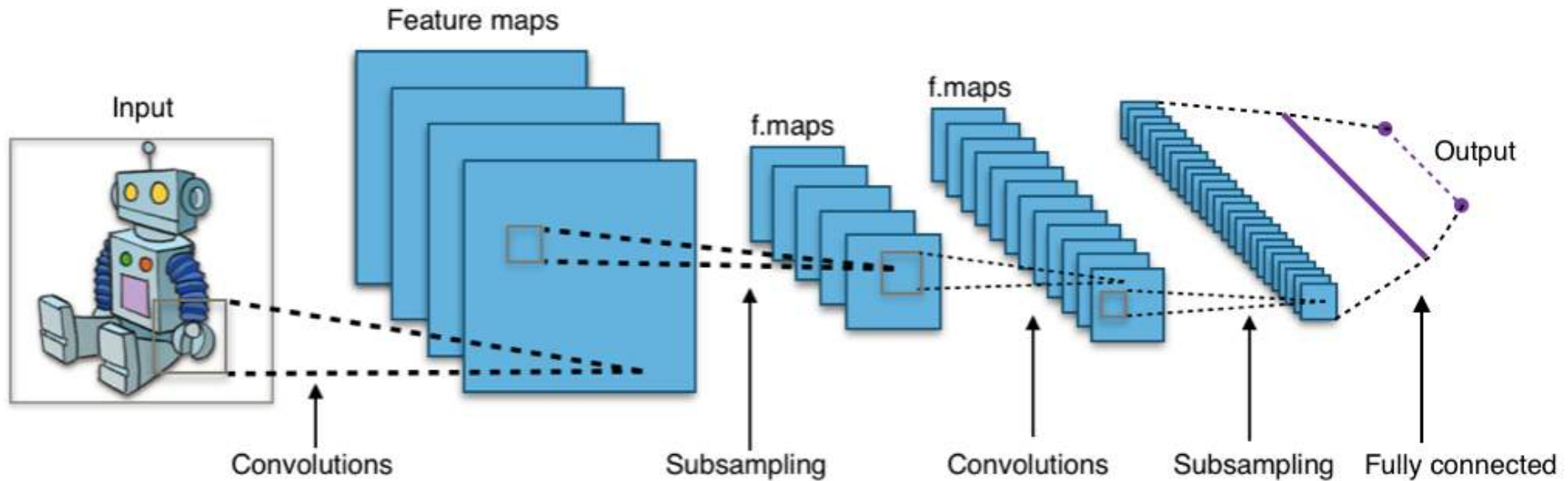


# Convolutional neural networks



# Overview of today's lecture

- Some notes on optimization.
- Convolutional neural networks.
- Training ConvNets.

# Slide credits

Most of these slides were adapted from:

- Noah Snavely (Cornell University).
- Fei-Fei Li (Stanford University).
- Andrej Karpathy (Stanford University).

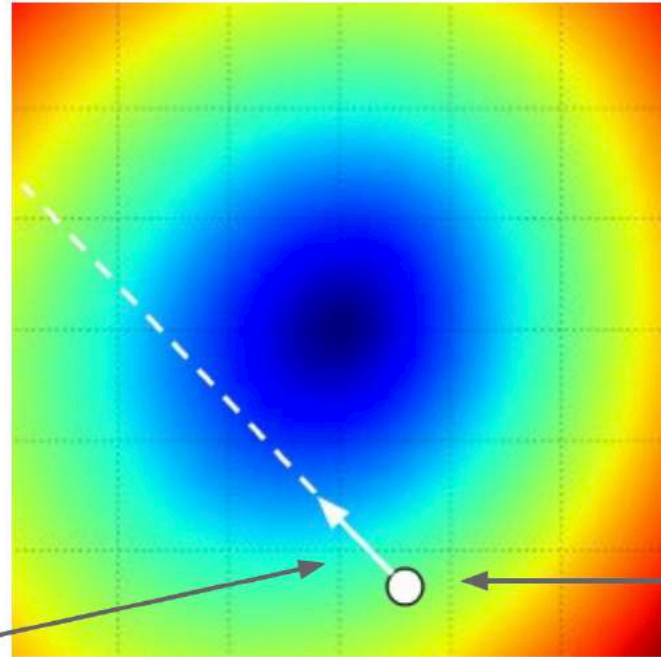
Some notes on optimization



# Summary

- Always use mini-batch gradient descent
- Incorrectly refer to it as “doing SGD” as everyone else  
(or call it batch gradient descent)
- The mini-batch size is a hyperparameter, but it is not very common to cross-validate over it (usually based on practical concerns, e.g. space/time efficiency)

# Learning rates



original  $\theta$

negative gradient direction

$$\theta \leftarrow \theta - \eta \frac{\partial \mathcal{L}}{\partial \theta}$$

Step size: learning rate

Too big: will miss the minimum

Too small: slow convergence

# Learning rate scheduling

- Use different learning rate at each iteration.
- Most common choice:

$$\eta_t = \frac{\eta_0}{\sqrt{t}}$$

Need to select initial learning rate  $\eta_0$ , important!!!

- More modern choice: Adaptive learning rates.

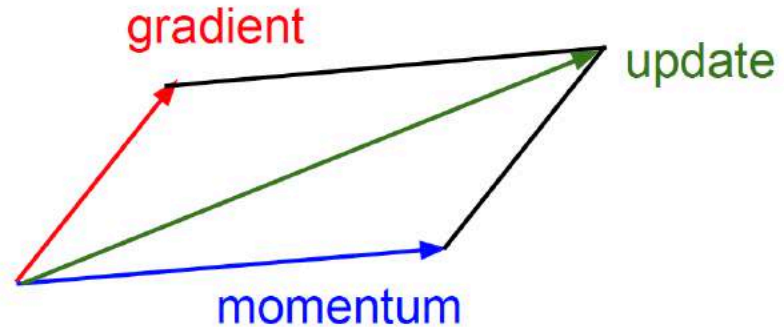
$$\eta_t = G \left( \left\{ \frac{\partial L}{\partial \theta} \right\}_{i=0}^t \right)$$

Many choices for G (Adam, Adagrad, Adadelata).

# Momentum Update

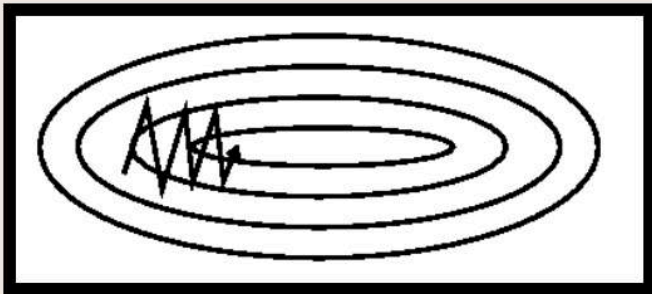
$$\theta \leftarrow \theta - \eta \frac{\partial \mathcal{L}}{\partial \theta}$$

$$\Delta\theta \leftarrow w \frac{\partial L}{\partial \theta} + (1 - w)\Delta\theta$$

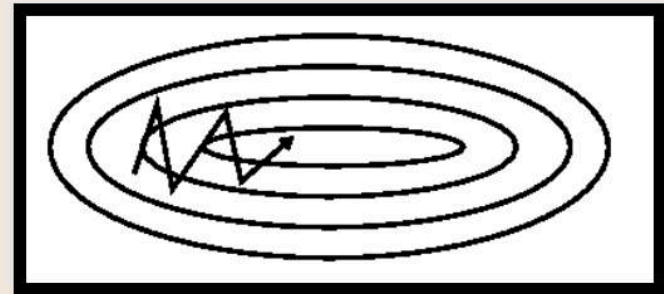


Take direction history into account!

```
weights_grad = evaluate_gradient(loss_fun, data, weights)
vel = vel * 0.9 - step_size * weights_grad
weights += vel
```



(Fig. 2a)



(Fig. 2b)

## Many other ways to perform optimization...

- Second order methods that use the Hessian (or its approximation): BFGS, **LBFGS**, etc.
- Currently, the lesson from the trenches is that well-tuned SGD+Momentum is very hard to beat for CNNs.
- No consensus on Adam etc.: Seem to give faster performance to worse local minima.

# Derivatives

- Given  $f(x)$ , where  $x$  is vector of inputs
  - Compute gradient of  $f$  at  $x$ :  $\nabla f(x)$

How do we do differentiation?

# Numerical differentiation

# Numerical differentiation

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h \frac{df(x)}{dx}$$



# Numerical differentiation

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h \frac{df(x)}{dx}$$

Numerical differentiation is:

- Approximate.
- Slow.
- Numerically unstable.
- Easy to write.

# Symbolic differentiation

# Symbolic differentiation

- What Mathematica does: Automatically derive *analytical* expressions for derivative.

# Symbolic differentiation

- What Mathematica does: Automatically derive *analytical* expressions for derivative.
- Often results in very redundant (and expensive to evaluate) expressions.

```
D[Log[1 + Exp[w * x + b]], w]
```

$$\text{Out[11]= } \frac{e^{b+wx} w}{1 + e^{b+wx}}$$

```
In[19]:= D[Log[1 + Exp[w2 * Log[1 + Exp[w1 * x + b1]] + b2]], w1]
```

$$\text{Out[19]= } \frac{e^{b_1+b_2+w_1 x+w_2 \text{Log}[1+e^{b_1+w_1 x}]} w_2 x}{(1 + e^{b_1+w_1 x}) \left(1 + e^{b_2+w_2 \text{Log}[1+e^{b_1+w_1 x}]}\right)}$$

- Often intractable.

# Automatic differentiation (autodiff)

# Automatic differentiation (autodiff)

- An autodiff system will convert the program into a sequence of **primitive operations** which have specified routines for computing derivatives.
- In this representation, backprop can be done in a completely mechanical way.

## Sequence of primitive operations:

Original program:

$$z = wx + b$$

$$y = \frac{1}{1 + \exp(-z)}$$

$$\mathcal{L} = \frac{1}{2}(y - t)^2$$

$$t_1 = wx$$

$$z = t_1 + b$$

$$t_3 = -z$$

$$t_4 = \exp(t_3)$$

$$t_5 = 1 + t_4$$

$$y = 1/t_5$$

$$t_6 = y - t$$

$$t_7 = t_6^2$$

$$\mathcal{L} = t_7/2$$

# In summary

- Numerical gradient: easy to implement, bad to use.
- Symbolic gradient: sometimes useful, often intractable.
- Automatic gradient: exact, fast, error-prone.

In practice: Use symbolic gradient for small/trivial programs. Almost always use analytic gradient, but check correctness of implementation with numerical gradient.

- This is called a gradient check.

# Convolutional Neural Networks



# Aside: “CNN” vs “ConvNet”

## Note:

- There are many papers that use either phrase, but
- “ConvNet” is the preferred term, since “CNN” clashes with other things called CNN



# Motivation



HOME ▾

MENU ▾

CONNECT

THE LATEST

POPULAR

MOST SHARED



MIT  
Technology  
Review

## 10 BREAKTHROUGH TECHNOLOGIES 2013

[Introduction](#)

[The 10 Technologies](#)

[Past Years](#)

### Deep Learning

With massive amounts of computational power, machines can now recognize objects and translate speech in real time. Artificial intelligence is finally getting smart. →

### Temporary Social Media

Messages that quickly self-destruct could enhance the privacy of online communications and make people freer to be spontaneous. →

### Prenatal DNA Sequencing

Reading the DNA of fetuses will be the next frontier of the genomic revolution. But do you really want to know about the genetic problems or musical aptitude of your unborn child? →

### Additive Manufacturing

Skeptical about 3-D printing? GE, the world's largest manufacturer, is on the verge of using the technology to make jet parts. →

### Baxter: The Blue-Collar Robot

Rodney Brooks's newest creation is easy to interact with, but the complex innovations behind the robot show just how hard it is to get along with people. →

### Memory Implants

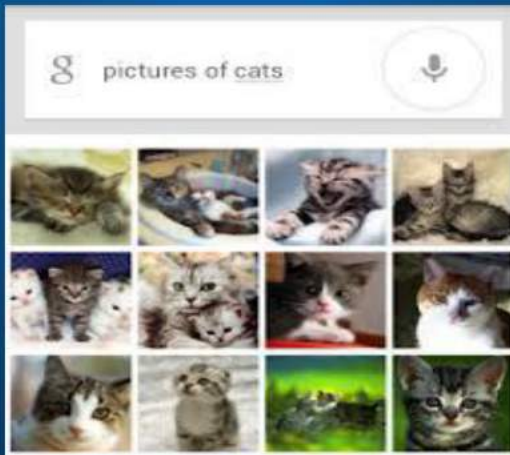
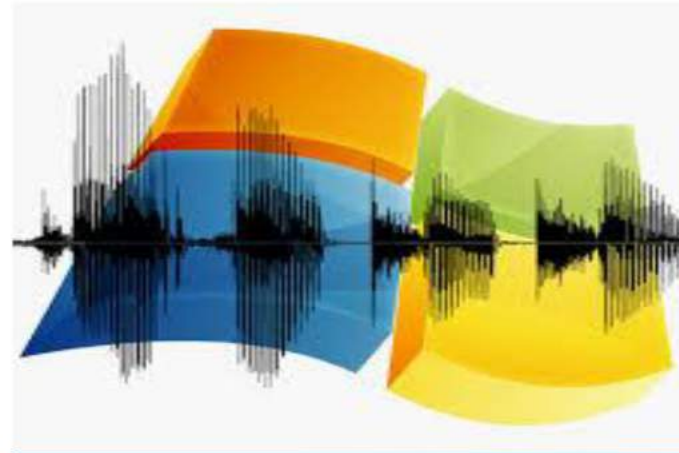
### Smart Watches

### Ultra-Efficient Solar

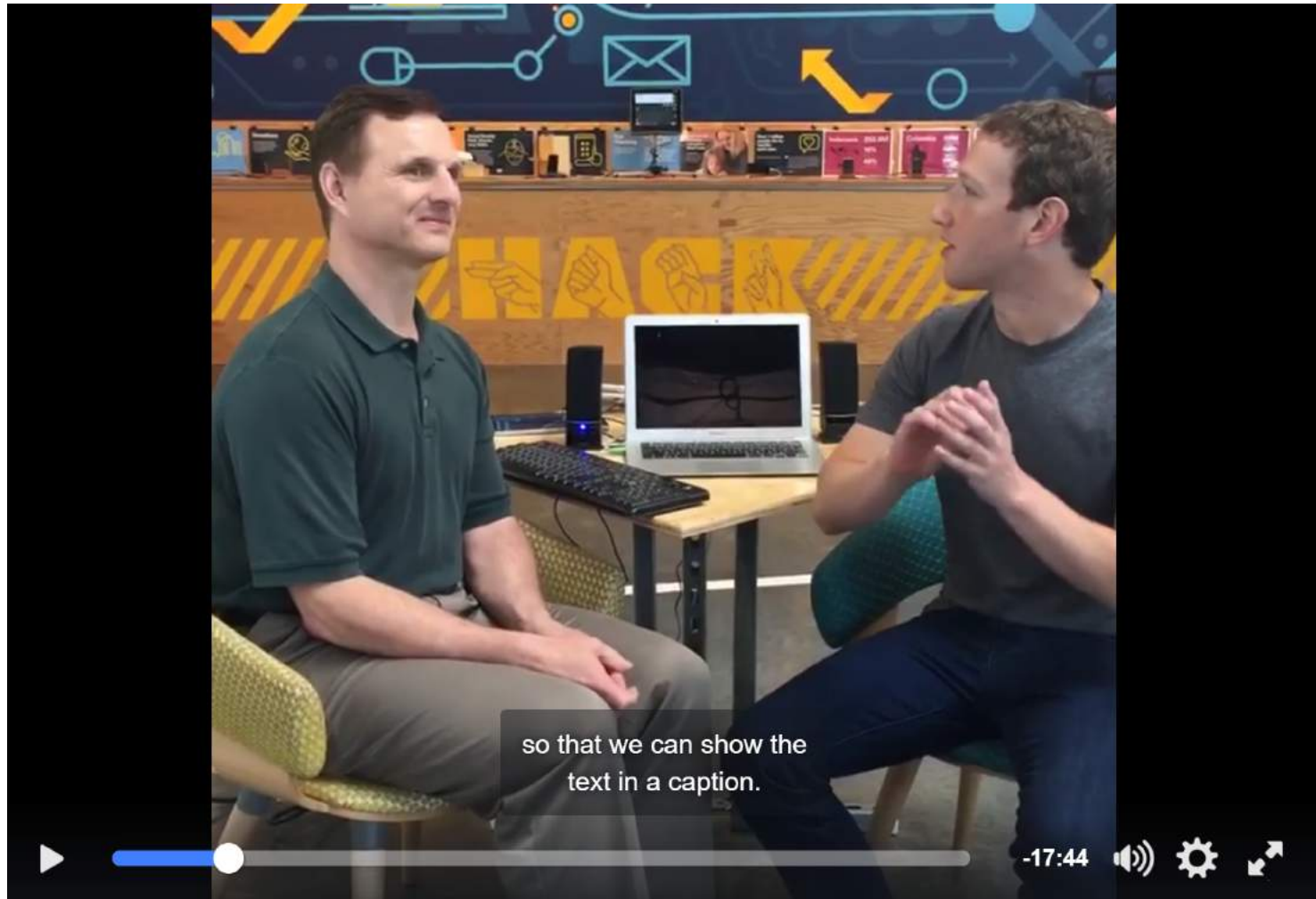
### Big Data from

### Supergrids

# Products



# Helping the Blind



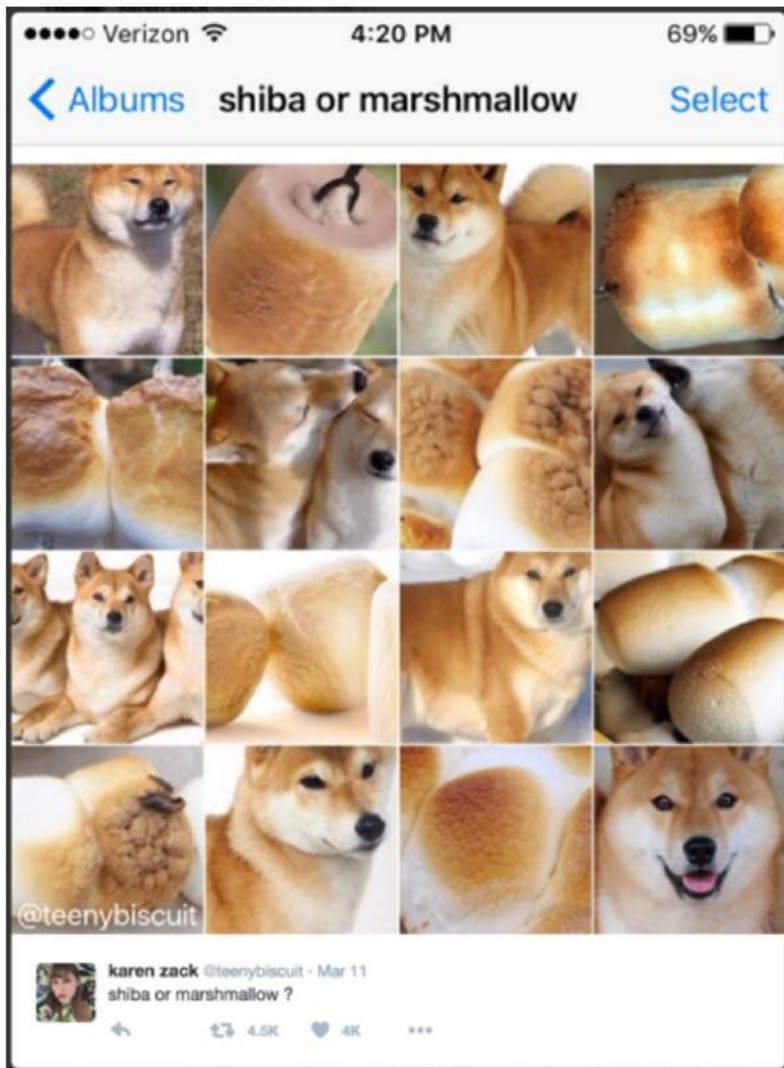
<https://www.facebook.com/zuck/videos/10102801434799001/>



# (Unrelated) Dog vs Food



# (Unrelated) Dog vs Food





# CNNs in 2012: “SuperVision” (aka “AlexNet”)

“AlexNet” — Won the ILSVRC2012 Challenge

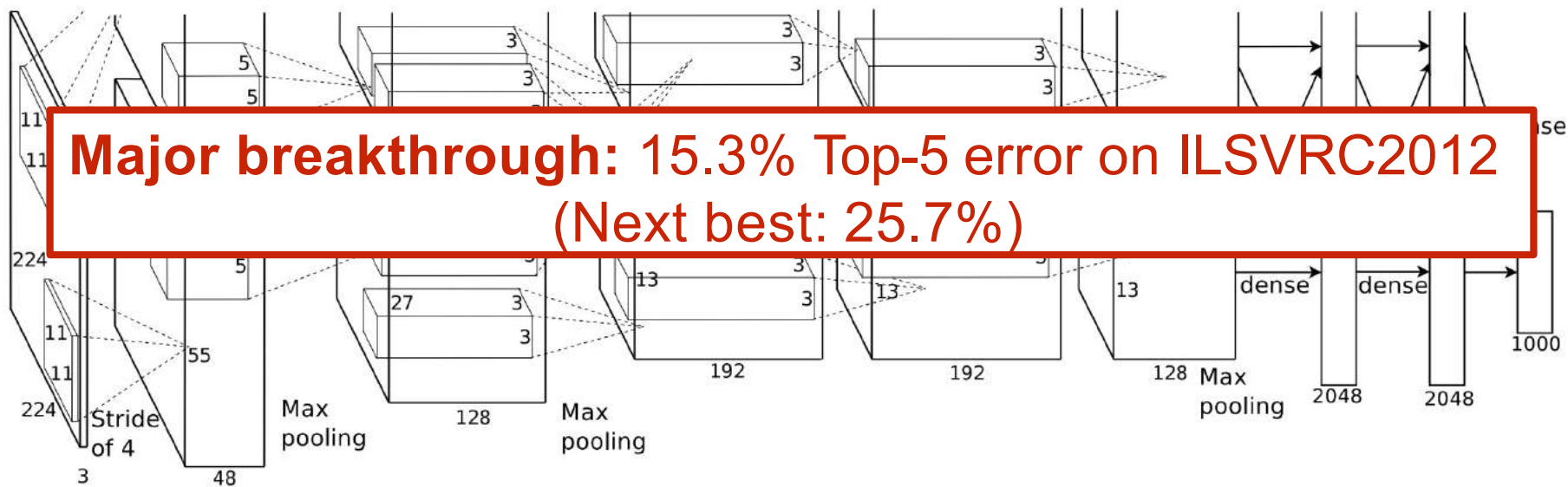
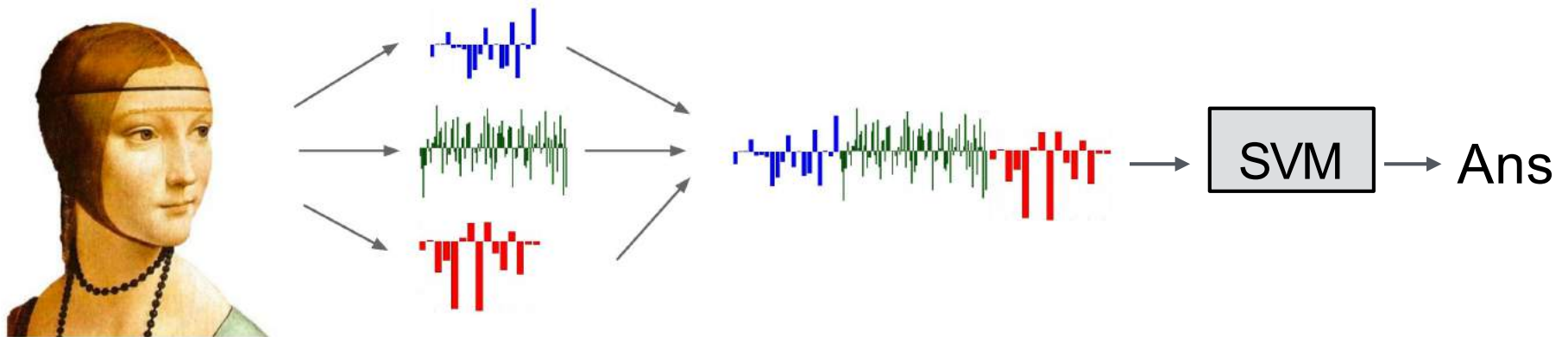


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

[Krizhevsky, Sutskever, Hinton. NIPS 2012]

# Recap: Before Deep Learning



*Input  
Pixels*

*Extract  
Features*

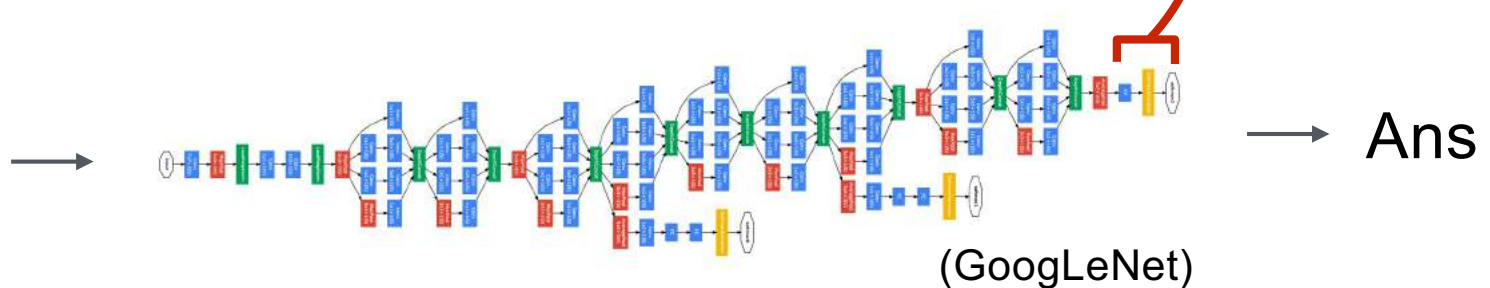
*Concatenate into  
a vector  $x$*

*Linear  
Classifier*



# The last layer of (most) CNNs are linear classifiers

This piece is just a linear classifier



*Input  
Pixels*

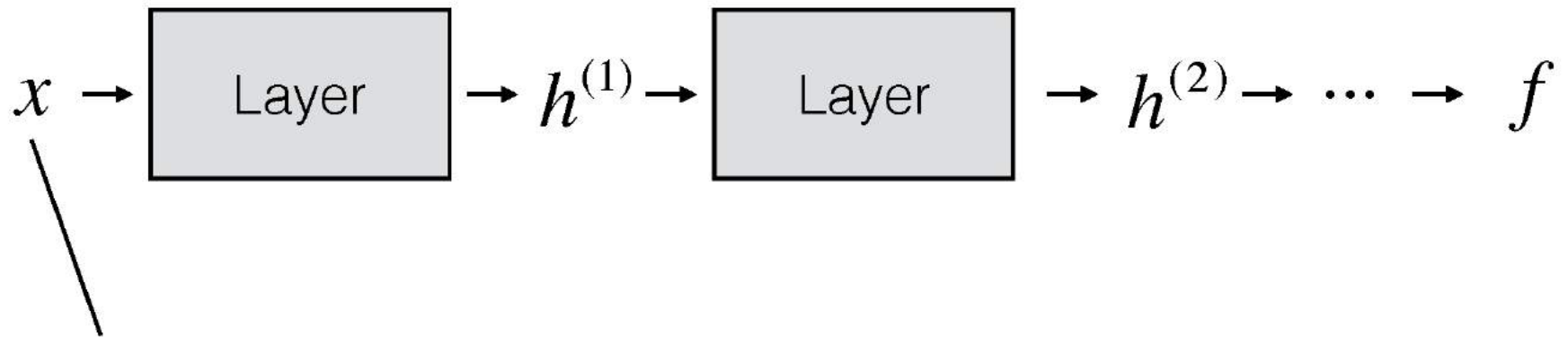
*Perform everything with a big neural  
network, trained end-to-end*

**Key:** perform enough processing so that by the time you get to the end of the network, the classes are linearly separable

# ConvNets

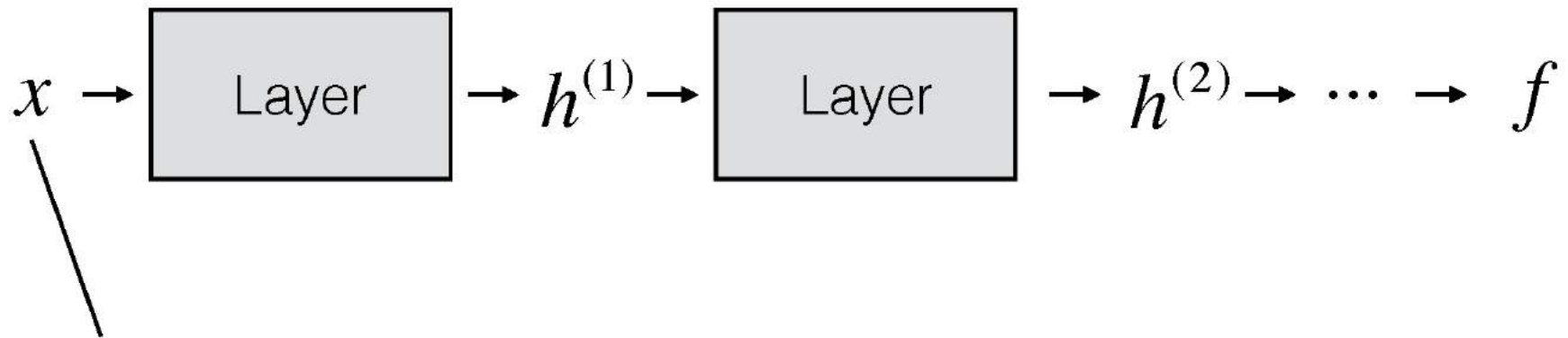
They're just neural networks with  
3D activations and weight sharing

# What shape should the activations have?



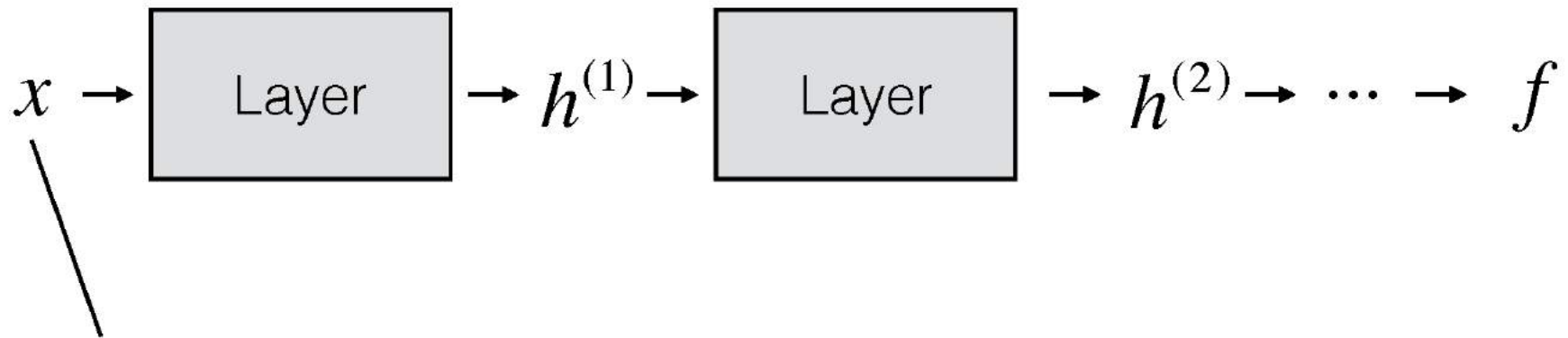
- The input is an image, which is 3D (RGB channel, height, width)

# What shape should the activations have?



- The input is an image, which is 3D (RGB channel, height, width)
- We could flatten it to a 1D vector, but then we lose structure

# What shape should the activations have?



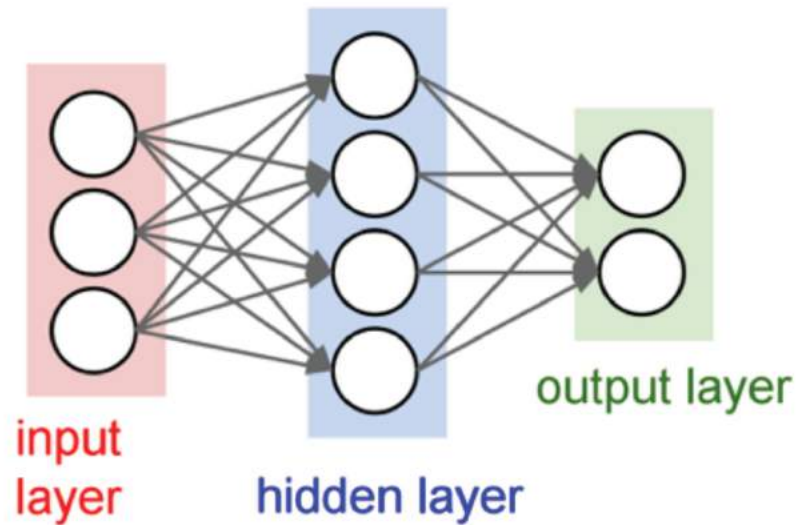
- The input is an image, which is 3D (RGB channel, height, width)
- We could flatten it to a 1D vector, but then we lose structure
- What about keeping everything in 3D?

