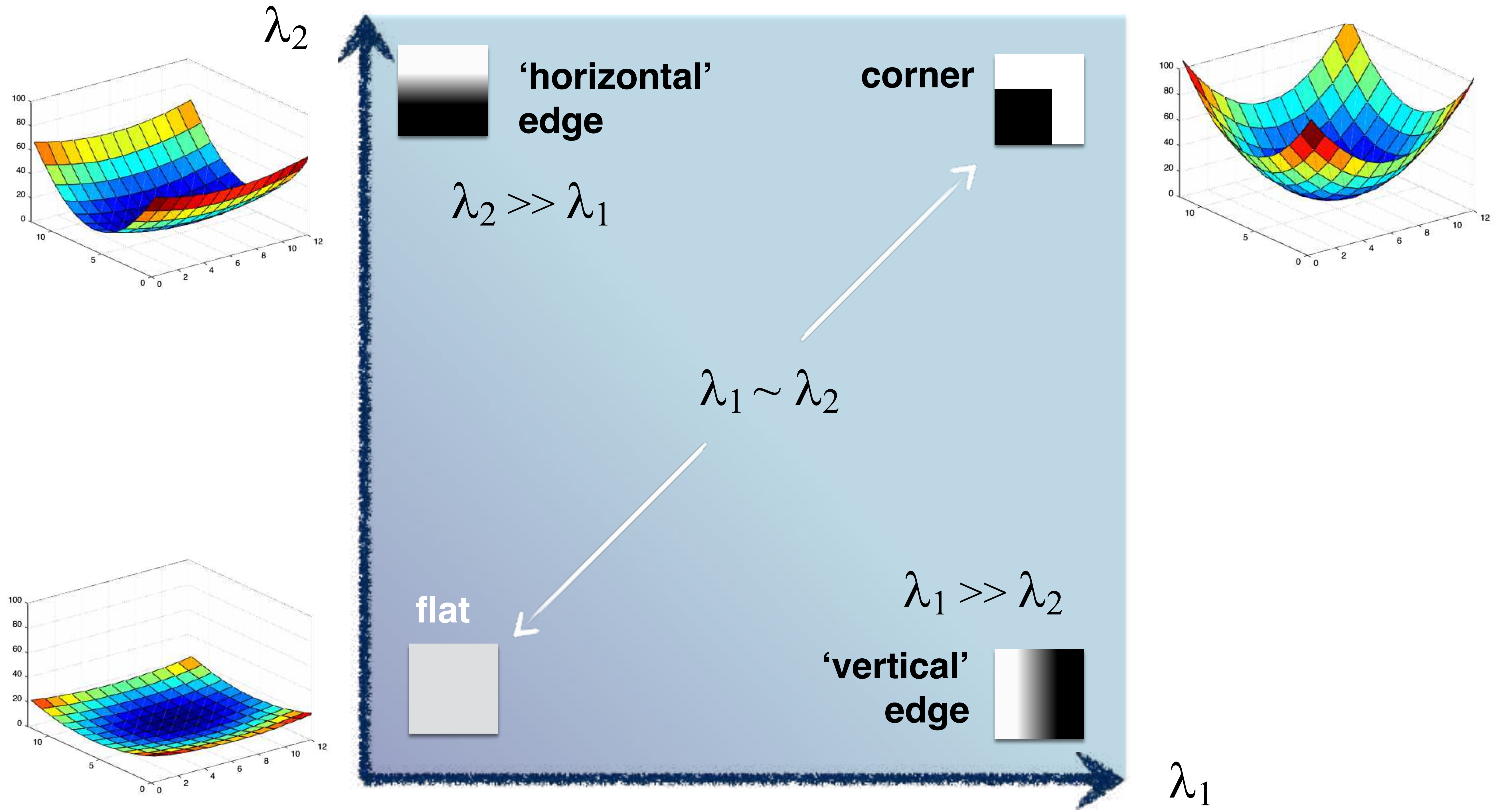
# interpreting eigenvalues



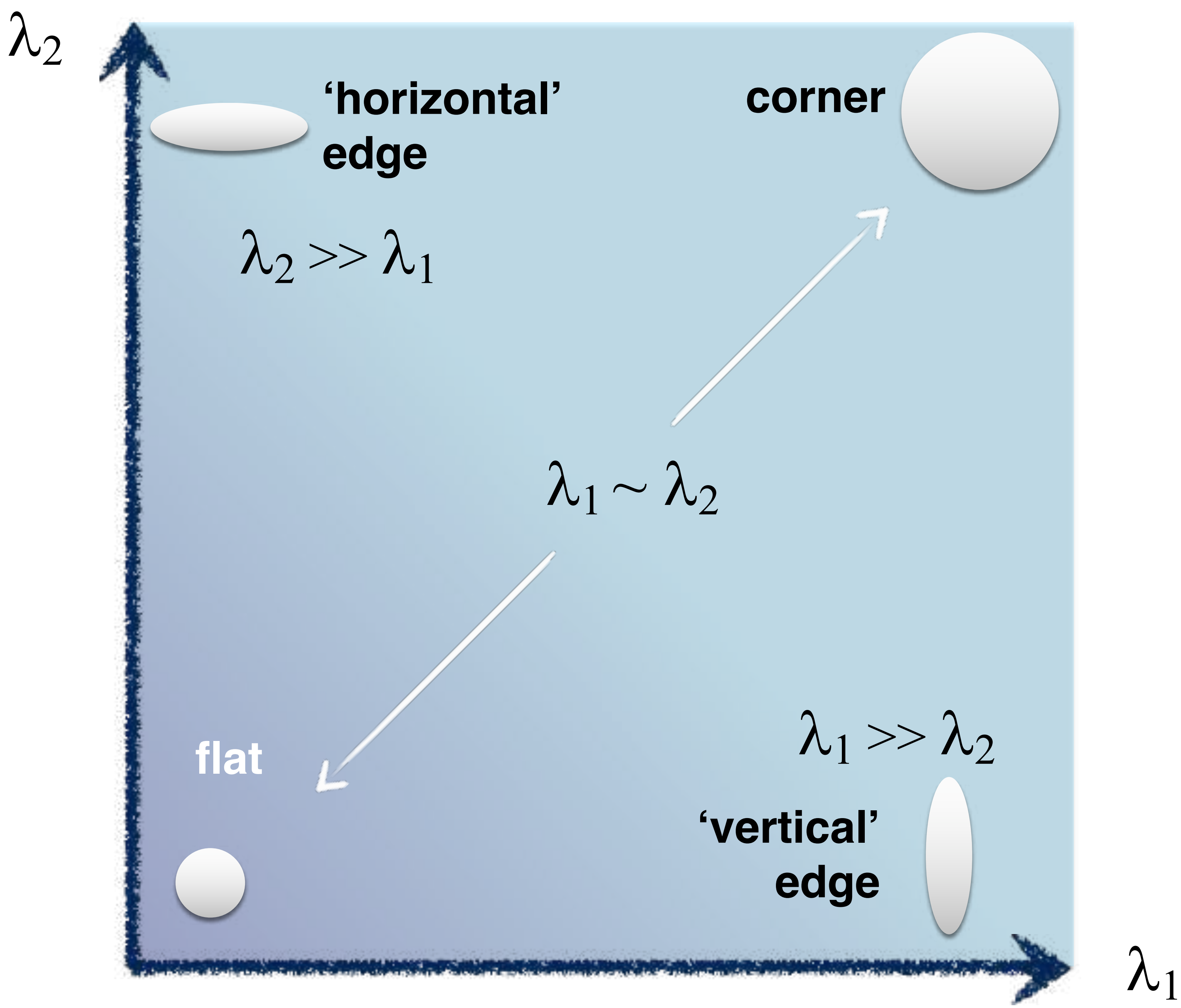
# interpreting eigenvalues



# interpreting eigenvalues



# interpreting eigenvalues



5. Use threshold on eigenvalues to detect corners

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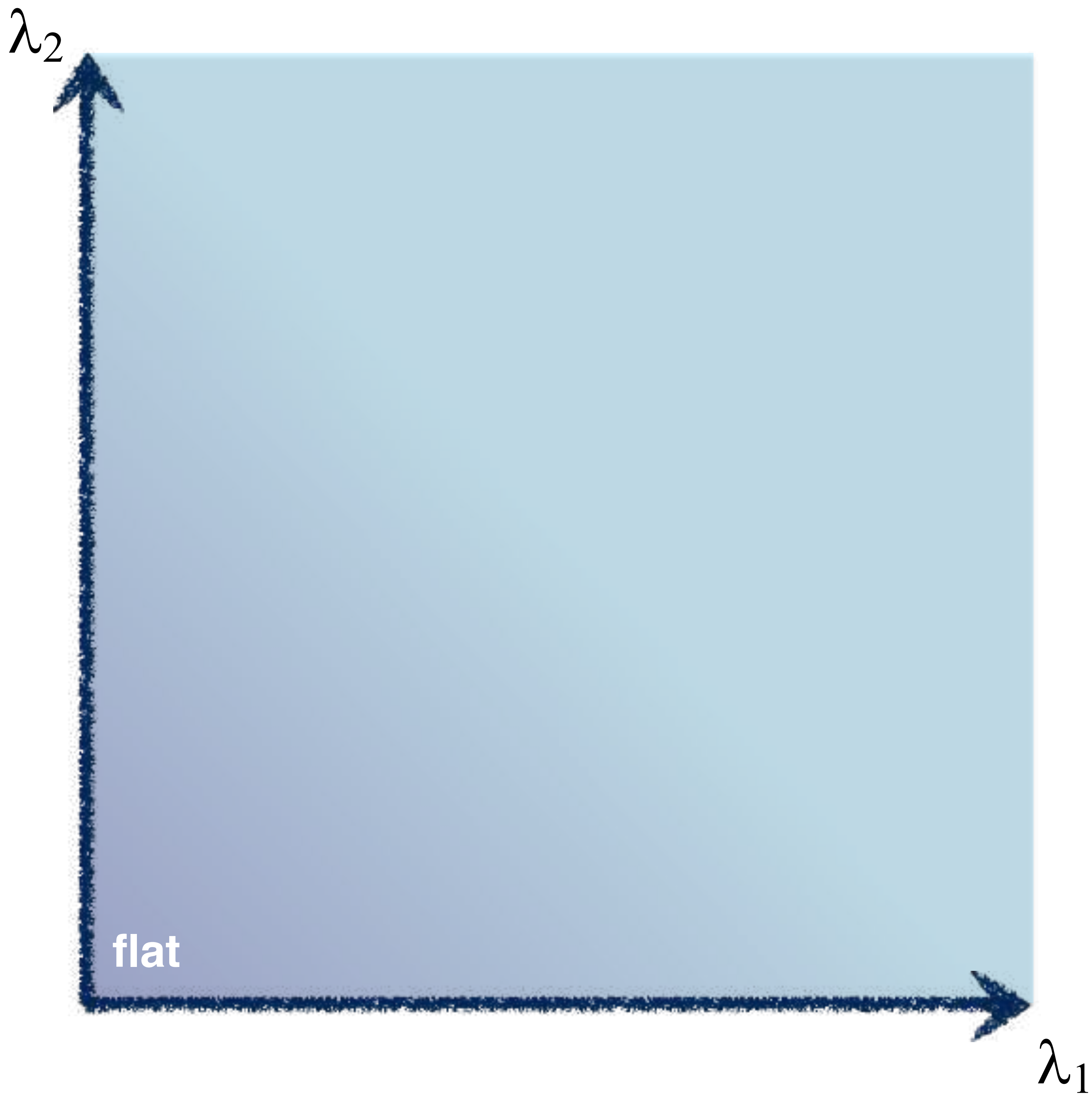
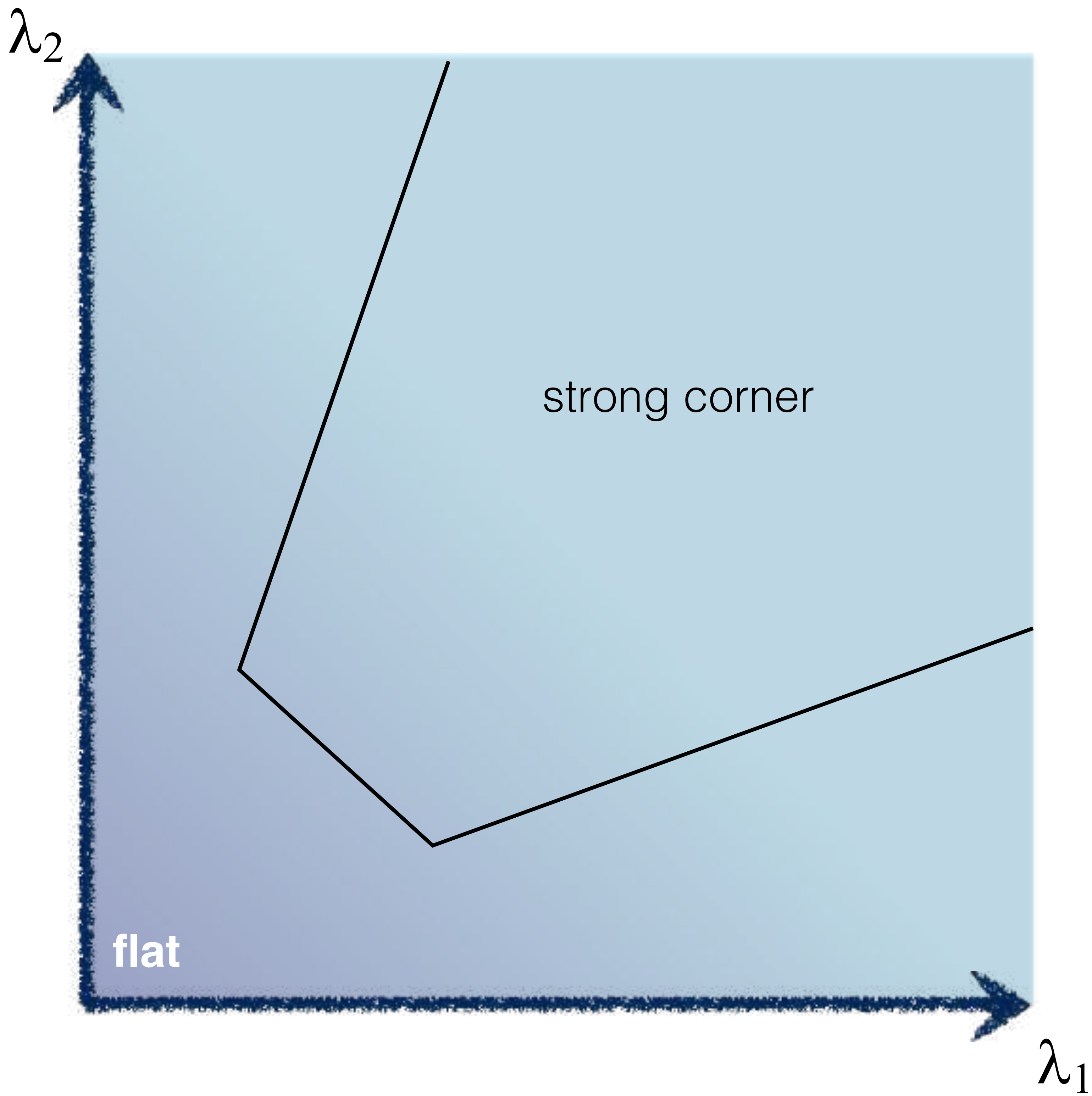



Think of a function to score 'cornerness'

# 5. Use threshold on eigenvalues to detect corners

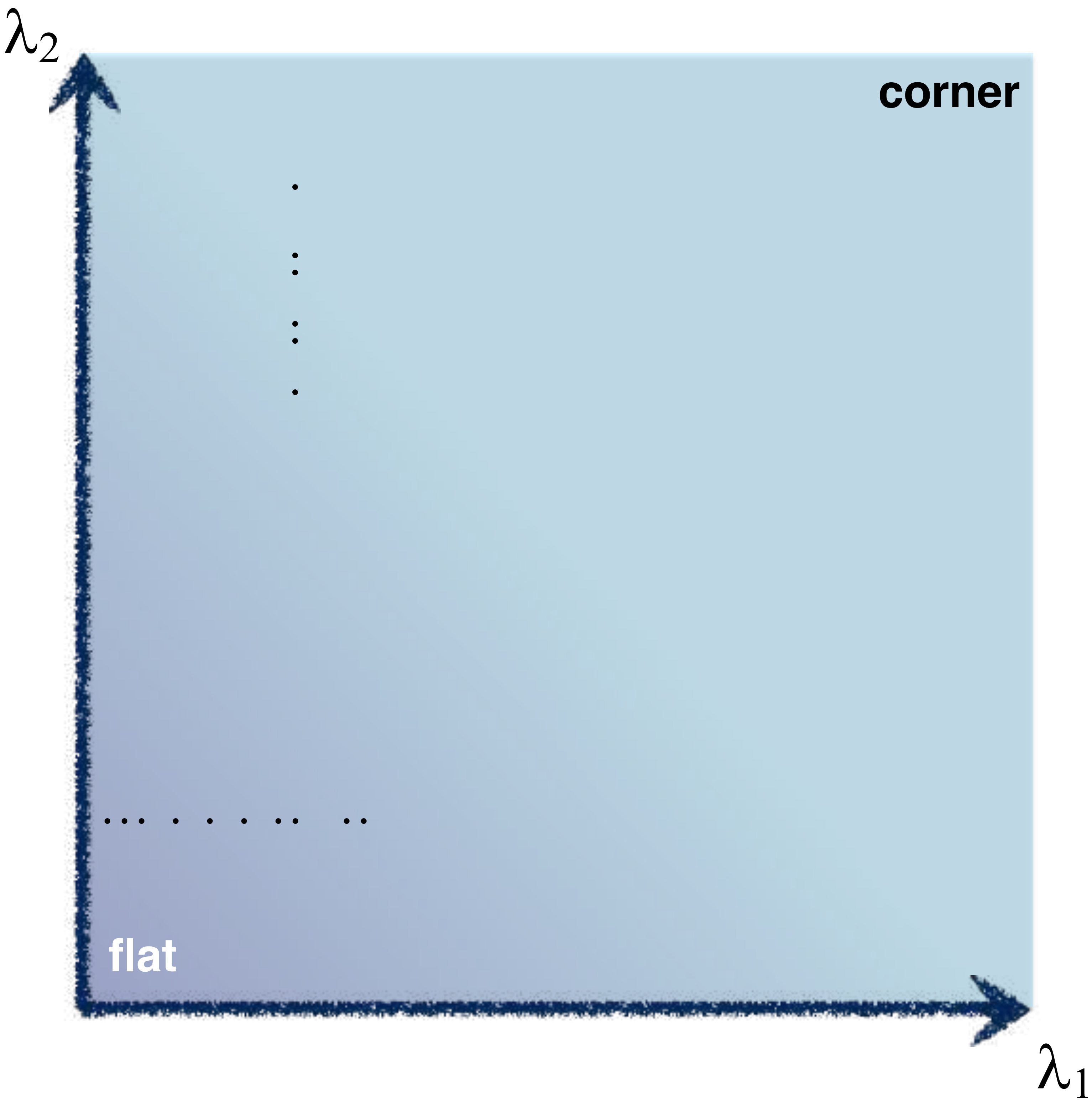



Think of a function to score ‘cornerness’

# 5. Use threshold on eigenvalues to detect corners

^

(a function of )

