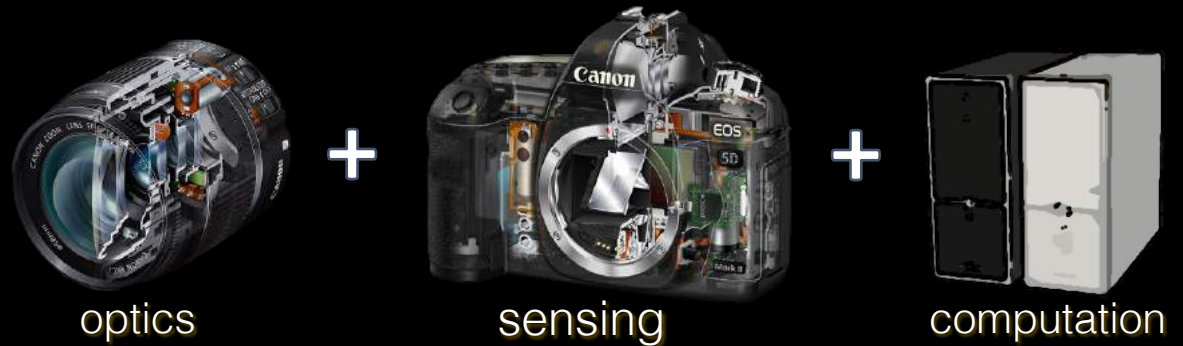


Special Topics: Computational Imaging

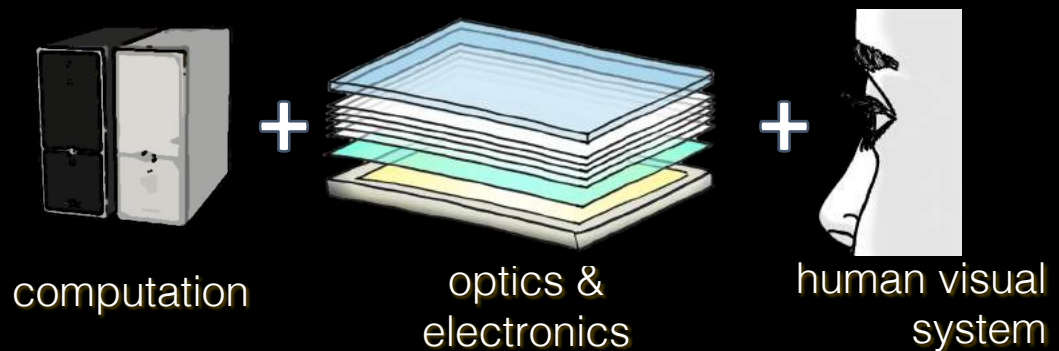
16-385 Computer Vision
Fall 2020, Lecture 27

computational imaging

Computational Cameras



Computational Displays

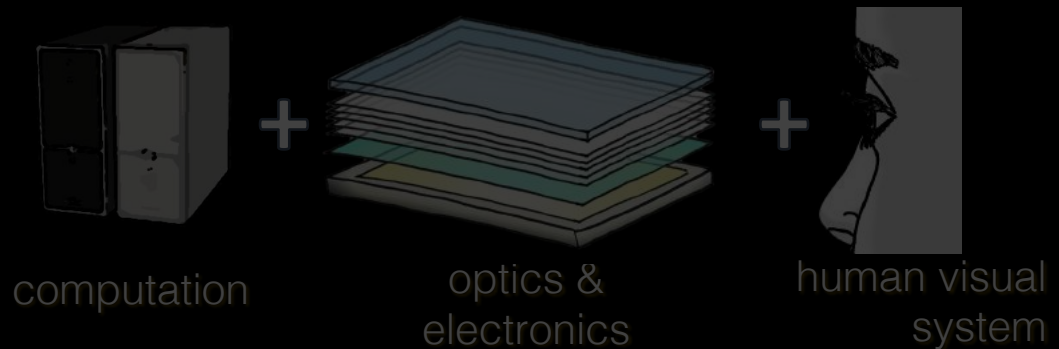


computational imaging

Computational Cameras



Computational Displays



computational imaging

Computational Cameras



HDR Imaging [Debevec, Nayar, ...]