# ConvNets

They're just neural networks with  
3D activations and weight sharing

# 3D Activations

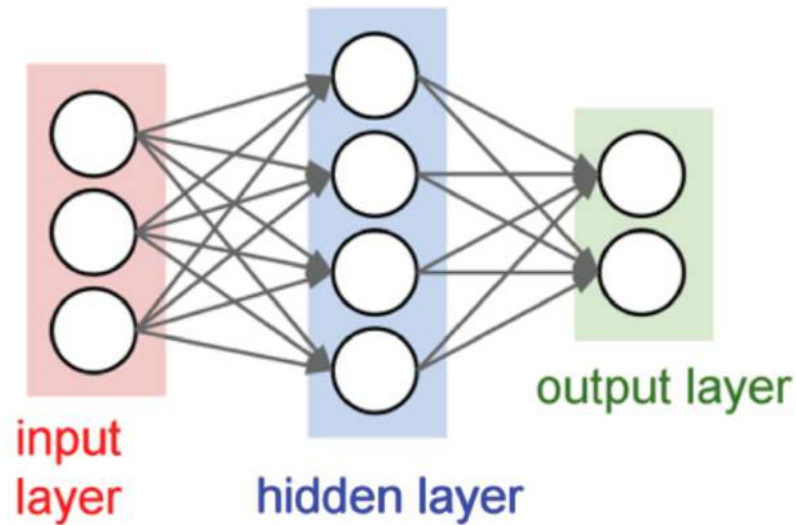
before:



**(1D vectors)**

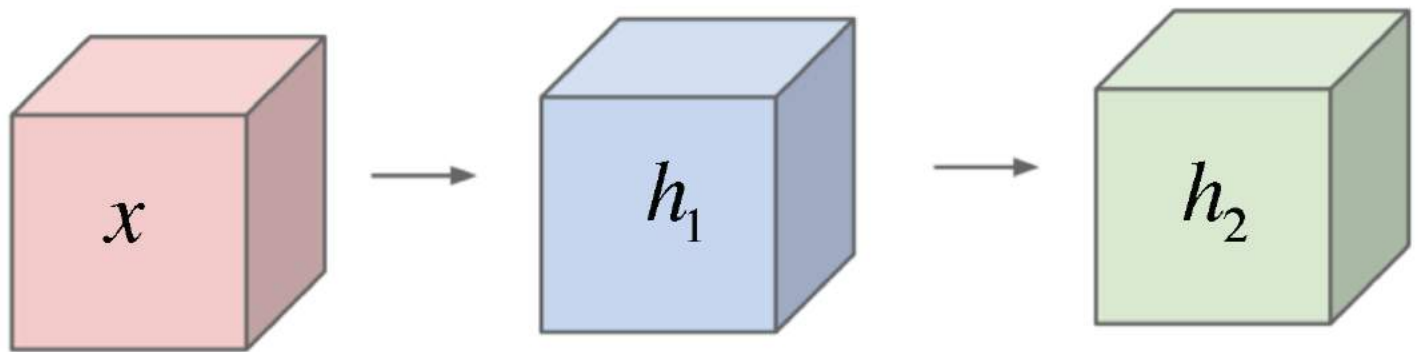
# 3D Activations

before:



**(1D vectors)**

now:

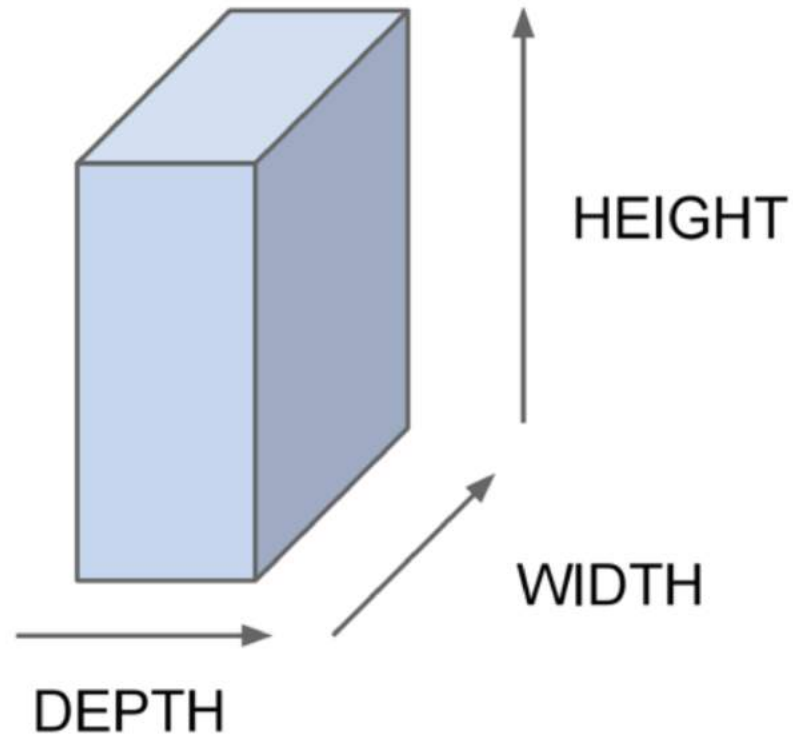


**(3D arrays)**



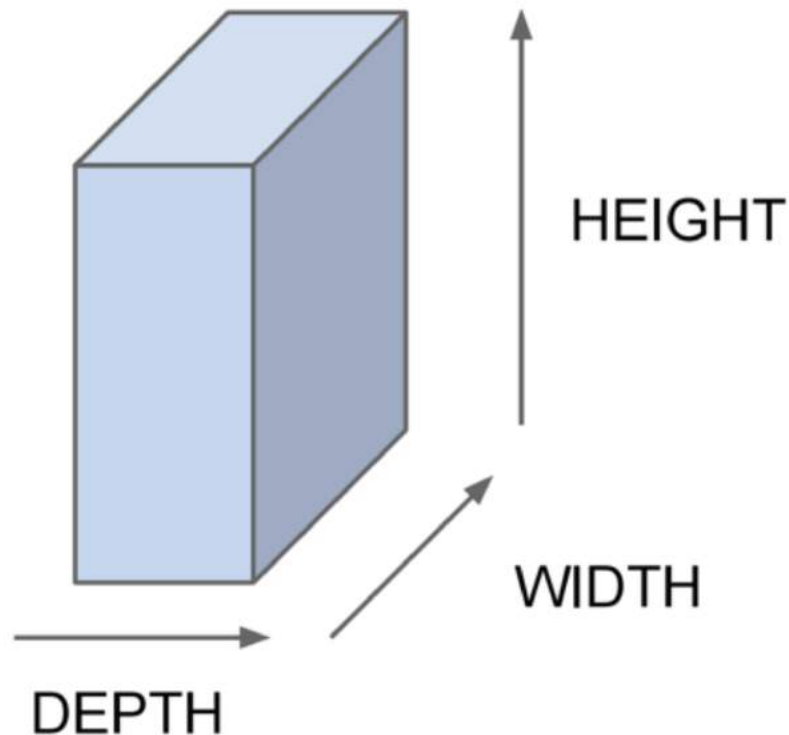
# 3D Activations

All Neural Net activations arranged in **3 dimensions**:



# 3D Activations

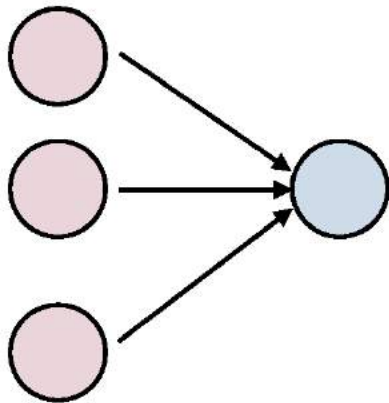
All Neural Net activations arranged in **3 dimensions**:



For example, a CIFAR-10 image is a  $3 \times 32 \times 32$  volume (3 depth — RGB channels, 32 height, 32 width)

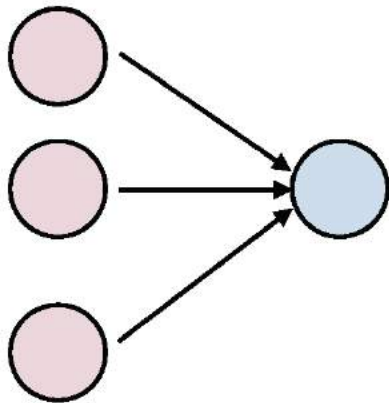
# 3D Activations

## 1D Activations:

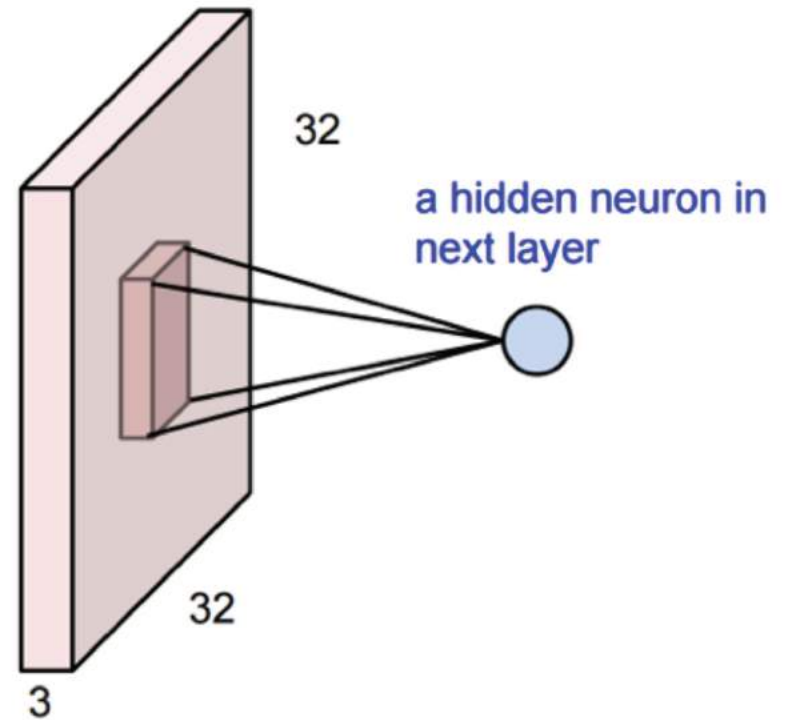


# 3D Activations

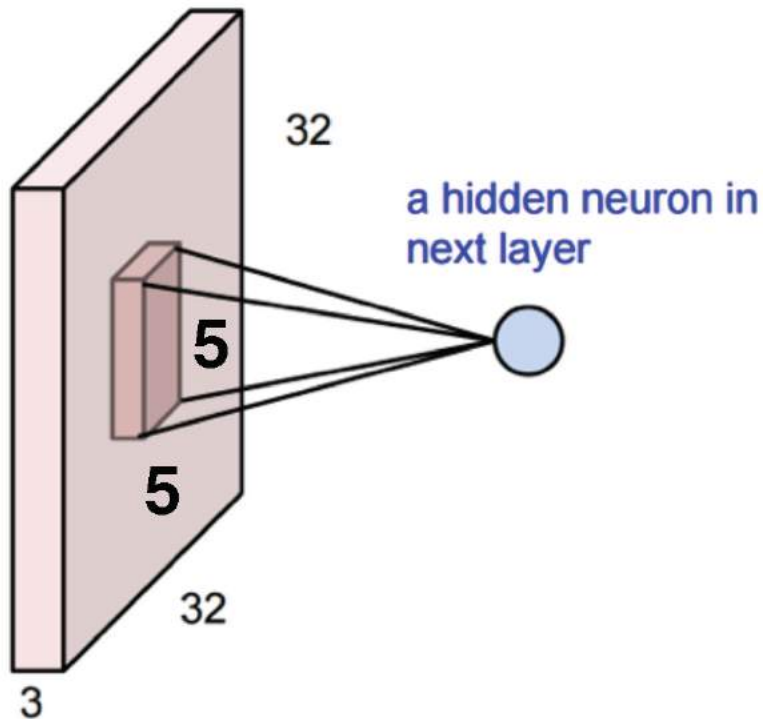
**1D Activations:**



**3D Activations:**

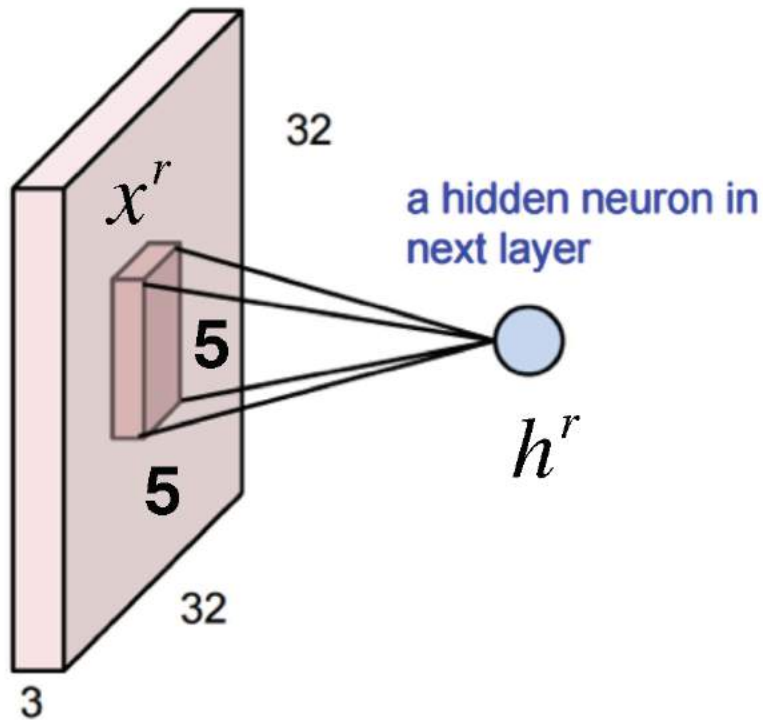


# 3D Activations



- The input is  $3 \times 32 \times 32$
- This neuron depends on a  $3 \times 5 \times 5$  chunk of the input
- The neuron also has a  $3 \times 5 \times 5$  set of weights and a bias (scalar)

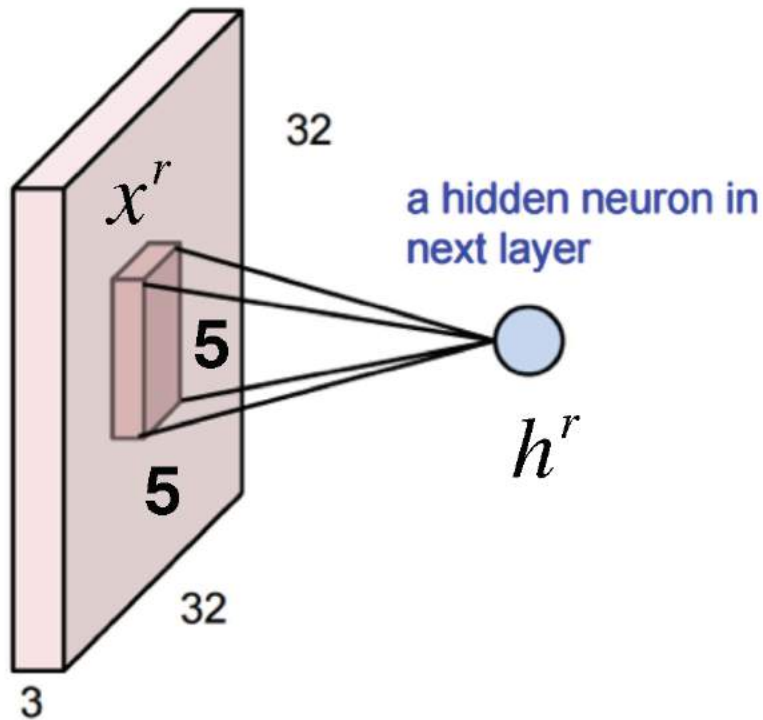
# 3D Activations



Example: consider the region of the input " $x^r$ "

With output neuron  $h^r$

# 3D Activations



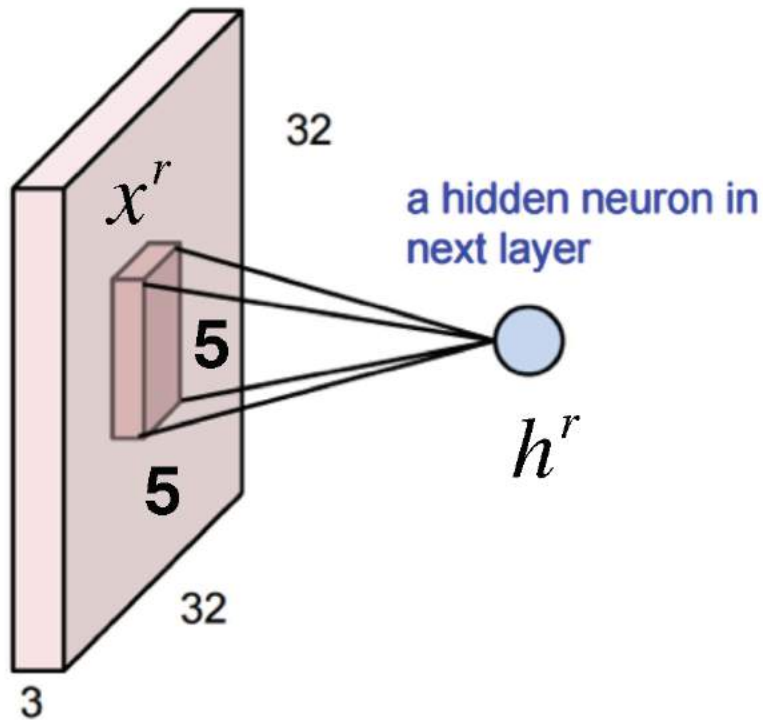
Example: consider the region of the input “ $x^r$ ”

With output neuron  $h^r$

Then the output is:

$$h^r = \sum_{ijk} x^r_{ijk} W_{ijk} + b$$

# 3D Activations



Example: consider the region of the input " $x^r$ "

With output neuron  $h^r$

Then the output is:

$$h^r = \sum_{ijk} x^r_{ijk} W_{ijk} + b$$



Sum over 3 axes



# 3D Activations

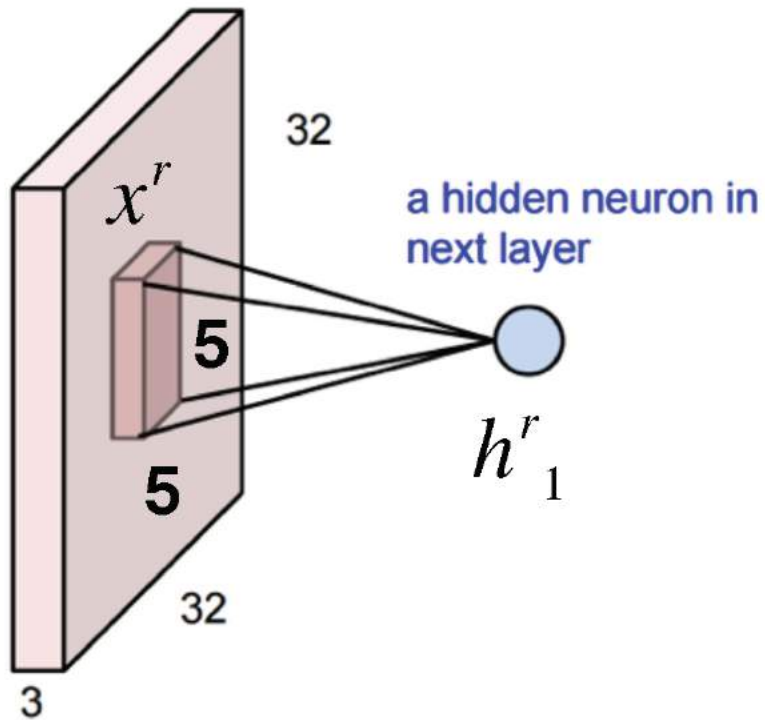


Figure: Andrej Karpathy

# 3D Activations

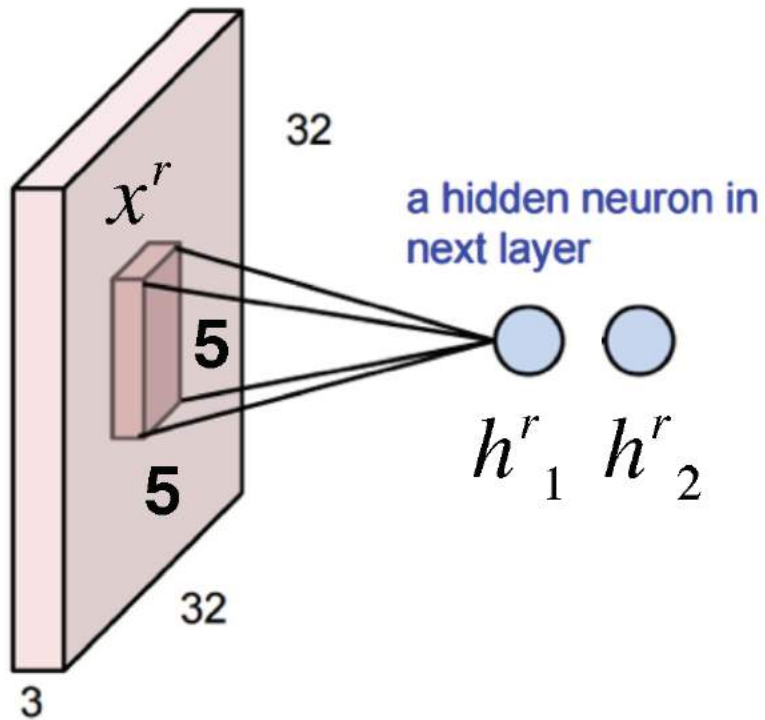
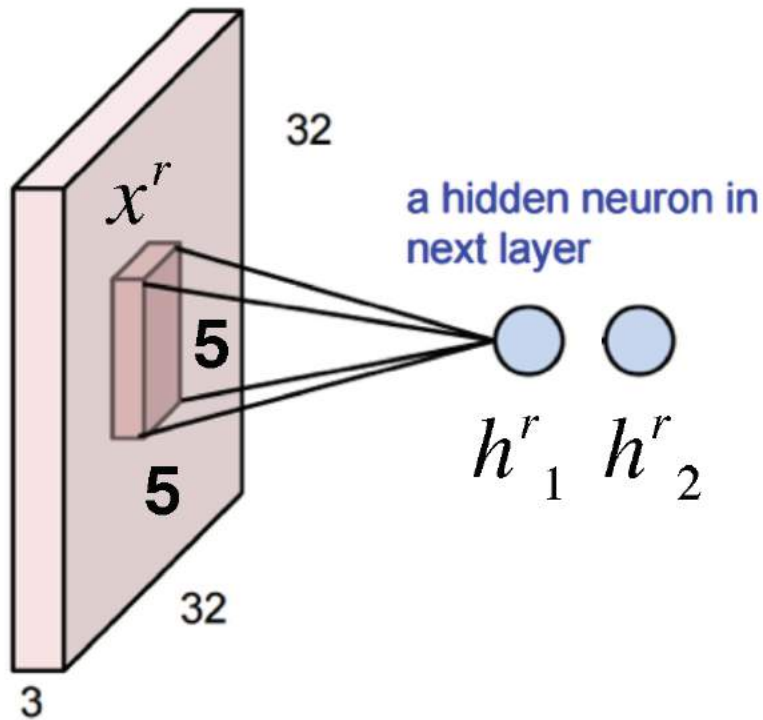