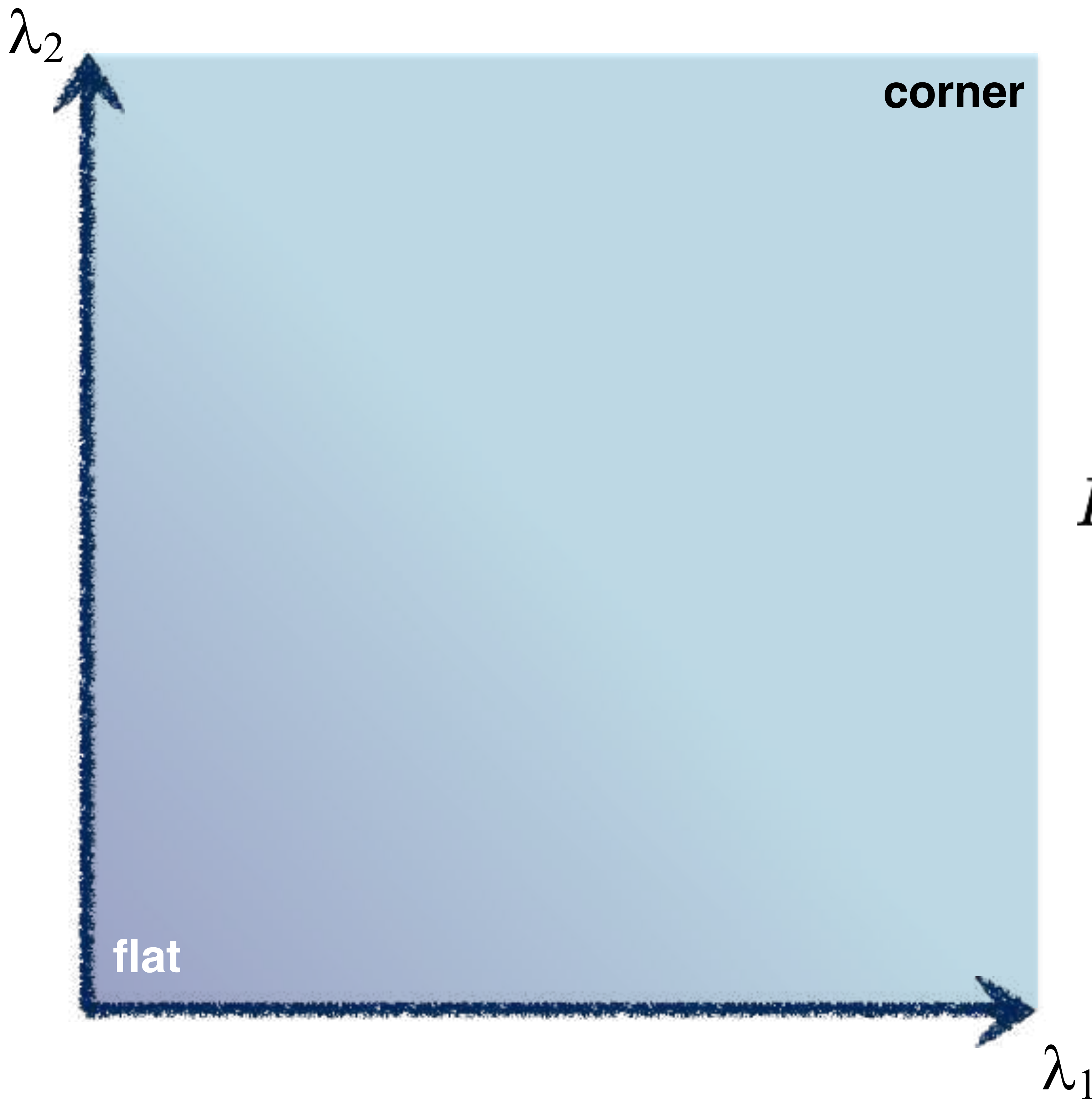



Use the smallest eigenvalue as the response function

$$R = \min(\lambda_1, \lambda_2)$$

# 5. Use threshold on eigenvalues to detect corners

^
(a function of )



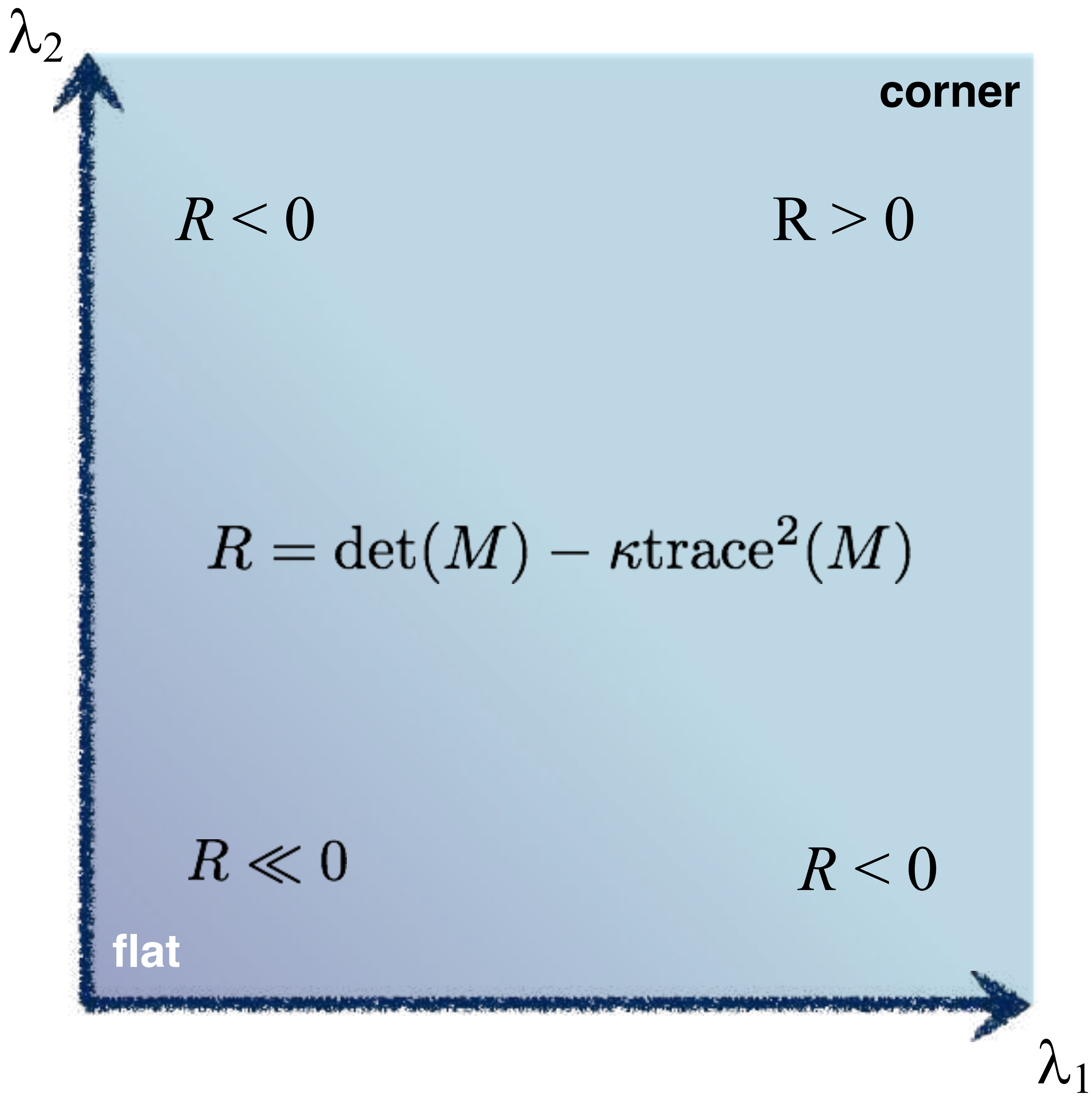

Eigenvalues need to be bigger than one.

$$R = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently…

# 5. Use threshold on eigenvalues to detect corners

^ (a function of )




$$\det M = \lambda_1 \lambda_2$$

$$\text{trace}\, M = \lambda_1 + \lambda_2$$

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

Harris & Stephens (1988)

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$R = \frac{\det(M)}{\operatorname{trace}(M) + \epsilon}$$

# Harris Detector

C.Harris and M.Stephens. “A Combined Corner and Edge Detector.”1988.

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

# Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." 1988.

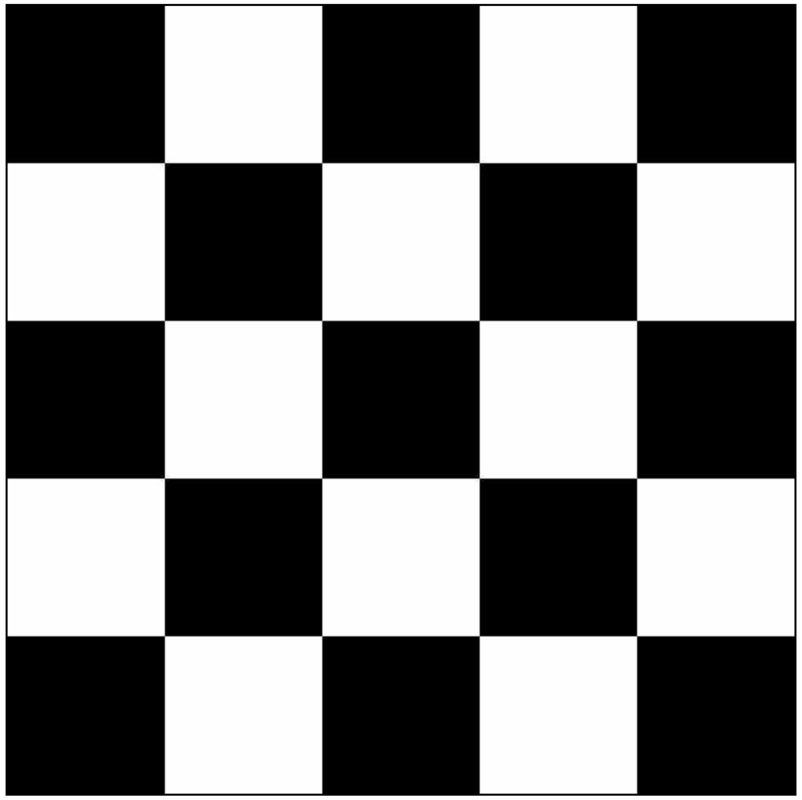
4. Define the matrix at each pixel

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

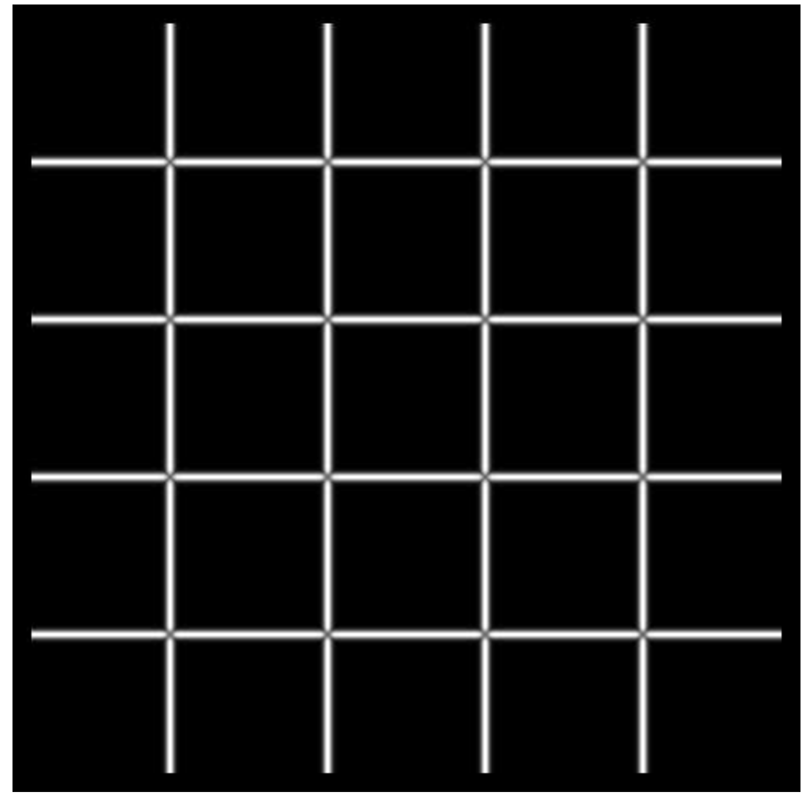
5. Compute the response of the detector at each pixel

$$R = \det M - k(\mathrm{trace} M)^2$$

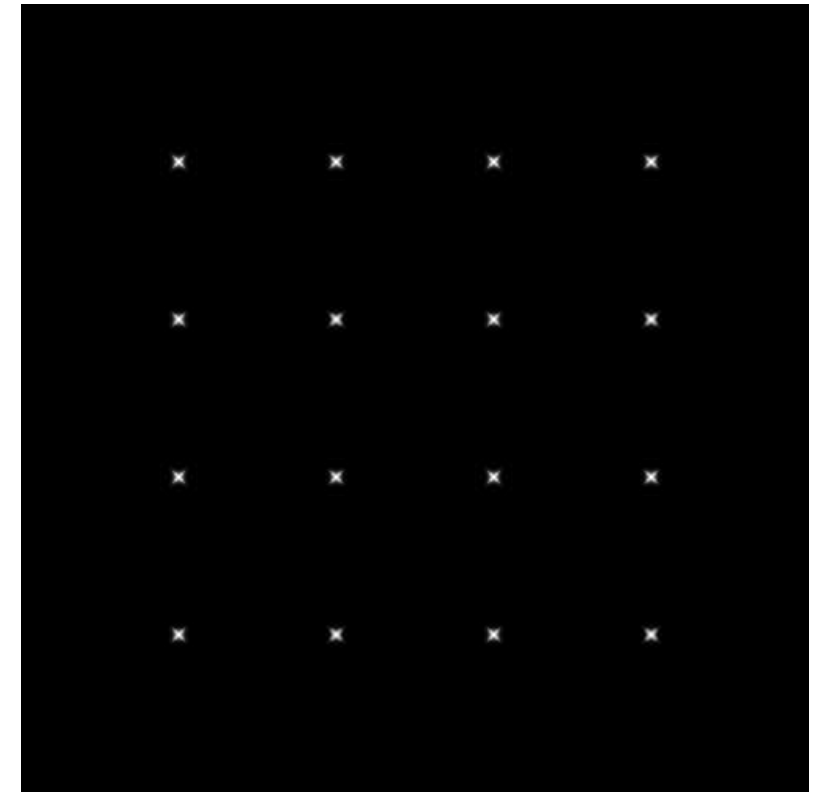
6. Threshold on value of R; compute non-max suppression.



$I$



$\lambda_{\max}$

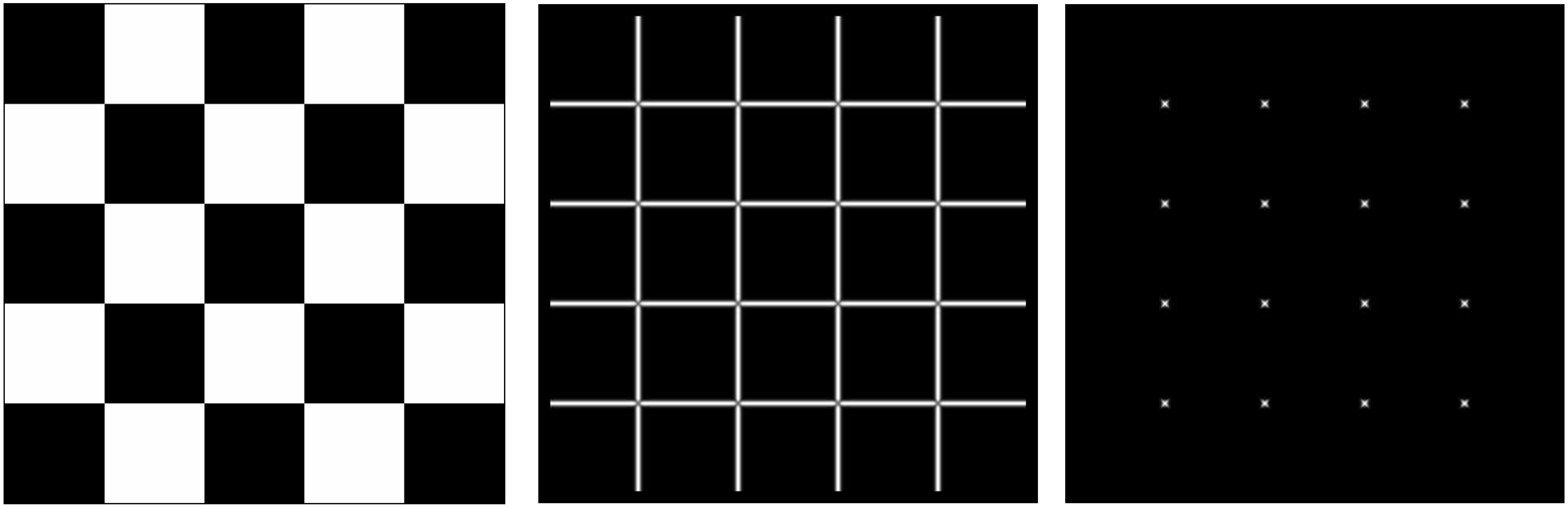


$\lambda_{\min}$

Yet another option:

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

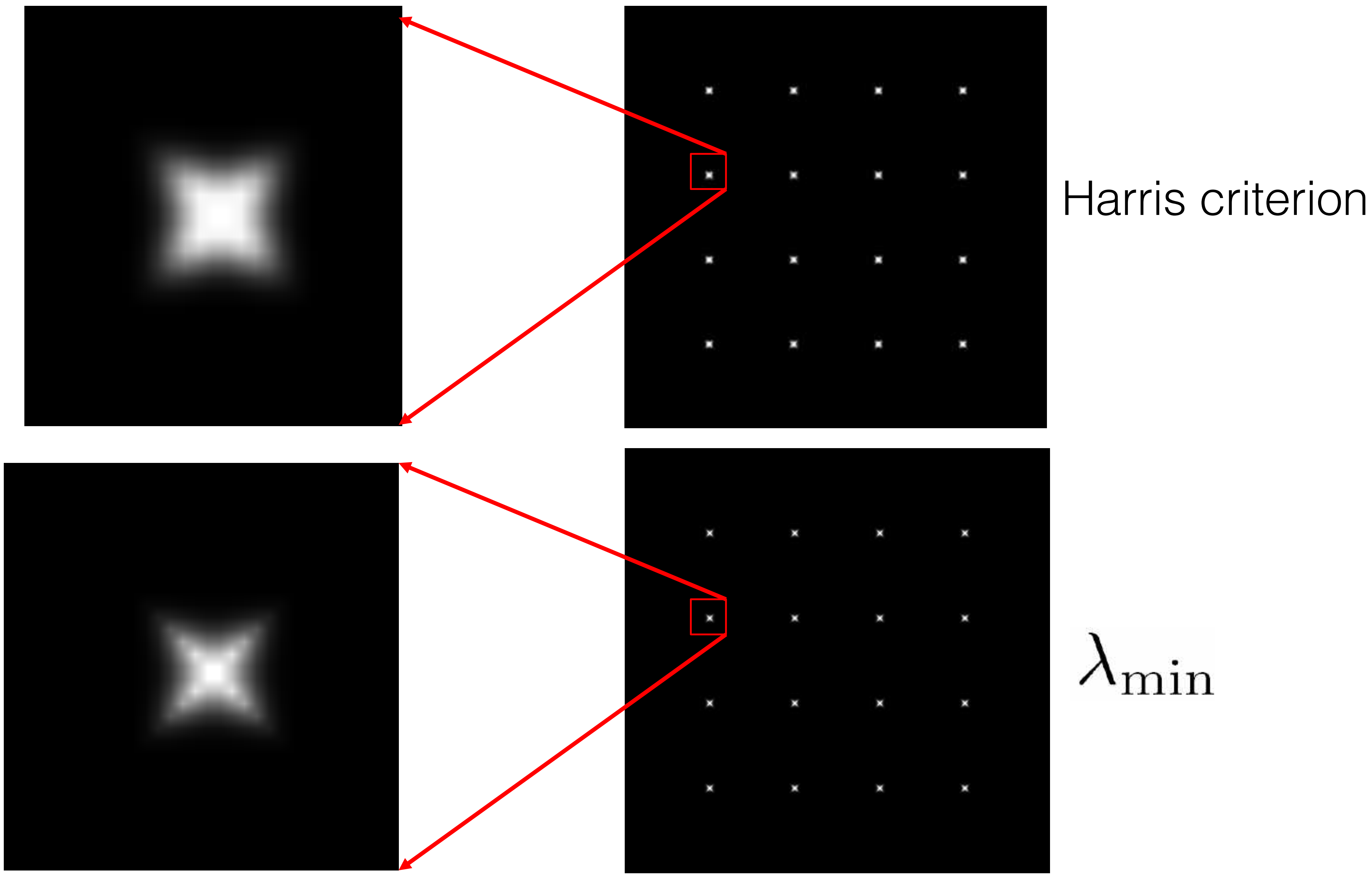
How do you write this equivalently using determinant and trace?