Super-resolution [Baker, ...]



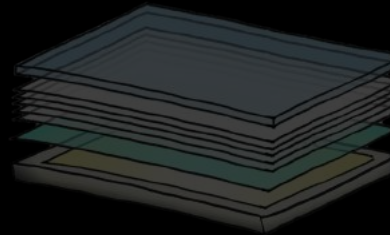
Light Fields [Levoy, ...]

Computational Displays



computation

+



optics &
electronics

+



human visual
system

computational imaging

Computational Cameras



HDR Imaging [Debevec, Nayar, ...]



Super-resolution [Baker, ...]



Light Fields [Levoy, ...]

Computational Displays



HDR Display [Seetzen, ...]



Super-resolution [Hirsch, Heide, ...]



Light Fields [Wetzstein, ...]

computational imaging

**Computational
Cameras**

+

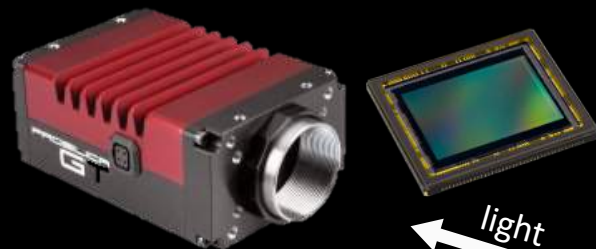
**Computational
Displays**

computational imaging

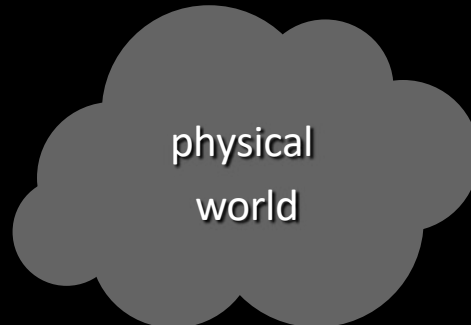
Computational
Cameras

+

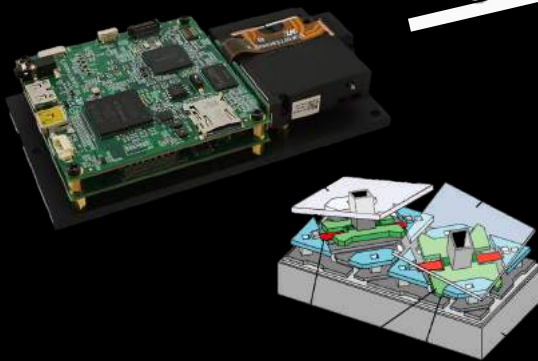
Computational
Displays



light



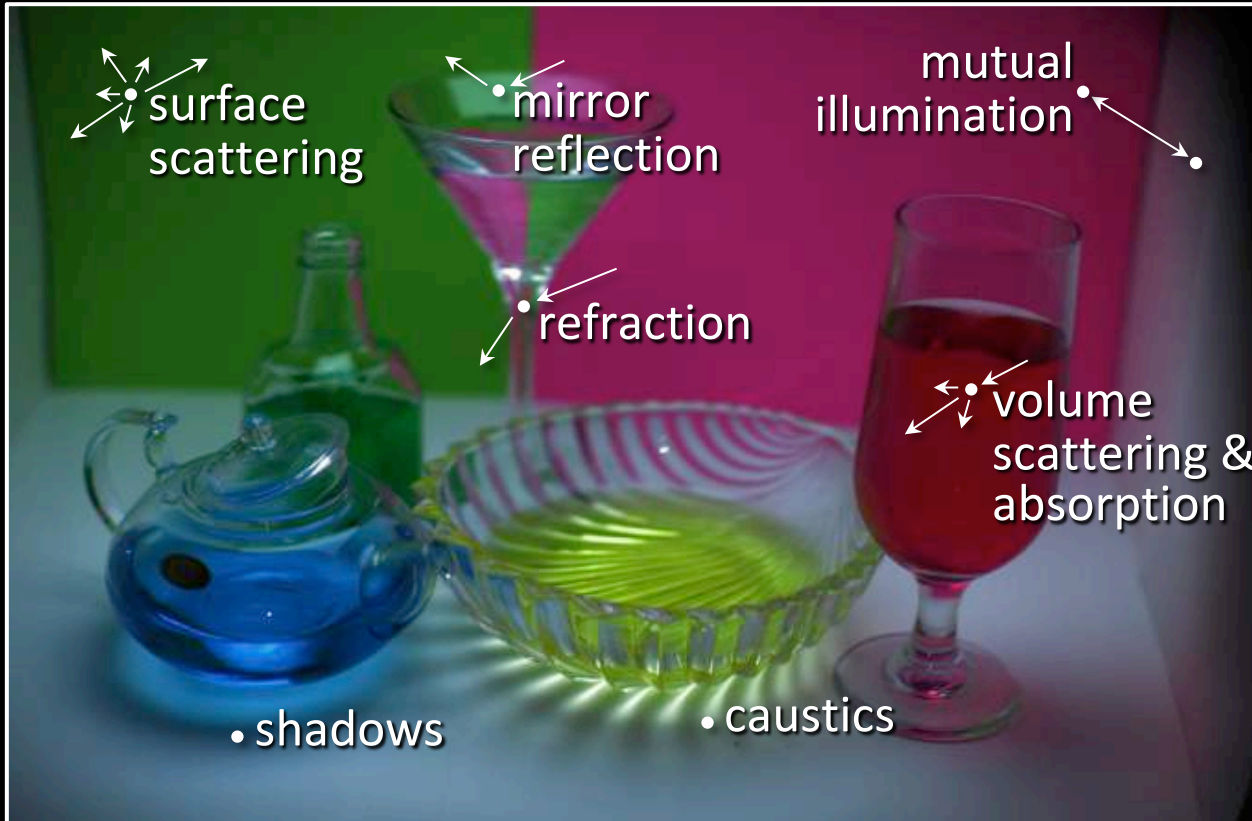