Figure: Andrej Karpathy

# 3D Activations

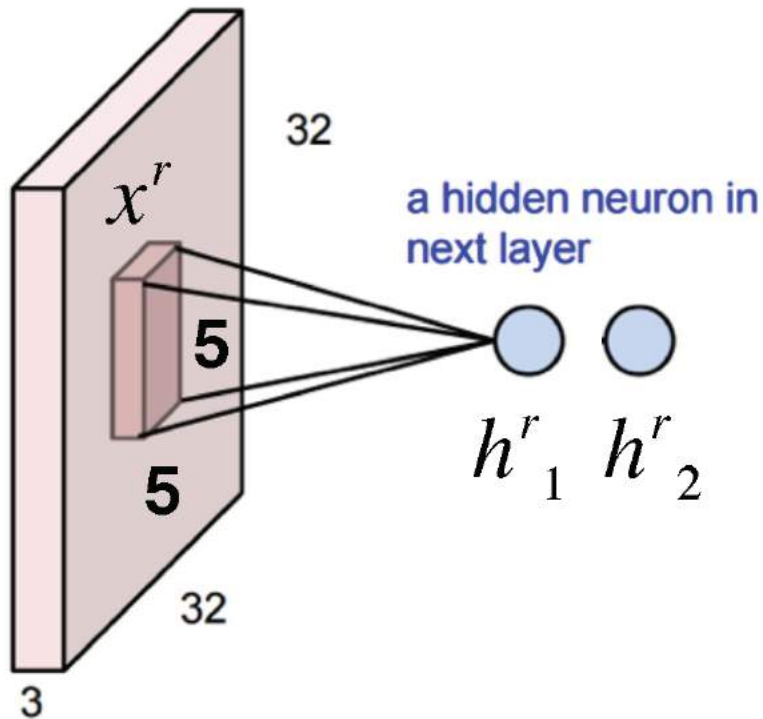


With **2** output neurons

$$h^r_1 = \sum_{ijk} x^r_{ijk} W_{1ijk} + b_1$$

$$h^r_2 = \sum_{ijk} x^r_{ijk} W_{2ijk} + b_2$$

# 3D Activations



With **2** output neurons

$$h^r_1 = \sum_{ijk} x^r_{ijk} W_{1ijk} + b_{1}$$

$$h^r_2 = \sum_{ijk} x^r_{ijk} W_{2ijk} + b_{2}$$

# 3D Activations

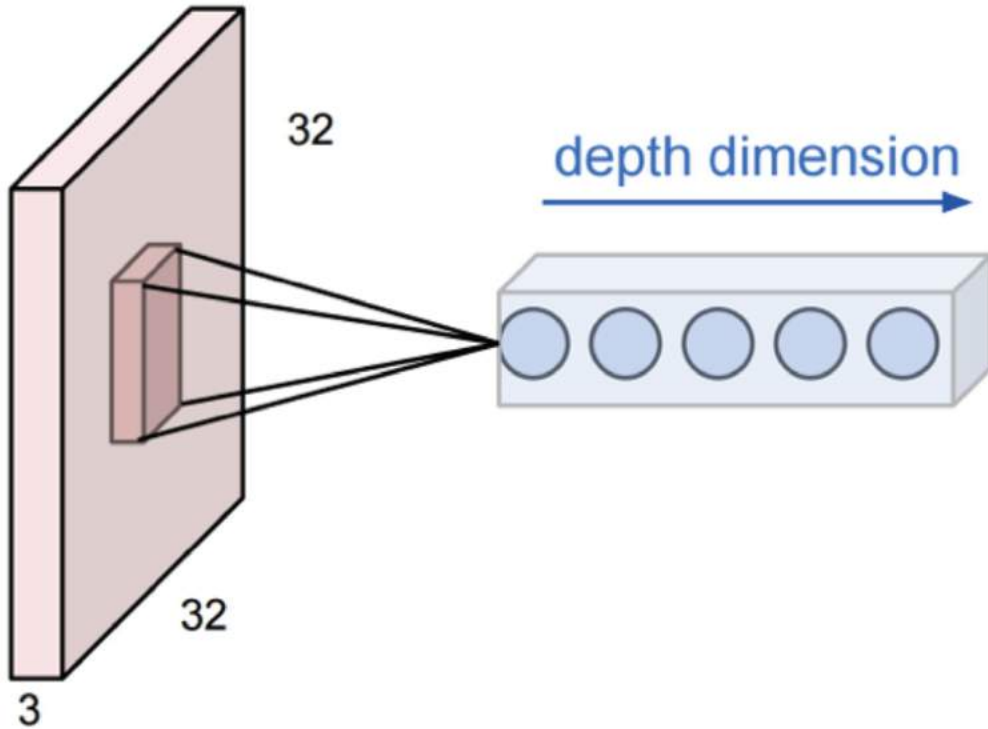
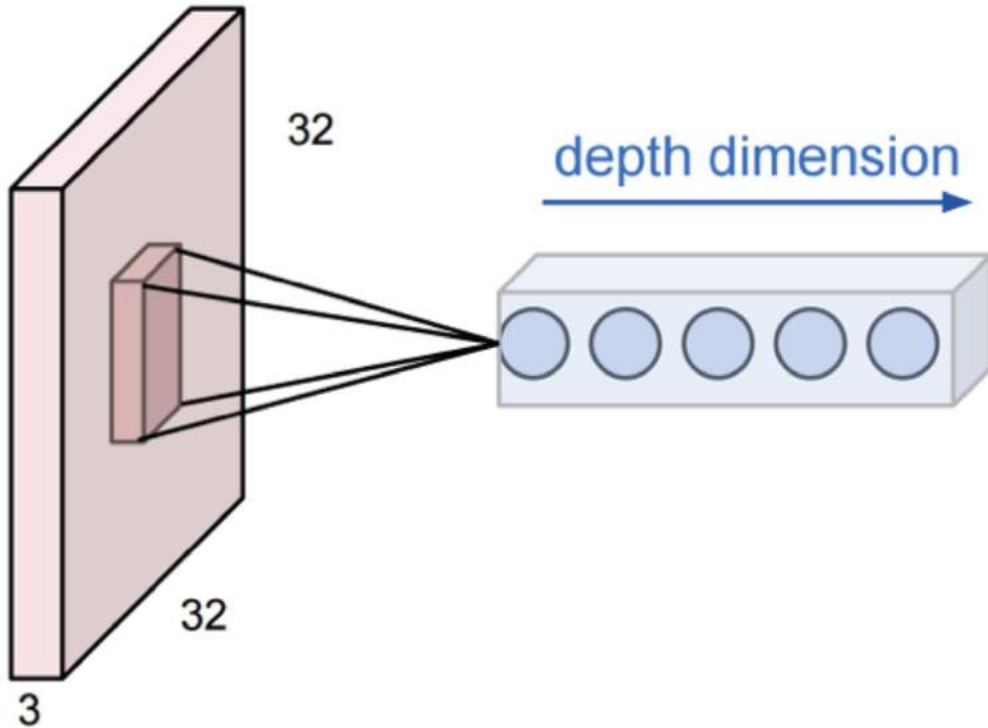


Figure: Andrej Karpathy

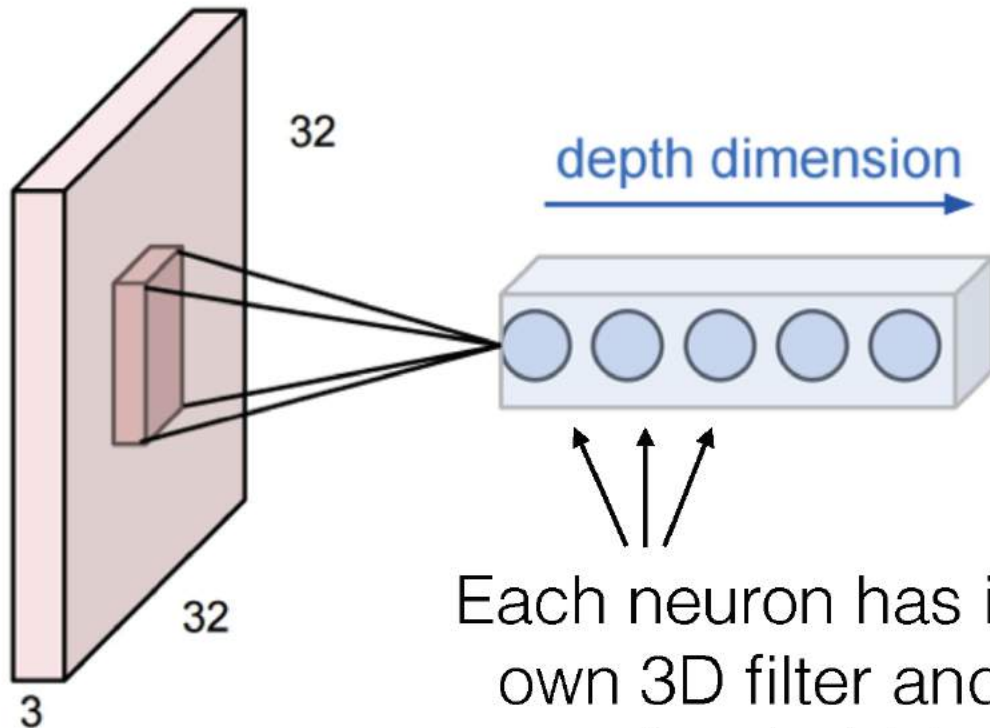
# 3D Activations



We can keep adding more outputs

These form a column in the output volume:  
[depth x 1 x 1]

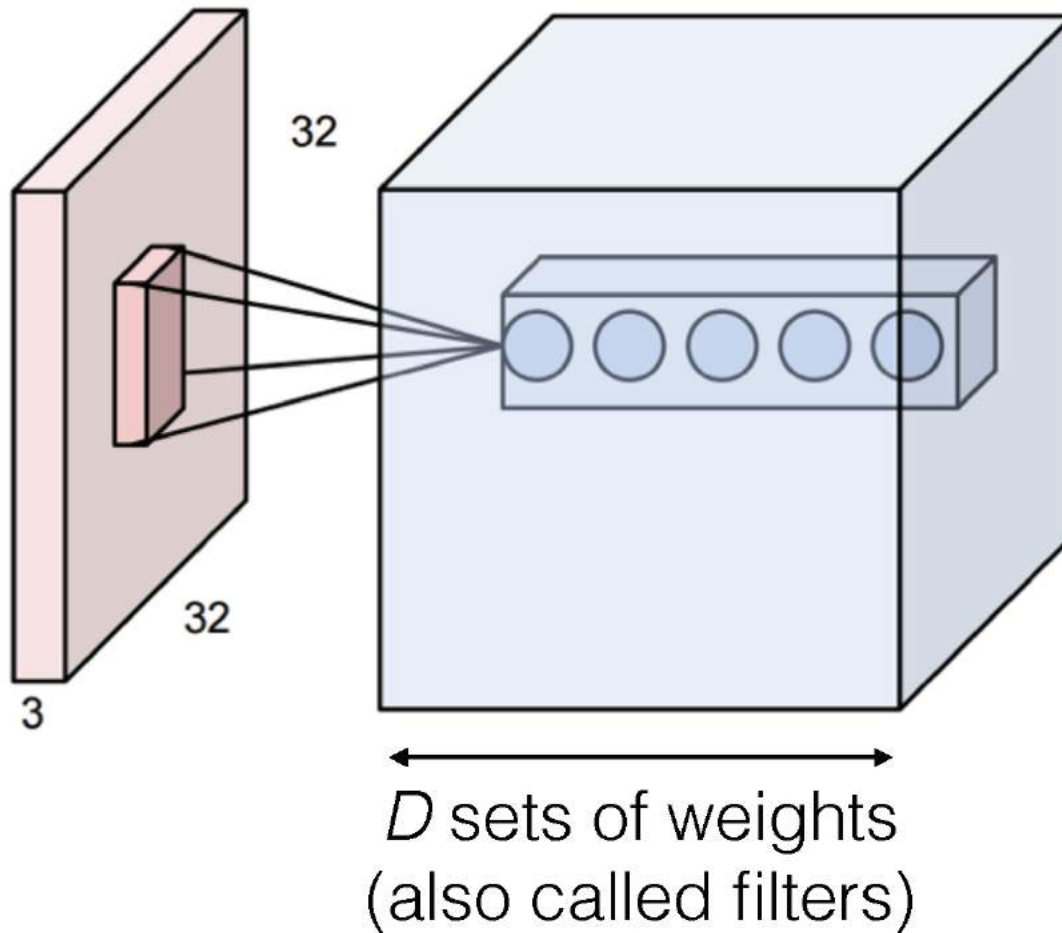
# 3D Activations



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[depth x 1 x 1]

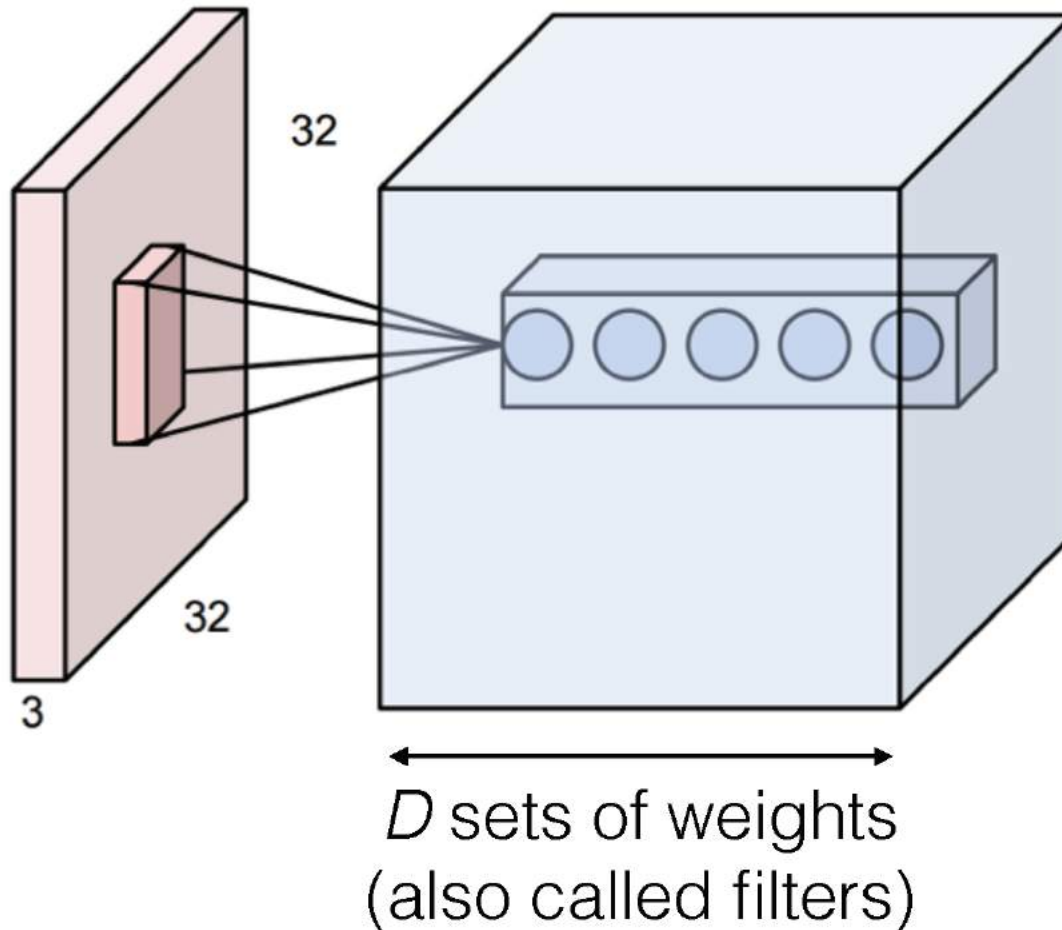
# 3D Activations



Now repeat this  
across the input



# 3D Activations



Now repeat this across the input

**Weight sharing:**  
Each filter shares the same weights (but each depth index has its own set of weights)

# 3D Activations

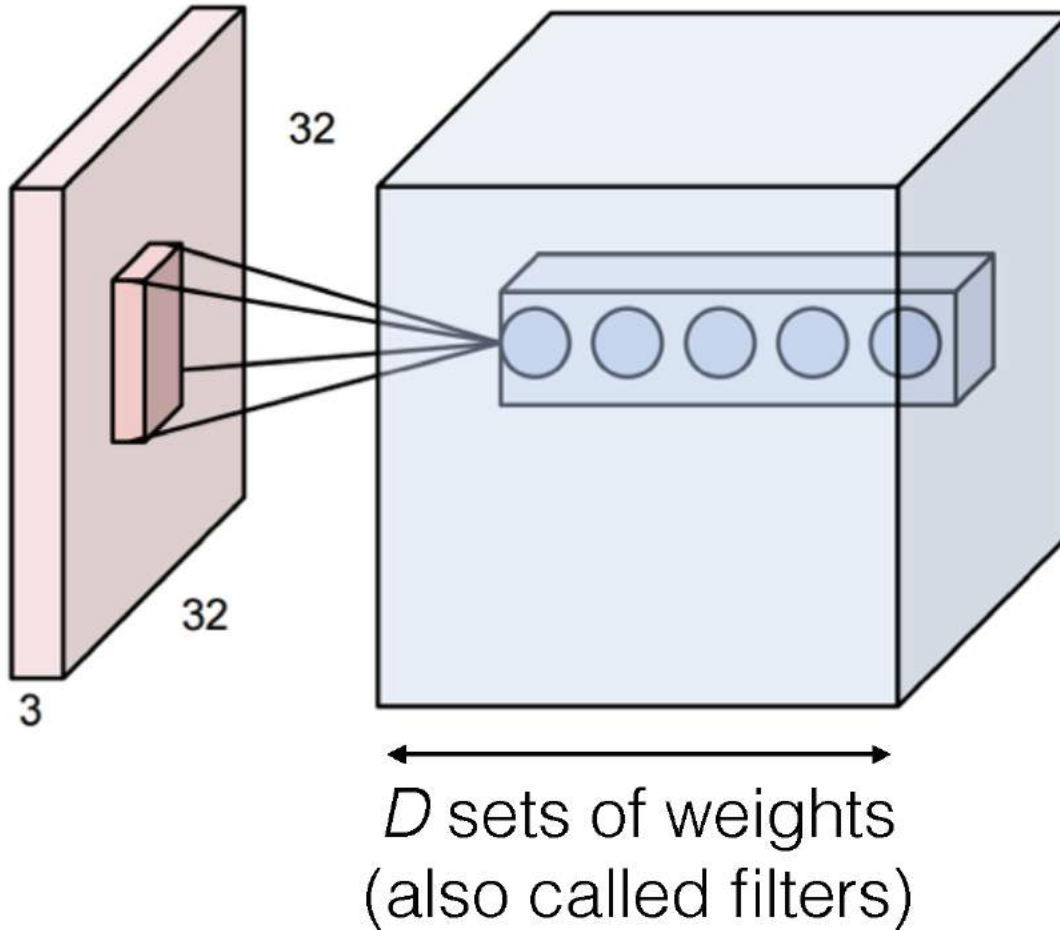
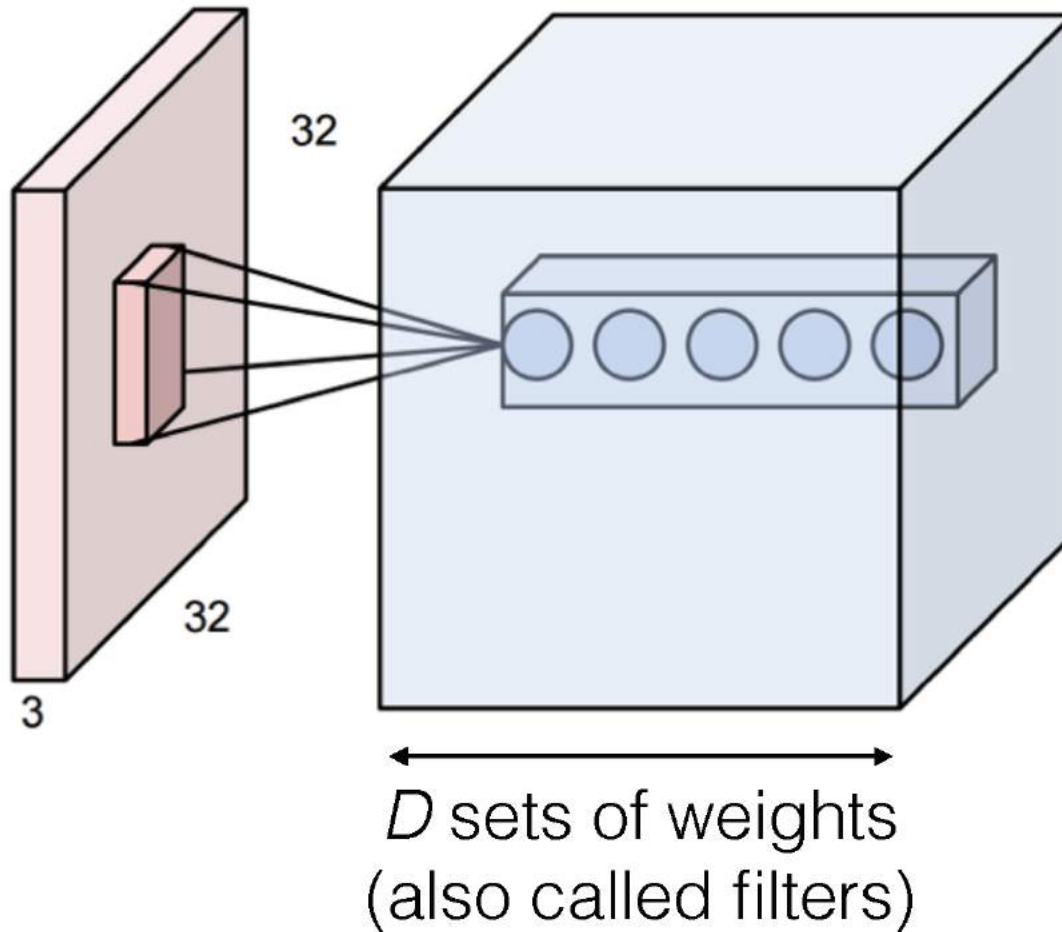


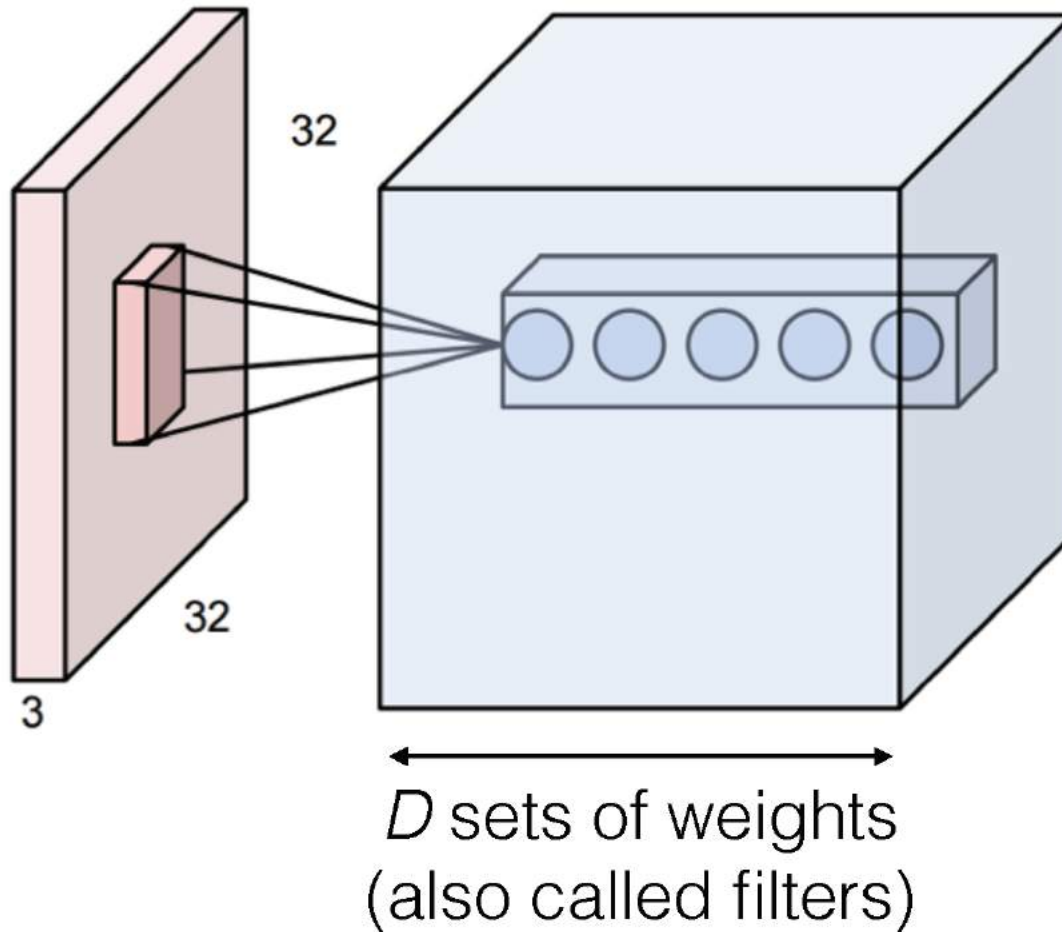
Figure: Andrej Karpathy

# 3D Activations



With weight sharing, this is called **convolution**

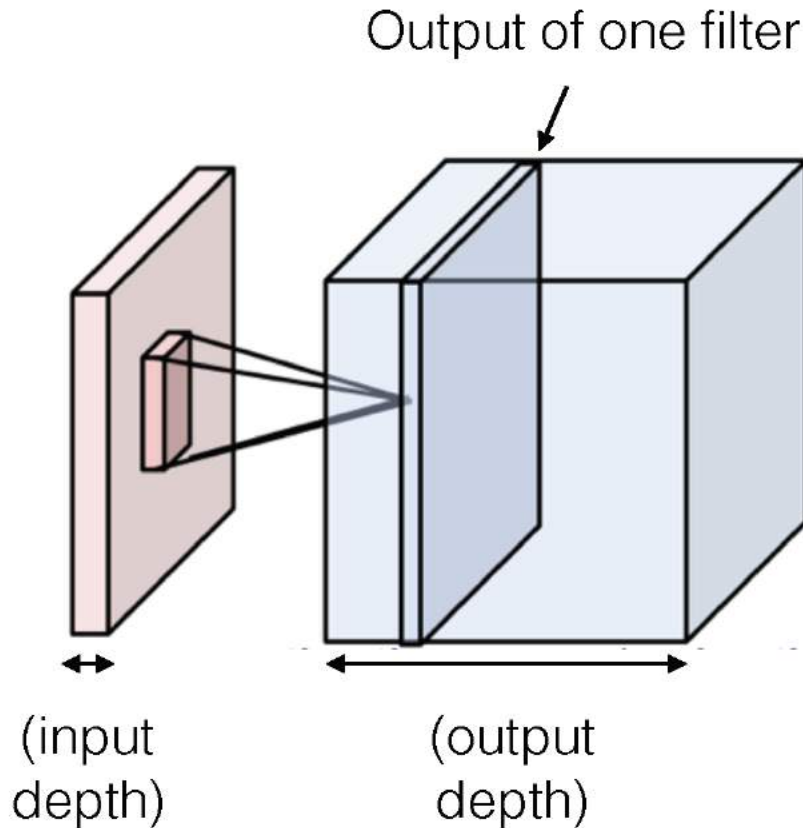
# 3D Activations



With weight sharing,  
this is called  
**convolution**

Without weight sharing,  
this is called a  
**locally connected layer**

# 3D Activations

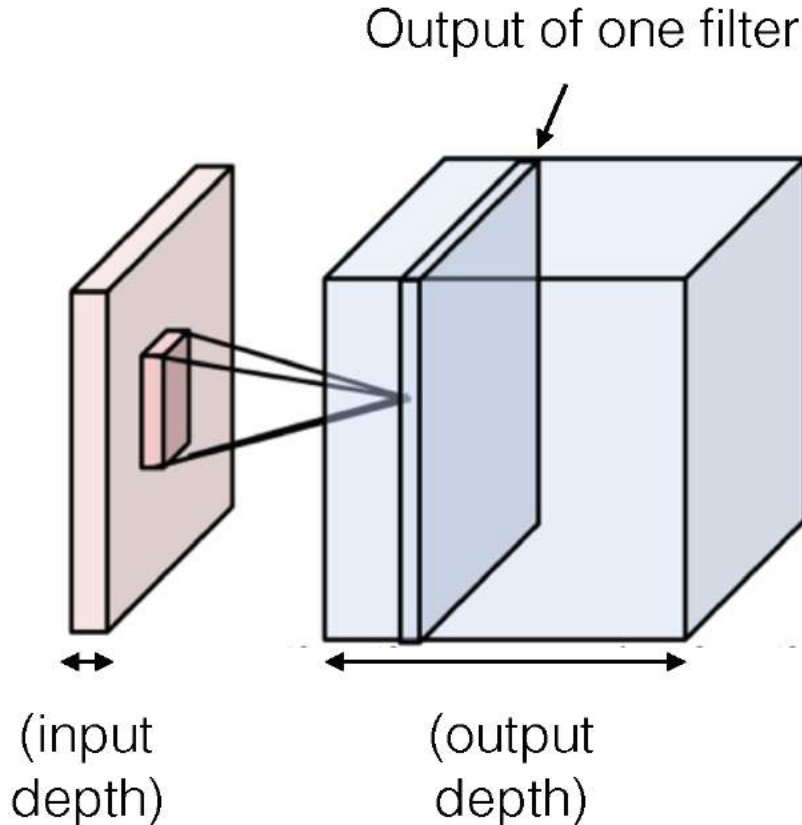


One set of weights gives one slice in the output

To get a 3D output of depth  $D$ , use  $D$  different filters

In practice, ConvNets use many filters ( $\sim 64$  to  $1024$ )

# 3D Activations



One set of weights gives one slice in the output

To get a 3D output of depth  $D$ , use  $D$  different filters

In practice, ConvNets use many filters ( $\sim 64$  to 1024)

All together, the weights are **4** dimensional:  
(output depth, input depth, kernel height, kernel width)



# 3D Activations

We can unravel the 3D cube and show each layer separately:

(Input)



one filter = one depth slice (or activation map) (32 filters, each 3x5x5)

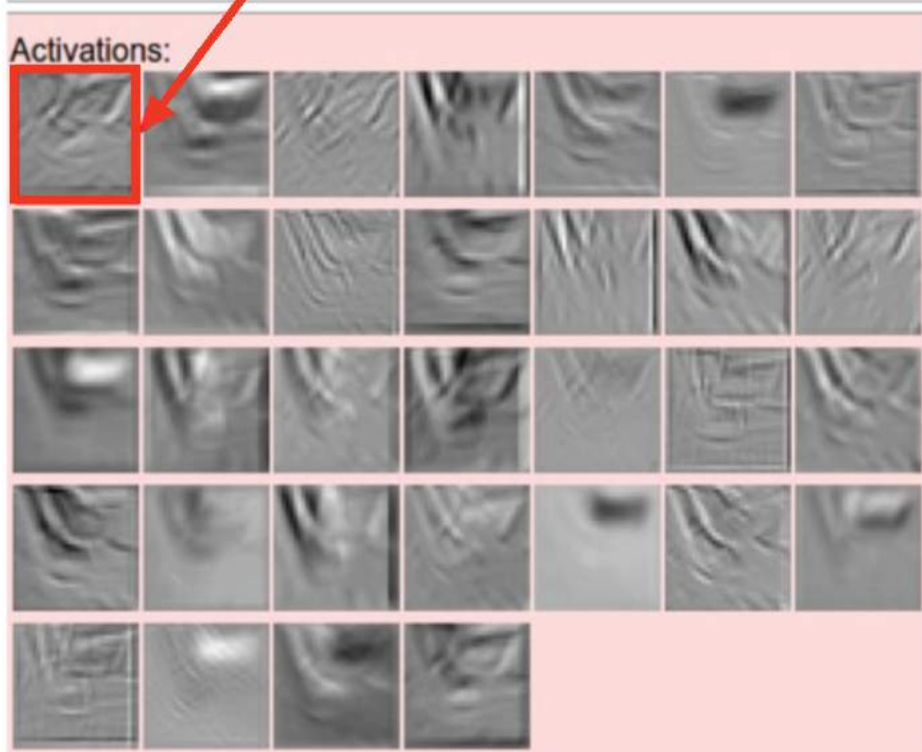


Figure: Andrej Karpathy

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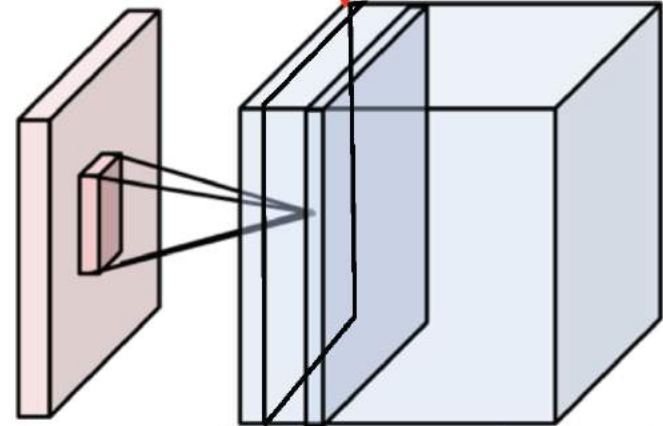
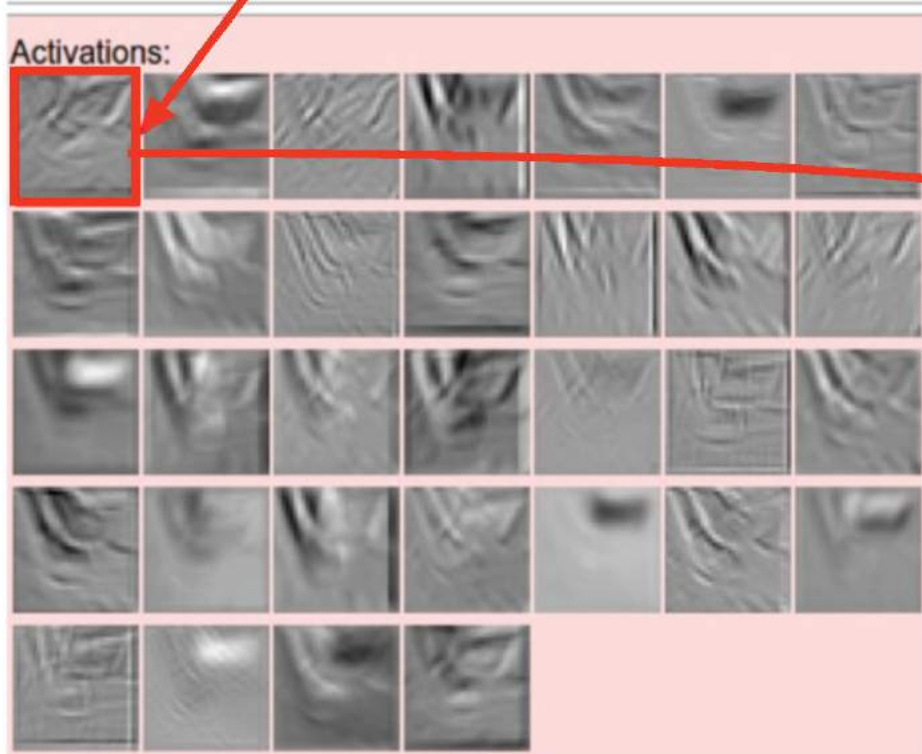