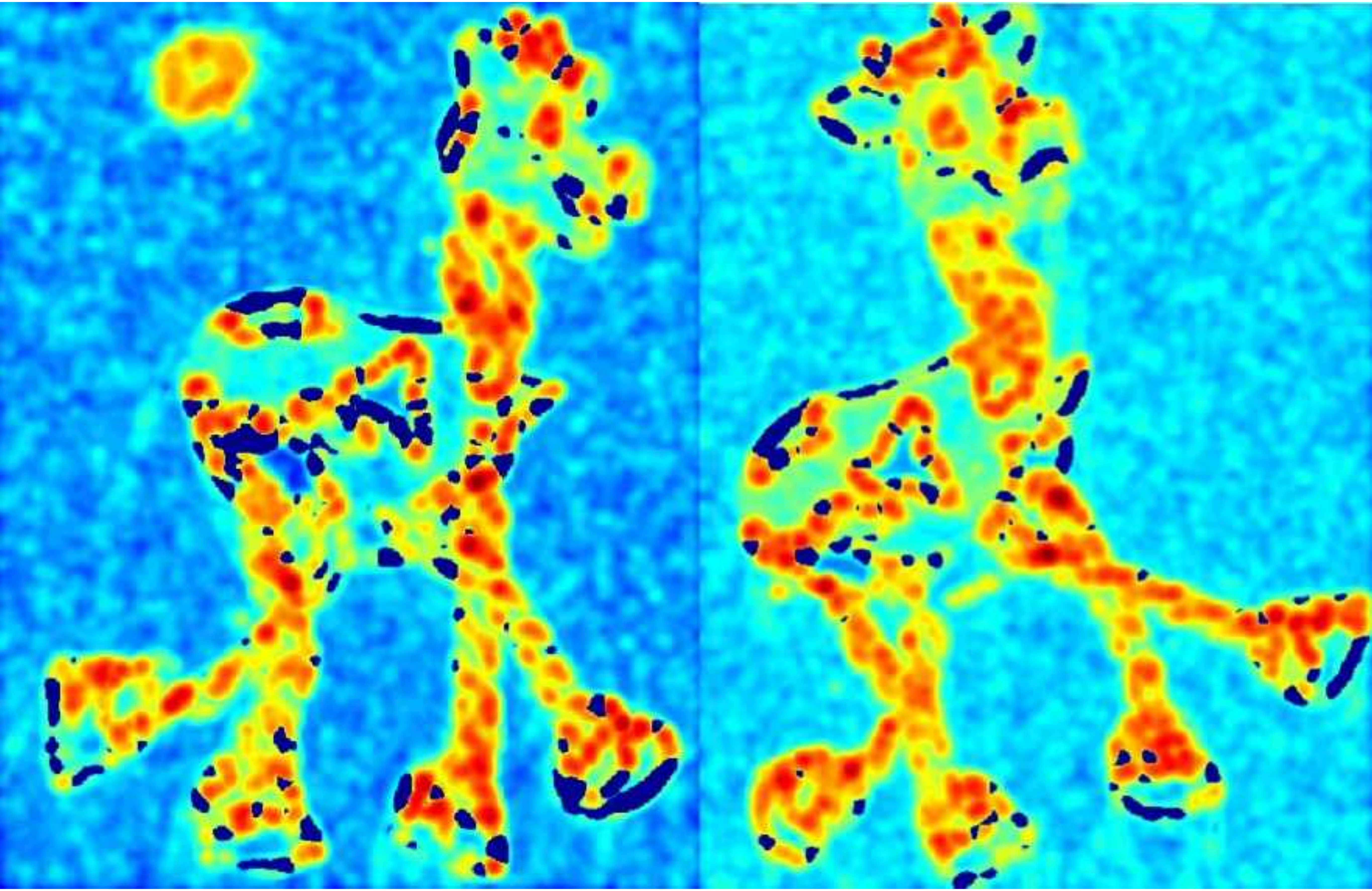
$I$ $\qquad$ $\lambda_{\max}$ $\qquad$ $\lambda_{\min}$

Yet another option: $$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{determinant(H)}{trace(H)}$$

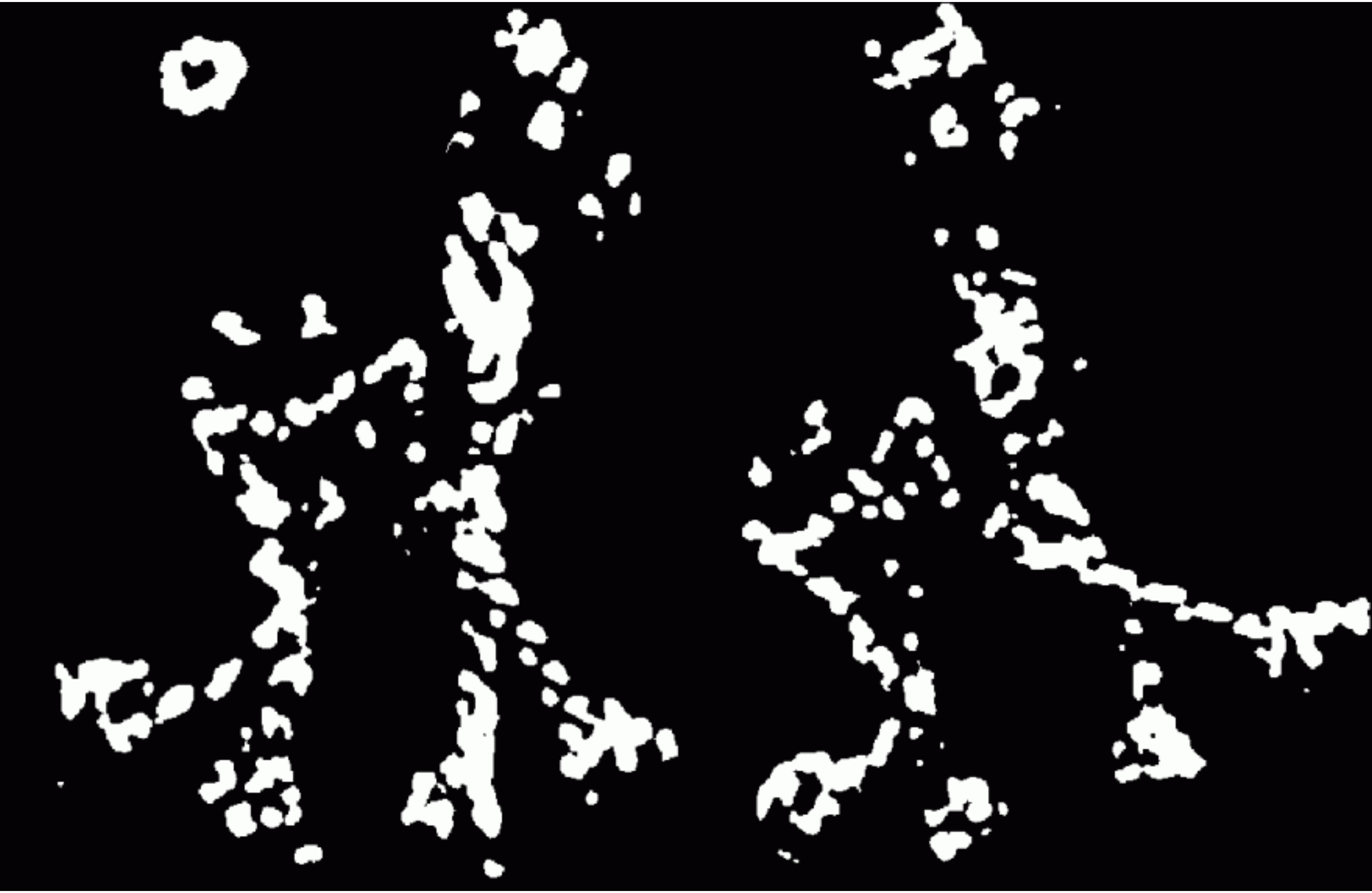
# Different criteria



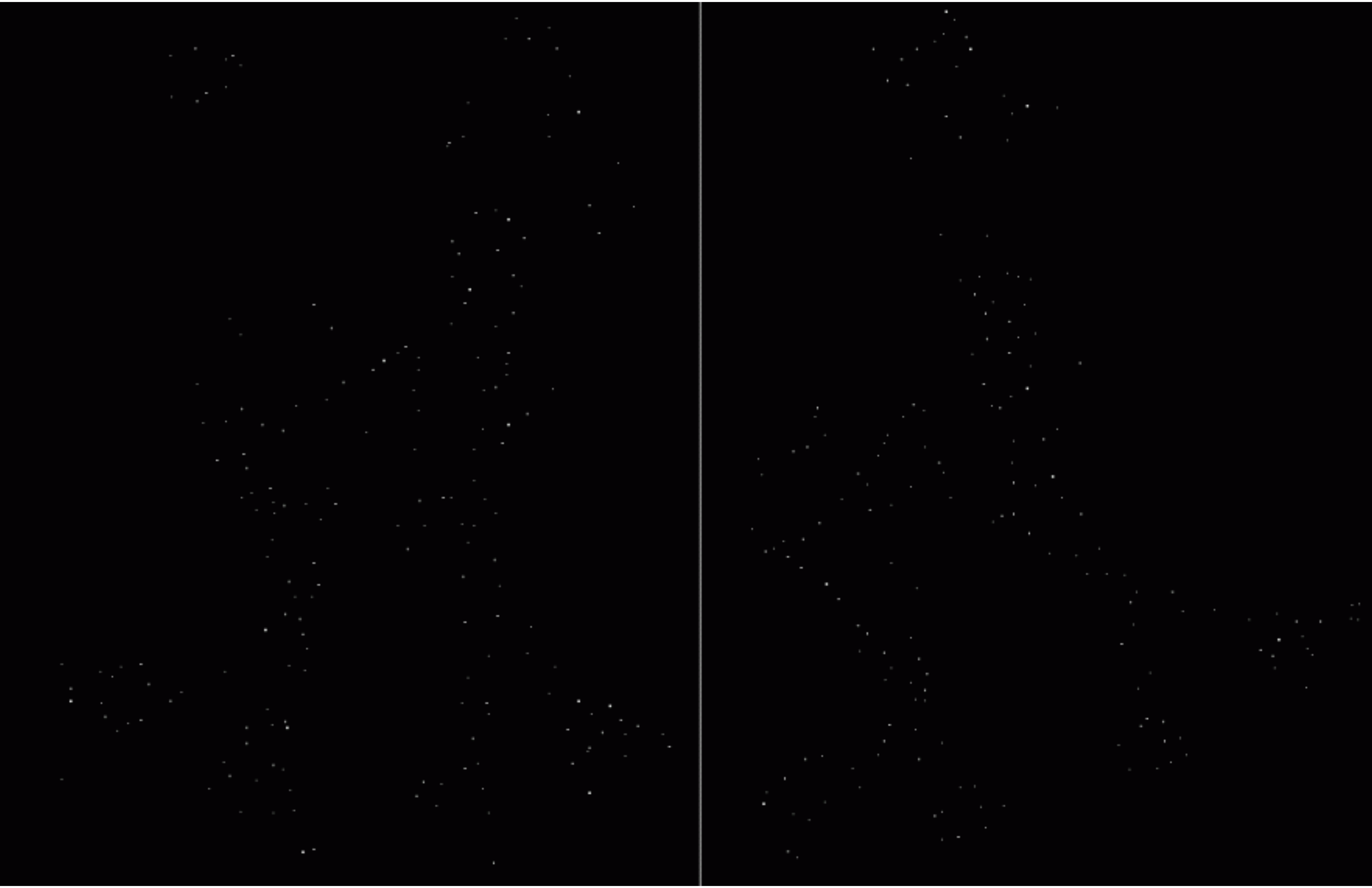

# Corner response



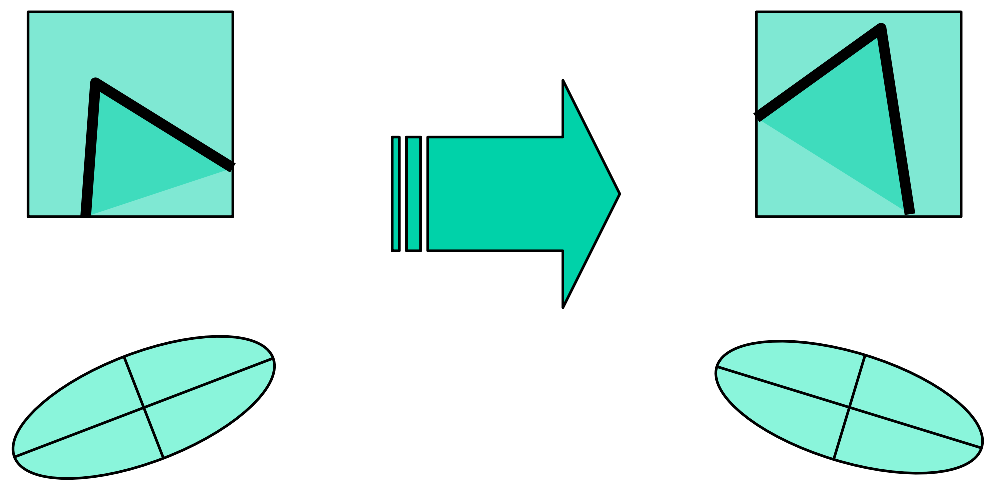
# Thresholded corner response



# Non-maximal suppression



# Harris corner response is invariant to rotation



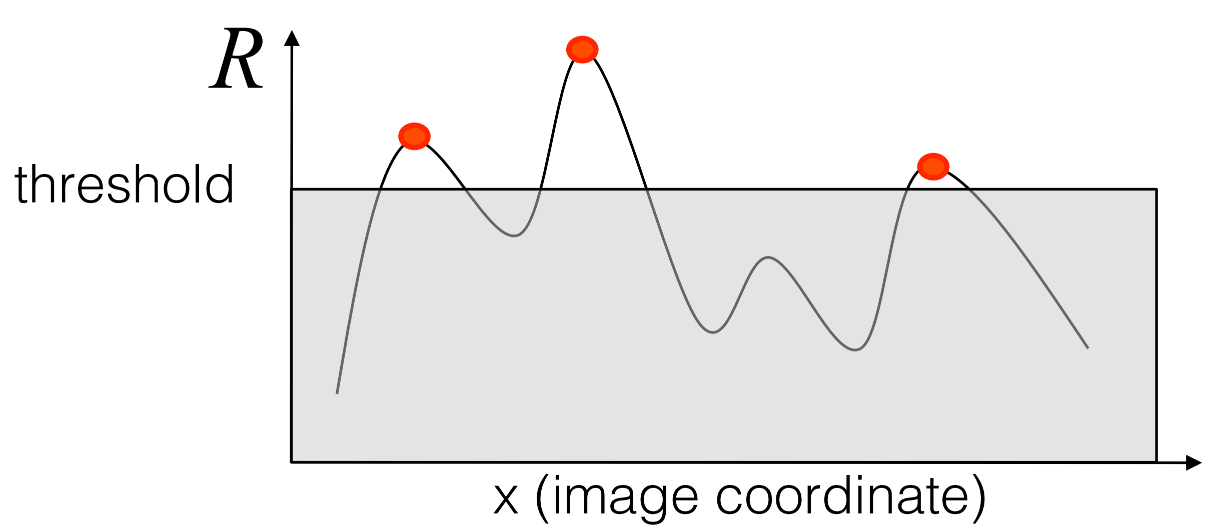
Ellipse rotates but its shape (**eigenvalues**) remains the same

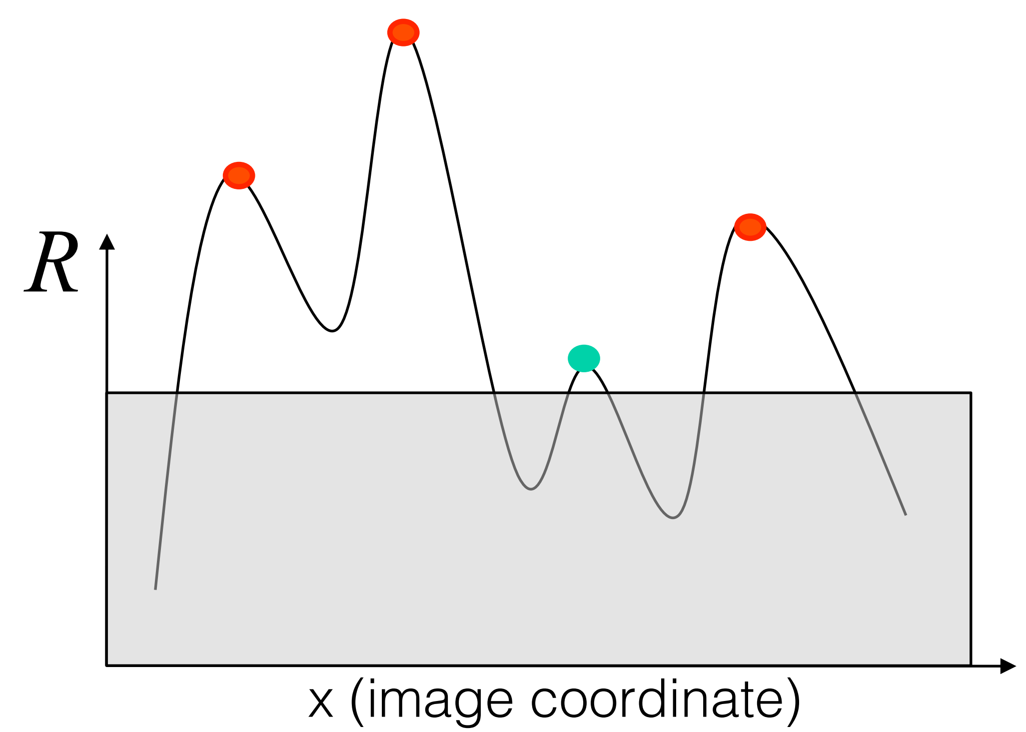
**Corner response R is invariant to image rotation**

# Harris corner response is invariant to intensity changes

Partial invariance to *affine intensity* change

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scale: $I \rightarrow a\,I$



The Harris detector is not invariant to changes in …

# The Harris corner detector is <u>not</u> invariant to scale

# Multi-scale detection

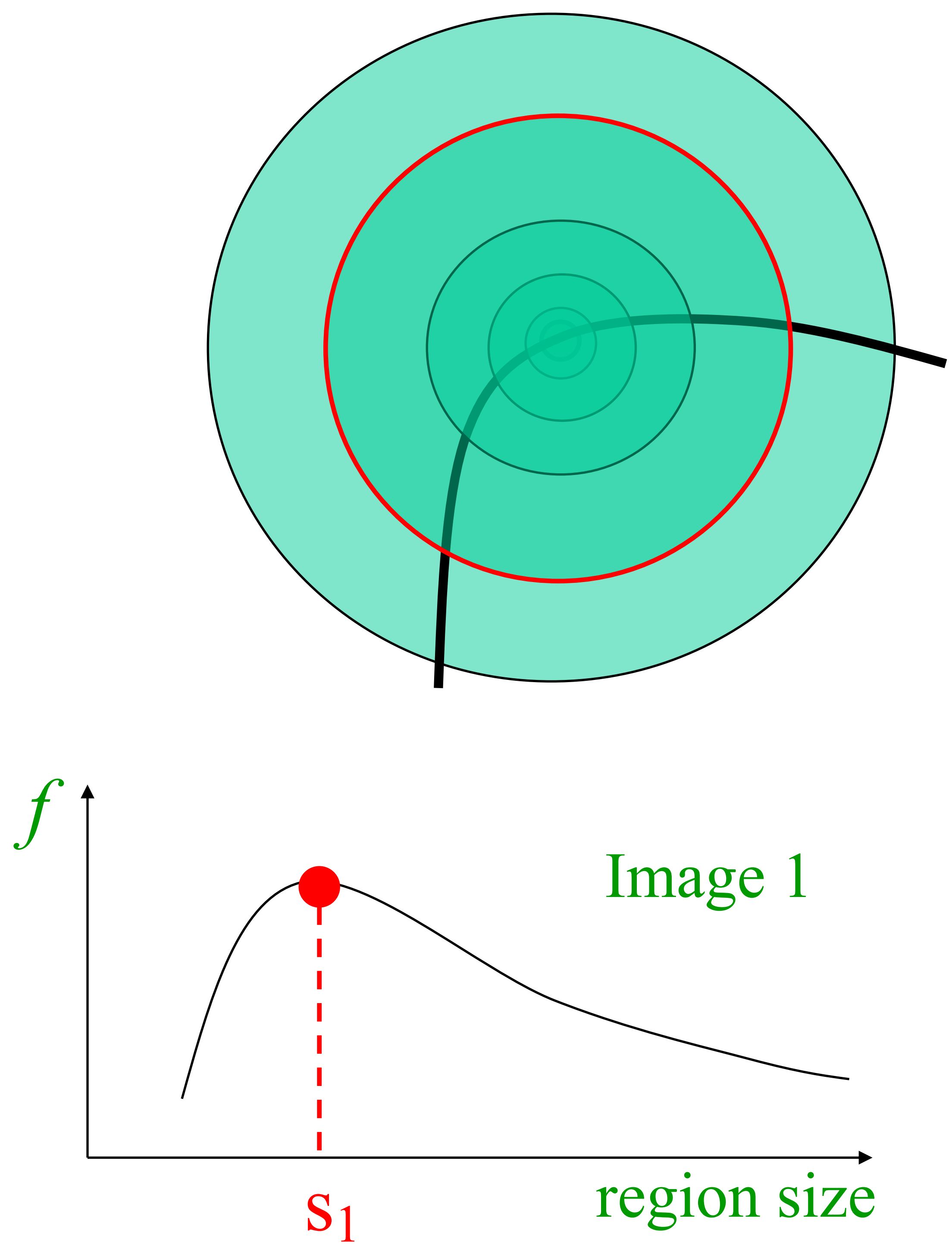
How can we make a feature detector scale-invariant?