light



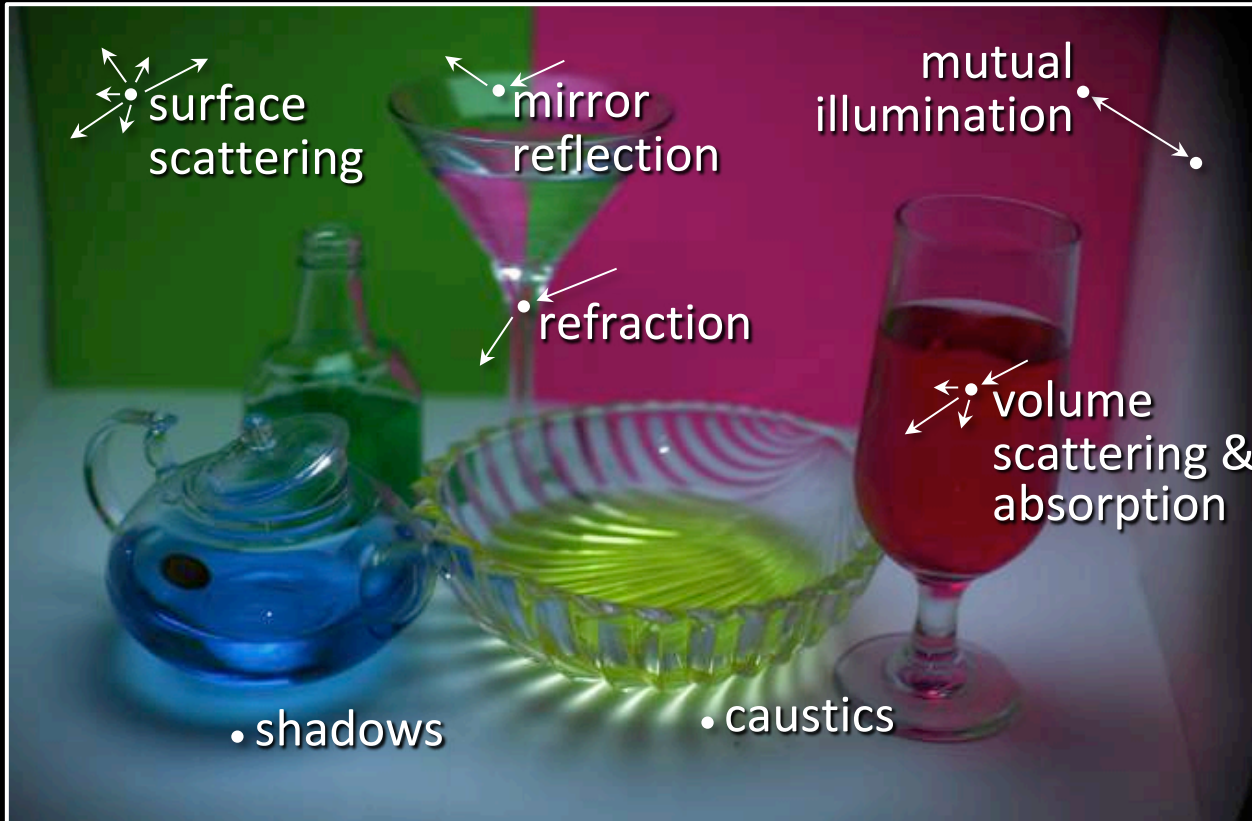
light transport in a general scene



light transport in a general scene

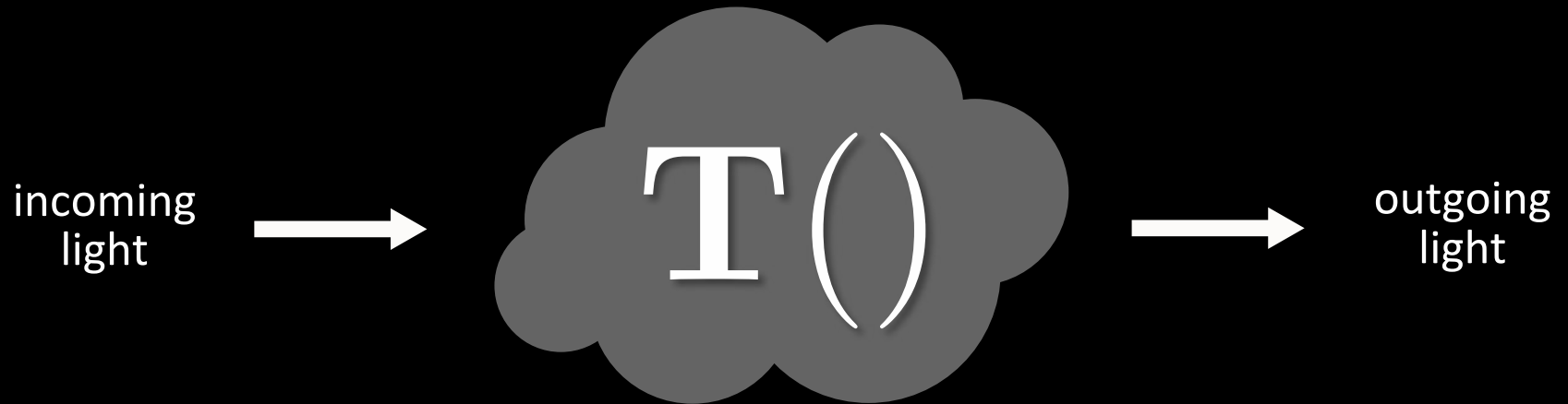


light transport in a general scene



computational light transport

computational light transport involves using controllable light sources & cameras to sample, acquire or analyze a scene's transport function