Figure: Andrej Karpathy



# 3D Activations

We can unravel the 3D cube and show each layer separately:

(Input)

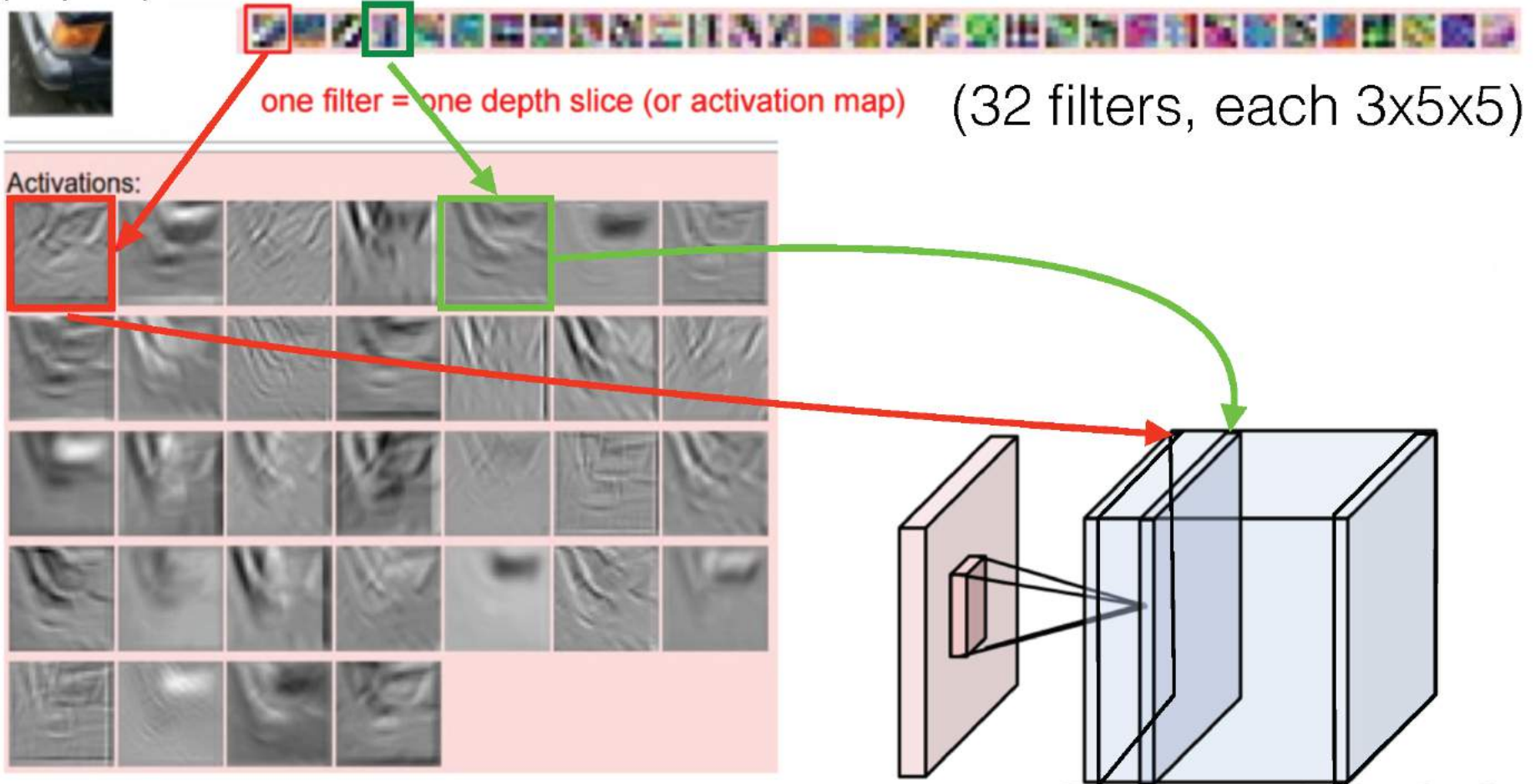
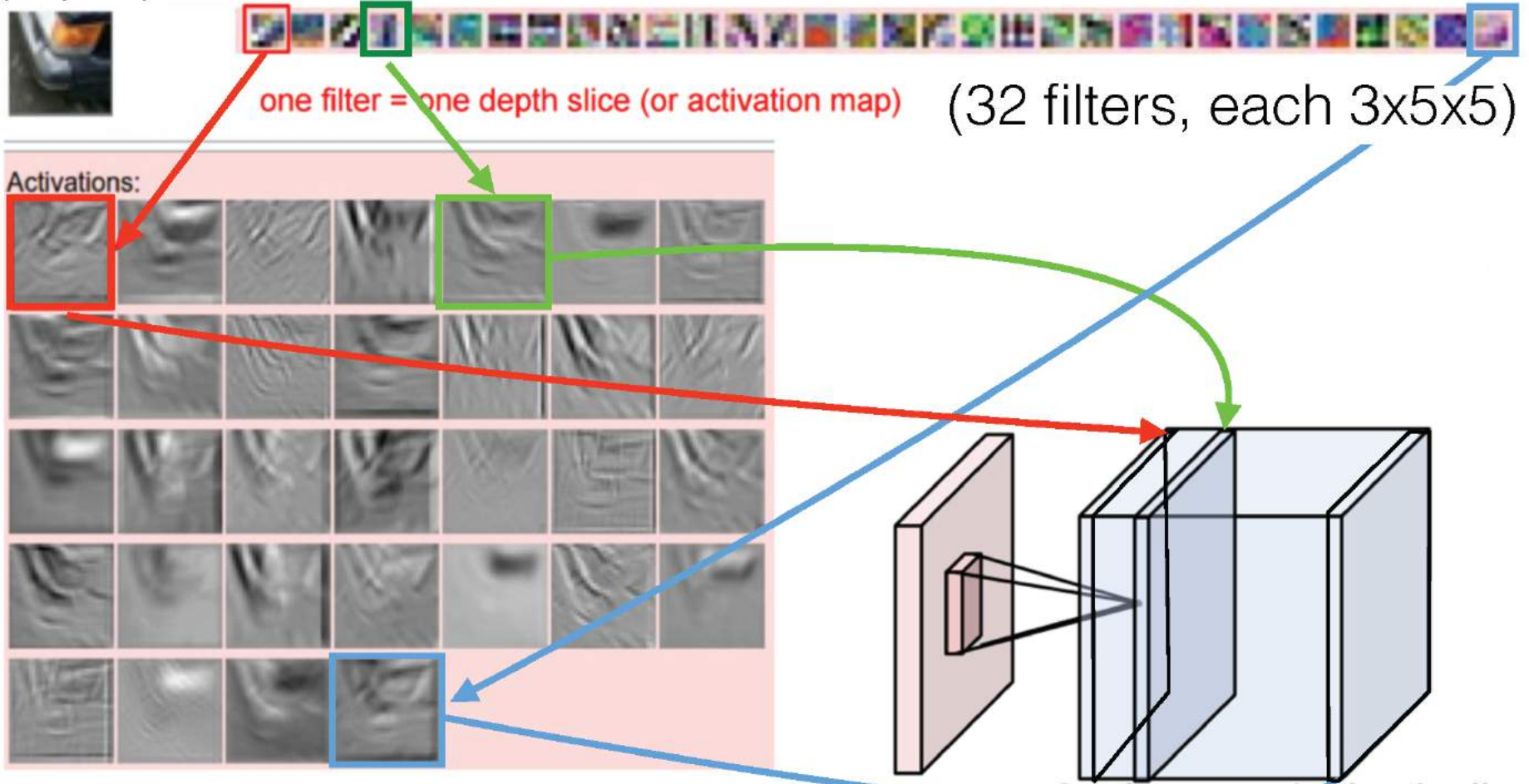


Figure: Andrej Karpathy

# 3D Activations

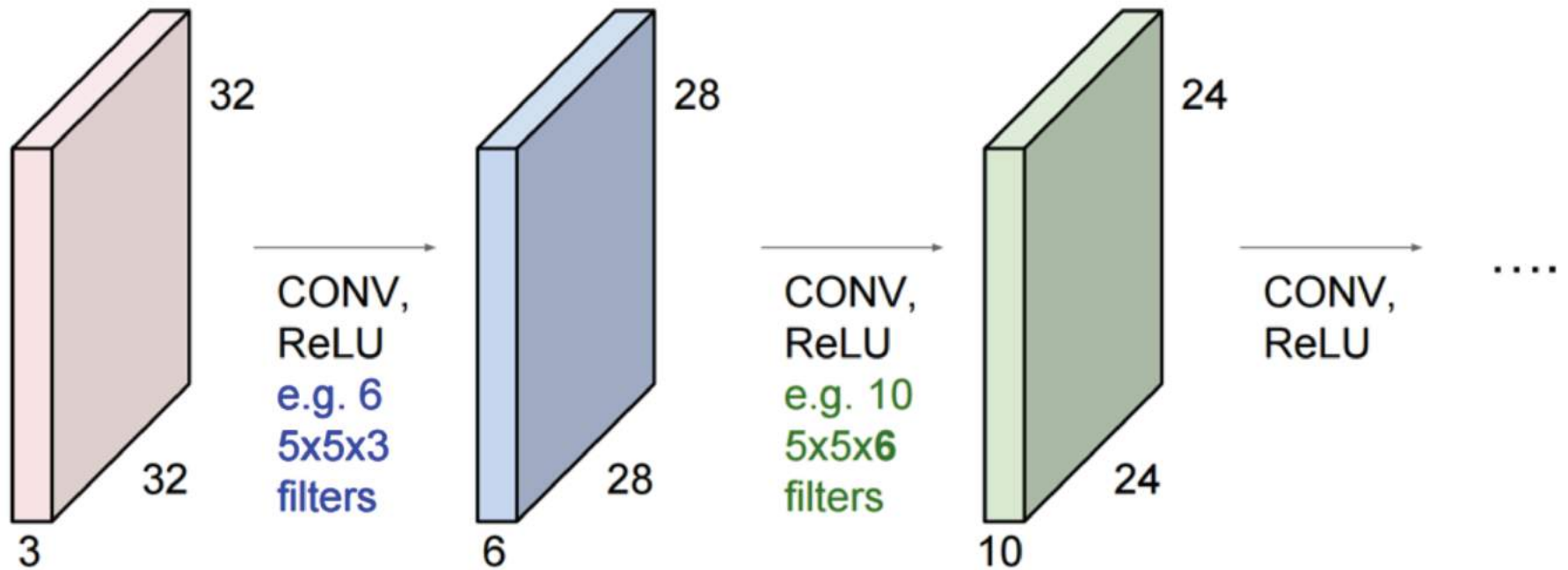
We can unravel the 3D cube and show each layer separately:

(Input)



# (Recap)

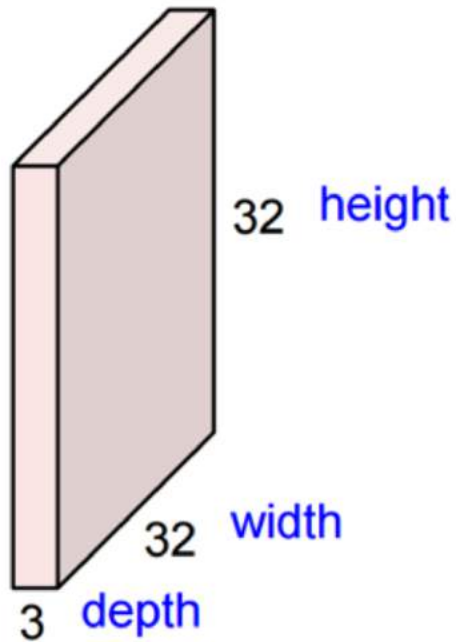
A **ConvNet** is a sequence of convolutional layers, interspersed with activation functions (and possibly other layer types)



(Recap)

# Convolution Layer

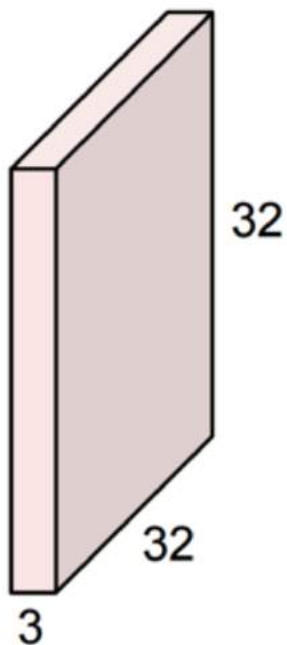
32x32x3 image



(Recap)

# Convolution Layer

32x32x3 image



5x5x3 filter

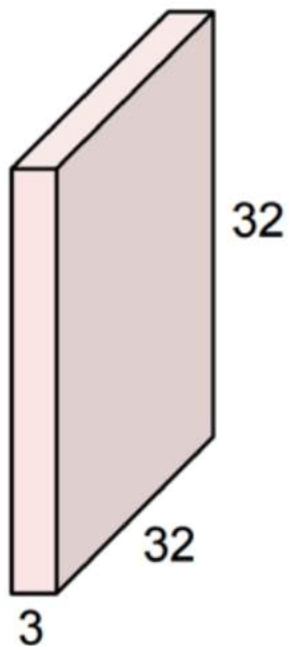


**Convolve** the filter with the image  
i.e. “slide over the image spatially,  
computing dot products”

# (Recap)

## Convolution Layer

32x32x3 image



Filters always extend the full depth of the input volume

5x5x3 filter

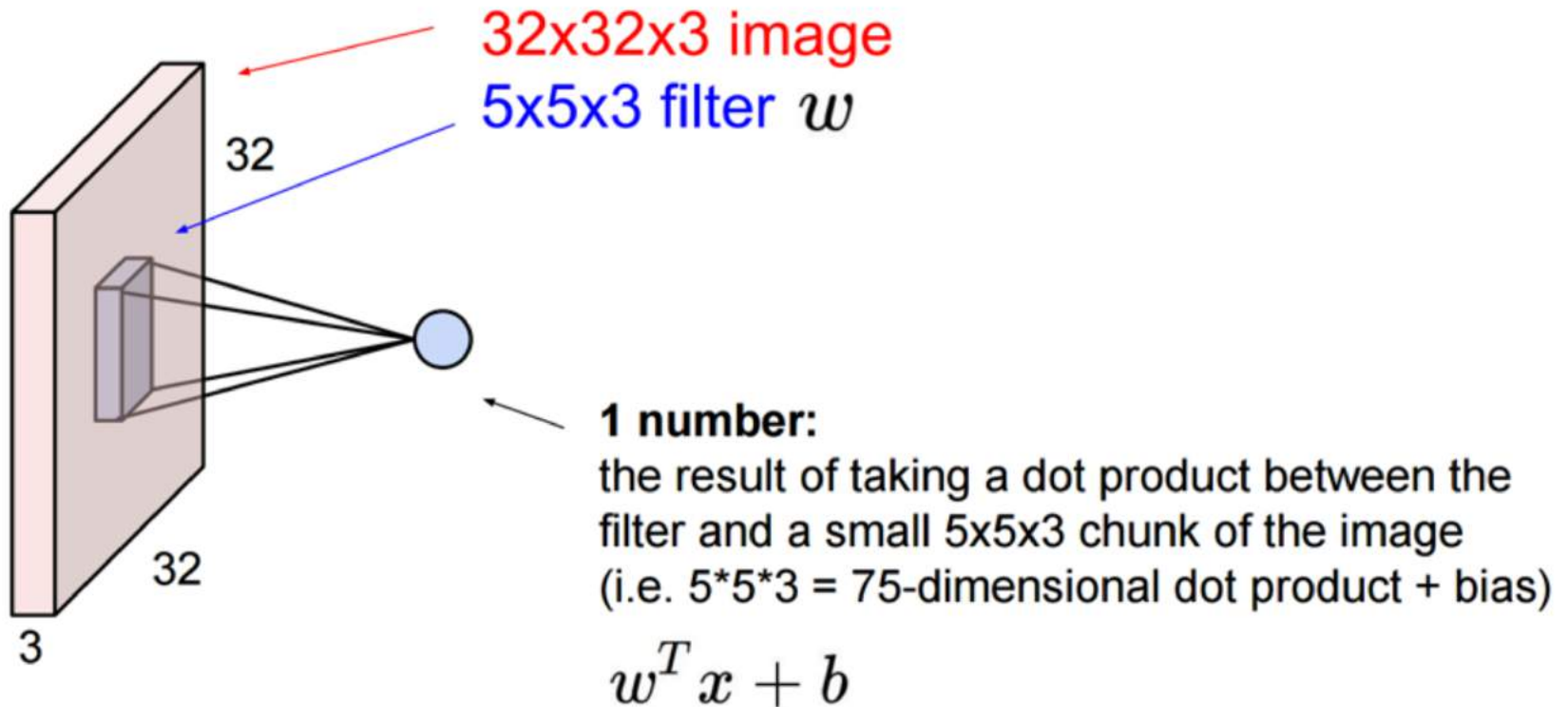


**Convolve** the filter with the image  
i.e. “slide over the image spatially,  
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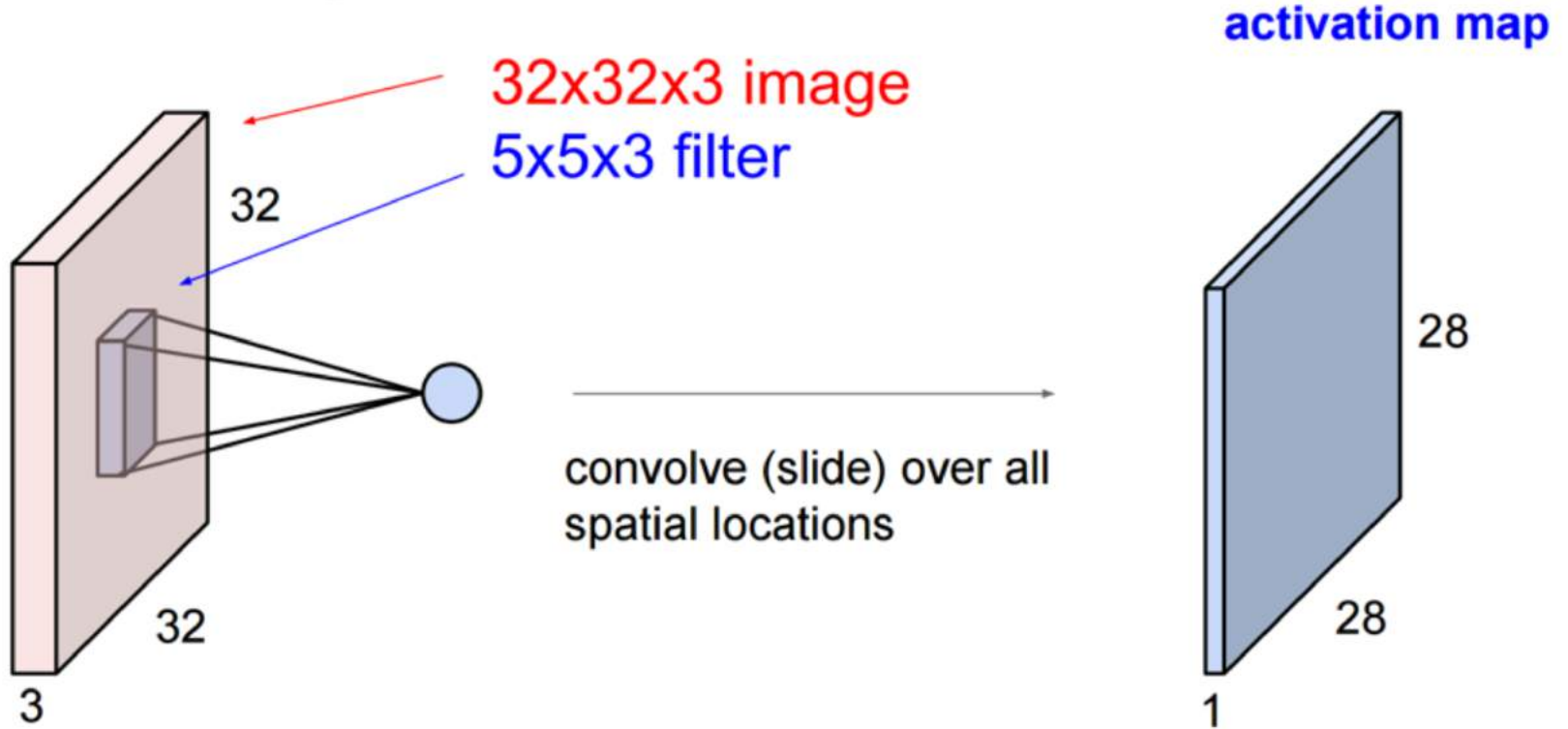
# (Recap)

## Convolution Layer



# (Recap)

## Convolution Layer

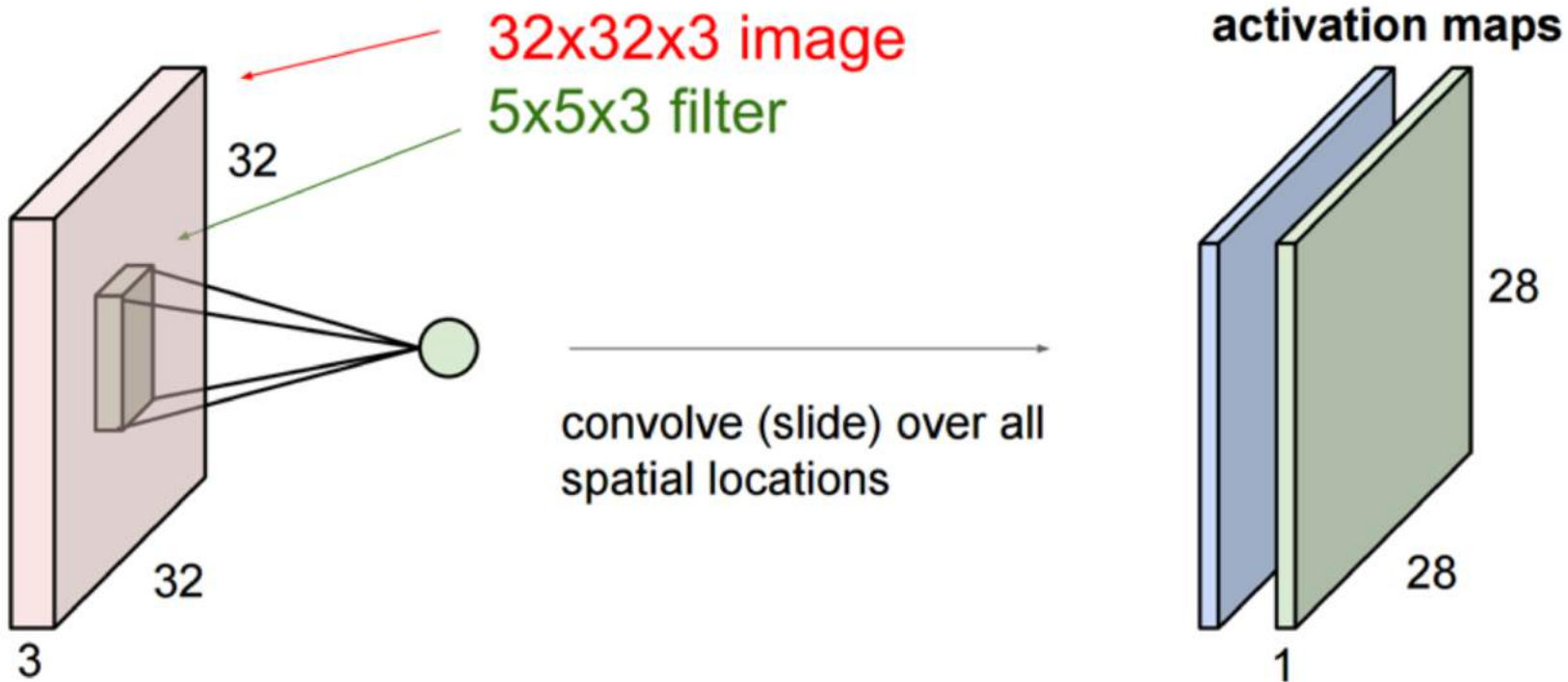




# (Recap)

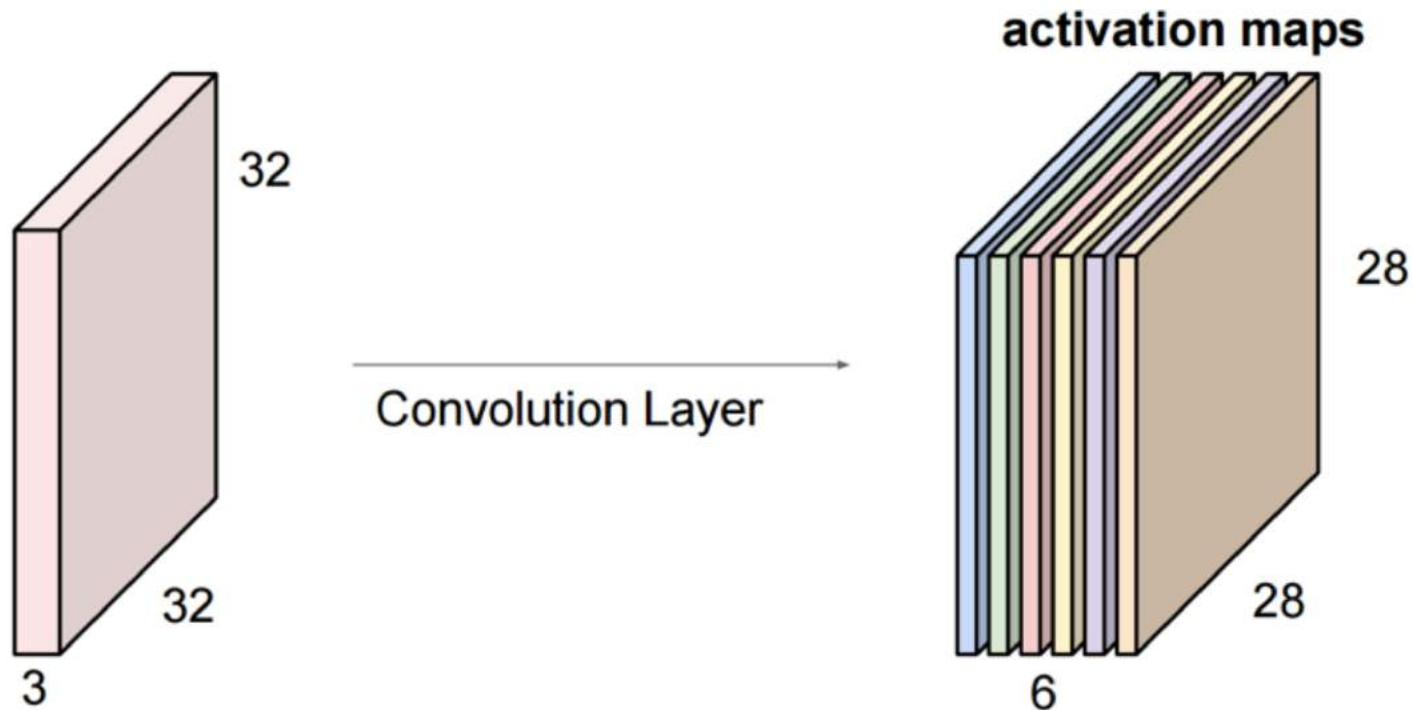
## Convolution Layer

consider a second, **green** filter



# (Recap)

For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



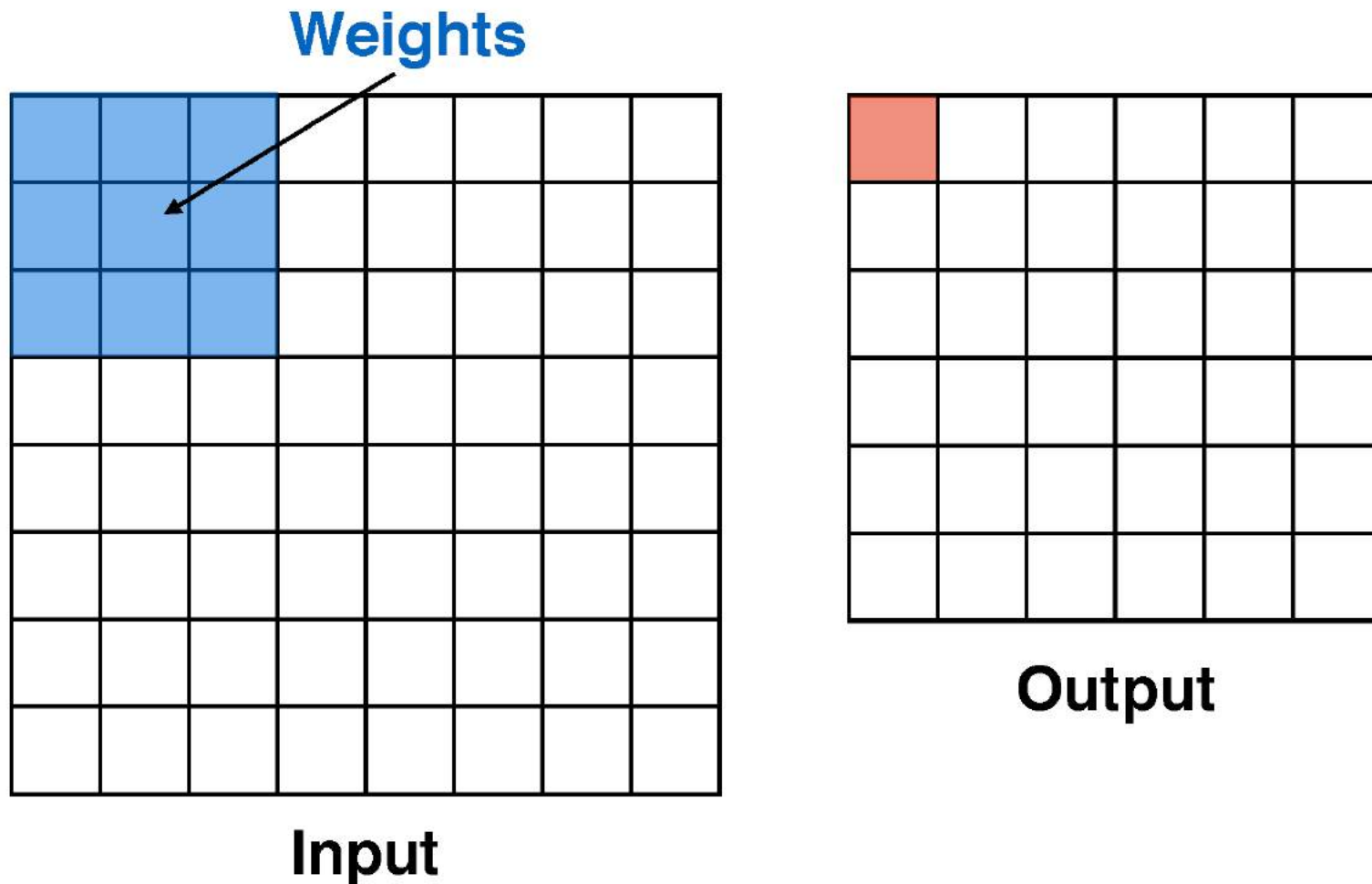
We stack these up to get a “new image” of size 28x28x6!