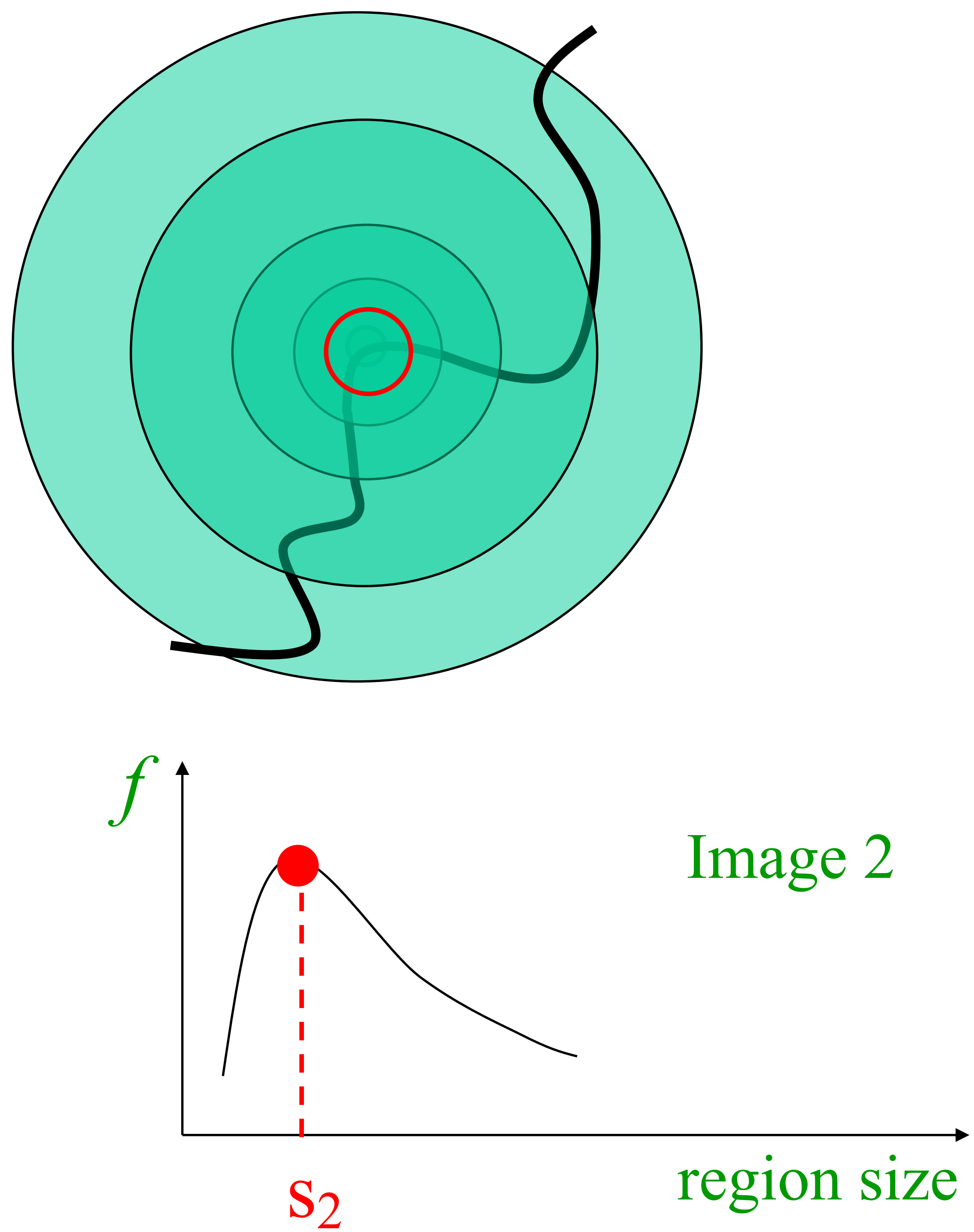
How can we automatically select the scale?

# Multi-scale blob detection

# Intuitively…

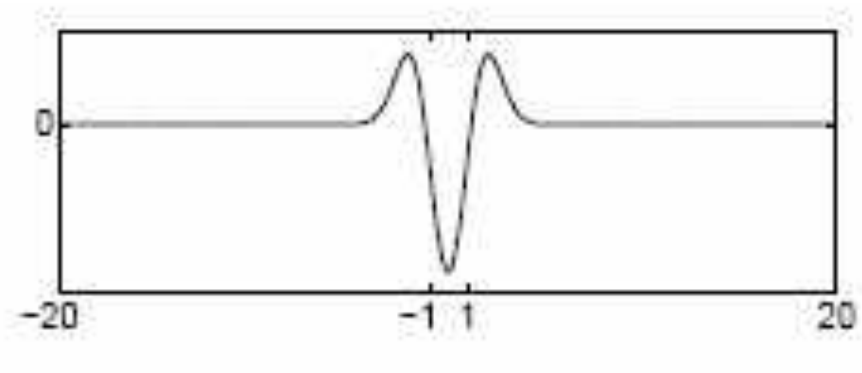
Find local maxima in both **position** and **scale**

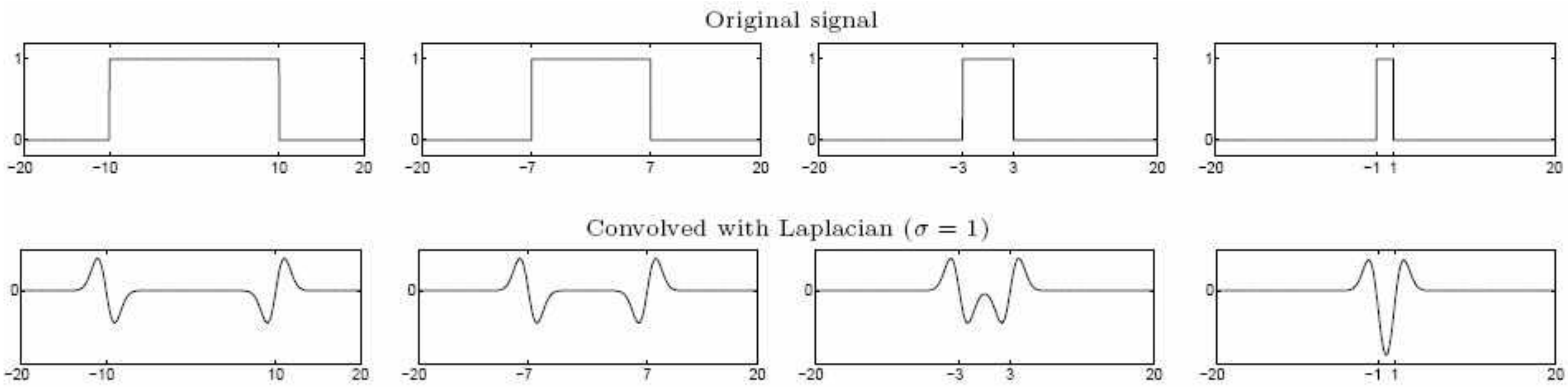


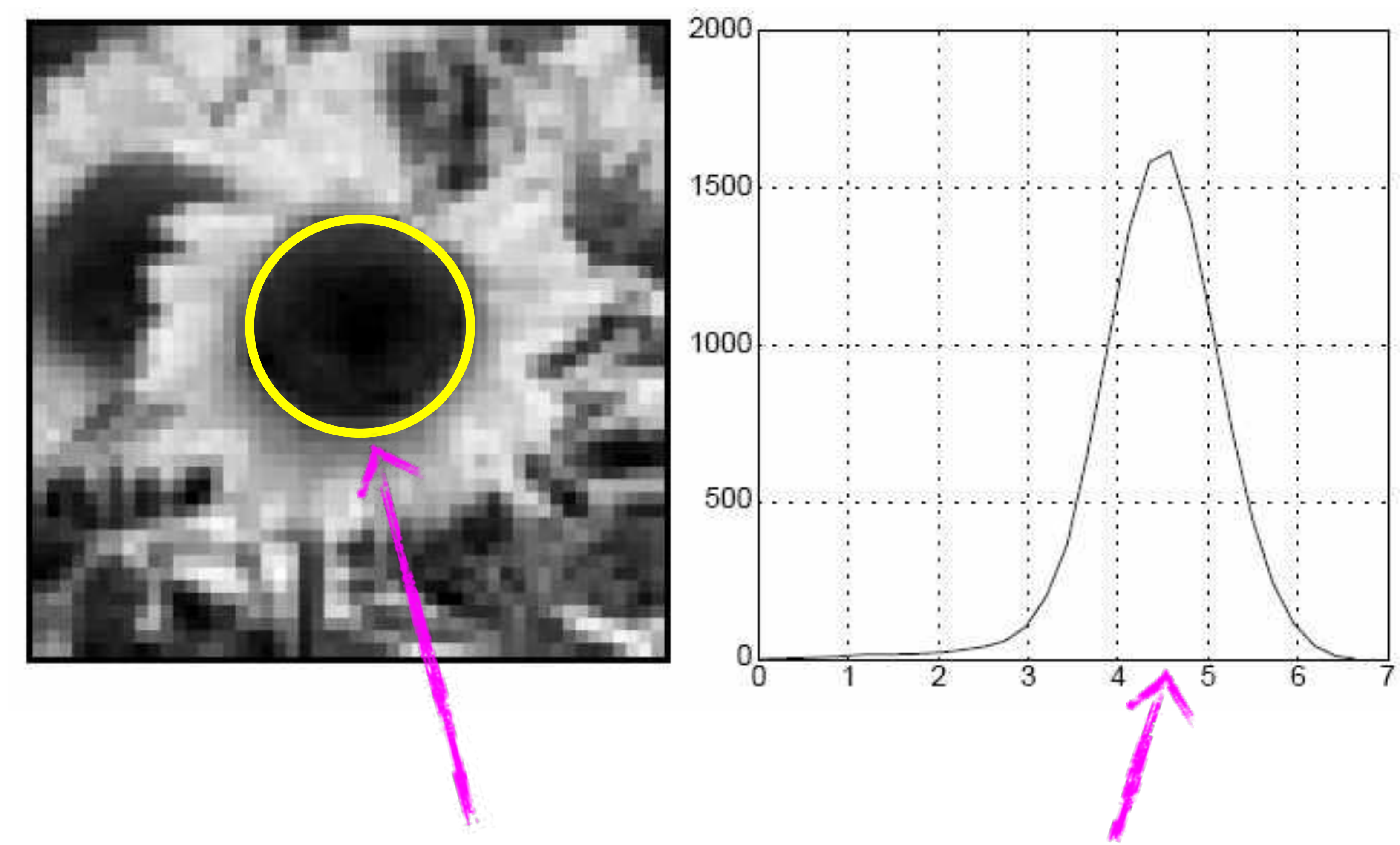
# Formally…

Laplacian filter



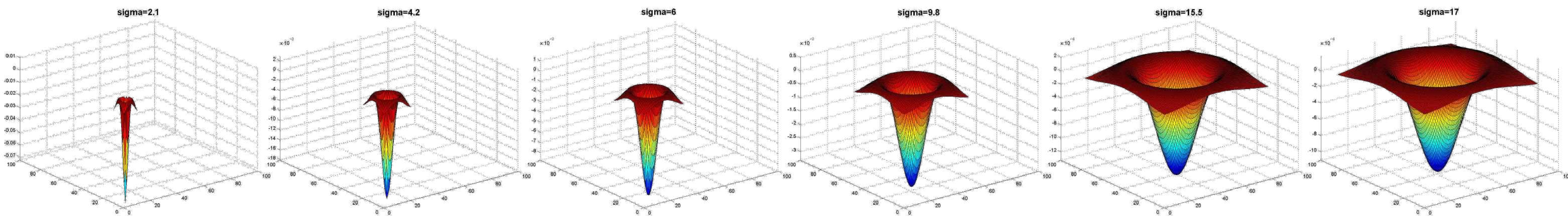

Highest response when the signal has the same **characteristic scale** as the filter characteristic scale - the scale that produces peak filter response




characteristic scale

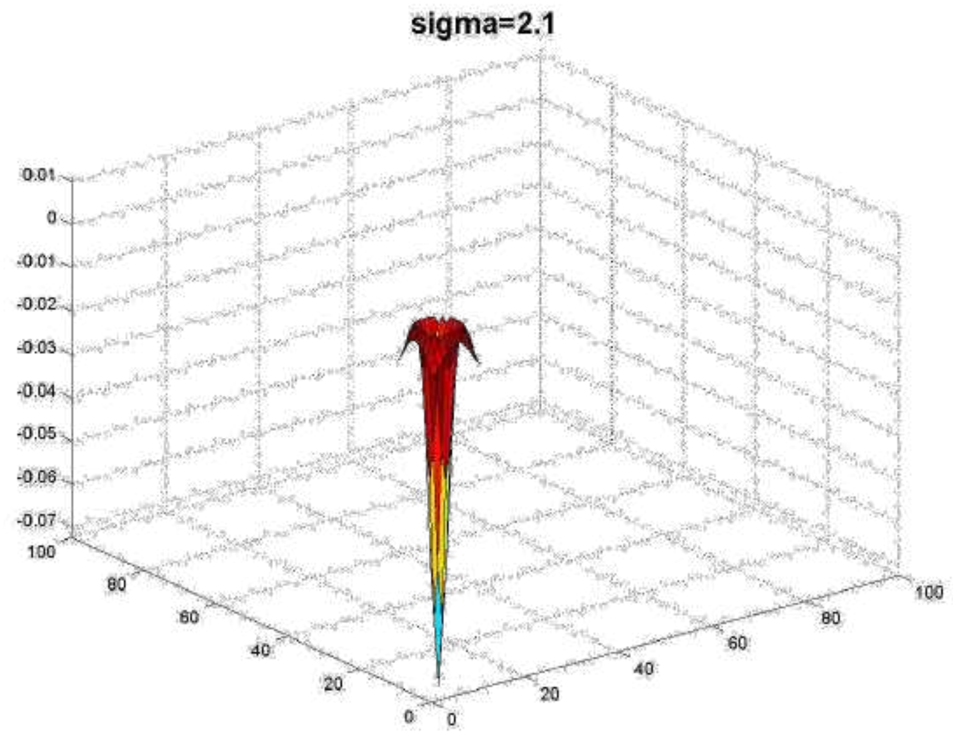
**we need to search over characteristic scales**