3D imaging for autonomous cars



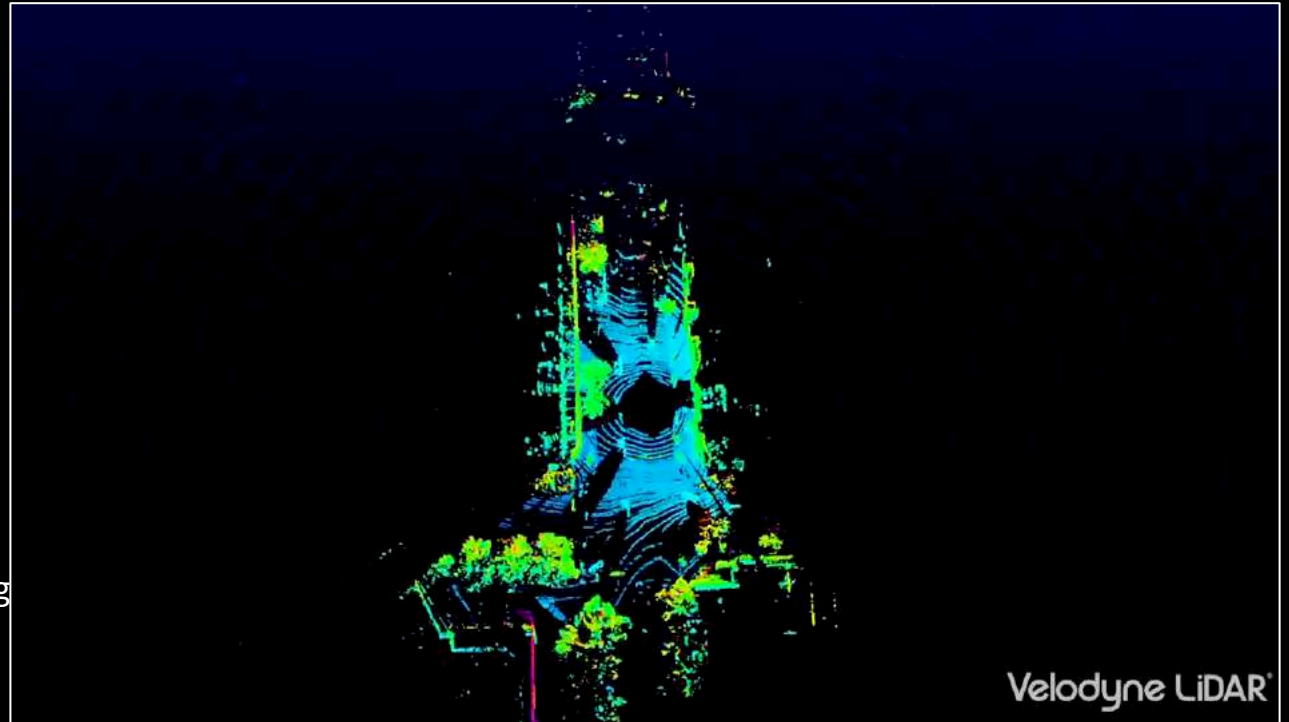
LIDAR (light detection and ranging)
Velodyne VLS-128