# Demos

- <http://cs231n.stanford.edu/>
- <http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html>

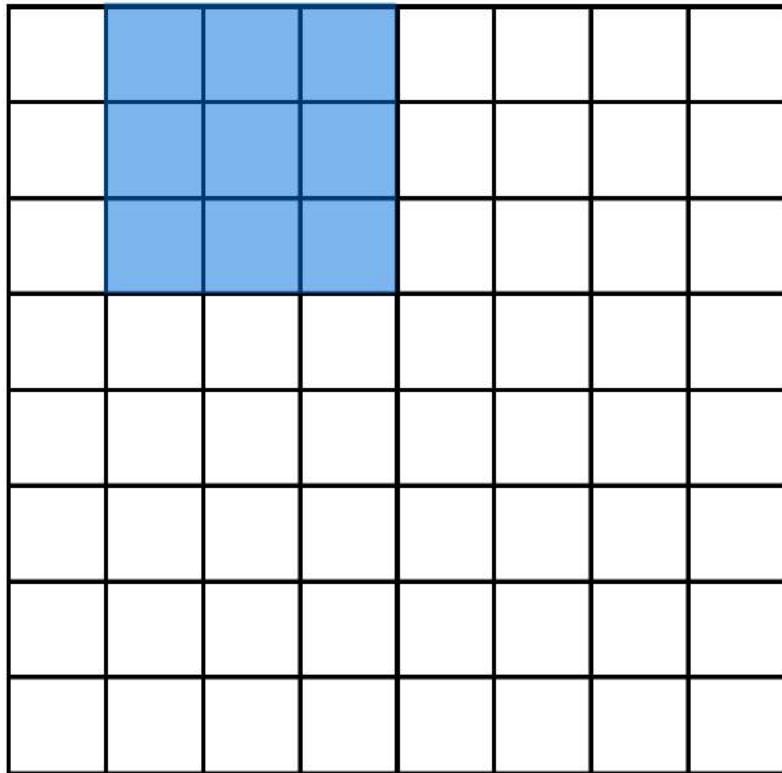
# Convolution: Stride

During convolution, the weights “slide” along the input to generate each output

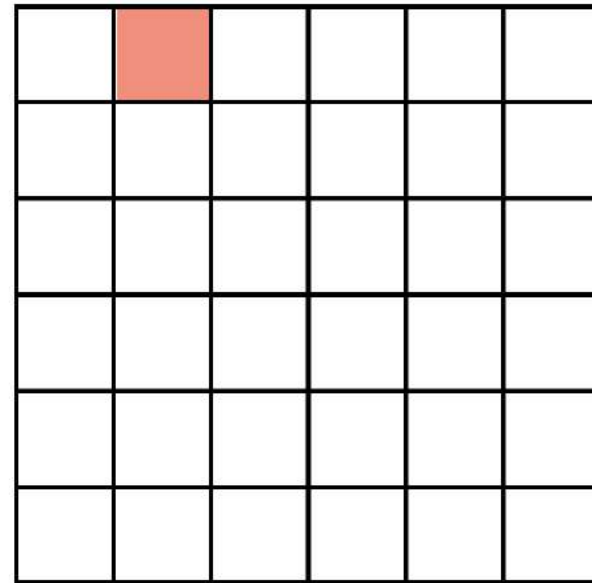


# Convolution: Stride

During convolution, the weights “slide” along the input to generate each output



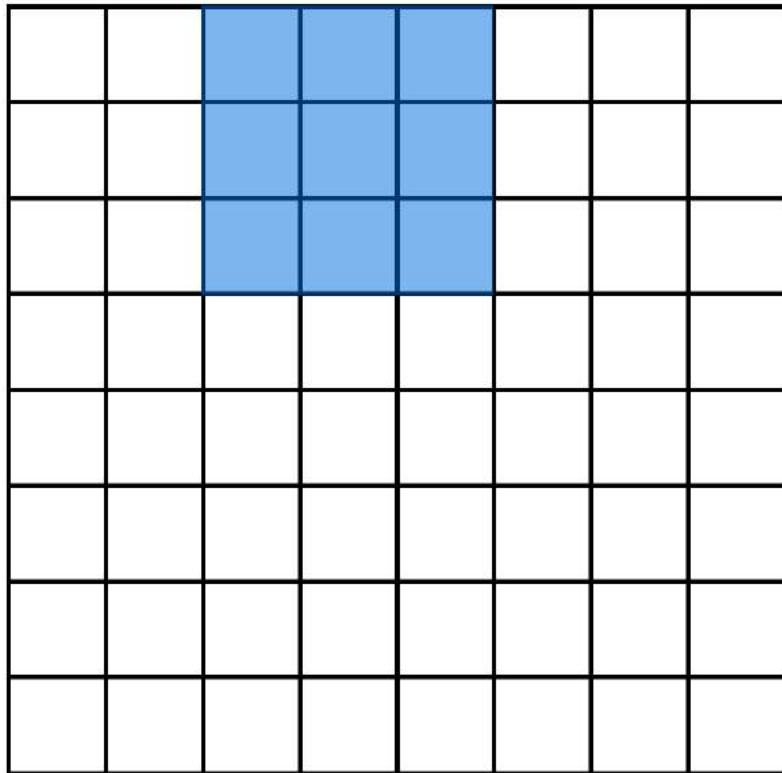
**Input**



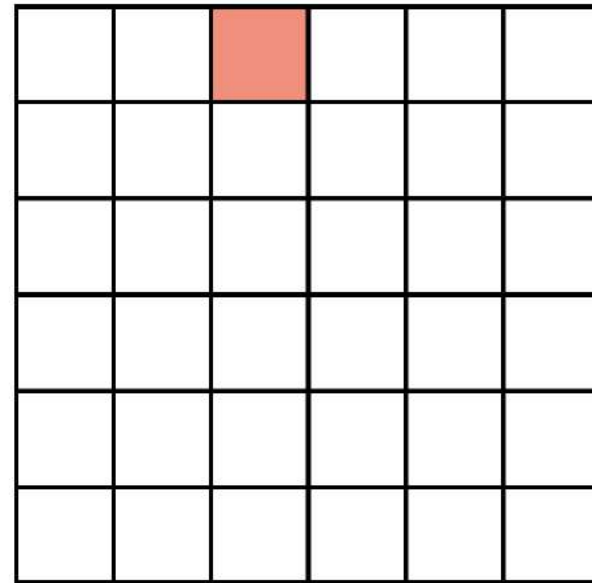
**Output**

# Convolution: Stride

During convolution, the weights “slide” along the input to generate each output



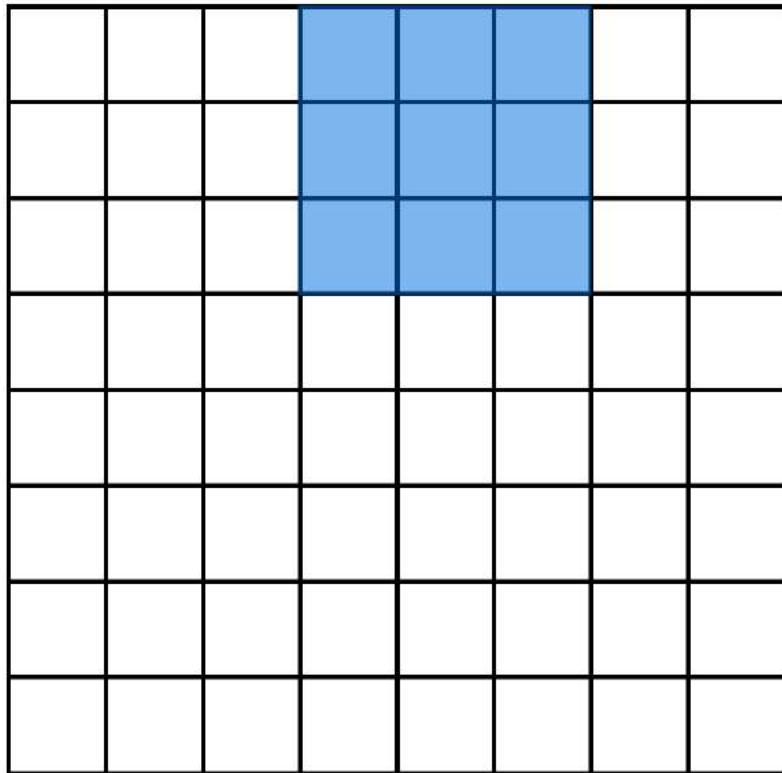
**Input**



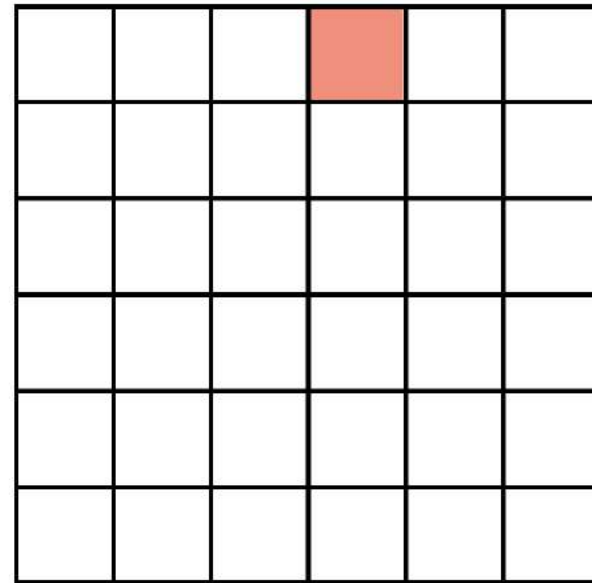
**Output**

# Convolution: Stride

During convolution, the weights “slide” along the input to generate each output



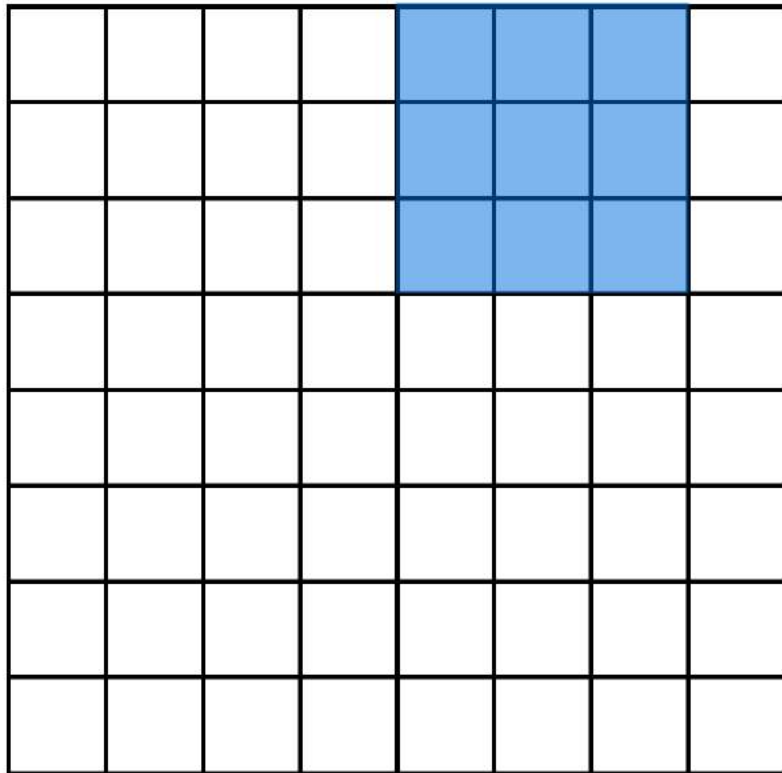
**Input**



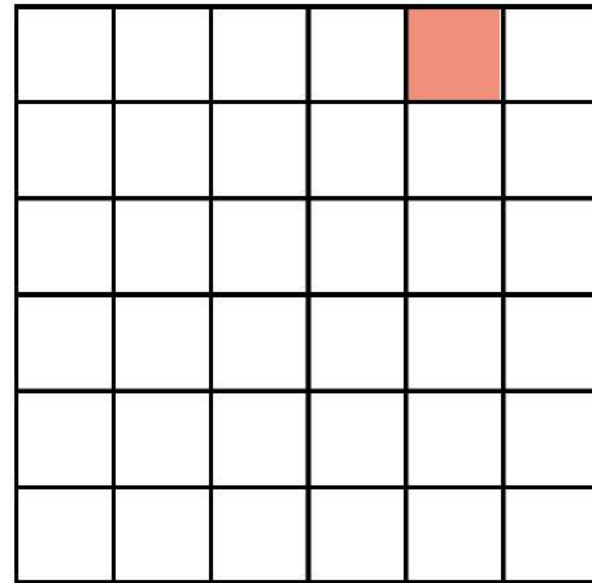
**Output**

# Convolution: Stride

During convolution, the weights “slide” along the input to generate each output



**Input**

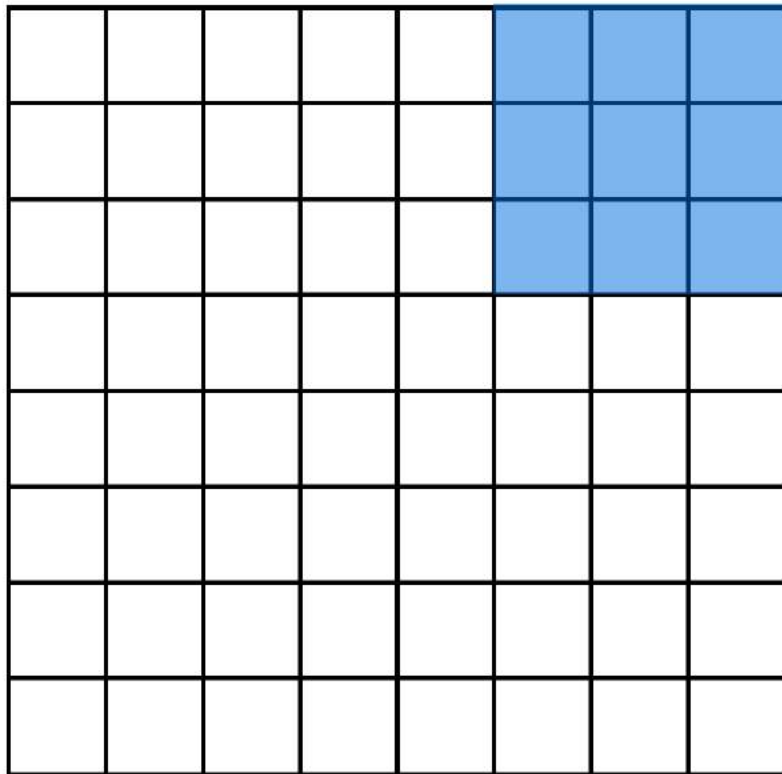


**Output**

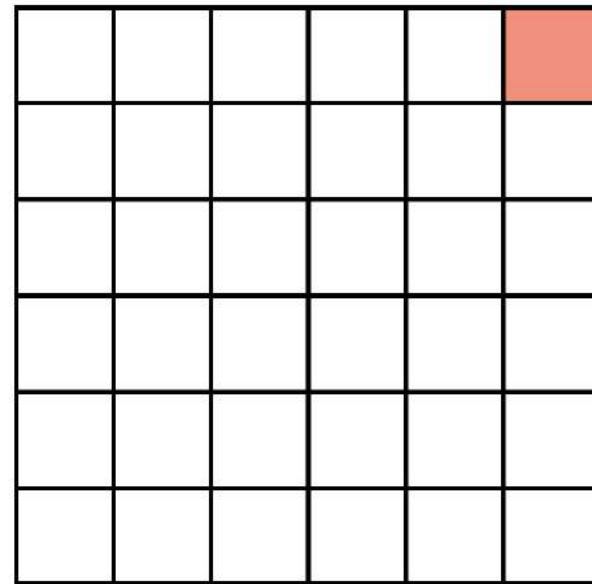


# Convolution: Stride

During convolution, the weights “slide” along the input to generate each output



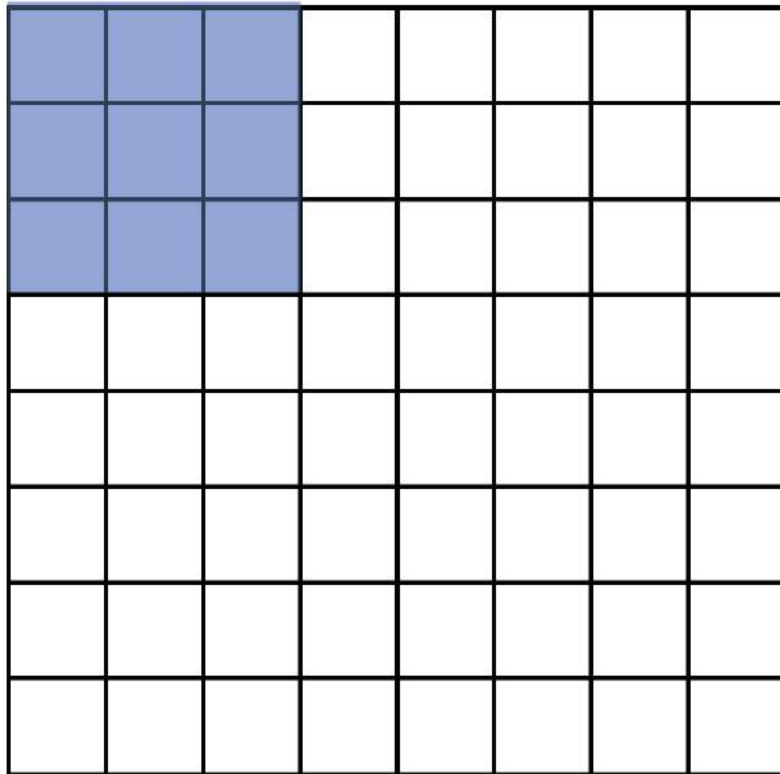
**Input**



**Output**

# Convolution: Stride

During convolution, the weights “slide” along the input to generate each output



**Input**

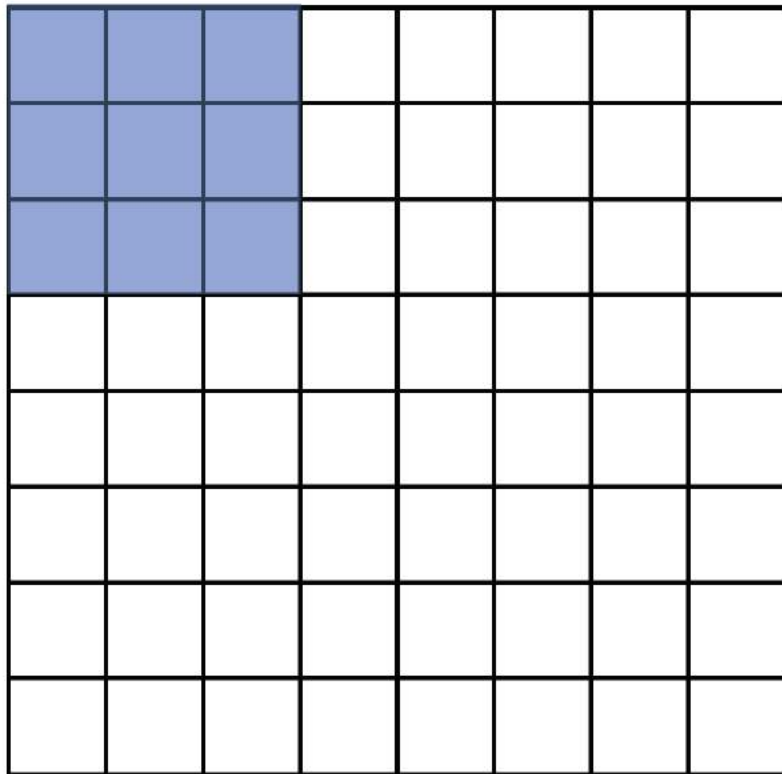
Recall that at each position, we are doing a **3D** sum:

$$h^r = \sum_{ijk} x^r_{ijk} W_{ijk} + b$$

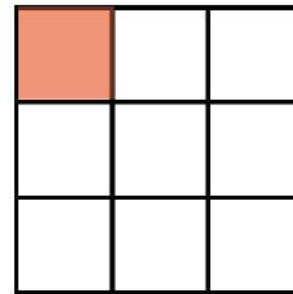
*(channel, row, column)*

# Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2



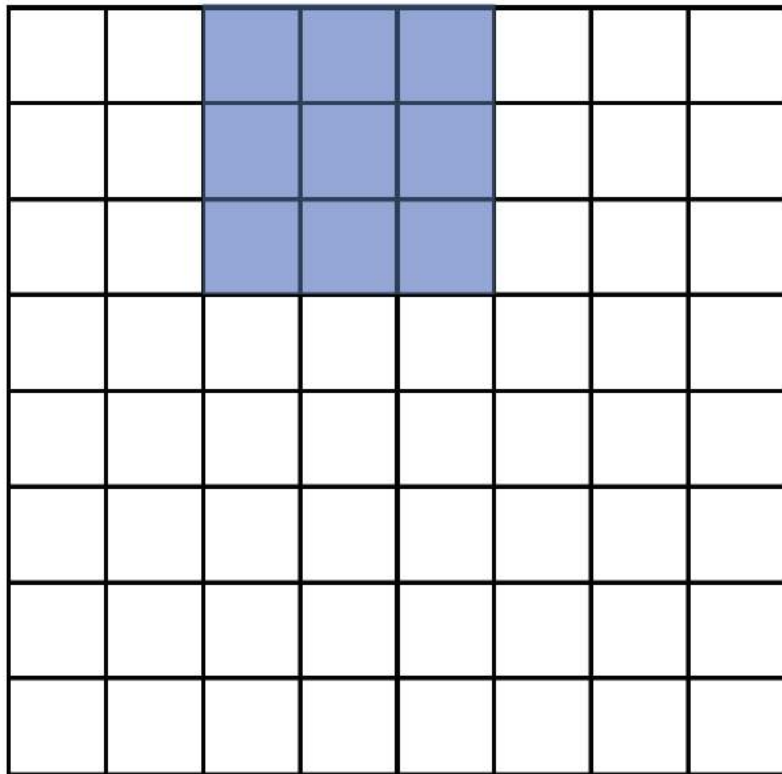
**Input**



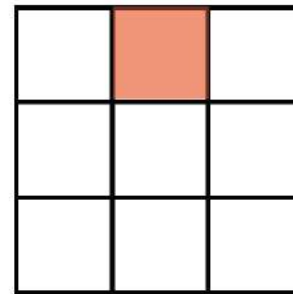
**Output**

# Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2



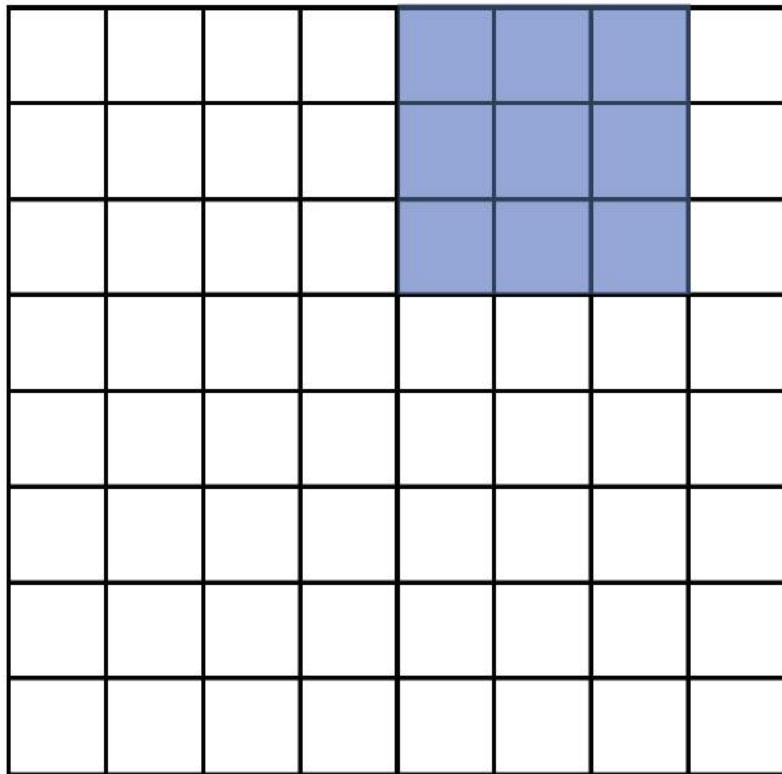
**Input**



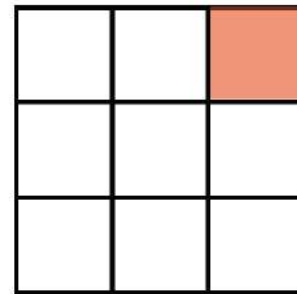
**Output**

# Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2



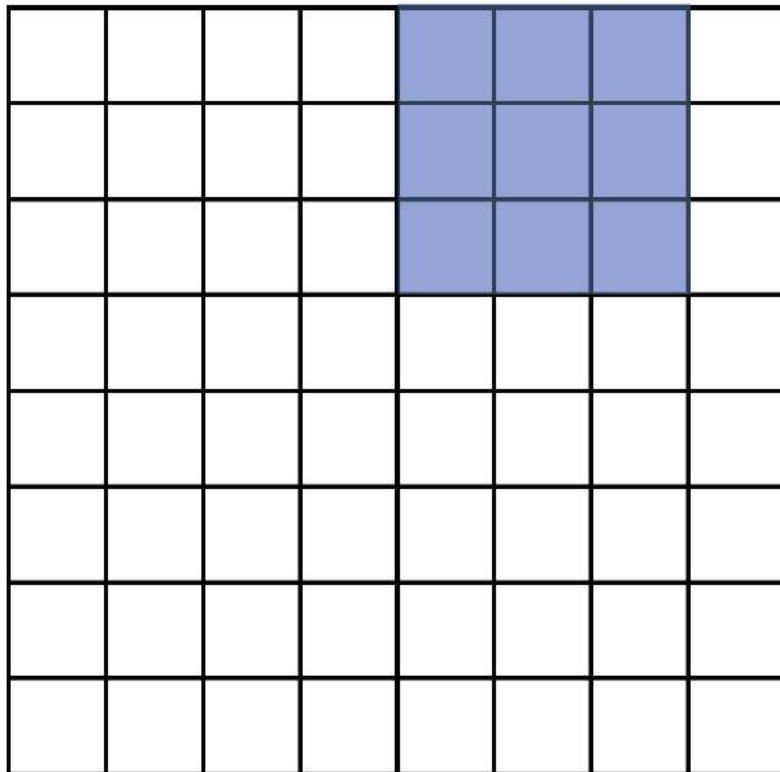
**Input**



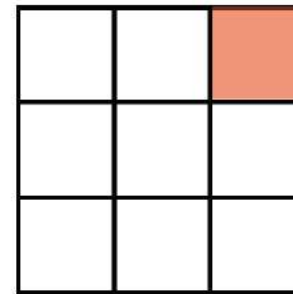
**Output**

# Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2



**Input**



**Output**

- Notice that with certain strides, we may not be able to cover all of the input
- The output is also half the size of the input

# Convolution: Padding

We can also pad the input with zeros.