# What happens if you apply different Laplacian filters?

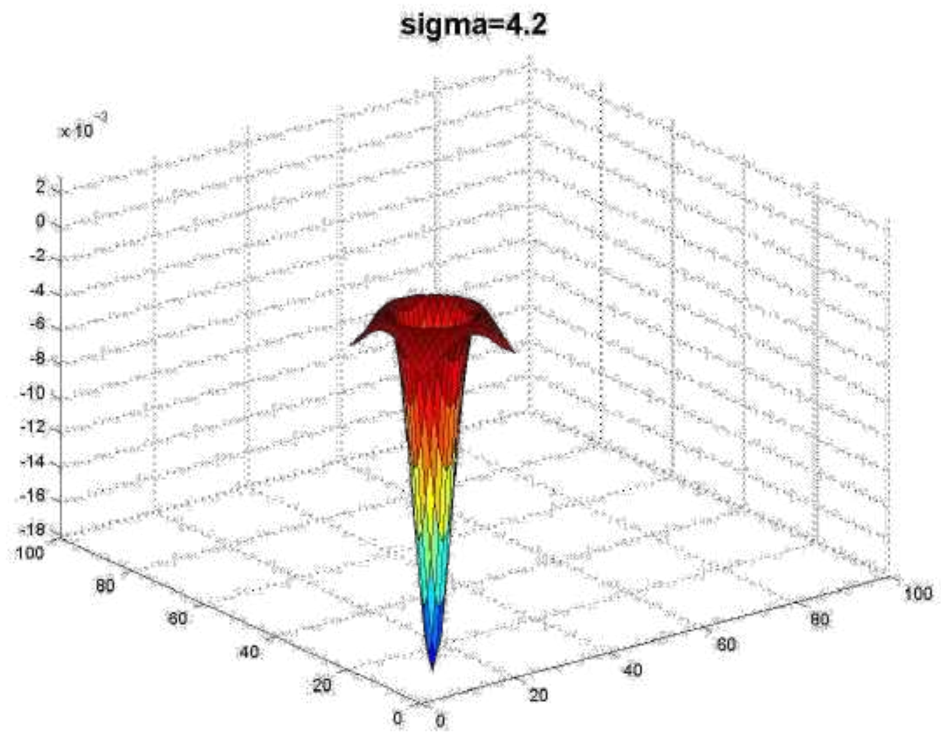



Full size

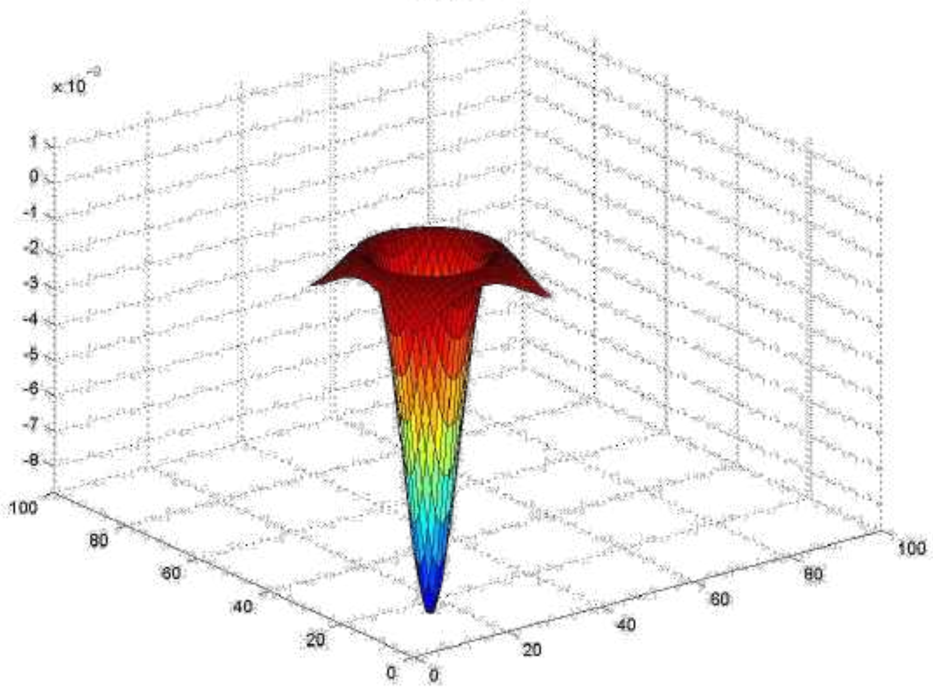
3/4 size

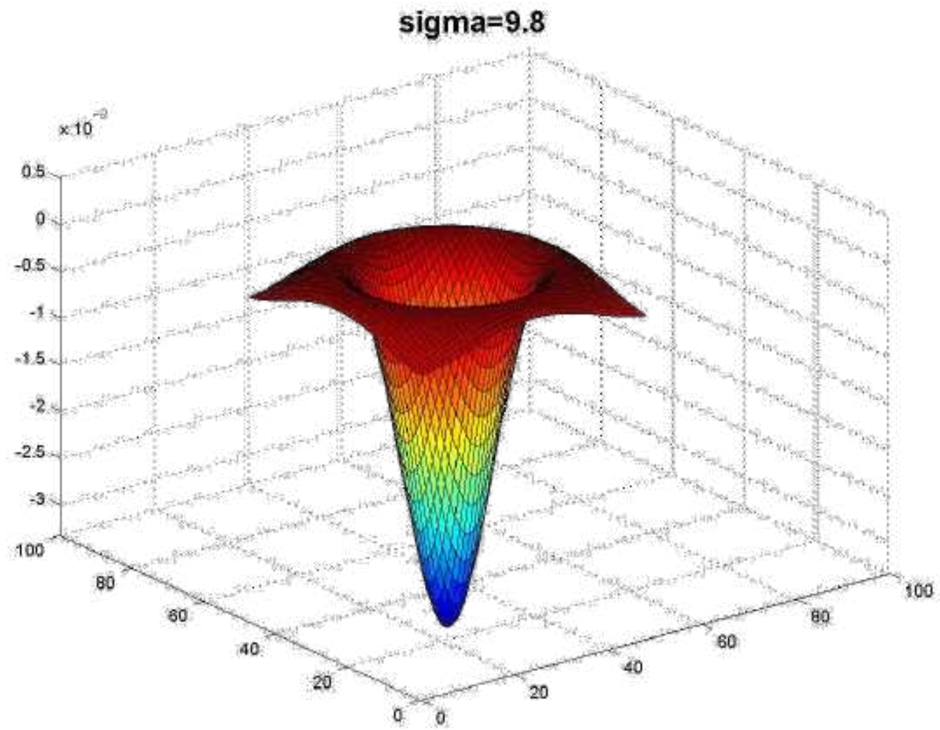


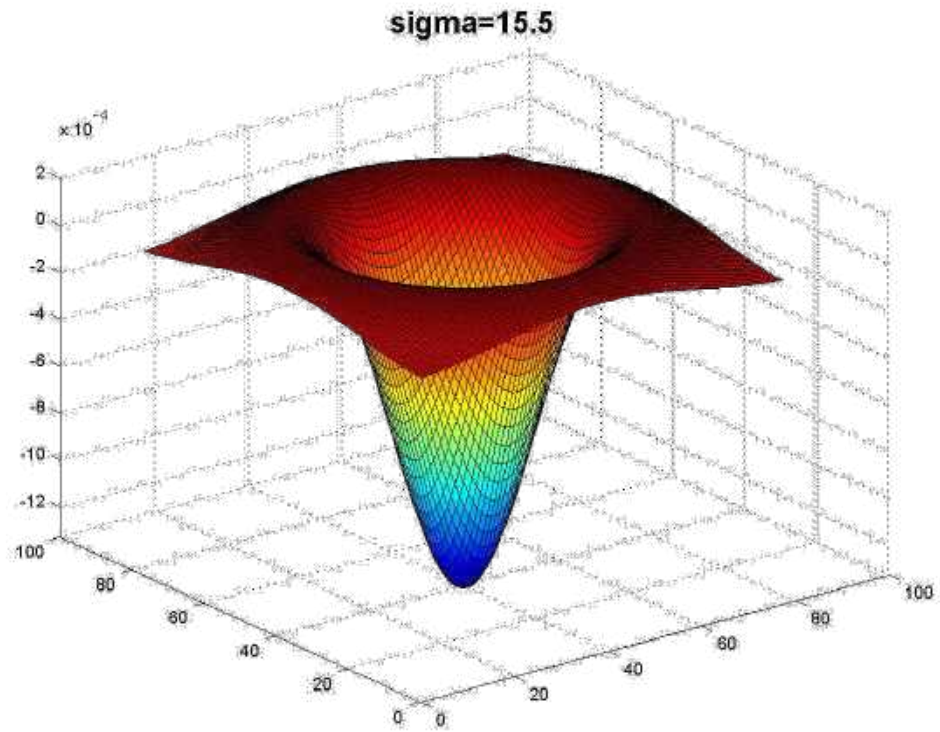

**jet** color scale
blue: low, red: high

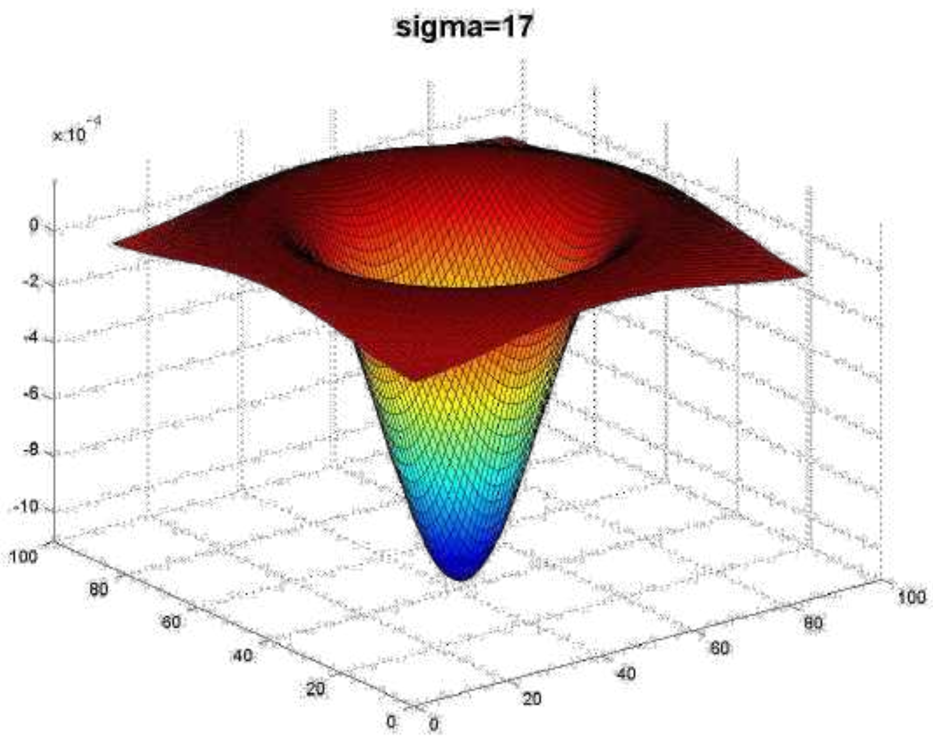

sigma=4.2
x 10^-3
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
100
80
60
40
20
0
0
20
40
60
80
100

sigma=6



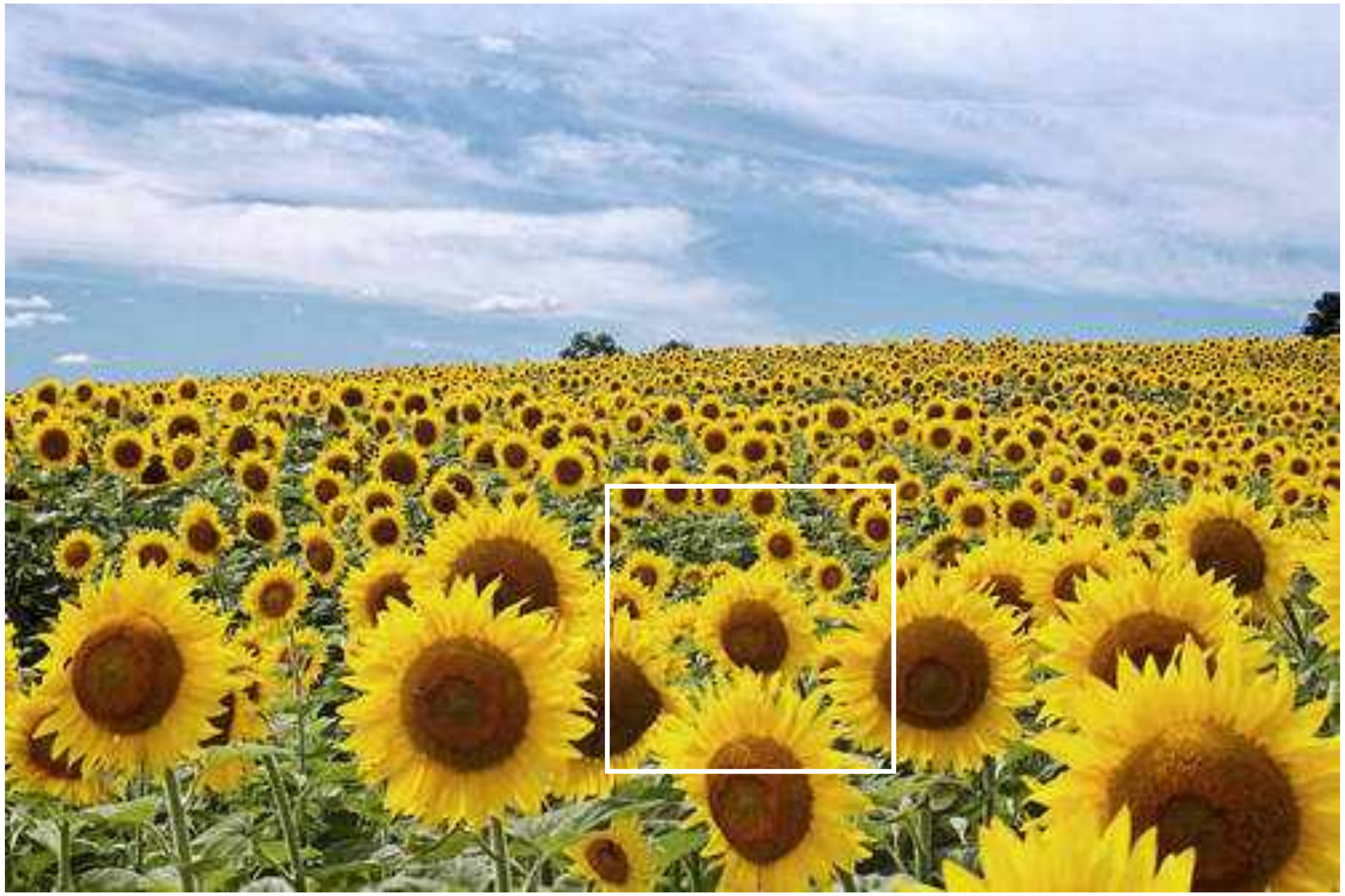


sigma=9.8
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
100
80
60
40
20
0
0
20
40
60
80
100


sigma=15.5
x 10^-4
2
0
-2
-4
-6
-8
-10
-12
100
80
60
40
20
0
0
20
40
60
80
100

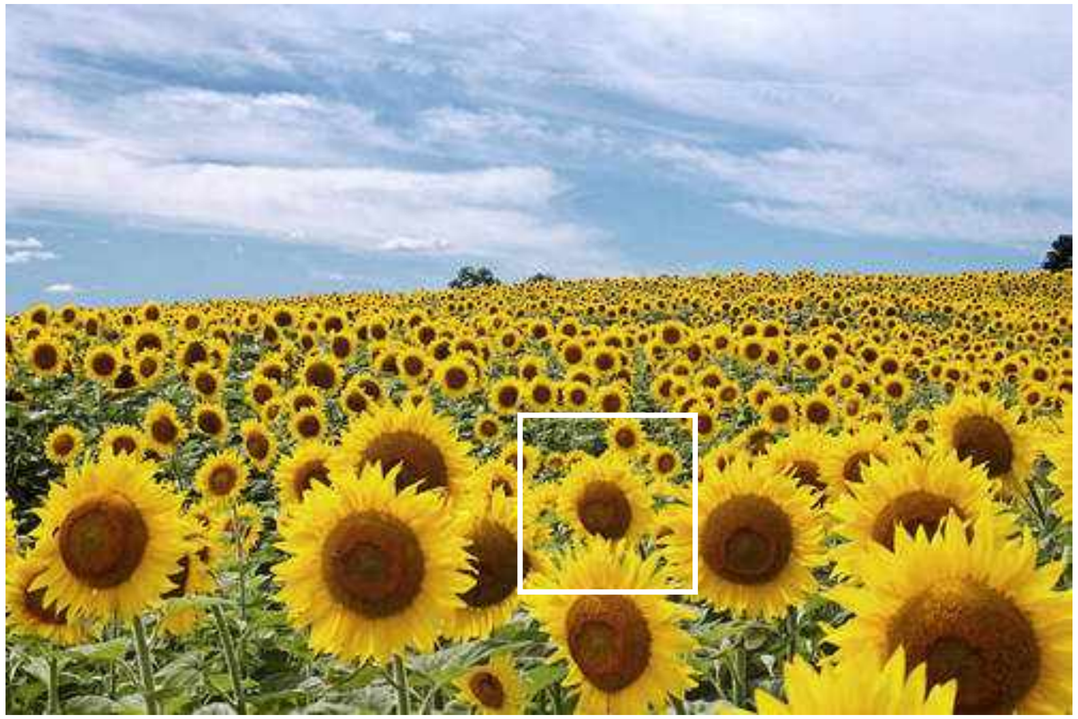

sigma=17
x 10^-4
0
-2
-4
-6
-8
-10
100
80
60
40
20
0
0
20
40
60
80
100

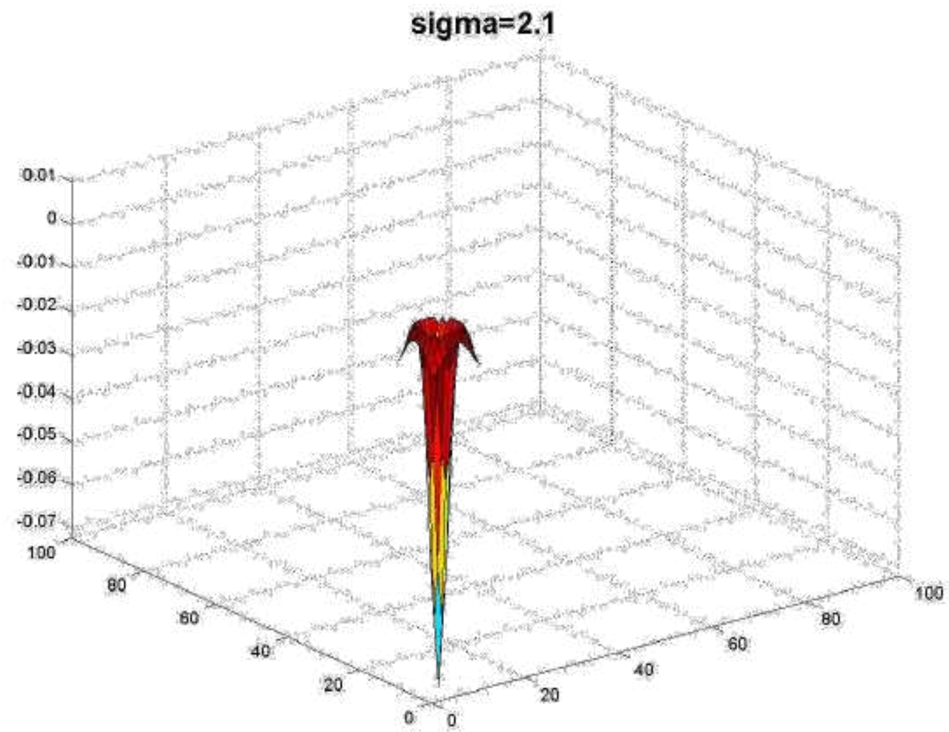
# What happened when you applied different Laplacian filters?