3D imaging for autonomous cars



LIDAR (light detection and ranging)
Velodyne VLS-128



3D imaging for smartphones



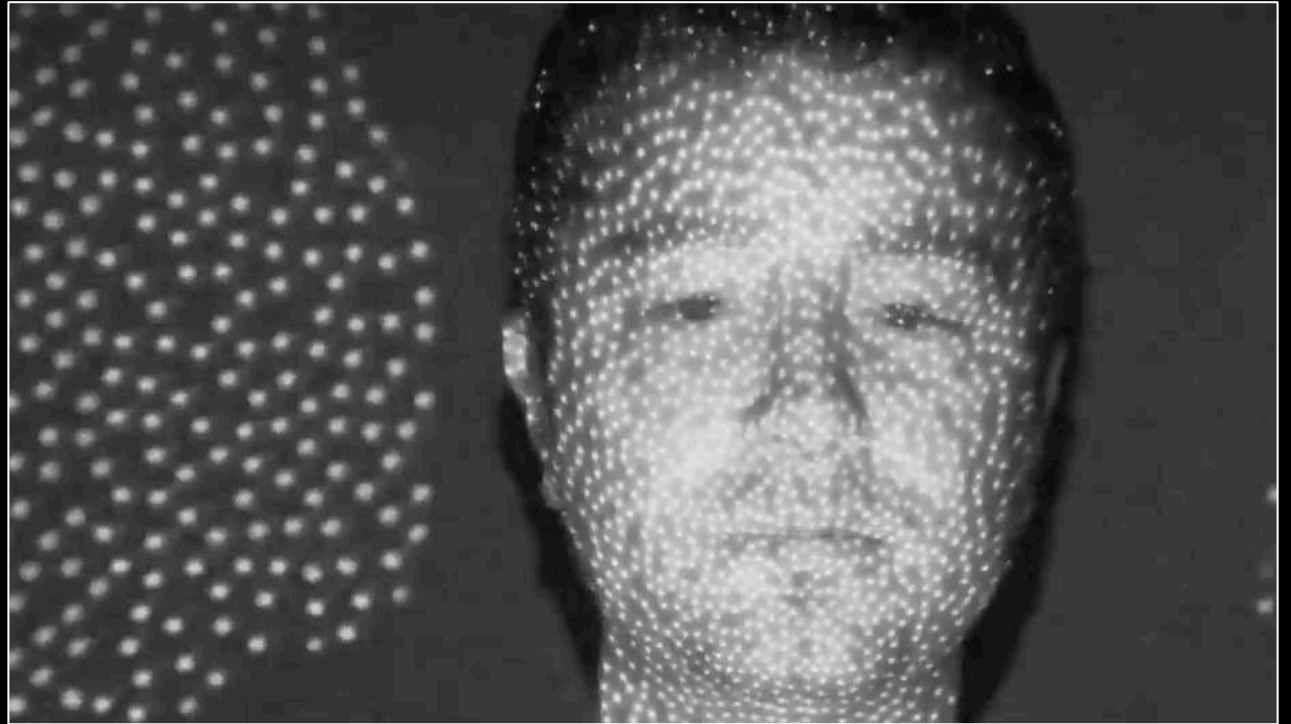
iPhone X



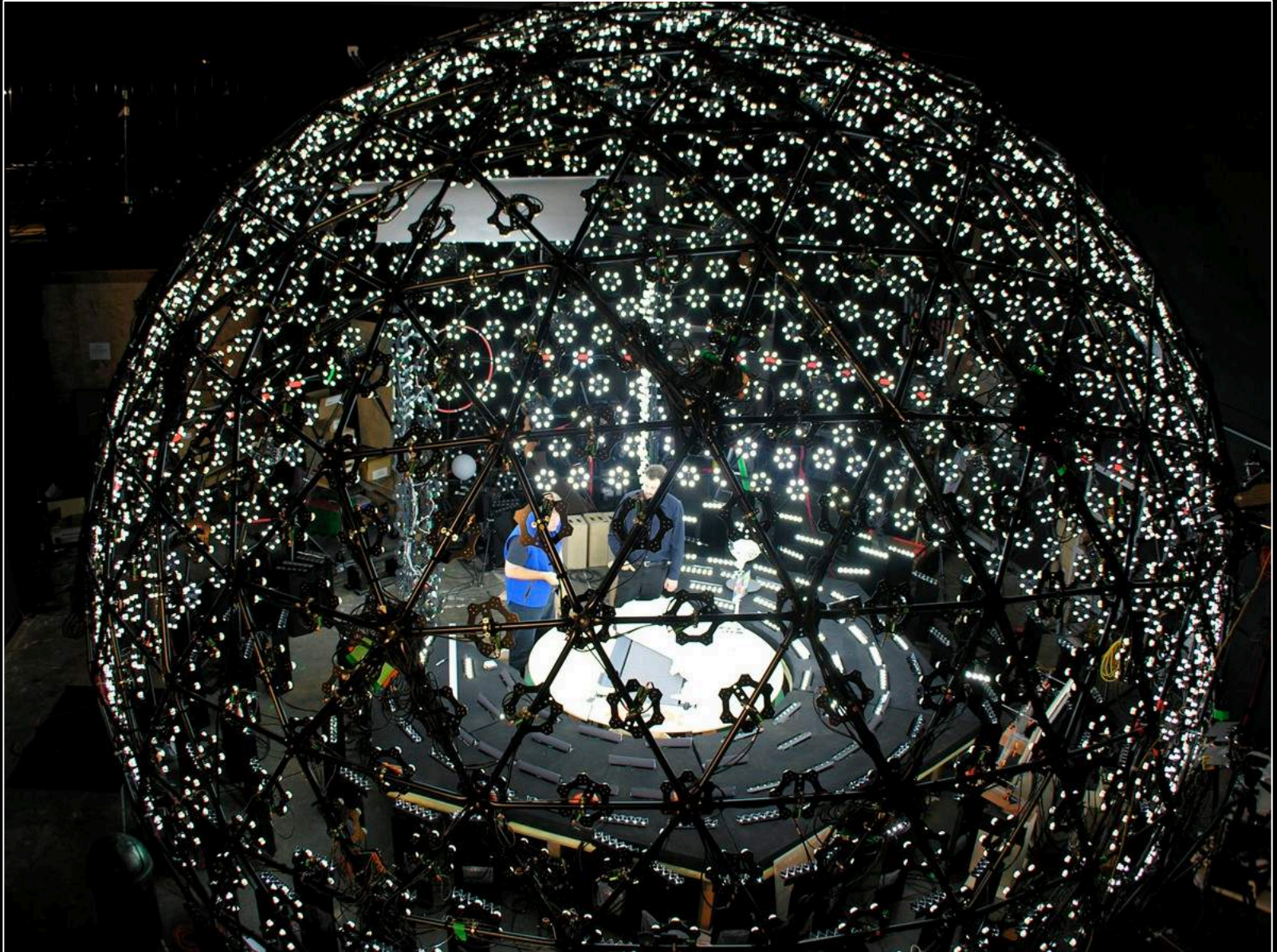