Here, **pad = 1**, **stride = 2**

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

**Input**


**Output**

# Convolution: Padding

We can also pad the input with zeros.

Here, **pad = 1**, **stride = 2**

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

**Input**


**Output**



# Convolution: Padding

We can also pad the input with zeros.

Here, **pad = 1, stride = 2**

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

**Input**


**Output**

# Convolution: Padding

We can also pad the input with zeros.

Here, **pad = 1**, **stride = 2**

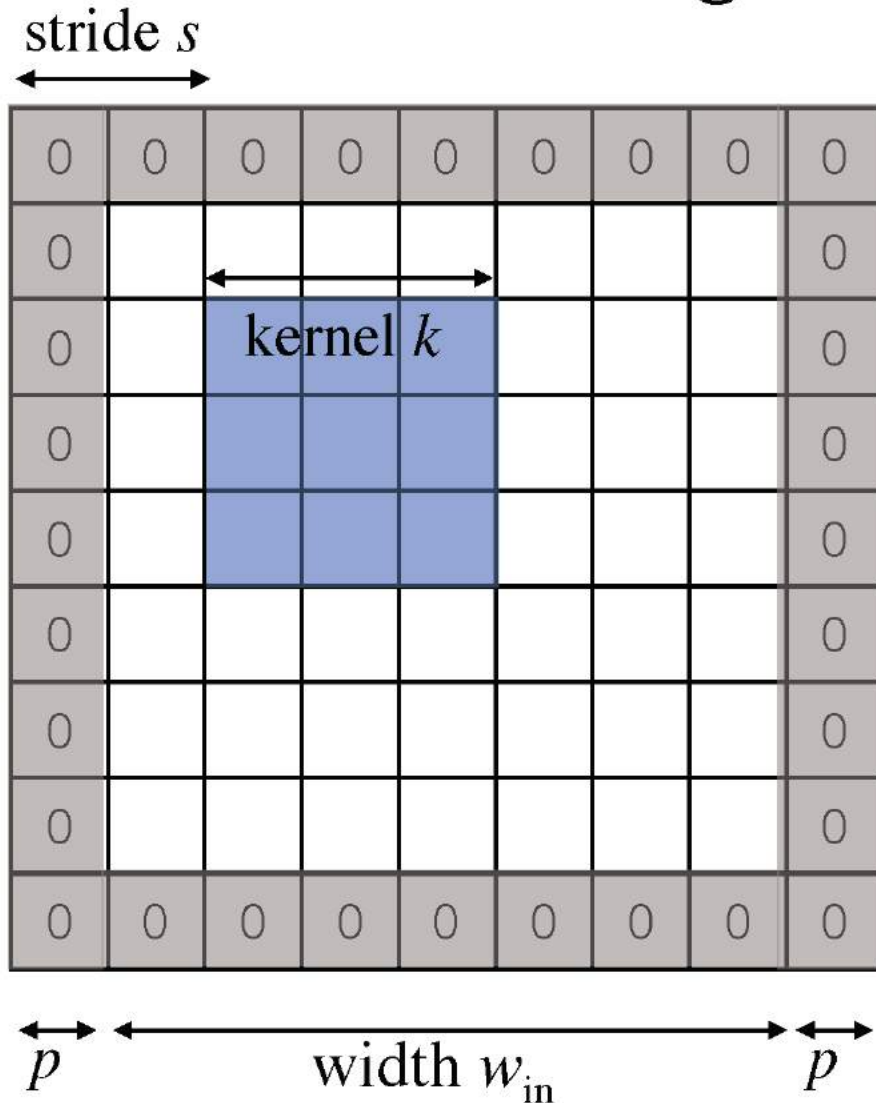
0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

**Input**


**Output**

# Convolution:

How big is the output?

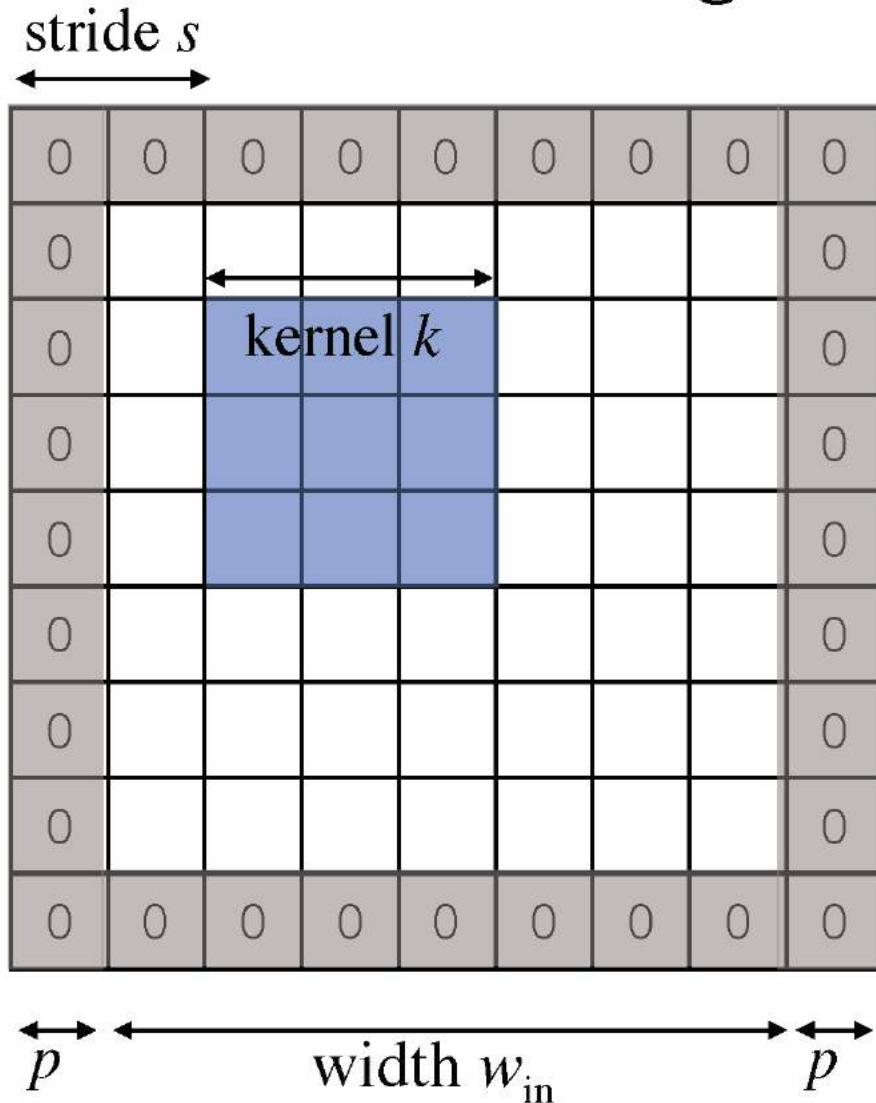


In general, the output has size:

$$w_{out} = \left\lfloor \frac{w_{in} + 2p - k}{s} \right\rfloor + 1$$

# Convolution:

How big is the output?



**Example:**  $k=3$ ,  $s=1$ ,  $p=1$

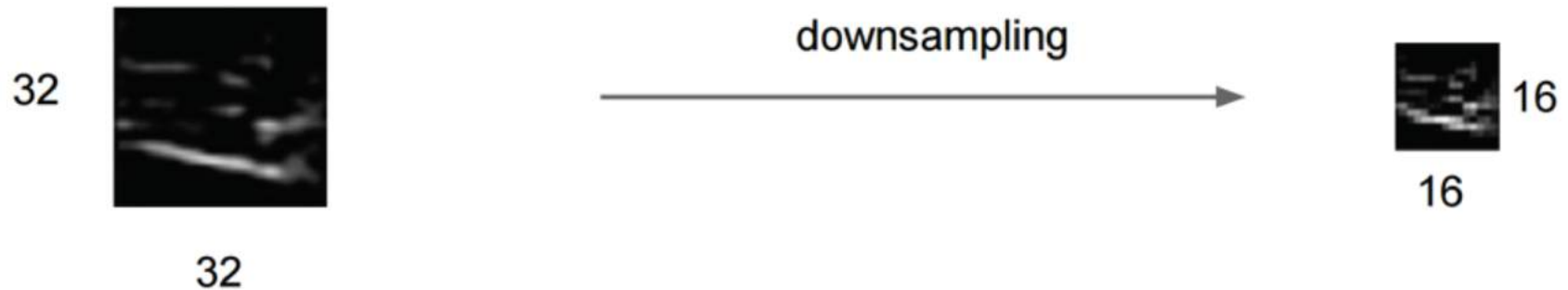
$$\begin{aligned}w_{out} &= \left\lfloor \frac{w_{in} + 2p - k}{s} \right\rfloor + 1 \\ &= \left\lfloor \frac{w_{in} + 2 - 3}{1} \right\rfloor + 1 \\ &= w_{in}\end{aligned}$$

VGGNet [Simonyan 2014]  
uses filters of this shape

# Pooling

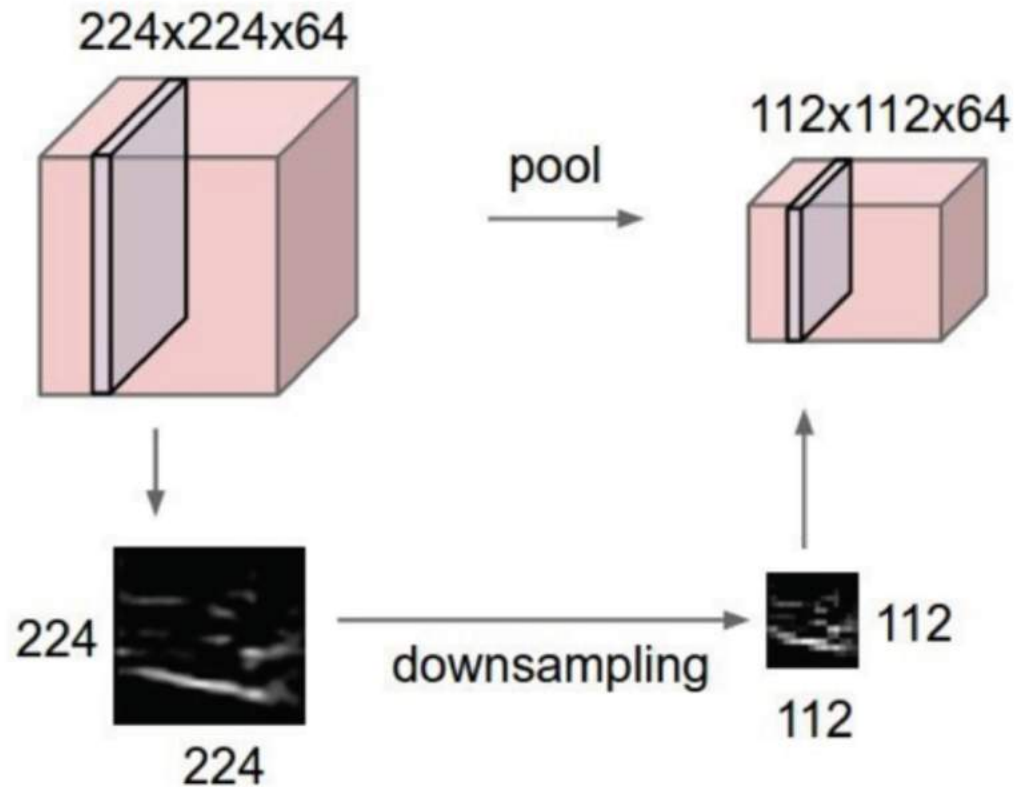
For most ConvNets, **convolution** is often followed by **pooling**:

- Creates a smaller representation while retaining the most important information
- The “max” operation is the most common
- Why might “avg” be a poor choice?

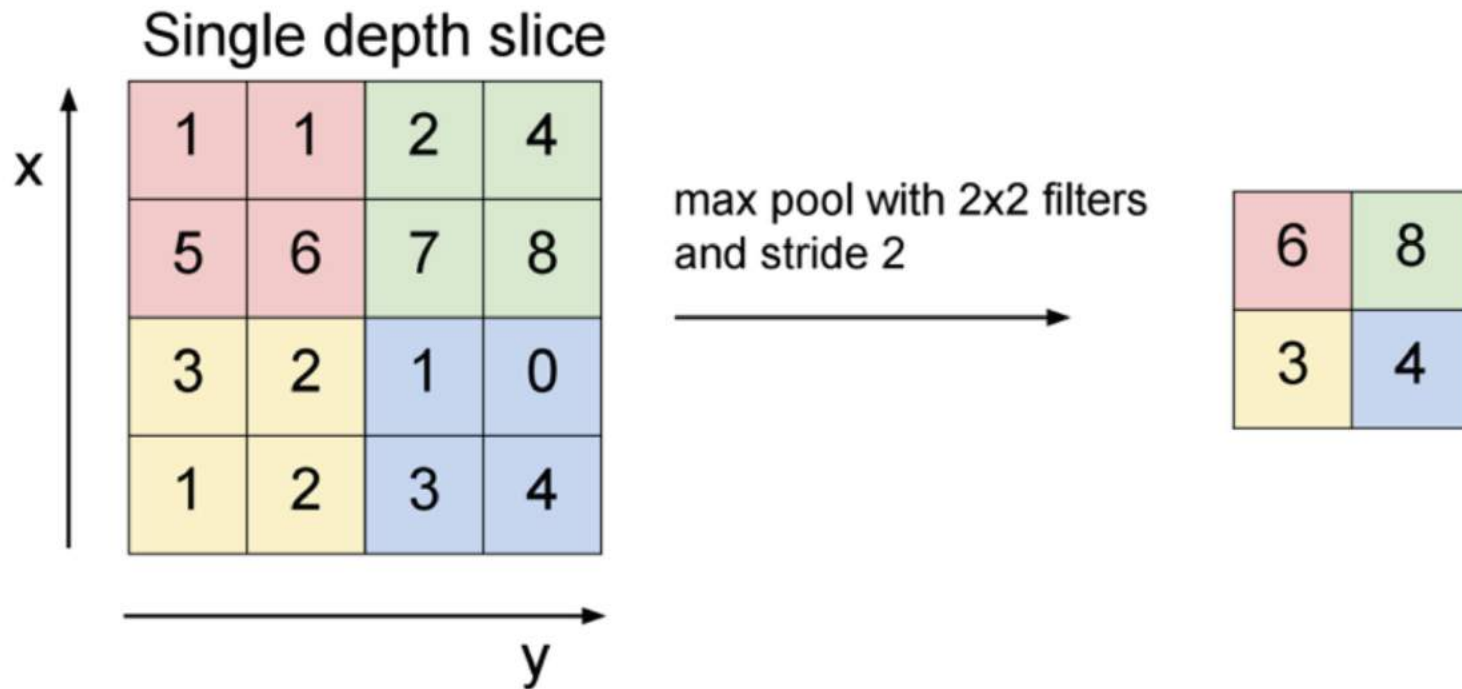


# Pooling

- makes the representations smaller and more manageable
- operates over each activation map independently:



# Max Pooling



What's the backprop rule for max pooling?

- In the forward pass, store the index that took the max
- The backprop gradient is the input gradient at that index



# Example ConvNet

CONV      CONV    POOL  
↓    ReLU   ↓    ReLU   ↓

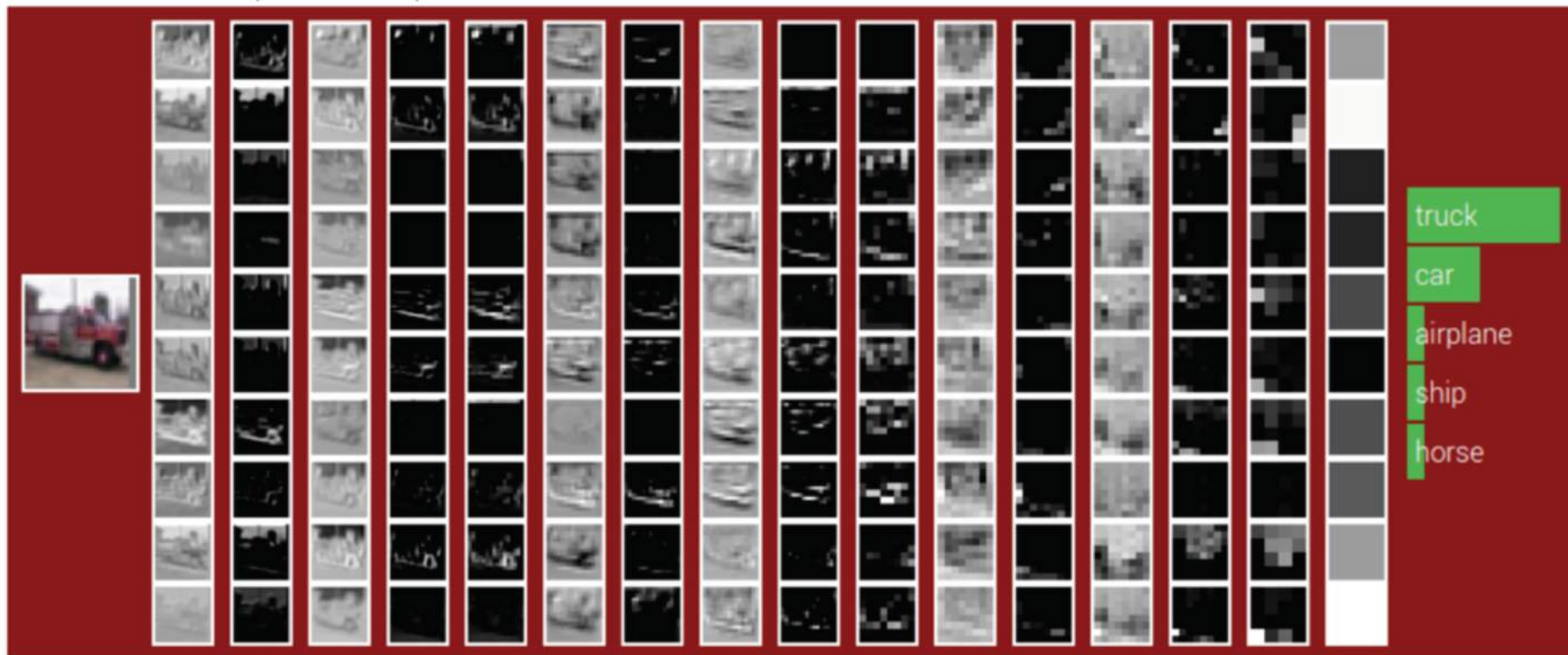


Figure: Andrej Karpathy



# Example ConvNet

CONV CONV POOL CONV CONV POOL CONV CONV POOL  
↓ ReLU ↓ ReLU ↓ ReLU ↓ ReLU ↓ ReLU ↓ ReLU ↓

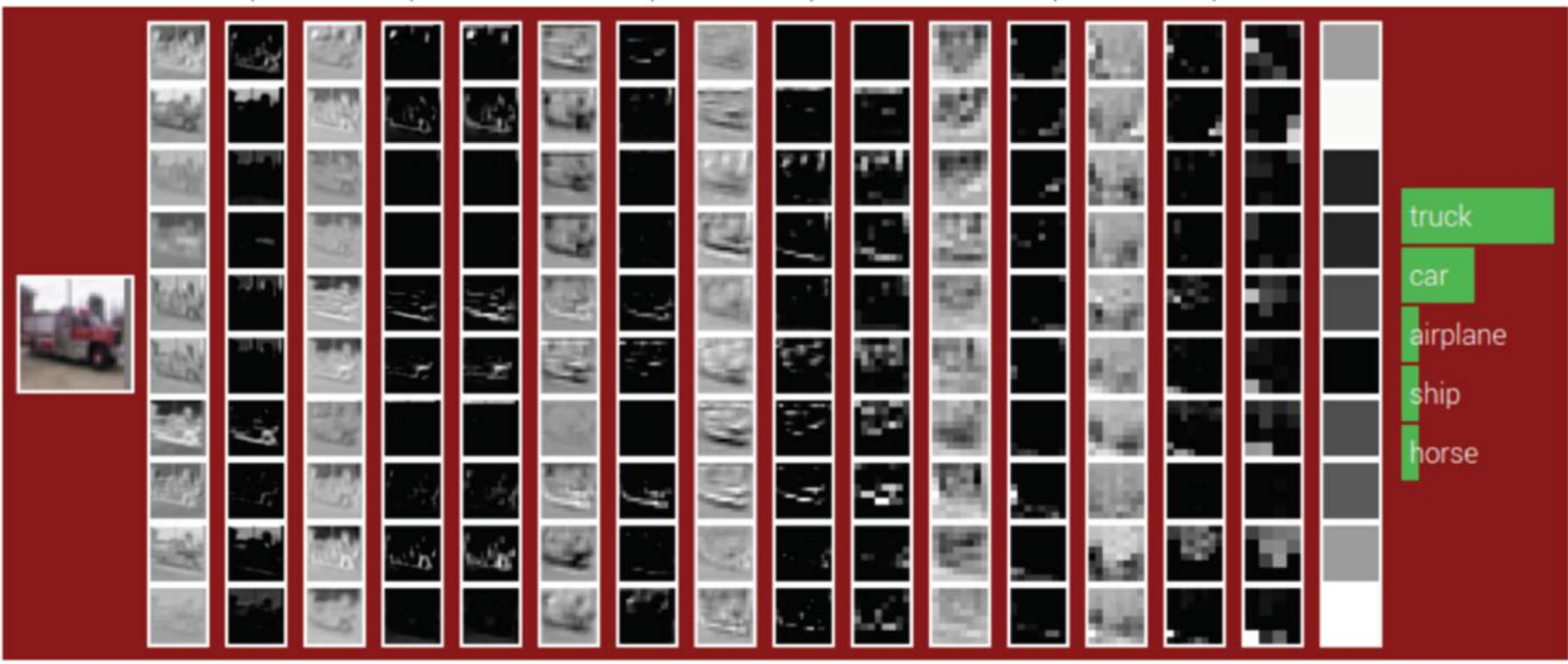
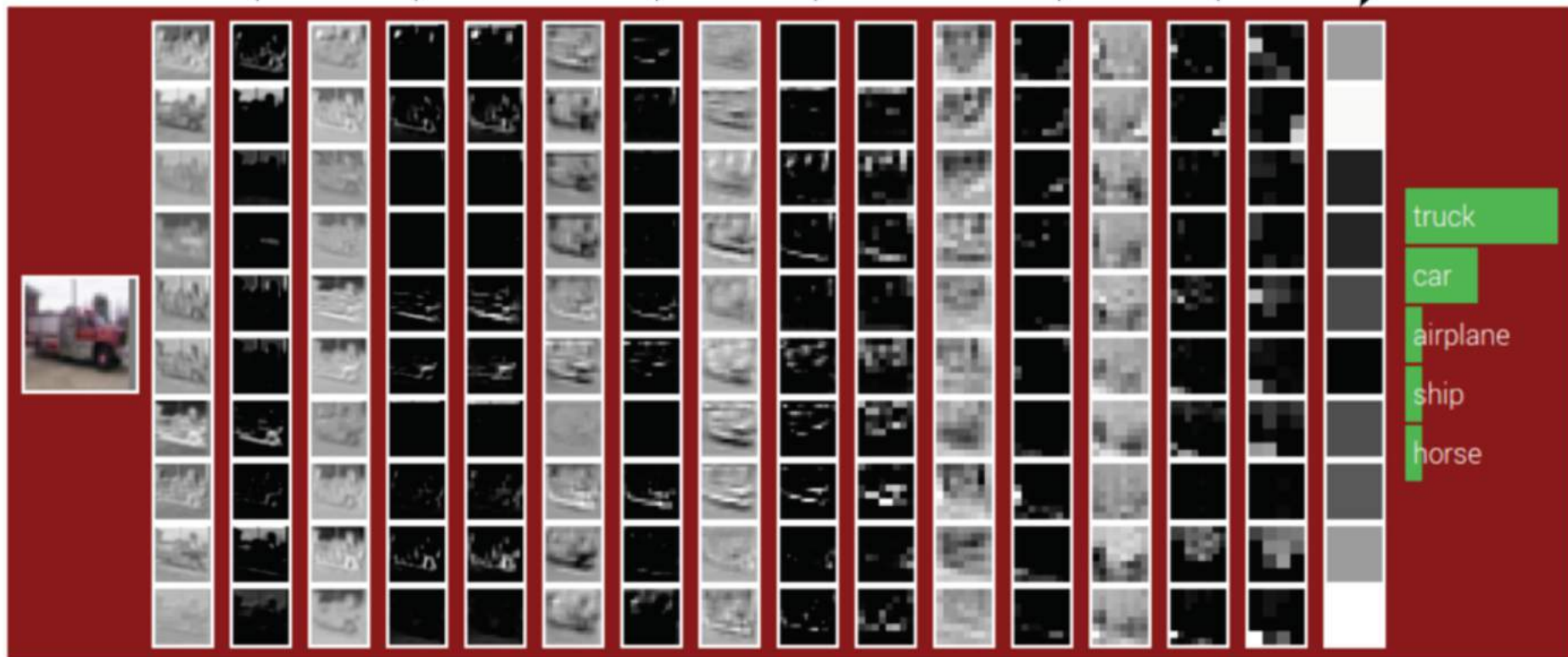


Figure: Andrej Karpathy



# Example ConvNet

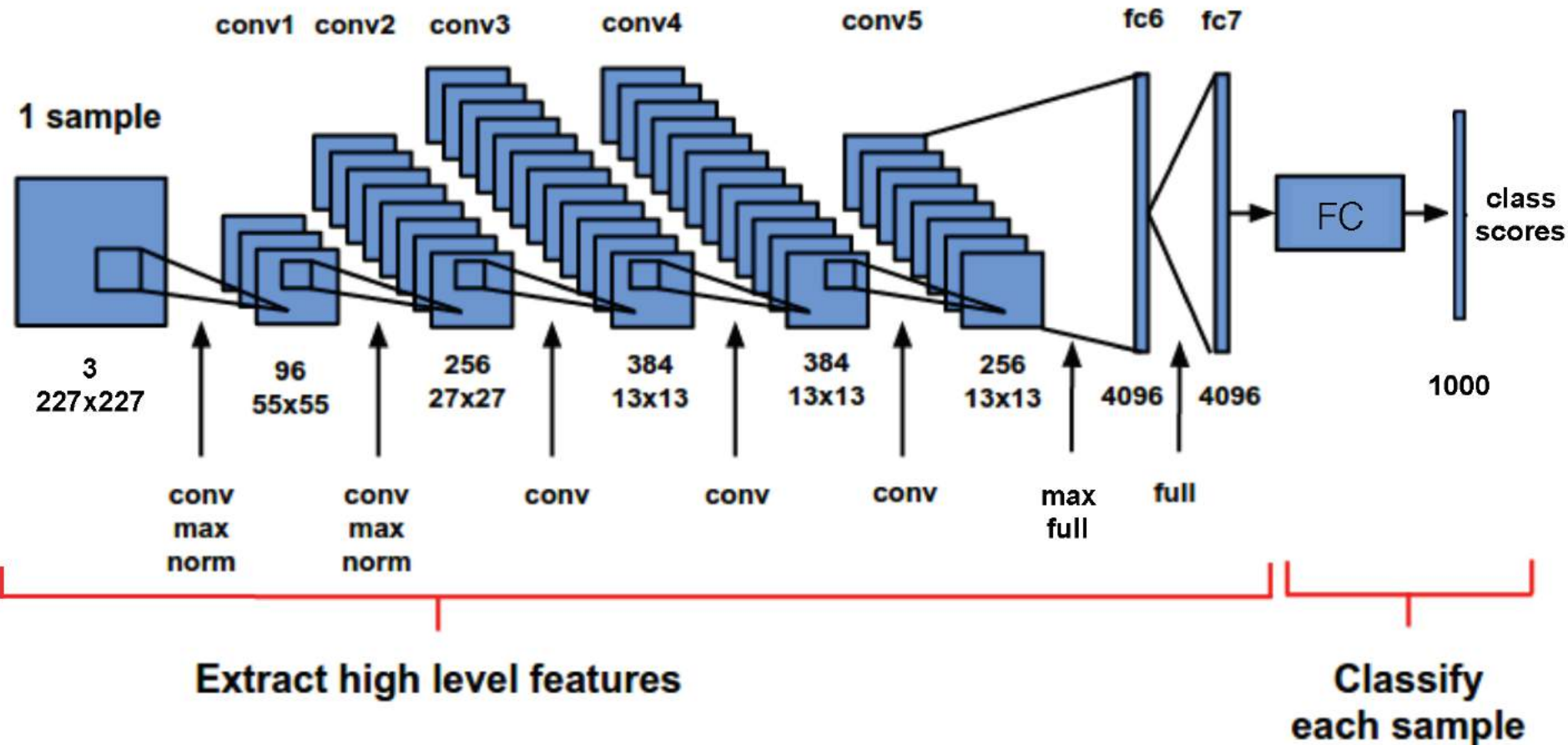
CONV    CONV    POOL    CONV    CONV    POOL    CONV    CONV    POOL    FC  
↓    ReLU   ↓    ReLU   ↓    ↓    ReLU   ↓    ReLU   ↓    ↓    ReLU   ↓    ReLU   ↓    ↓    (Fully-connected)