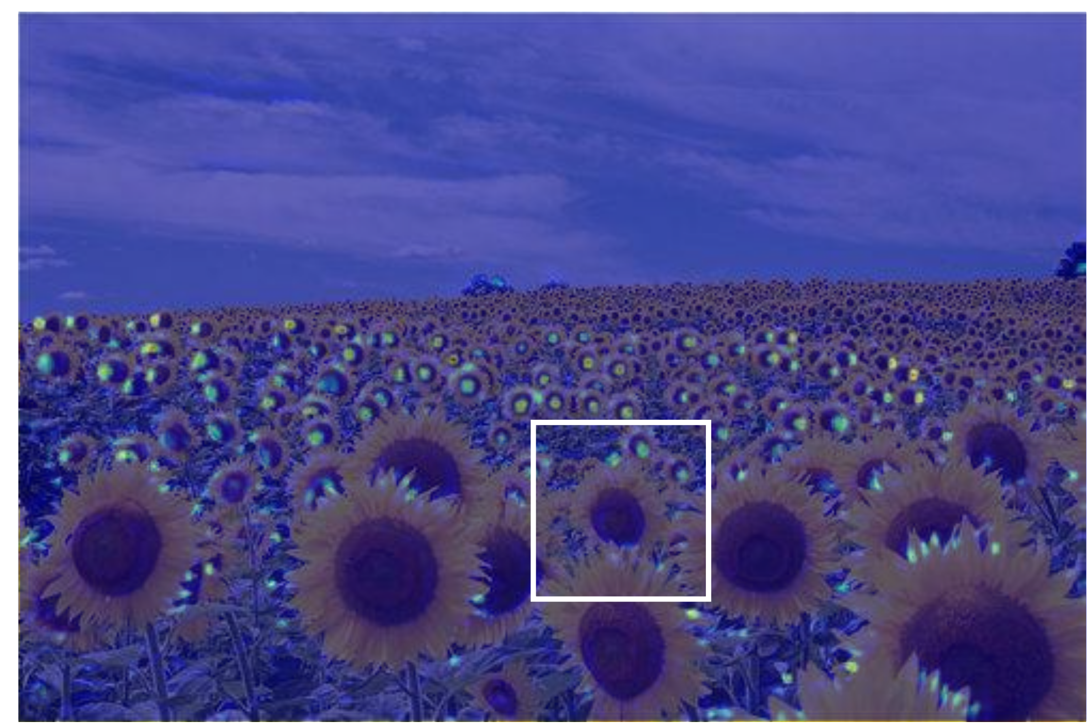
Full size

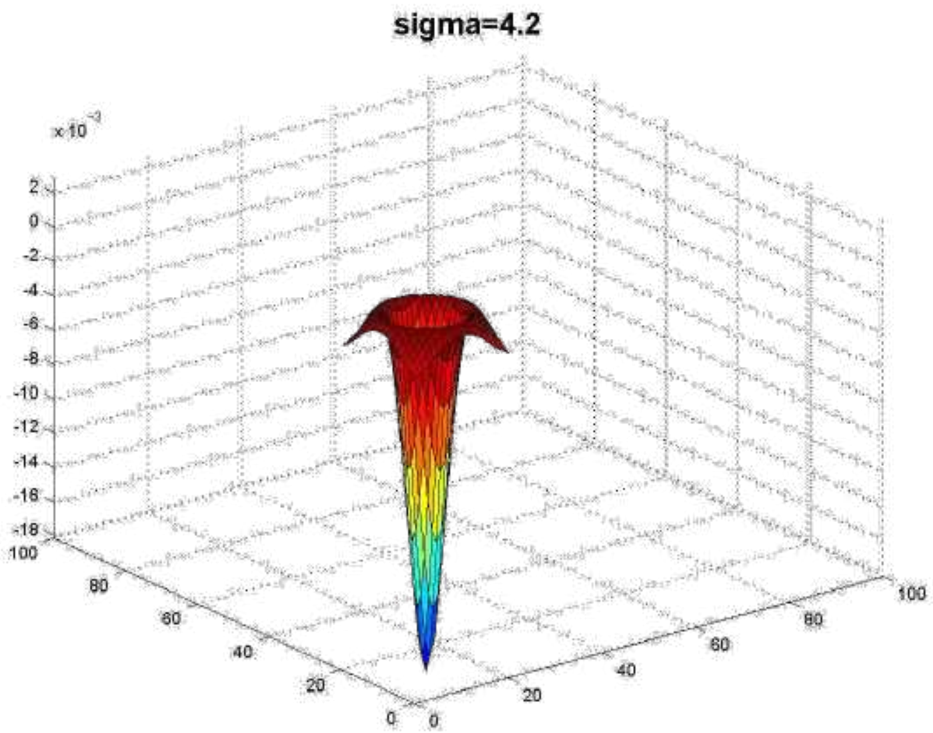


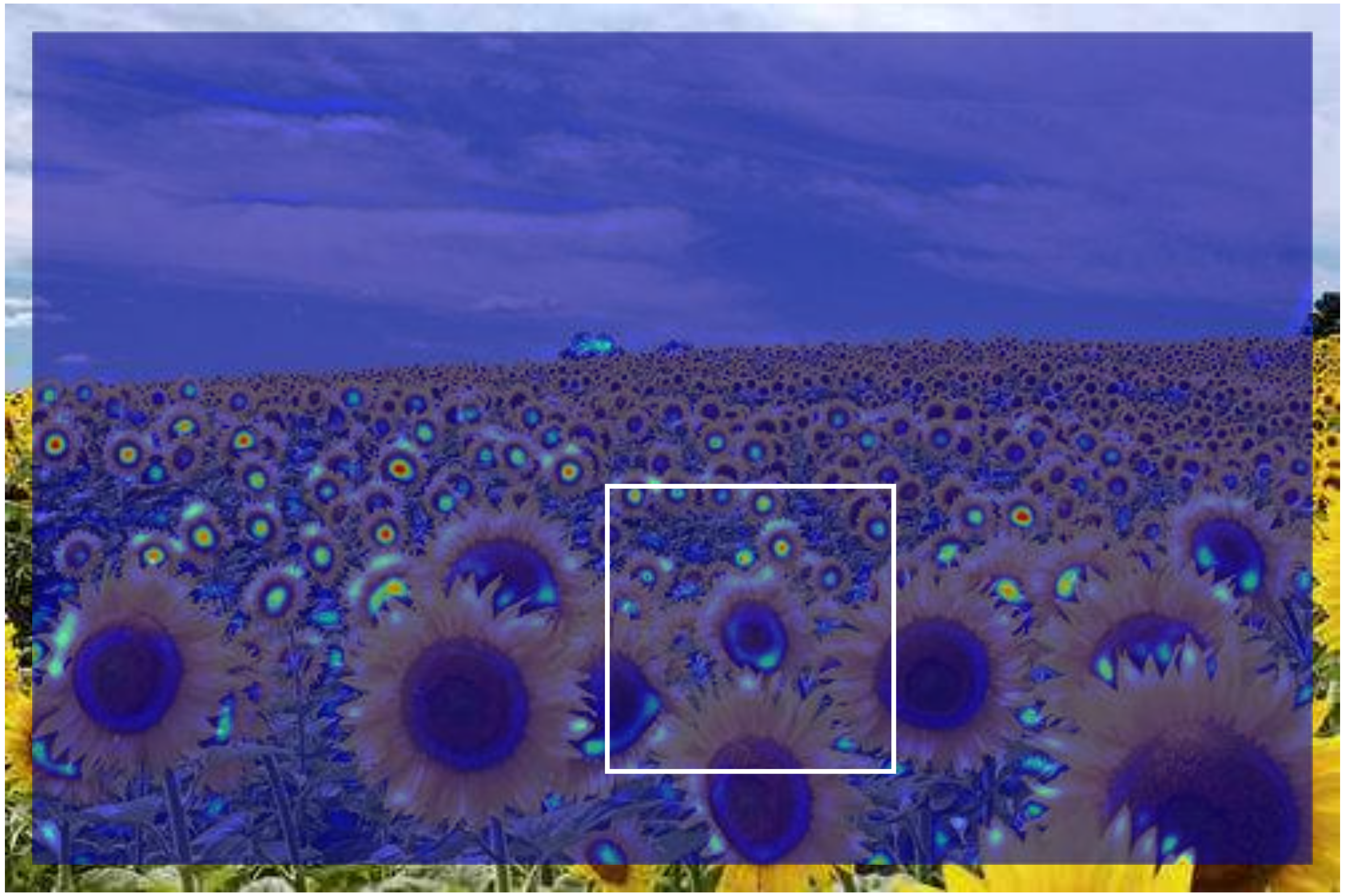
3/4 size

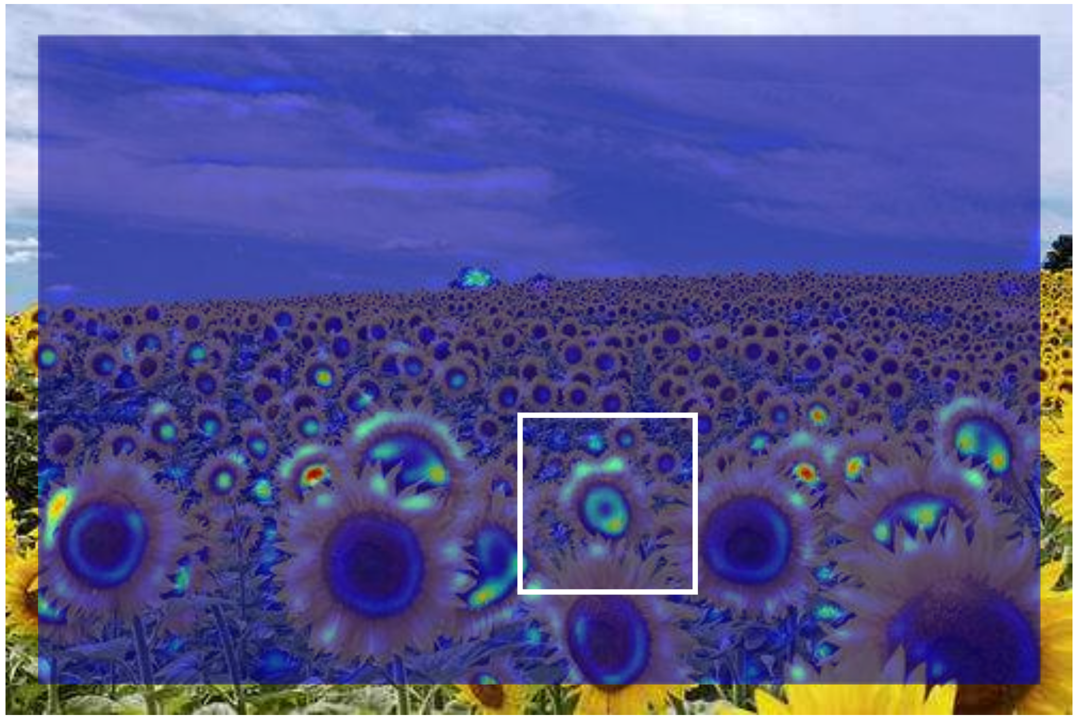


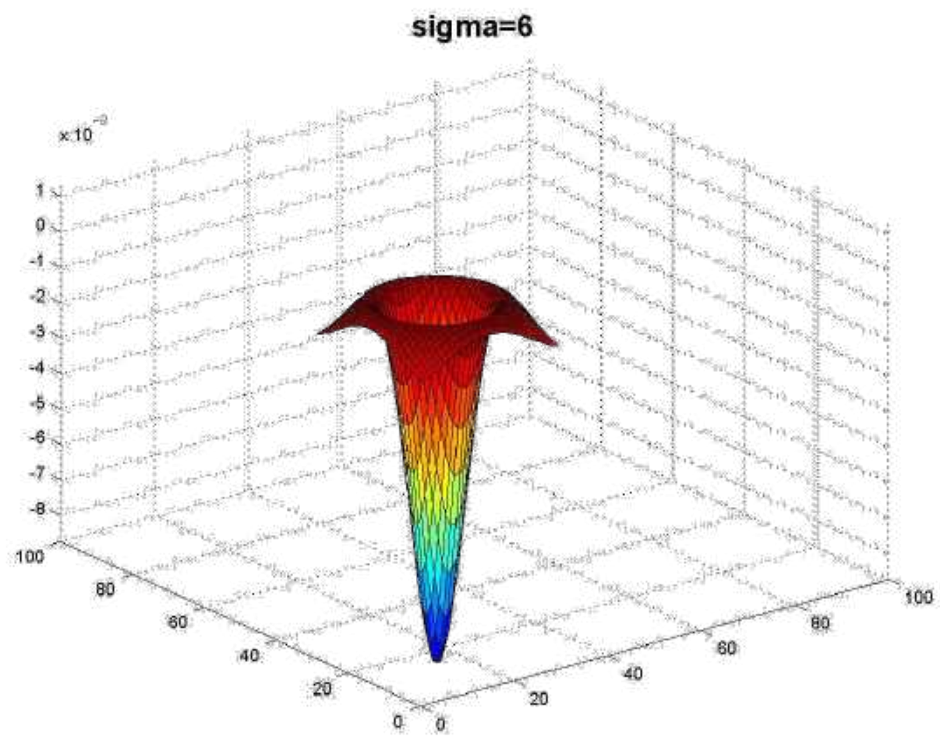

sigma=2.1
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
100
80
60
40
20
0
0
20
40
60
80
100

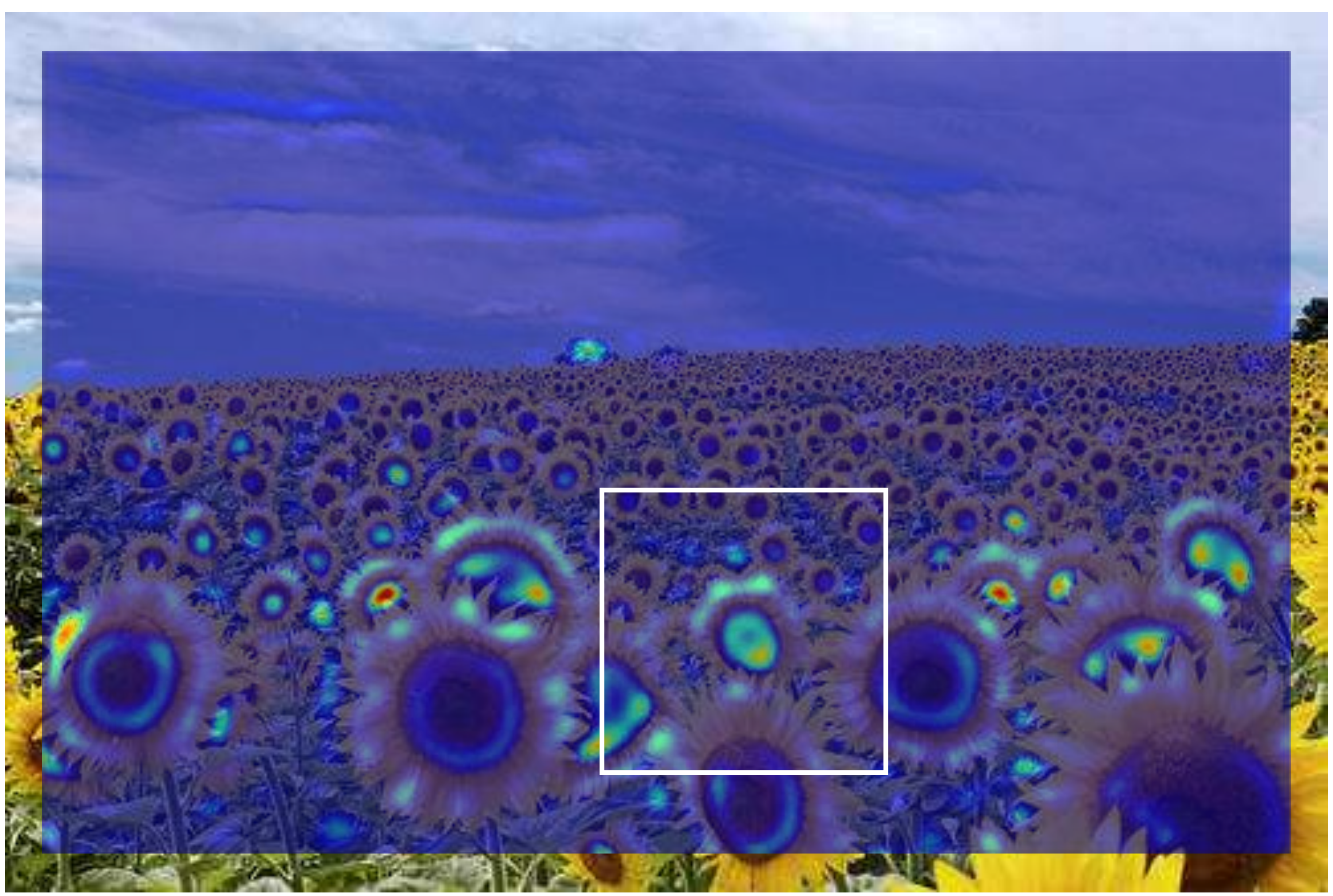


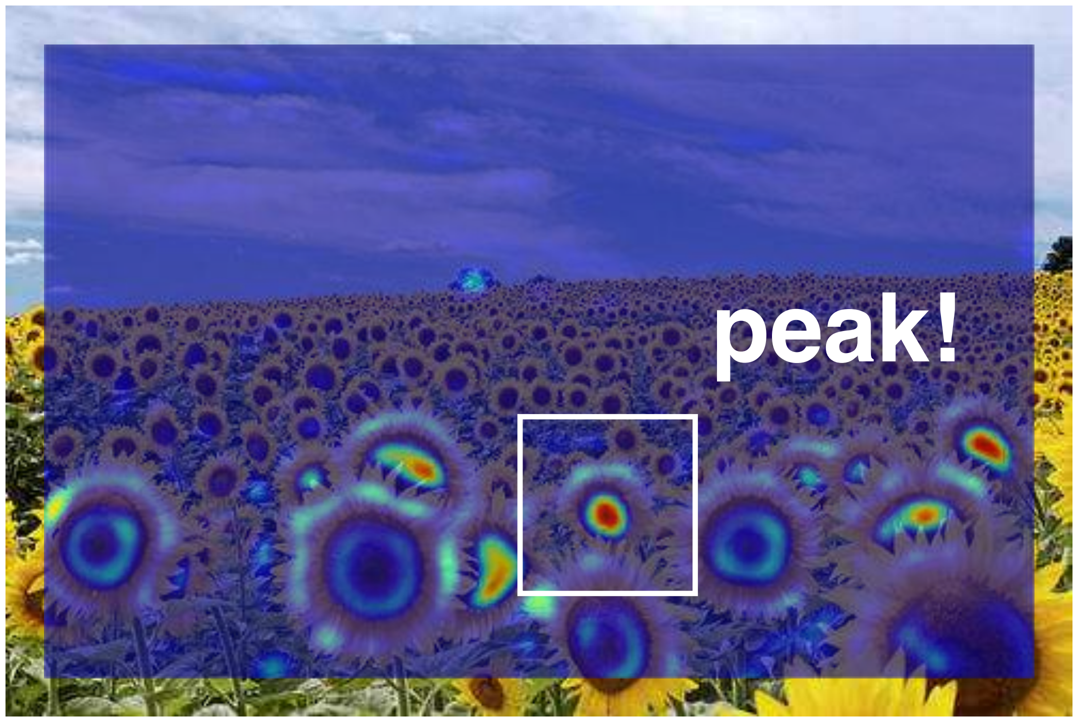

sigma=4.2

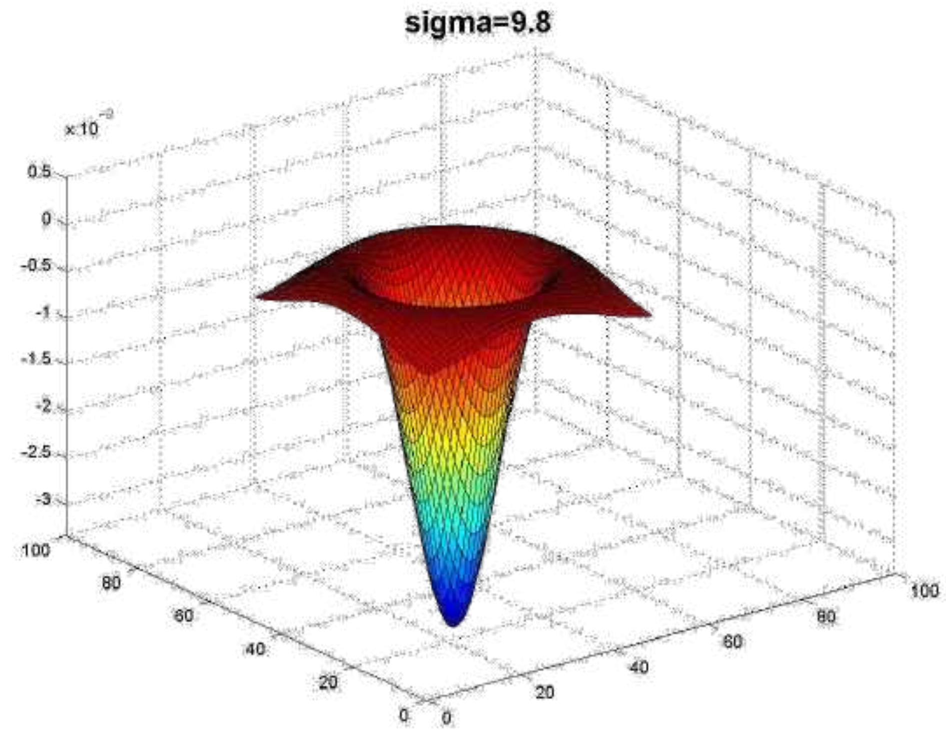


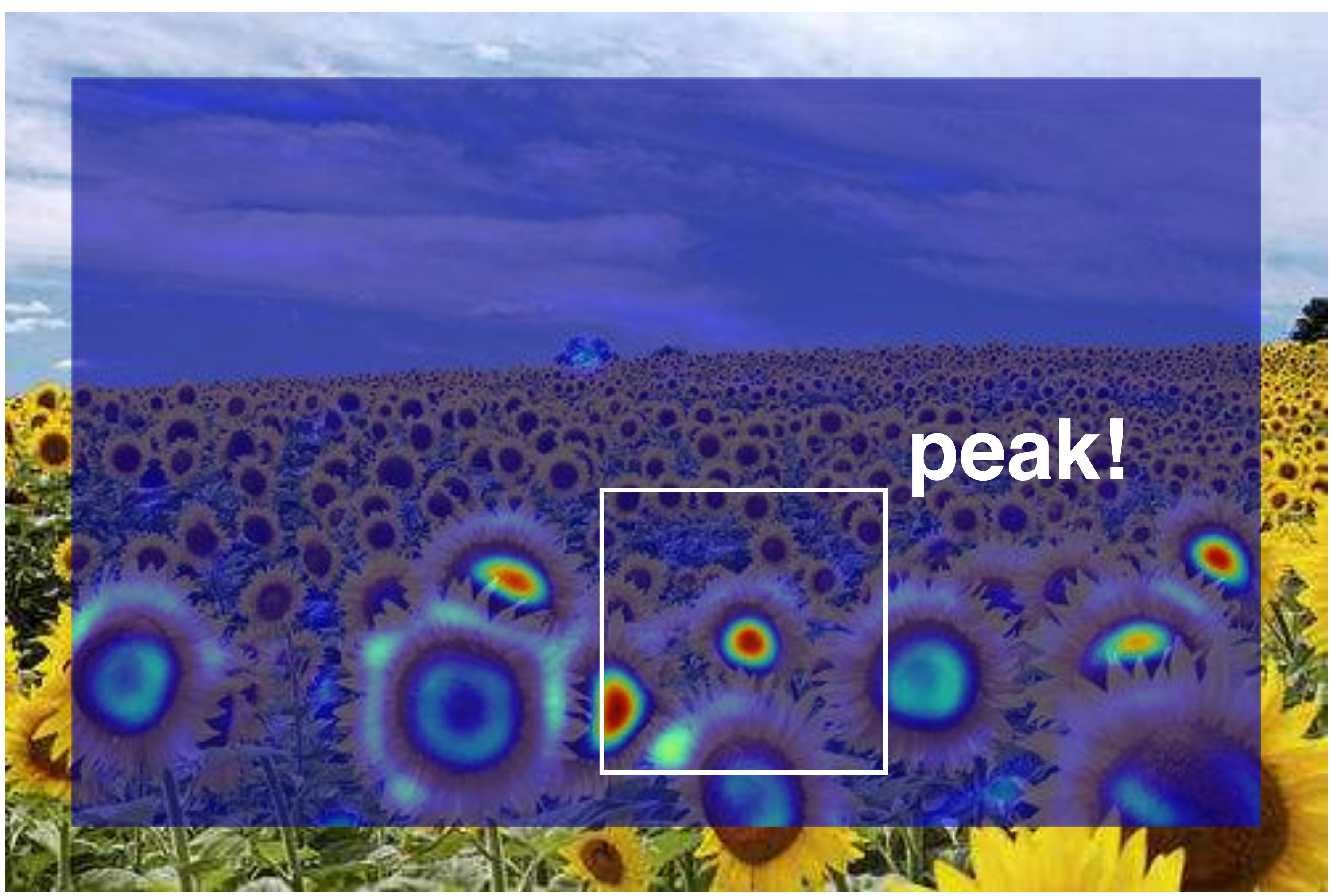


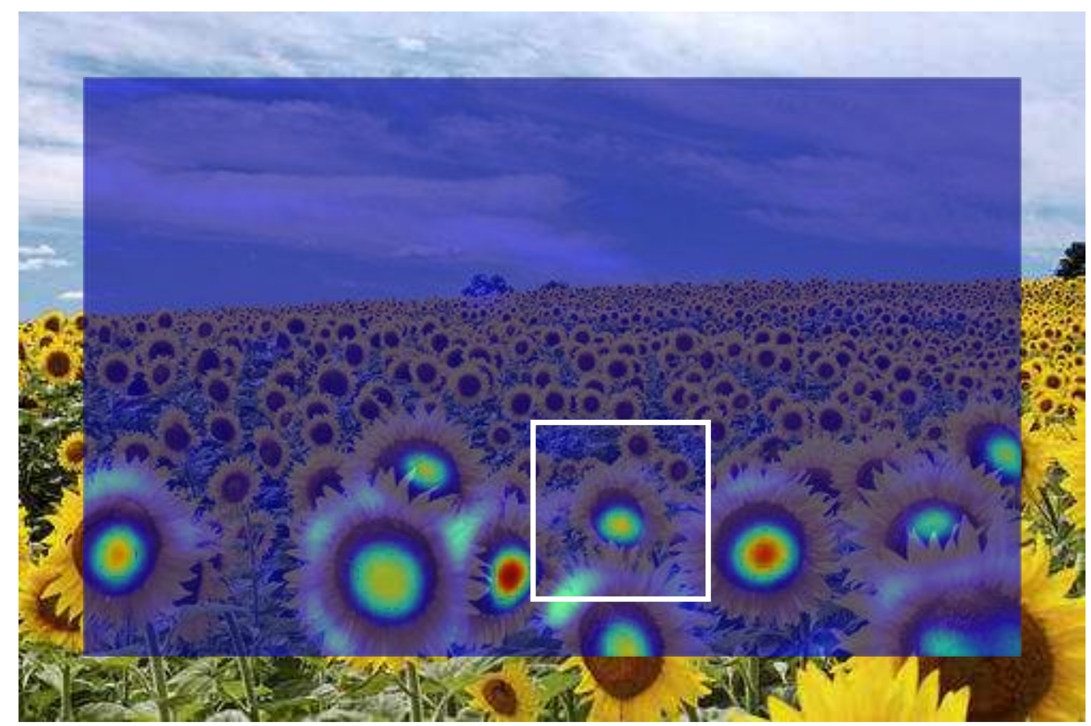

sigma=6

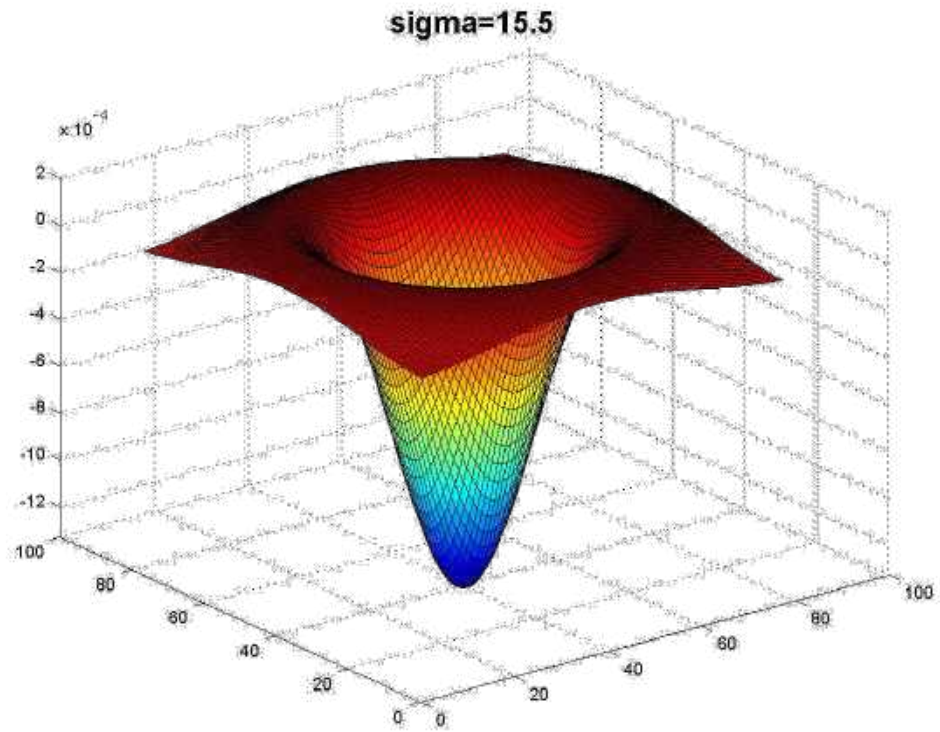


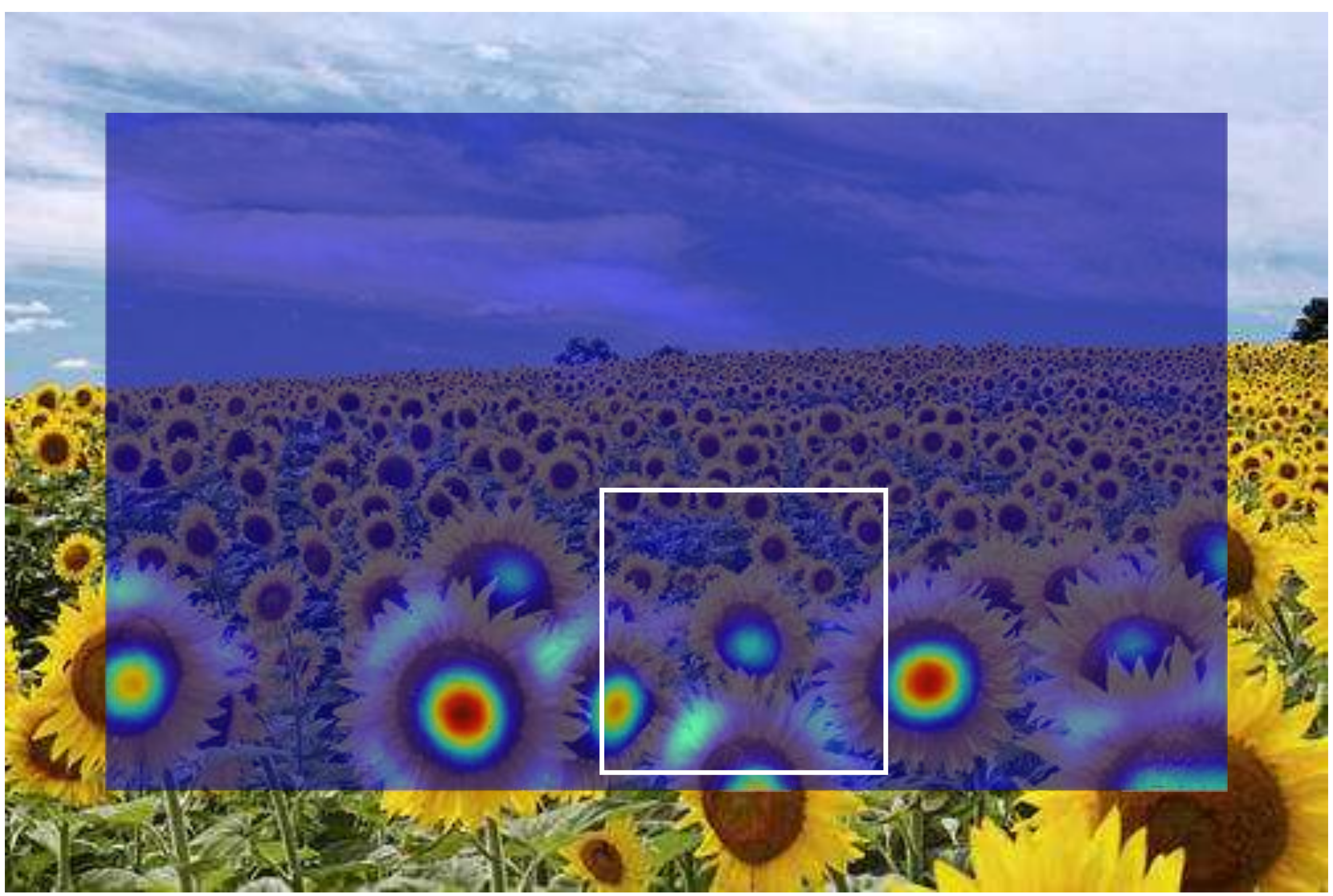

peak!

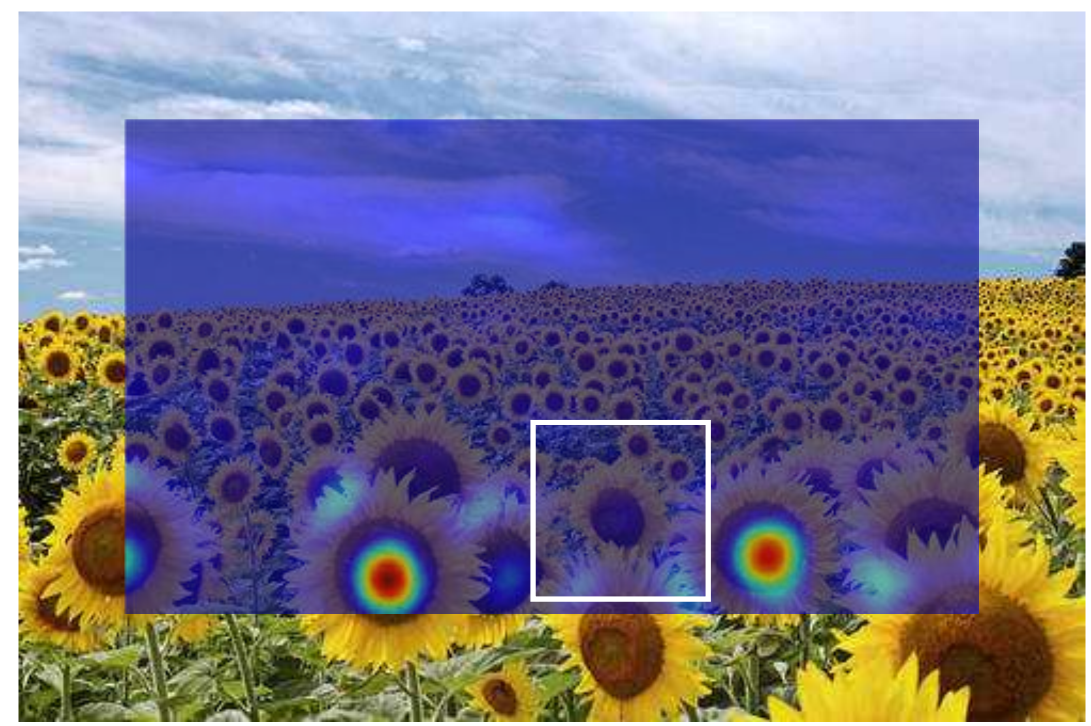

sigma=9.8

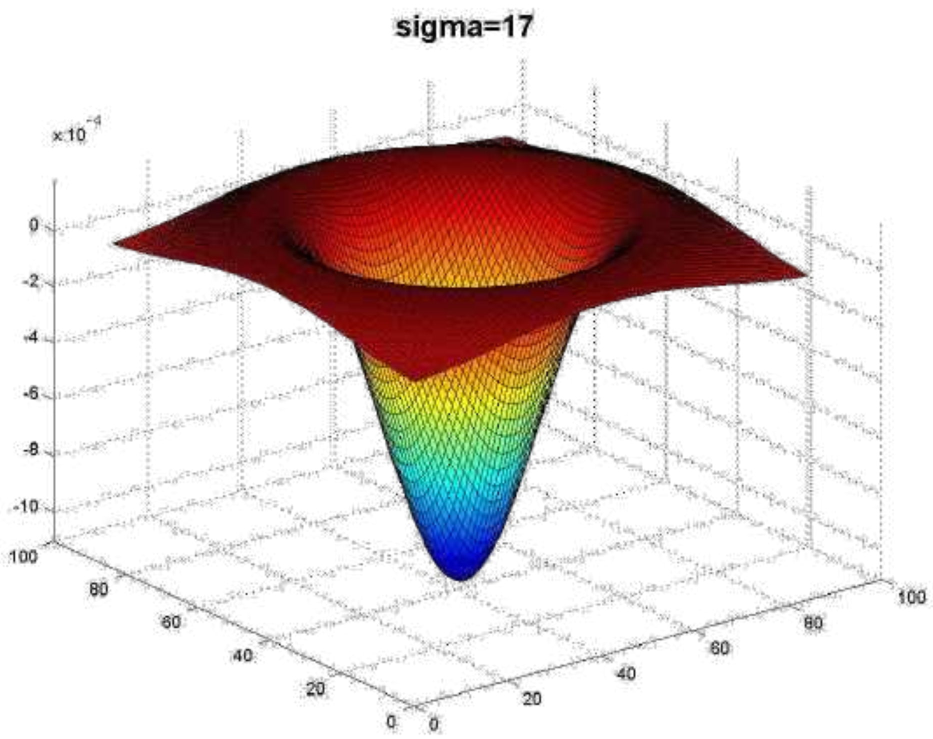

peak!




sigma=15.5
x 10⁻⁴
2
0
-2
-4
-6
-8
-10
-12
100
80
60
40
20
0
0
20
40
60
80
100






sigma=17
0
-2
-4
-6
-8
-10
100
80
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20
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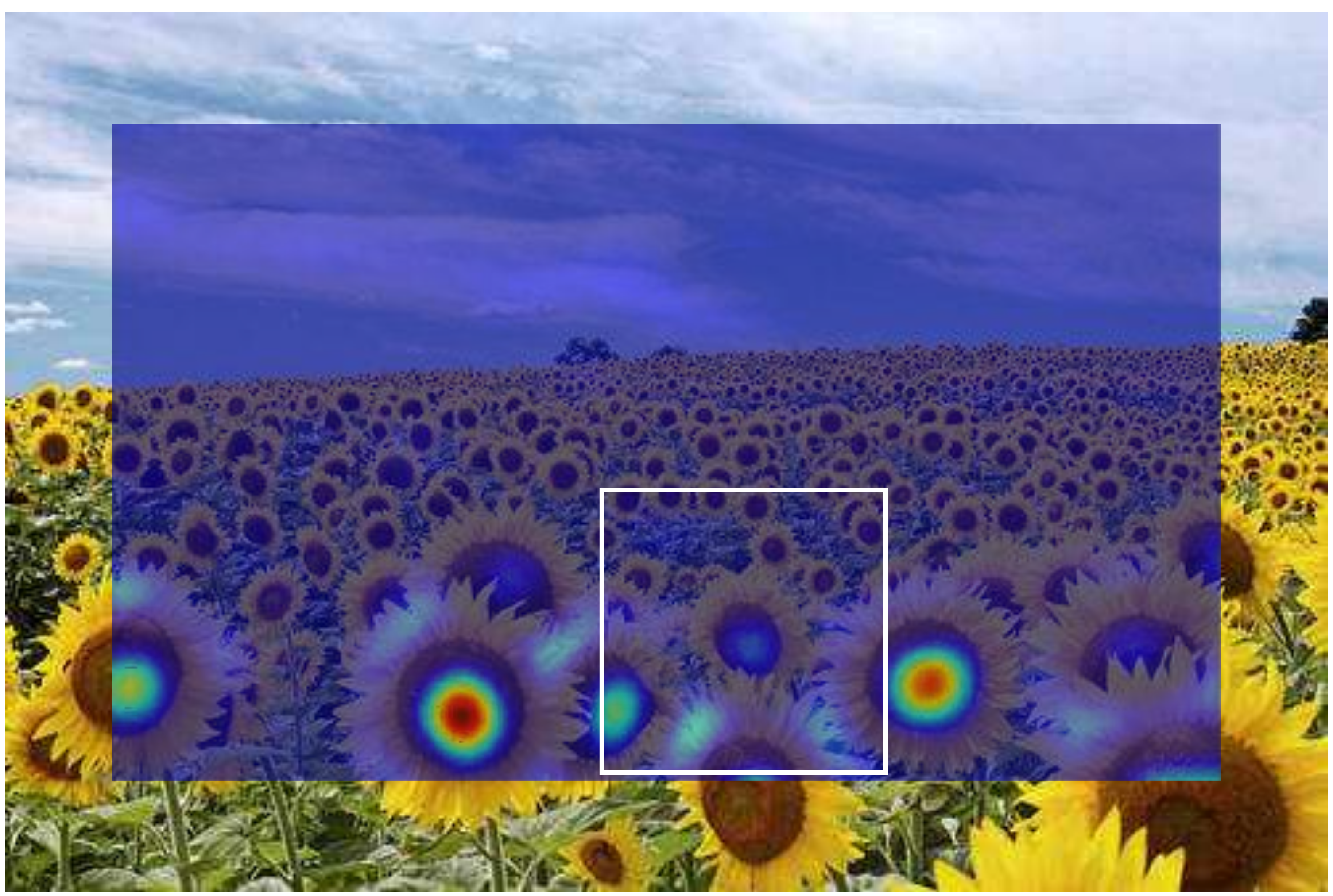
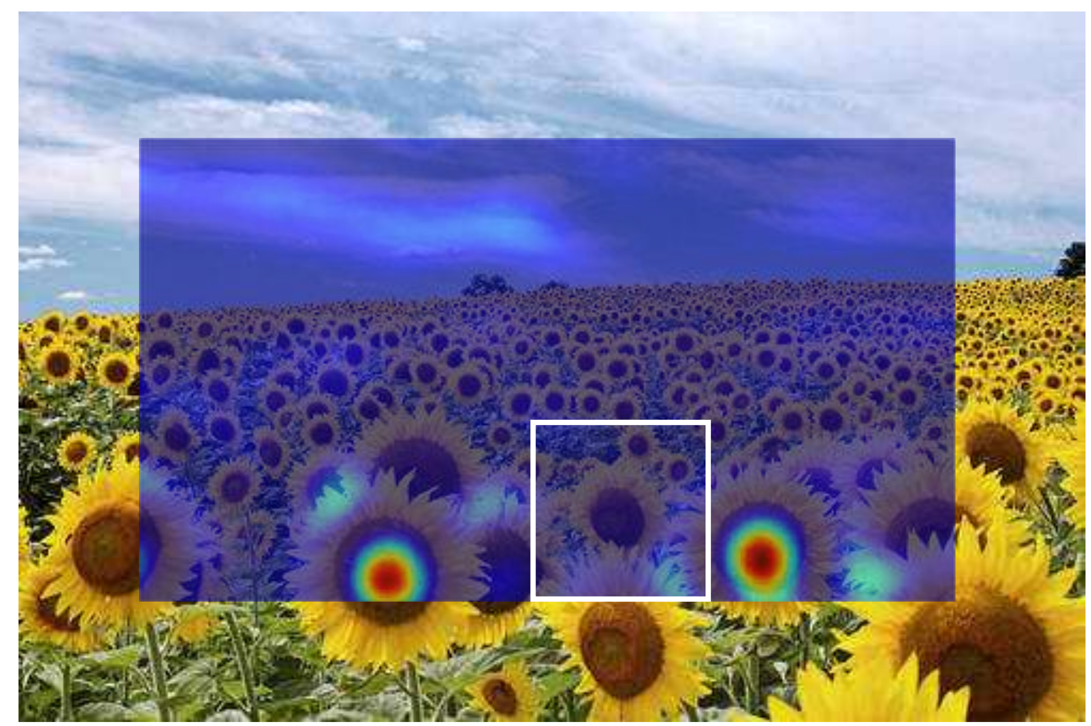
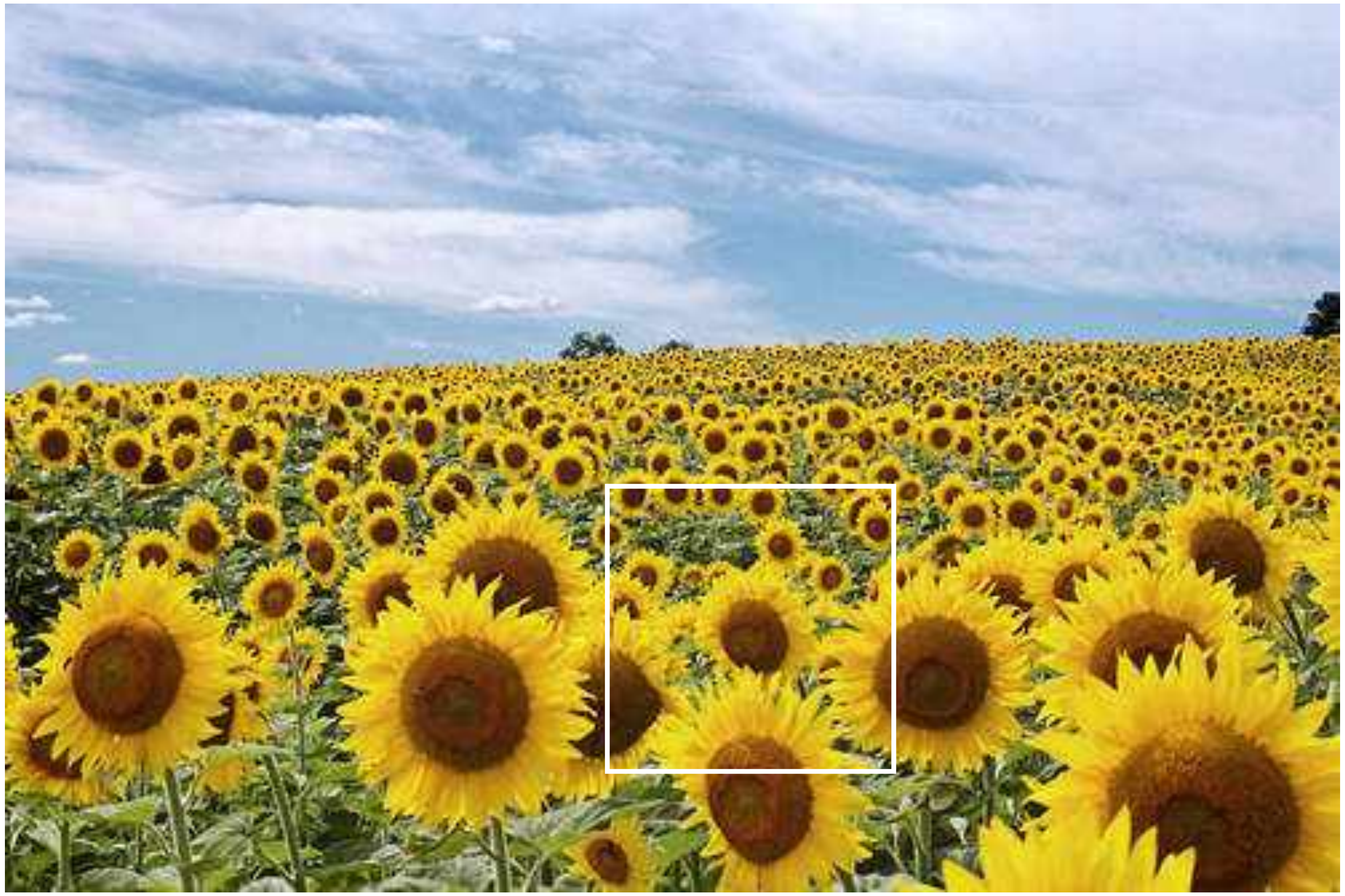
0
20
40
60
80
100