10x3x3 conv filters, stride 1, pad 1  
2x2 pool filters, stride 2

Figure: Andrej Karpathy

# Example: AlexNet [Krizhevsky 2012]



“max”: max pooling

“norm”: local response normalization

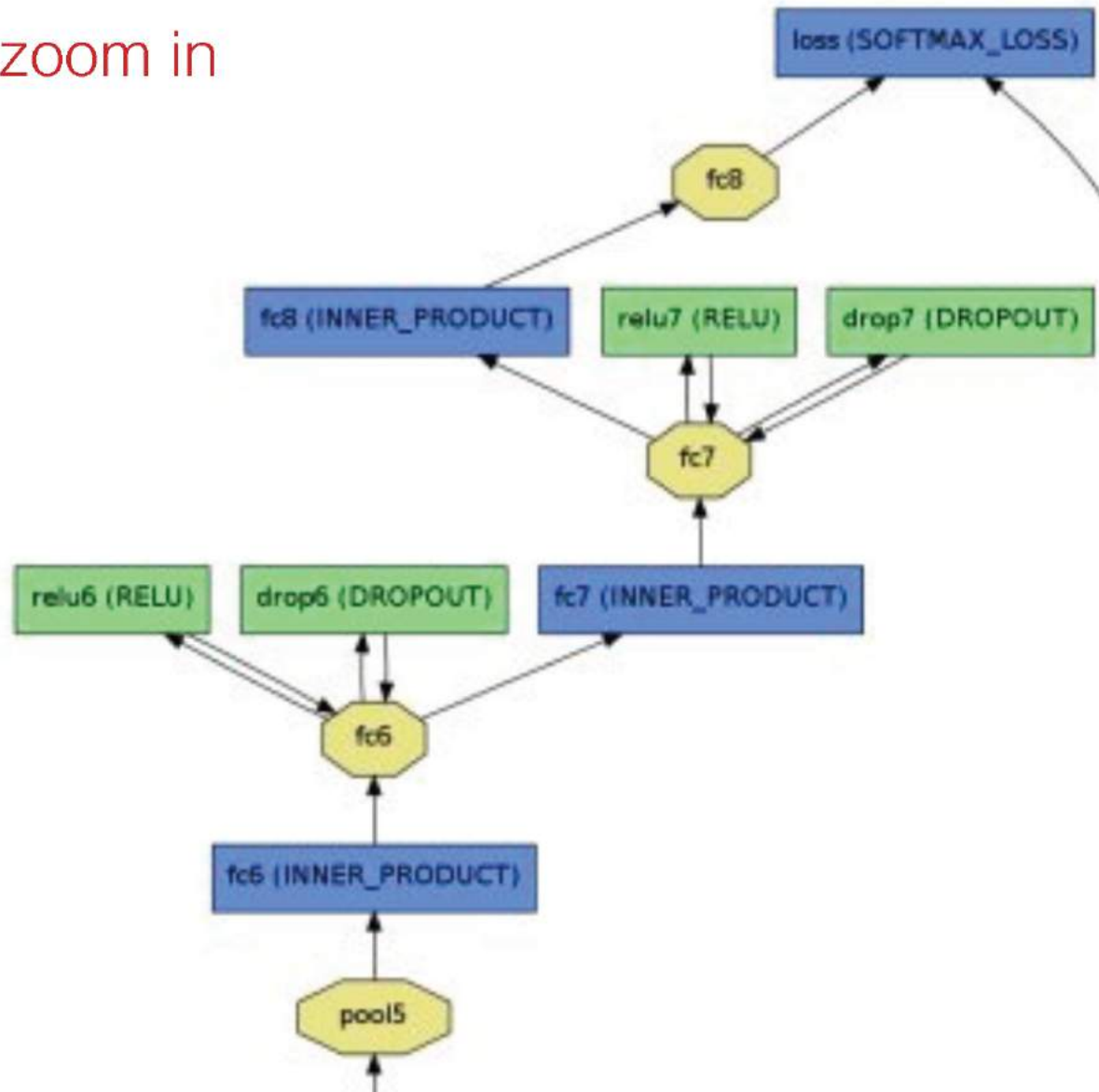
“full”: fully connected

Figure: [Karnowski 2015] (with corrections)

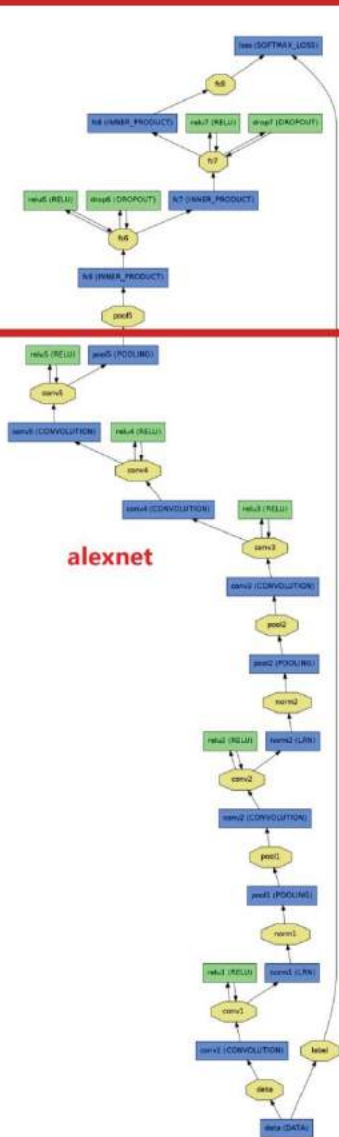


# Example: AlexNet [Krizhevsky 2012]

zoom in



alexnet



# Training ConvNets

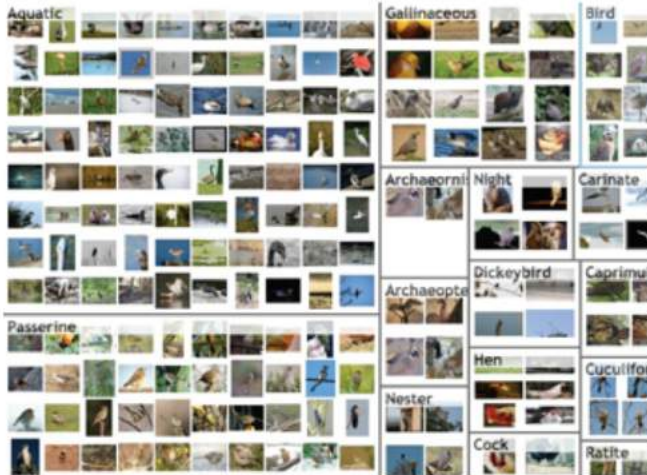
# How do you actually train these things?

## Roughly speaking:

Gather  
labeled data

Find a ConvNet  
architecture

Minimize  
the loss



# Training a convolutional neural network

- Split and preprocess your data
- Choose your network architecture
- Initialize the weights
- Find a learning rate and regularization strength
- Minimize the loss and monitor progress
- Fiddle with knobs



# Mini-batch Gradient Descent

## **Loop:**

1. Sample a batch of training data (~100 images)
2. Forwards pass: compute loss (avg. over batch)
3. Backwards pass: compute gradient
4. Update all parameters

**Note:** usually called “stochastic gradient descent” even though SGD has a batch size of 1

# Regularization

**Regularization reduces overfitting:**

$$L = L_{\text{data}} + L_{\text{reg}} \quad L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2$$

$\lambda = 0.001$



$\lambda = 0.01$



$\lambda = 0.1$

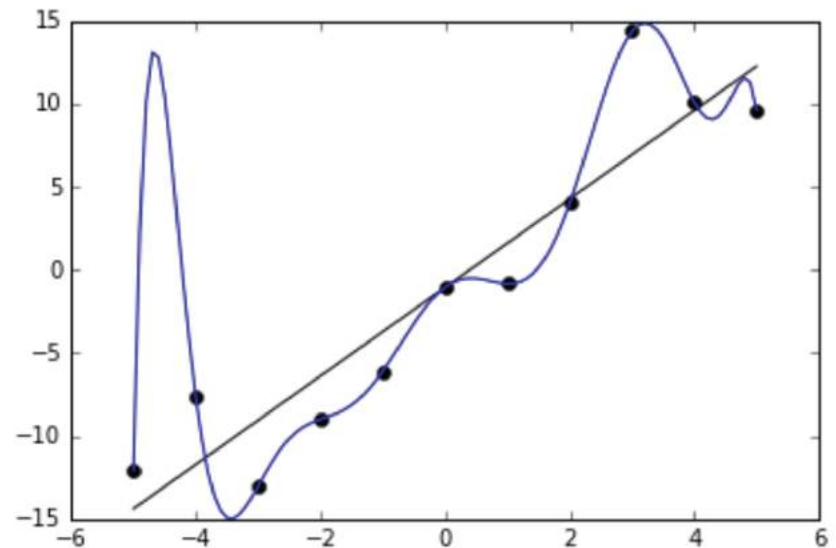


# Overfitting

**Overfitting:** modeling noise in the training set instead of the “true” underlying relationship

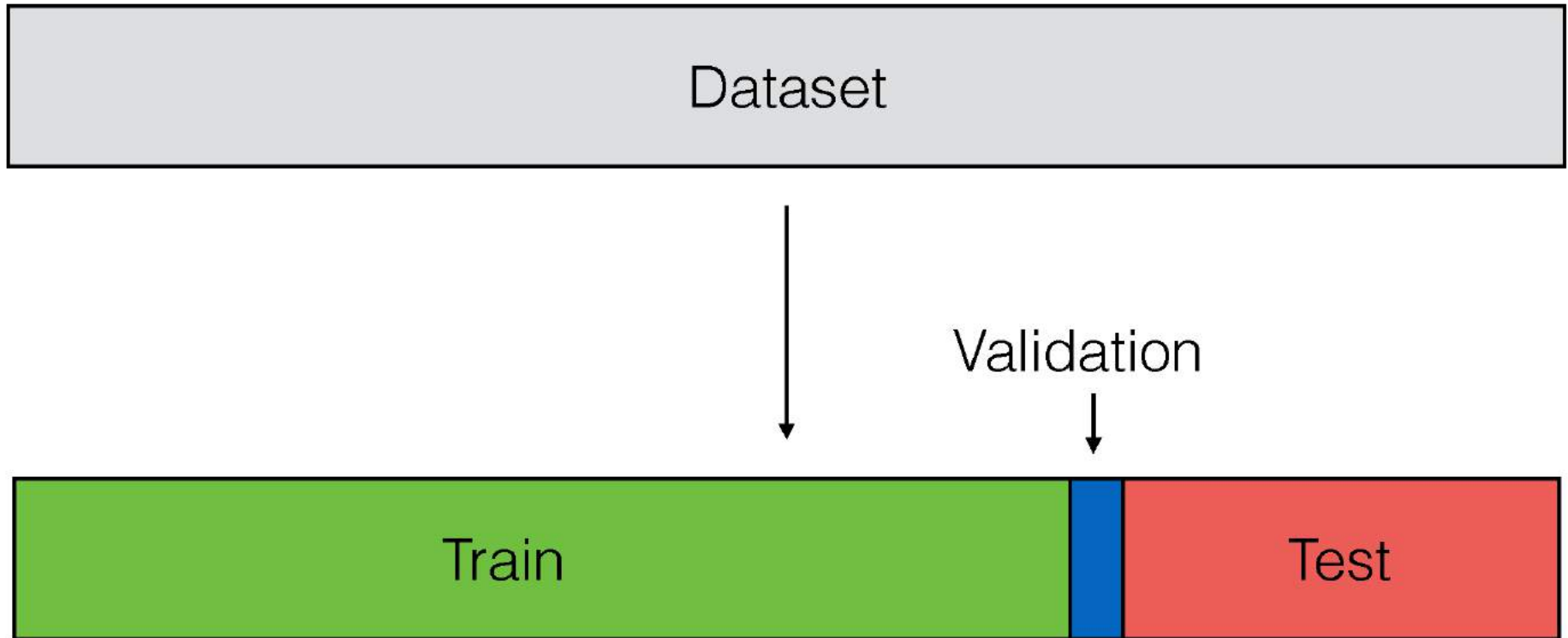
**Underfitting:** insufficiently modeling the relationship in the training set

**General rule:** models that are “bigger” or have more capacity are more likely to overfit



# (0) Dataset split

**Split your data into “train”, “validation”, and “test”:**



# (0) Dataset split



**Train:** gradient descent and fine-tuning of parameters

**Validation:** determining hyper-parameters (learning rate, regularization strength, etc) and picking an architecture

**Test:** estimate real-world performance  
(e.g. accuracy = fraction correctly classified)

# (0) Dataset split



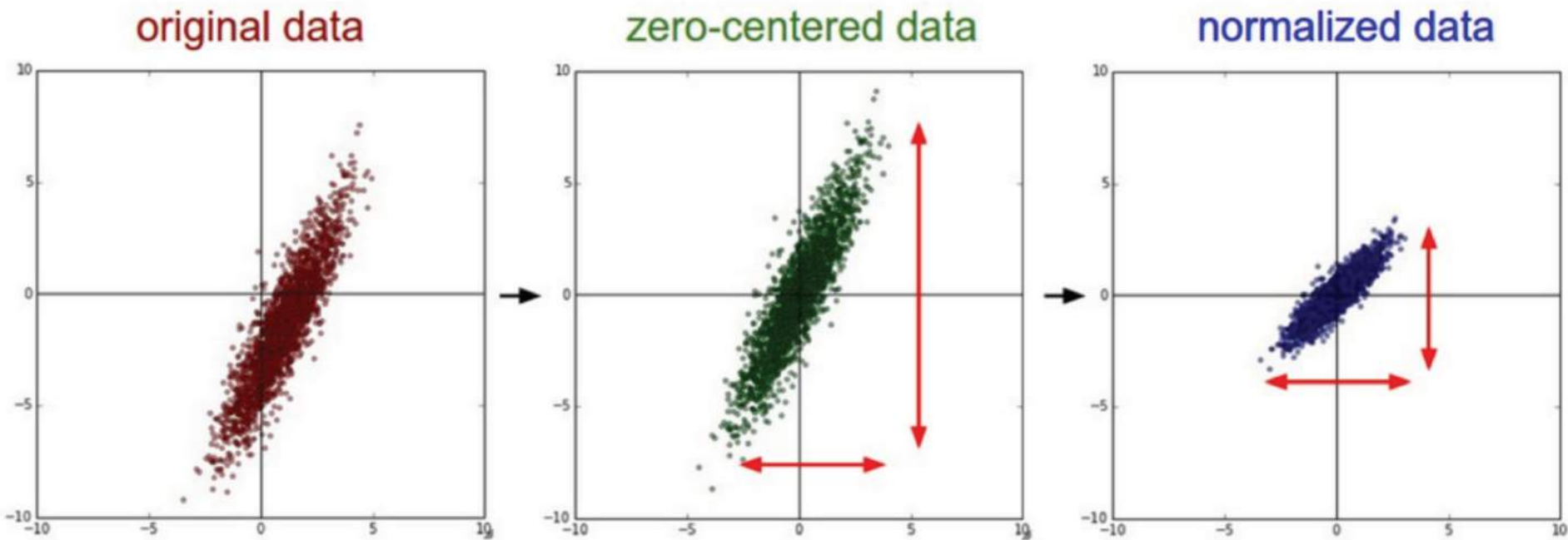
## **Be careful with false discovery:**

To avoid false discovery, once we have used a test set once, we should *not use it again* (but nobody follows this rule, since it's expensive to collect datasets)

Instead, try and avoid looking at the test score until the end

# (1) Data preprocessing

**Preprocess the data so that learning is better conditioned:**

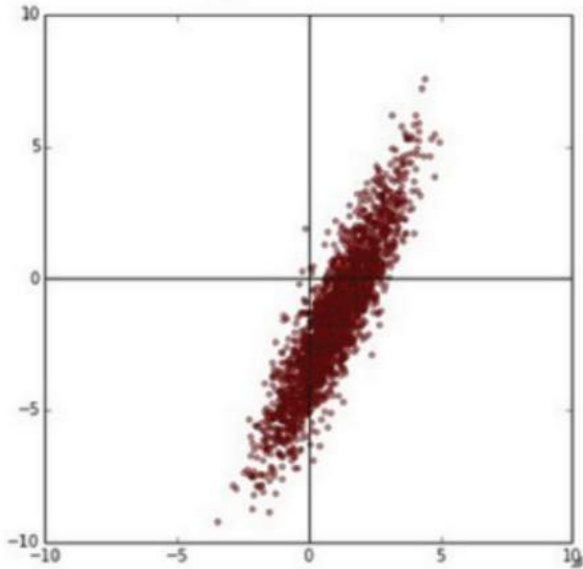


*Figure: Andrej Karpathy*

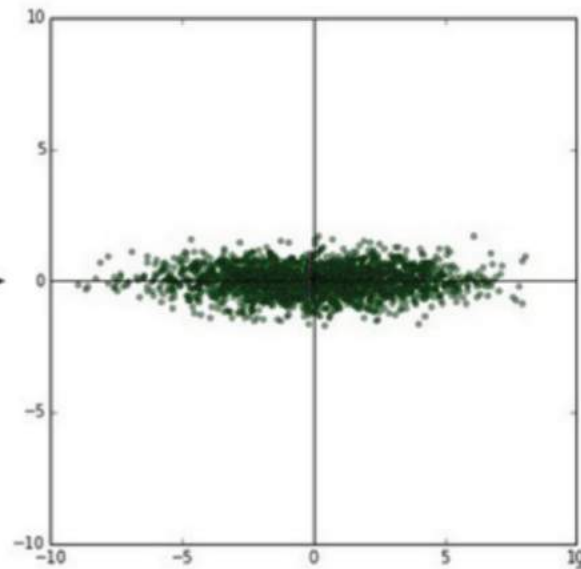
# (1) Data preprocessing

In practice, you may also see **PCA** and **Whitening** of the data:

original data

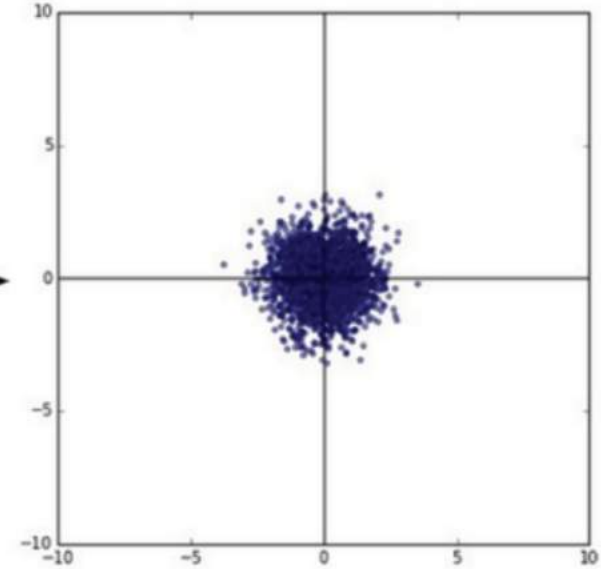


decorrelated data



(data has diagonal covariance matrix)

whitened data



(covariance matrix is the identity matrix)



# (1) Data preprocessing

For ConvNets, typically only the mean is subtracted.



An input image (256x256)



Minus sign



The mean input image

A per-channel mean also works (one value per R,G,B).

# (1) Data preprocessing

**Augment the data** — extract random crops from the input, with slightly jittered offsets. Without this, typical ConvNets (e.g. [Krizhevsky 2012]) overfit the data.



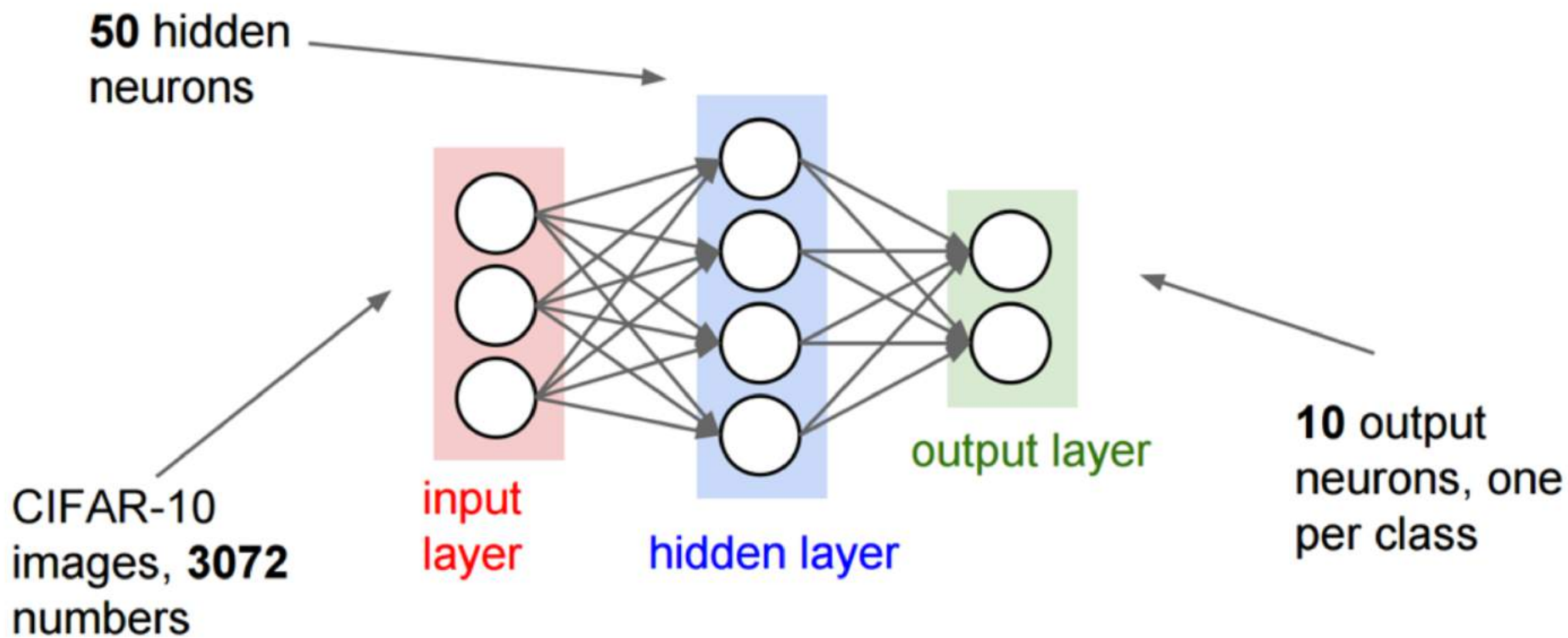
**E.g.** 224x224 patches  
extracted from 256x256 images

Randomly reflect horizontally

Perform the augmentation live  
during training

# (2) Choose your architecture

**Toy example: one hidden layer of size 50**



# (3) Initialize your weights

**Set the weights to small random numbers:**

```
W = np.random.randn(D, H) * 0.001
```

(matrix of small random numbers drawn from a Gaussian distribution)

(the magnitude is important and this is not optimal — more on this later)

**Set the bias to zero (or small nonzero):**

```
b = np.zeros(H)
```



### (3) Check that the loss is reasonable

```
def init_two_layer_model(input_size, hidden_size, output_size):  
    # initialize a model  
    model = {}  
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)  
    model['b1'] = np.zeros(hidden_size)  
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)  
    model['b2'] = np.zeros(output_size)  
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes  
loss, grad = two_layer_net(X_train, model, y_train, 0.0) # disable regularization  
print loss
```

returns the loss and the  
gradient for all parameters

### (3) Check that the loss is reasonable

```
def init_two_layer_model(input_size, hidden_size, output_size):  
    # initialize a model  
    model = {}  
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)  
    model['b1'] = np.zeros(hidden_size)  
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)  
    model['b2'] = np.zeros(output_size)  
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes  
loss, grad = two_layer_net(X_train, model, y_train, 1e3) # crank up regularization  
print loss
```

 loss went up, good. (sanity check)

## (4) Overfit a small portion of the data

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples ←
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
                                  model, two_layer_net,
                                  num_epochs=200, reg=0.0,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = False,
                                  learning_rate=1e-3, verbose=True)
```

### Details:

'sgd': vanilla gradient descent (no momentum etc)

learning\_rate\_decay = 1: constant learning rate

sample\_batches = False (full gradient descent, no batches)

epochs = 200: number of passes through the data









# (4) Find a learning rate

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e-6, verbose=True)
```

```
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best validation accuracy: 0.192000
```

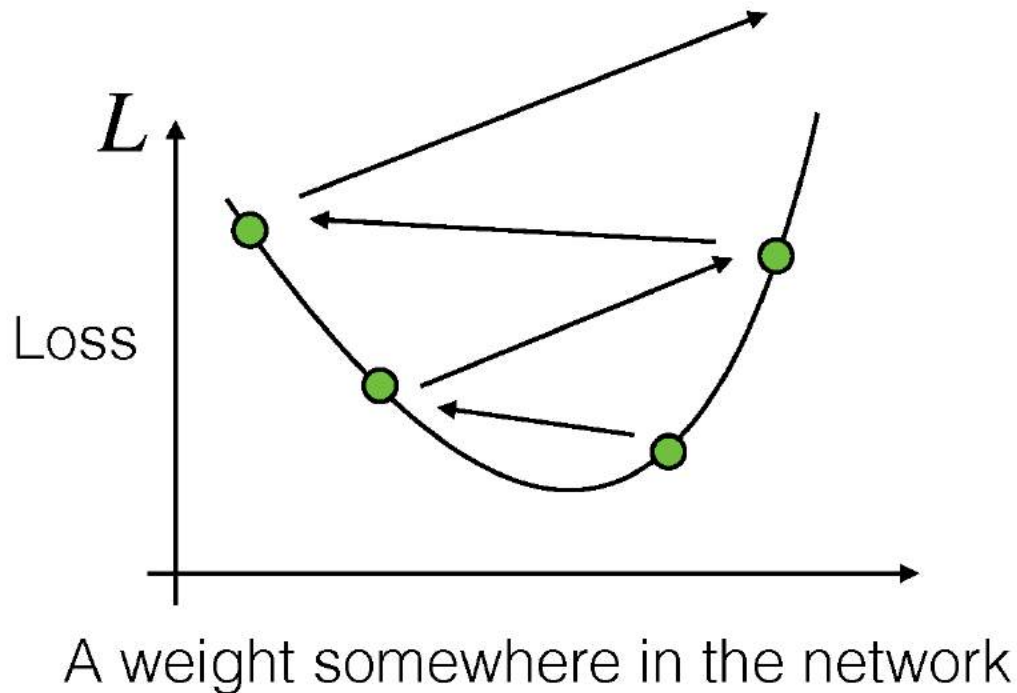
**Loss barely changes**

**Why is the accuracy 20%?**

(learning rate is too low or regularization too high)

# (4) Find a learning rate

Learning rate:  $1e6$  — what could go wrong?