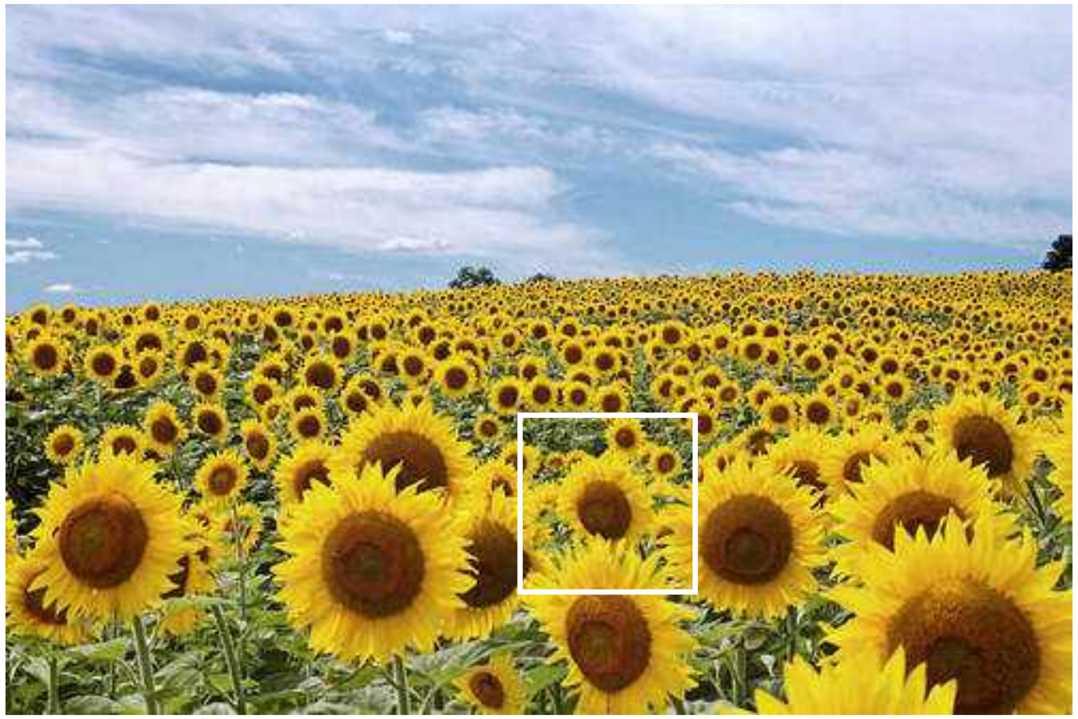


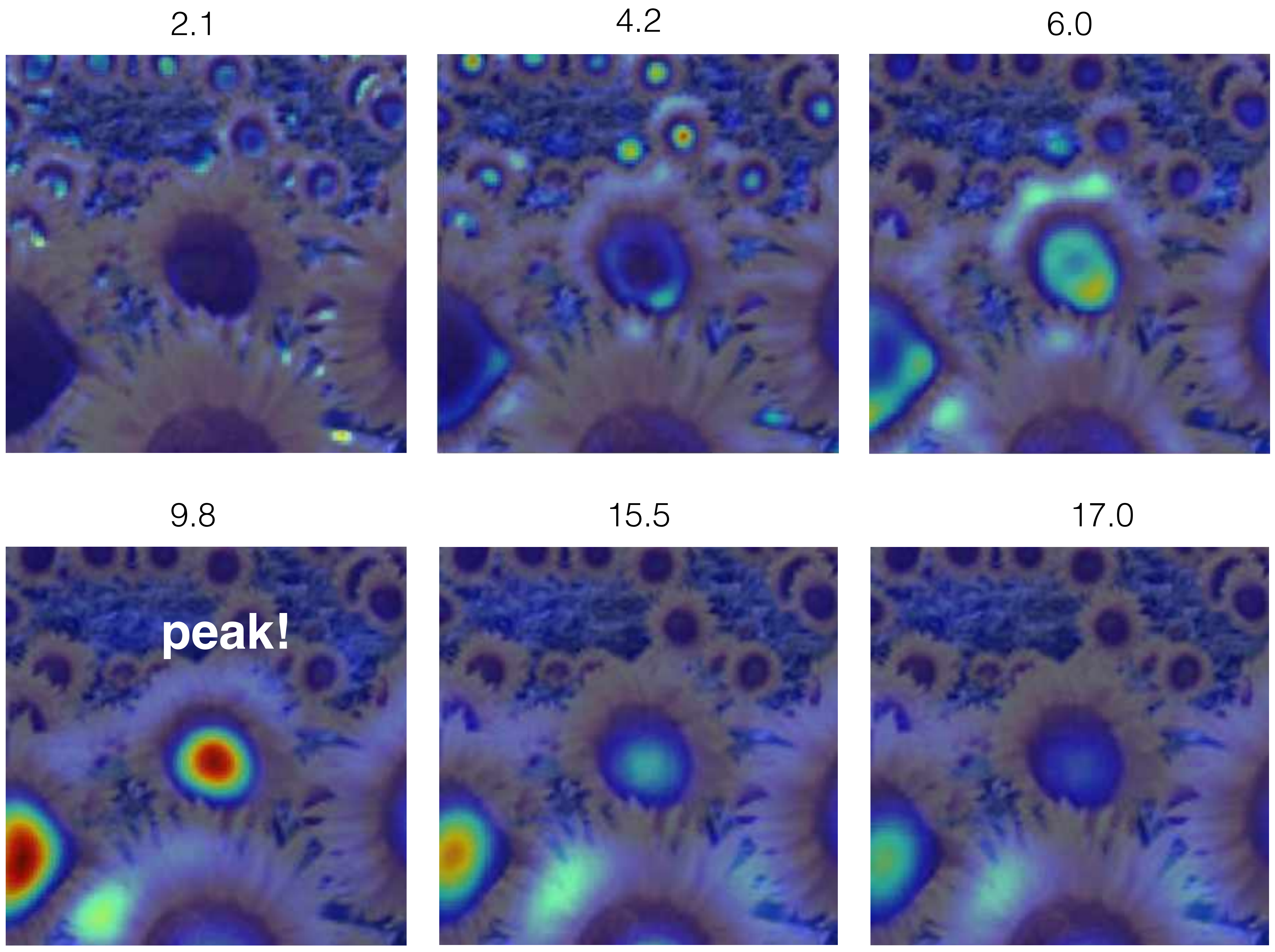
What happened when you applied different Laplacian filters?

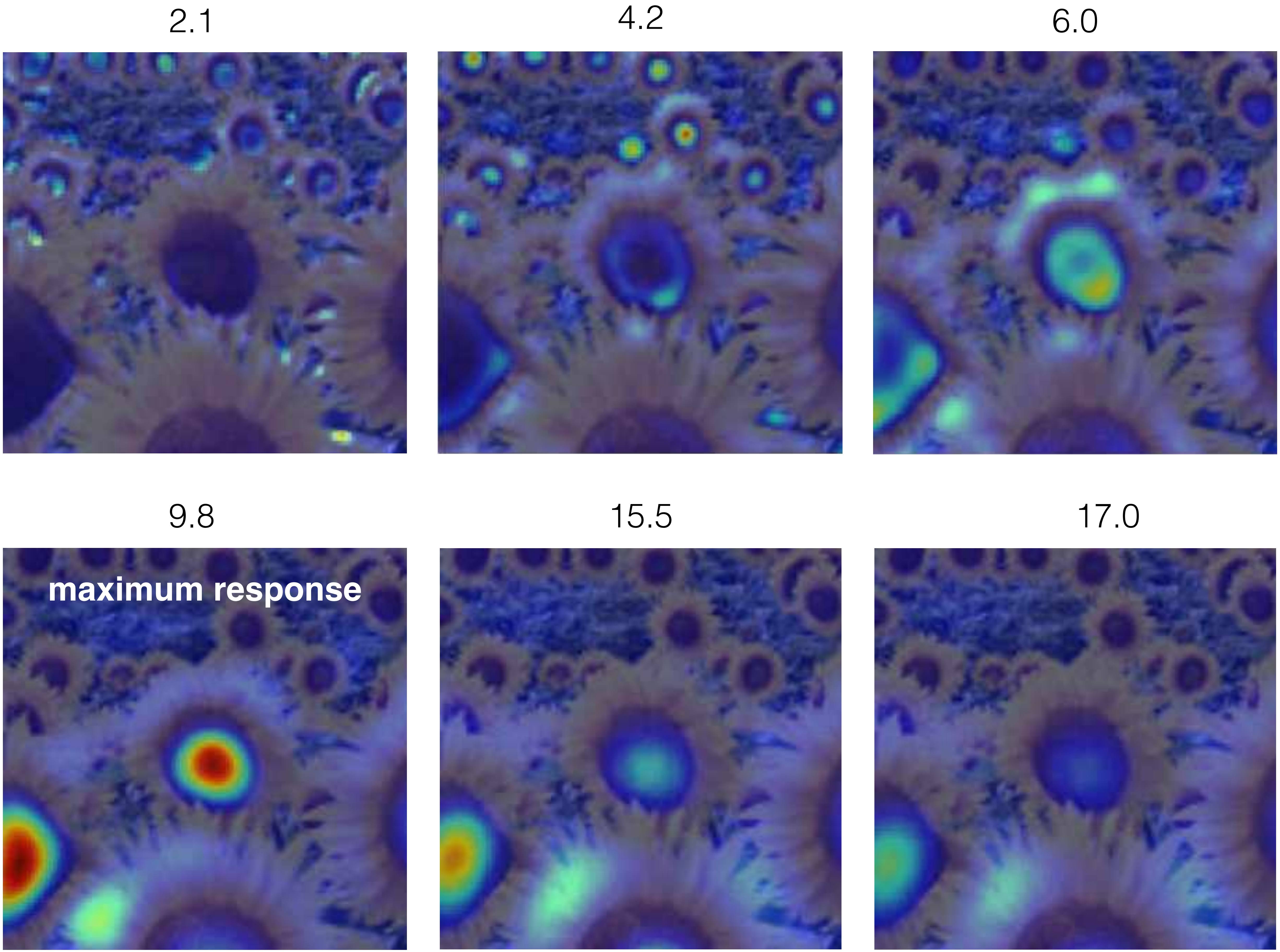
Full size



3/4 size

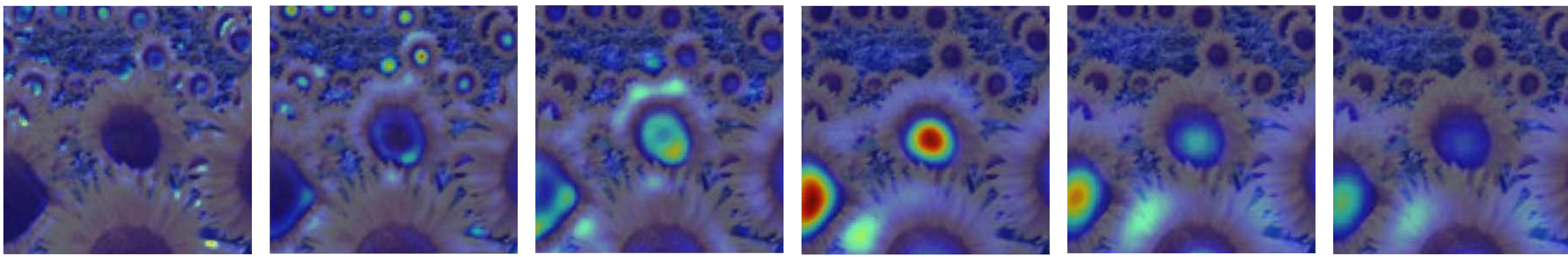



2.1
4.2
6.0
9.8
peak!
15.5
17.0


2.1
4.2
6.0
9.8
maximum response
15.5
17.0

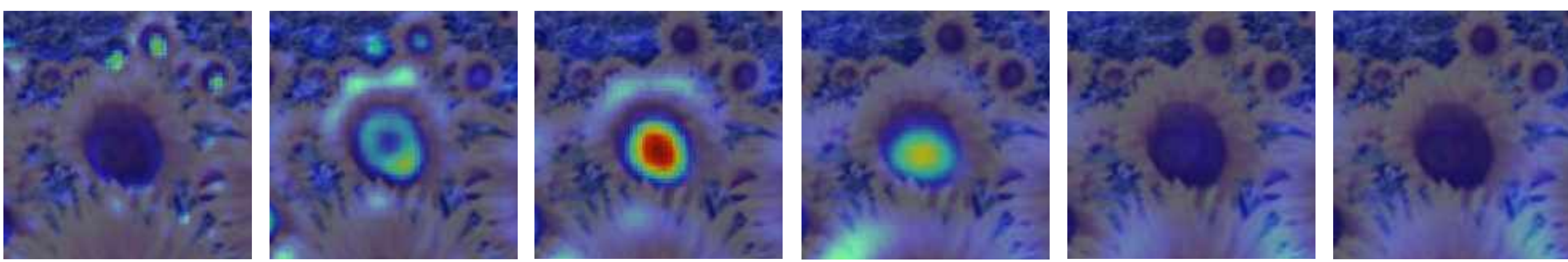
# optimal scale

2.1 4.2 6.0 9.8 15.5 17.0



Full size image

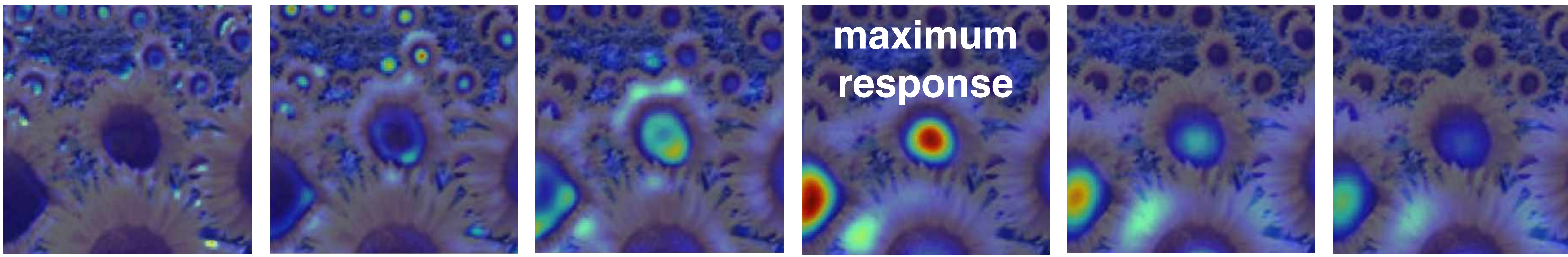
2.1 4.2 6.0 9.8 15.5 17.0



3/4 size image

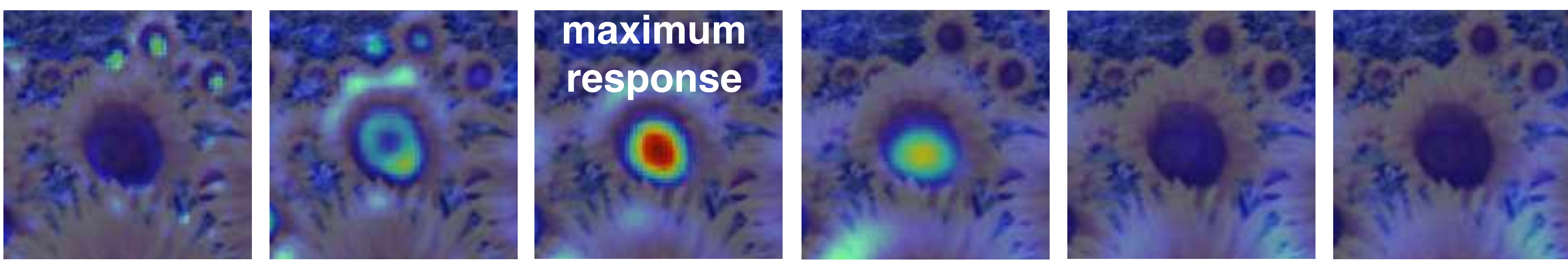
# optimal scale

2.1 | 4.2 | 6.0 | 9.8 | 15.5 | 17.0

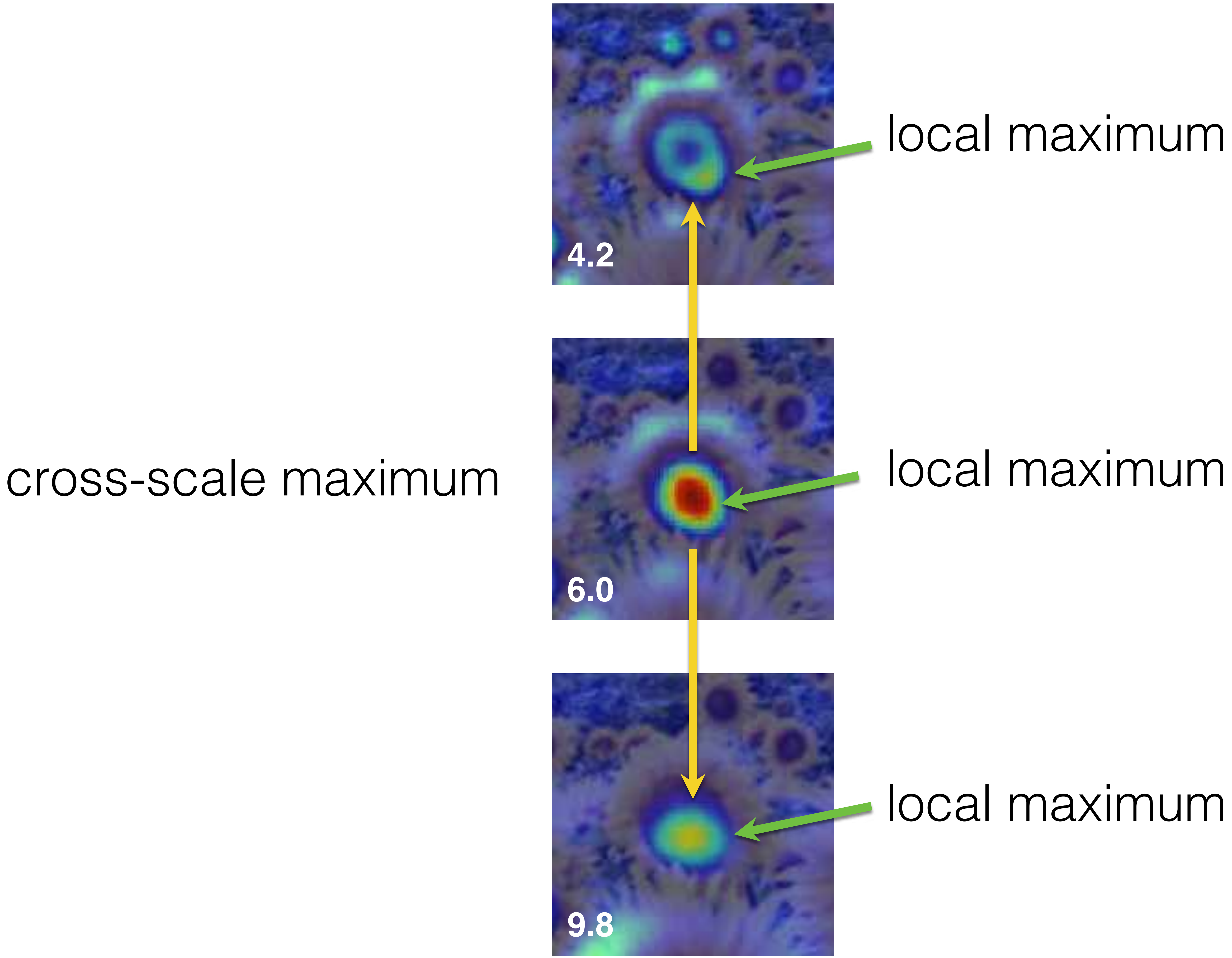



Full size image

2.1 | 4.2 | 6.0 | 9.8 | 15.5 | 17.0




3/4 size image


local maximum
4.2
cross-scale maximum
local maximum
6.0
local maximum
9.8

How would you implement scale selection?

# Implementation

```
For each level of the Gaussian pyramid

      compute feature response (e.g. Harris, Laplacian)


For each level of the Gaussian pyramid

     if local maximum and cross-scale

     save scale and location of feature
```

$(x, y, s)$