# (4) Find a learning rate

## **Coarse to fine search**

First stage: only a few epochs (passes through the data) to get a rough idea

Second stage: longer running time, finer search

**Tip:** if loss  $> 3 * \text{original loss}$ , quit early  
(learning rate too high)

# (4) Find a learning rate

**Normally, you don't have the budget for lots of cross-validation** —> visualize as you go

## Plot the loss

For very small learning rates, the loss decreases linearly and slowly

*(Why linearly?)*

Larger learning rates tend to look more exponential

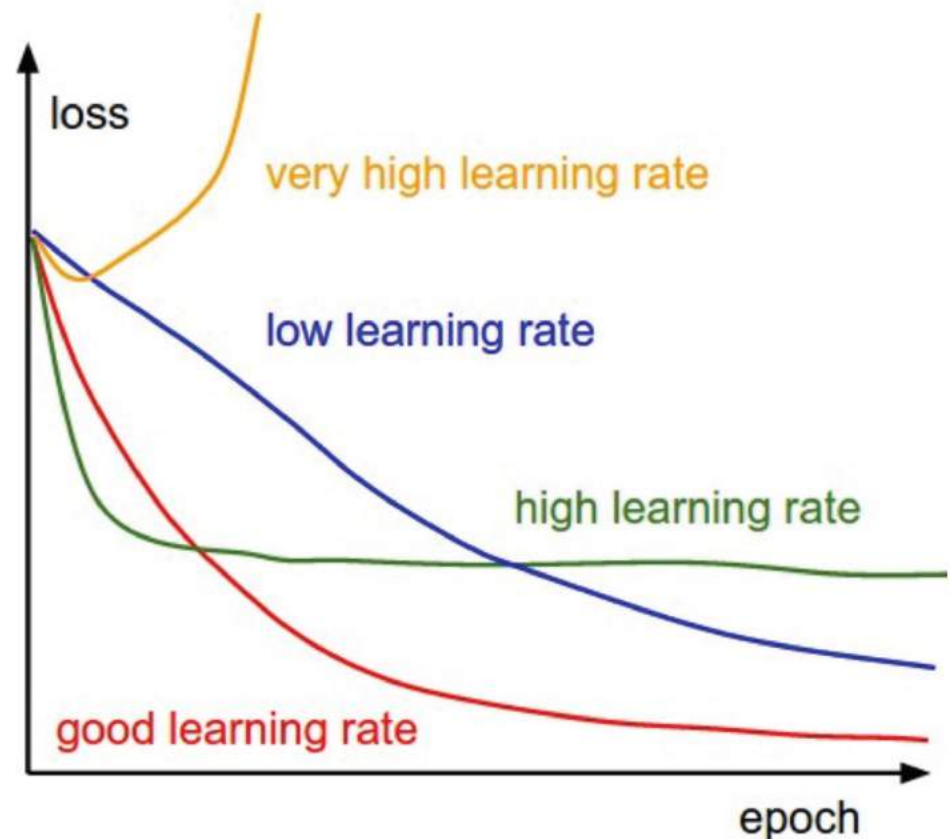


Figure: Andrej Karpathy



# (4) Find a learning rate

**Normally, you don't have the budget for lots of cross-validation** —> visualize as you go

## Typical training loss:

*Why is it varying so rapidly?*

The width of the curve is related to the batchsize — if too noisy, increase the batch size

Possibly too linear  
(learning rate too small)

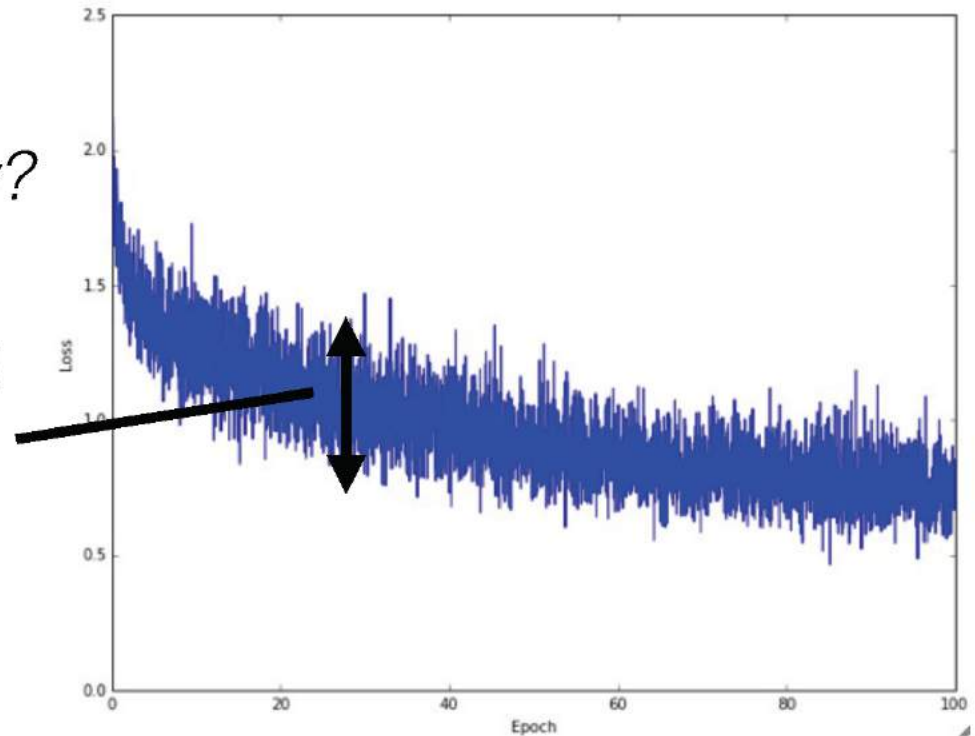
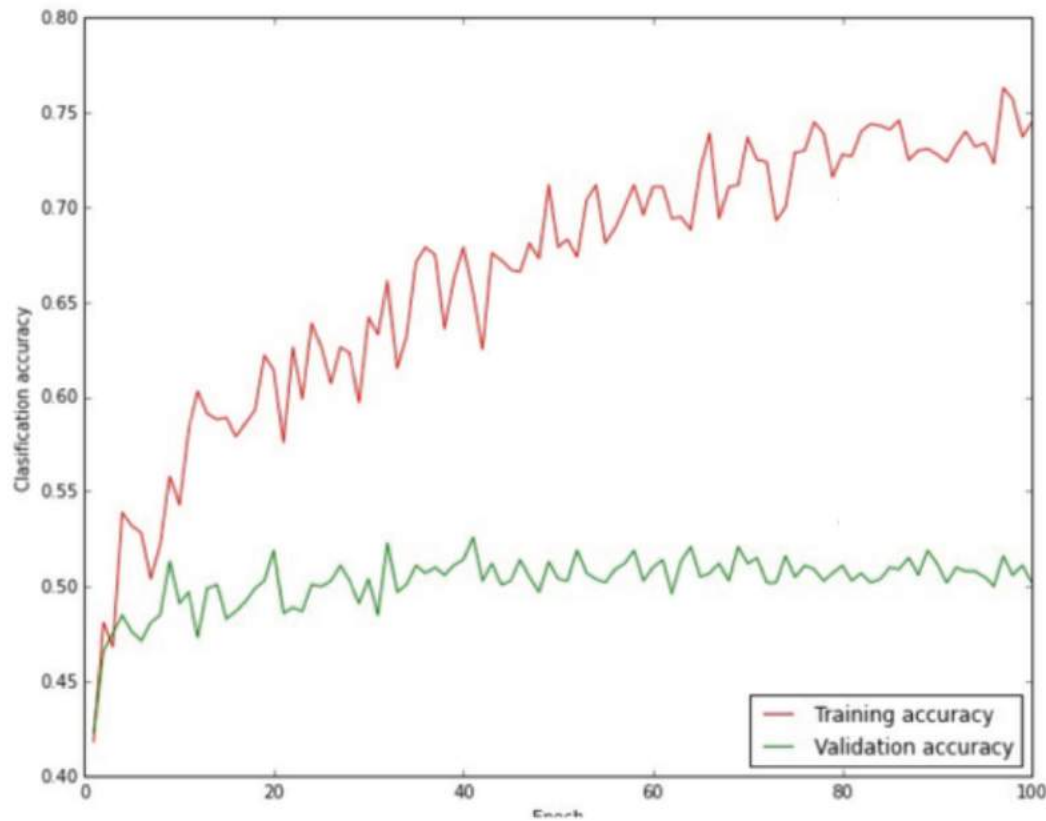


Figure: Andrej Karpathy

# (4) Find a learning rate

## Visualize the accuracy



**Big gap:** overfitting  
(increase regularization)

**No gap:** underfitting  
(increase model capacity,  
make layers bigger  
or decrease regularization)

# (4) Find a learning rate

## Visualize the weights

Noisy weights: possibly regularization not strong enough

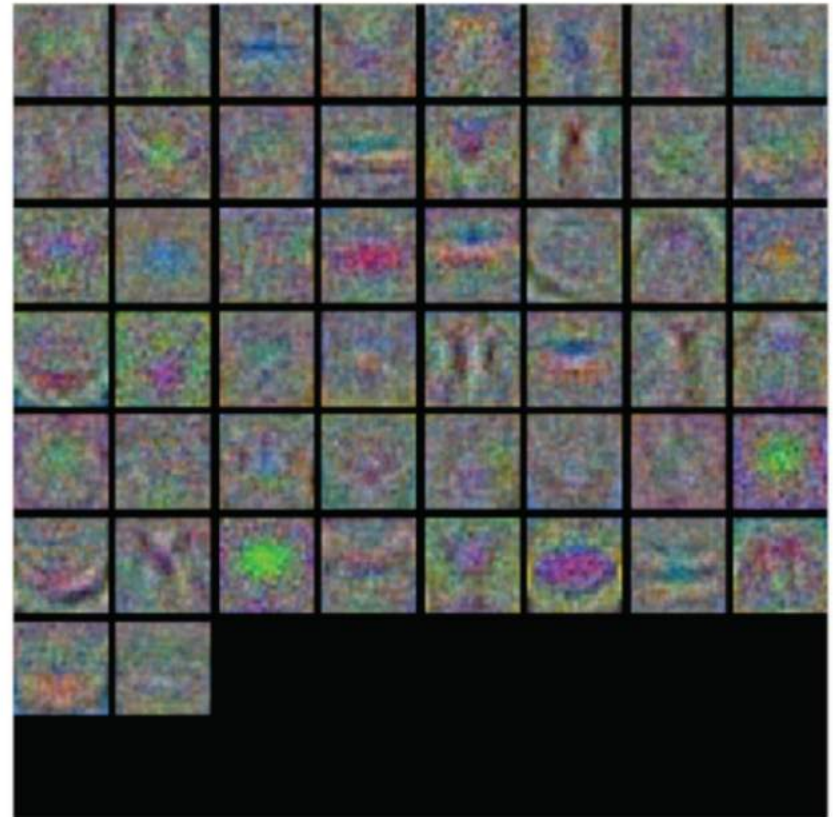
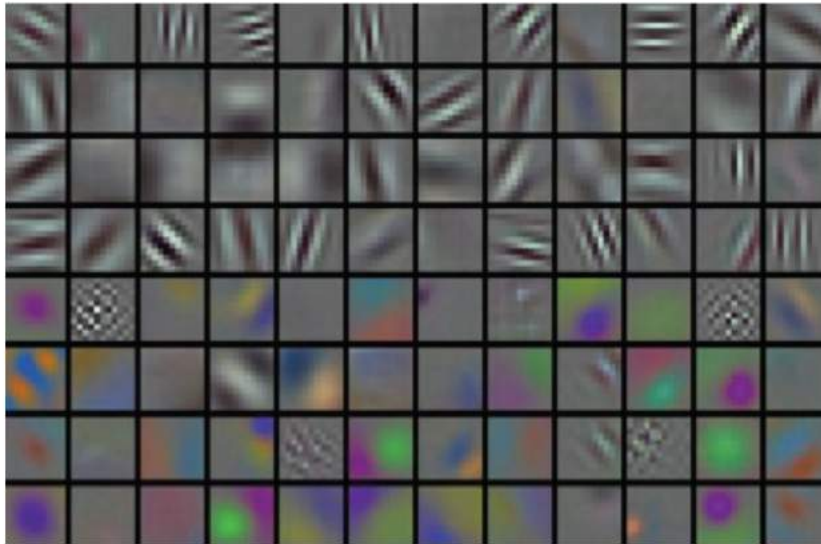


Figure: Andrej Karpathy

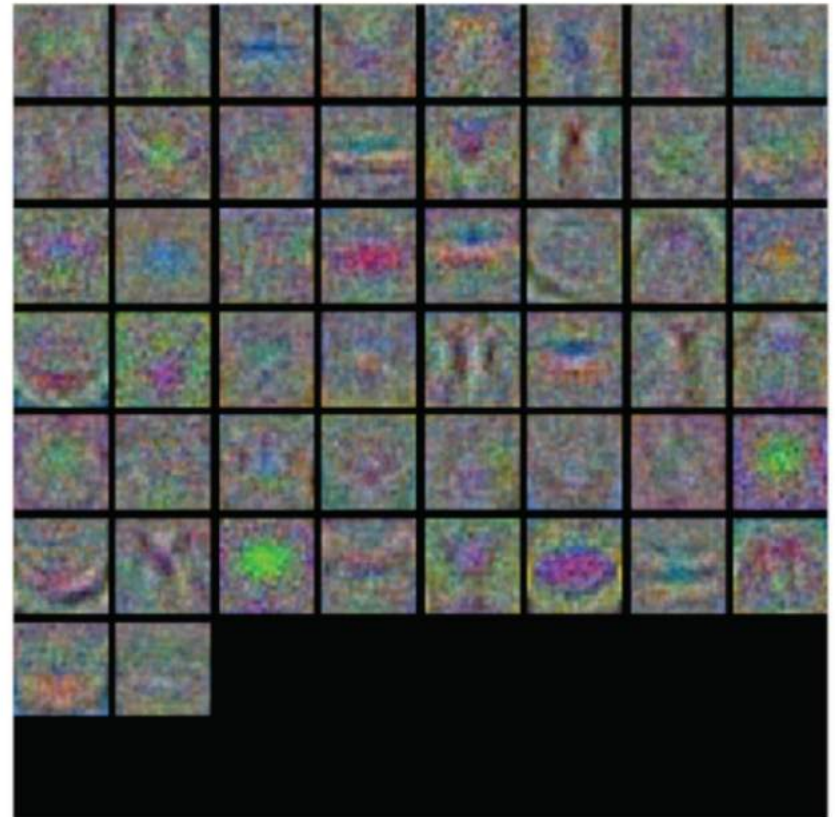


# (4) Find a learning rate

## Visualize the weights



Nice clean weights:  
training is proceeding well



# Learning rate schedule

**How do we change the learning rate over time?**

**Various choices:**

- Step down by a factor of 0.1 every 50,000 mini-batches (used by SuperVision [Krizhevsky 2012])
- Decrease by a factor of 0.97 every epoch (used by GoogLeNet [Szegedy 2014])
- Scale by  $\sqrt{1-t/\text{max\_t}}$  (used by BVLC to re-implement GoogLeNet)
- Scale by  $1/t$
- Scale by  $\exp(-t)$

# Summary of things to fiddle

- Network architecture
- Learning rate, decay schedule, update type
- Regularization (L2, L1, maxnorm, dropout, ...)
- Loss function (softmax, SVM, ...)
- Weight initialization

Neural network  
parameters



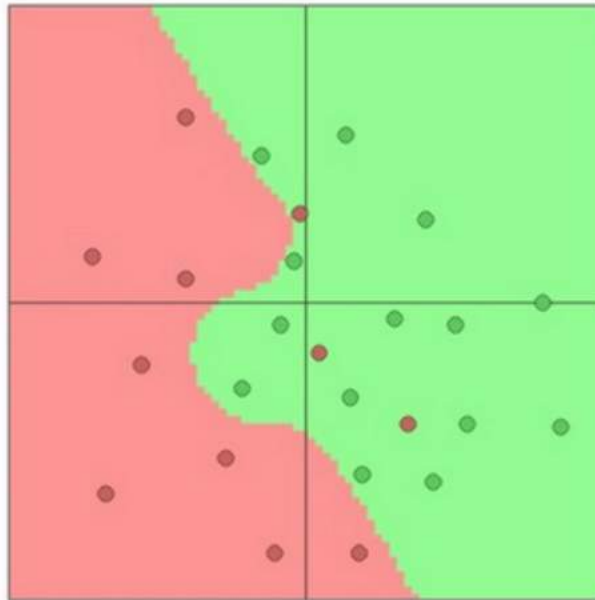
# (Recall) Regularization reduces overfitting

$$L = L_{\text{data}} + L_{\text{reg}} \quad L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2$$

$\lambda = 0.001$



$\lambda = 0.01$



$\lambda = 0.1$



# Example Regularizers

## L2 regularization

$$L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2$$

(L2 regularization encourages small weights)

## L1 regularization

$$L_{\text{reg}} = \lambda \|W\|_1 = \lambda \sum_{ij} |w_{ij}|$$

(L1 regularization encourages sparse weights: weights are encouraged to reduce to exactly zero)

## “Elastic net”

$$L_{\text{reg}} = \lambda_1 \|W\|_1 + \lambda_2 \|W\|_2^2$$

(combine L1 and L2 regularization)

## Max norm

Clamp weights to some max norm

$$\|W\|_2^2 \leq c$$

# “Weight decay”

**Regularization is also called “weight decay” because the weights “decay” each iteration:**

$$L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2 \quad \longrightarrow \quad \frac{\partial L}{\partial W} = \lambda W$$

Gradient descent step:

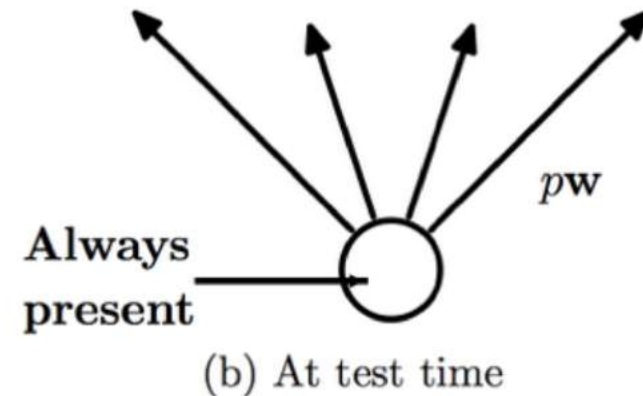
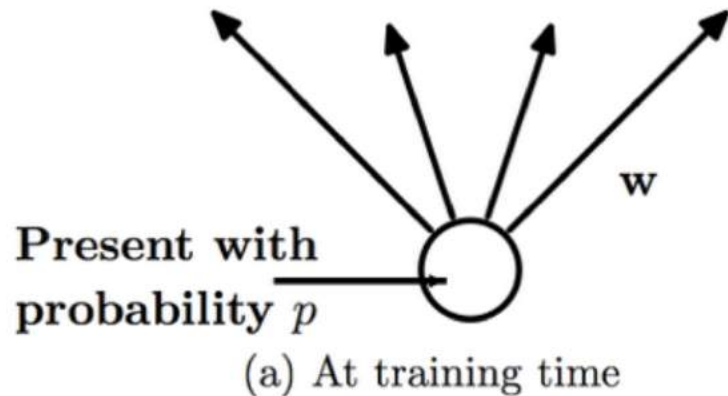
$$W \leftarrow W - \alpha \lambda W - \frac{\partial L_{\text{data}}}{\partial W}$$

Weight decay:  $\alpha \lambda$  (weights always decay by this amount)

**Note:** biases are sometimes excluded from regularization

# Dropout

**Simple but powerful technique to reduce overfitting:**

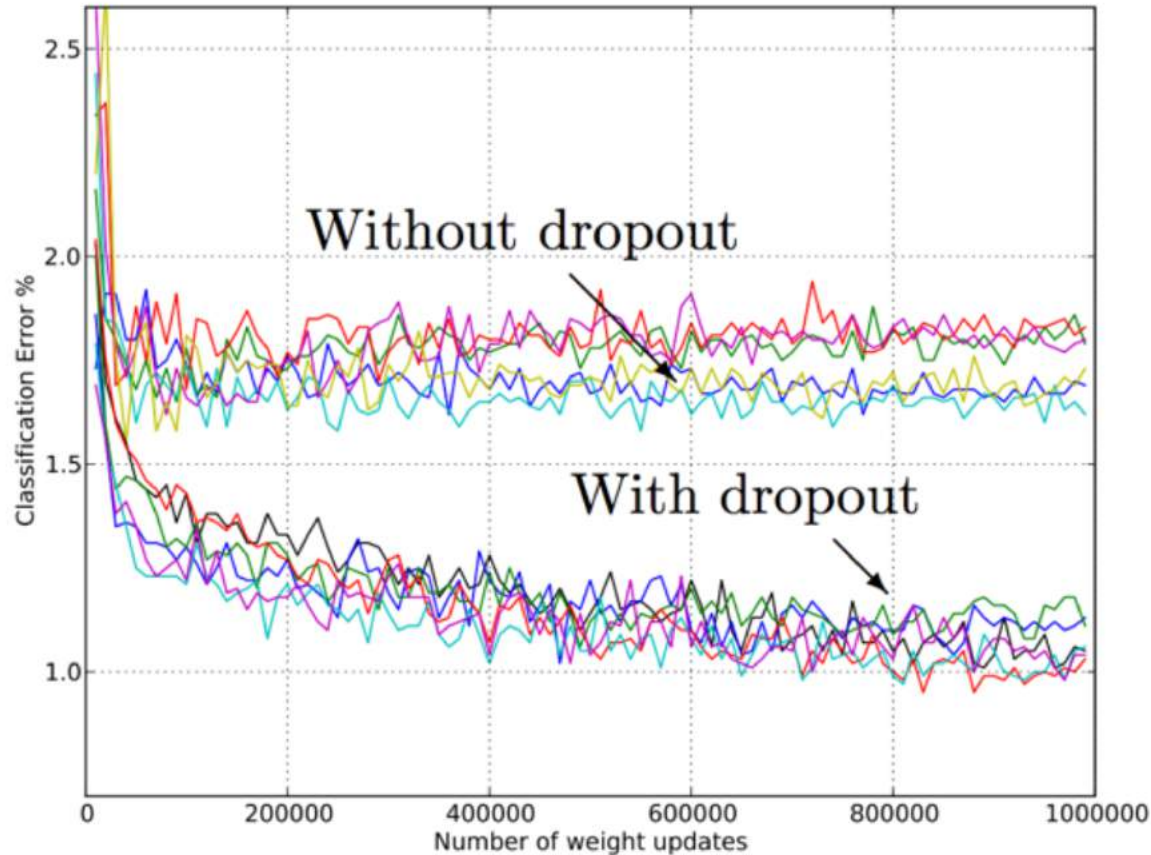


[Srivasta et al, “Dropout: A Simple Way to Prevent Neural Networks from Overfitting”, JMLR 2014]



# Dropout

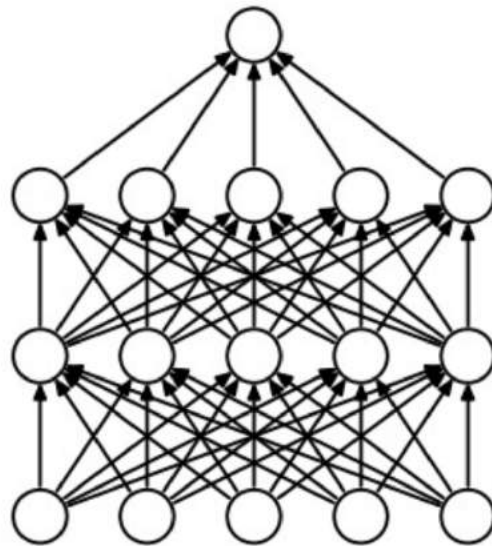
**Simple but powerful technique to reduce overfitting:**



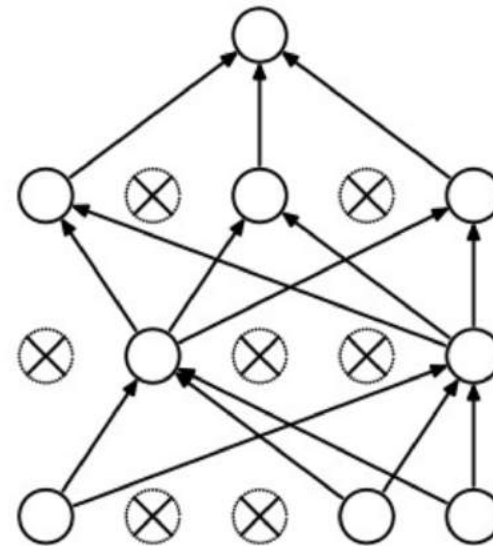
[Srivasta et al, "Dropout: A Simple Way to Prevent Neural Networks from Overfitting", JMLR 2014]

# Dropout

**Simple but powerful technique to reduce overfitting:**



(a) Standard Neural Net



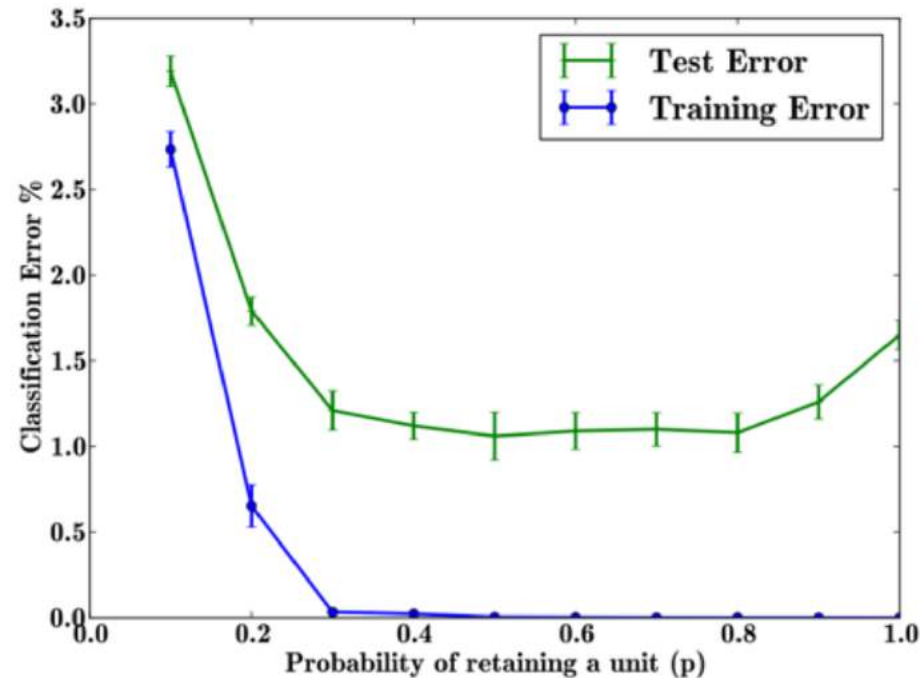
(b) After applying dropout.

**Note:** Dropout can be interpreted as an approximation to taking the geometric mean of an ensemble of exponentially many models

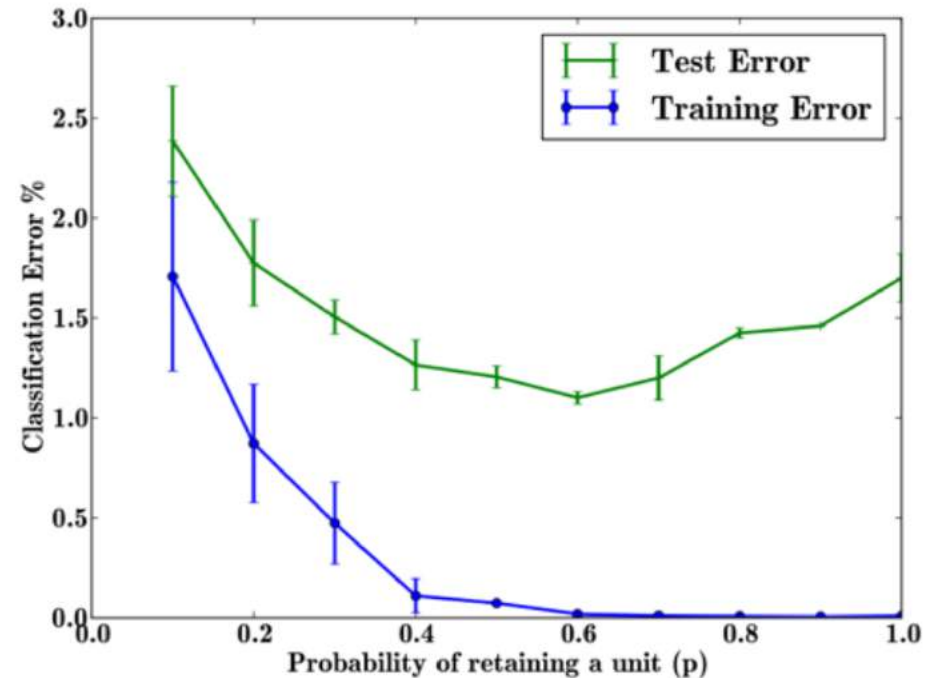
[Srivasta et al, “Dropout: A Simple Way to Prevent Neural Networks from Overfitting”, JMLR 2014]

# Dropout

How much dropout? Around  $p = 0.5$



(a) Keeping  $n$  fixed.



(b) Keeping  $pn$  fixed.

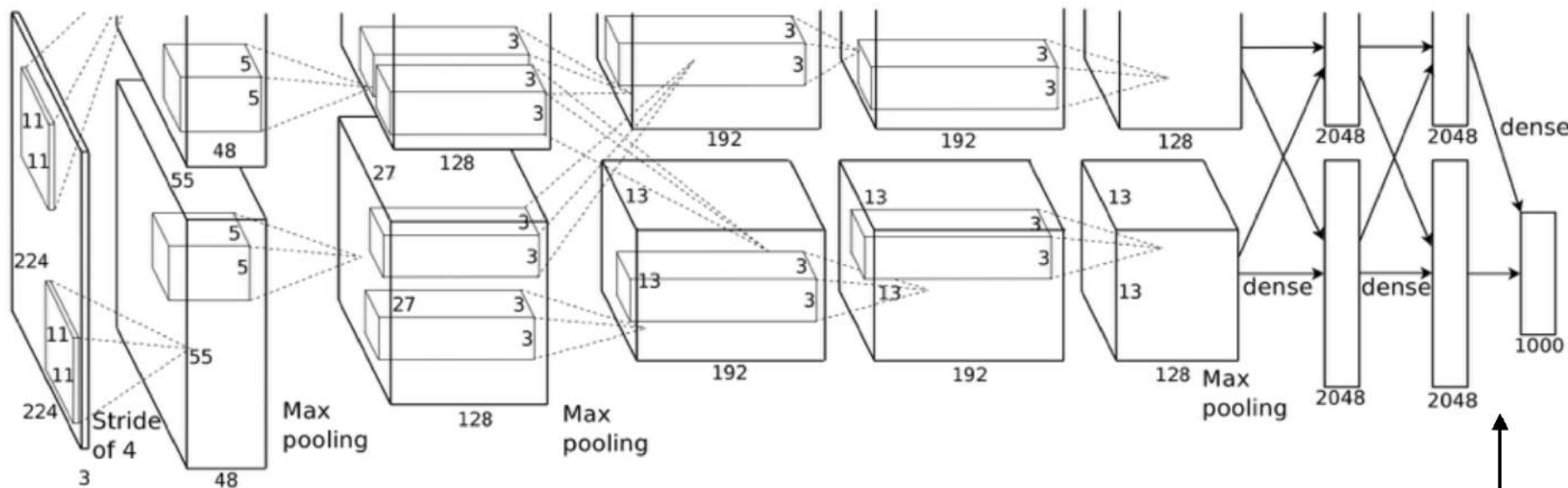
[Srivasta et al, "Dropout: A Simple Way to Prevent Neural Networks from Overfitting", JMLR 2014]

# Dropout

## Case study: [Krizhevsky 2012]

*“Without dropout, our network exhibits substantial overfitting.”*

Dropout here



**But not here — why?**

[Krizhevsky et al, “ImageNet Classification with Deep Convolutional Neural Networks”, NIPS 2012]

# Dropout

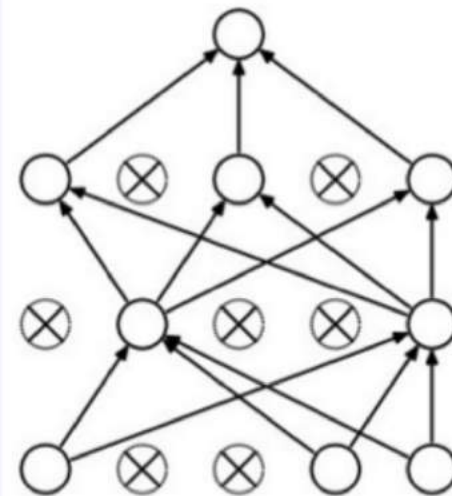
```
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
    """ X contains the data """

    # forward pass for example 3-layer neural network
    H1 = np.maximum(0, np.dot(W1, X) + b1)
    U1 = np.random.rand(*H1.shape) < p # first dropout mask
    H1 *= U1 # drop!
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
    U2 = np.random.rand(*H2.shape) < p # second dropout mask
    H2 *= U2 # drop!
    out = np.dot(W3, H2) + b3

    # backward pass: compute gradients... (not shown)
    # perform parameter update... (not shown)
```

Example forward pass with a 3-layer network using dropout



(note, here  $X$  is a single input)



# Dropout

**Test time:** scale the activations

Expected value of a neuron  $h$  with dropout:

$$E[h] = ph + (1 - p)0 = ph$$

```
def predict(X):  
    # ensembled forward pass  
    H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations  
    H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations  
    out = np.dot(W3, H2) + b3
```

We want to keep the same expected value

# Summary

- Preprocess the data (subtract mean, sub-crops)
- Initialize weights carefully
- Use Dropout
- Use SGD + Momentum
- Fine-tune from ImageNet
- Babysit the network as it trains