3D imaging for smartphones



iPhone X



Paul Debevec's light stage 6



https://www.fxguide.com/fxfeatured/light_stage_6/

Paul Debevec's light stage 6

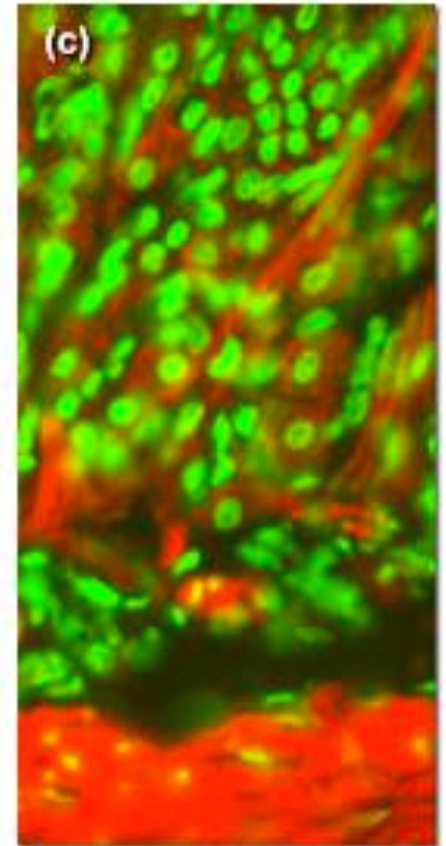
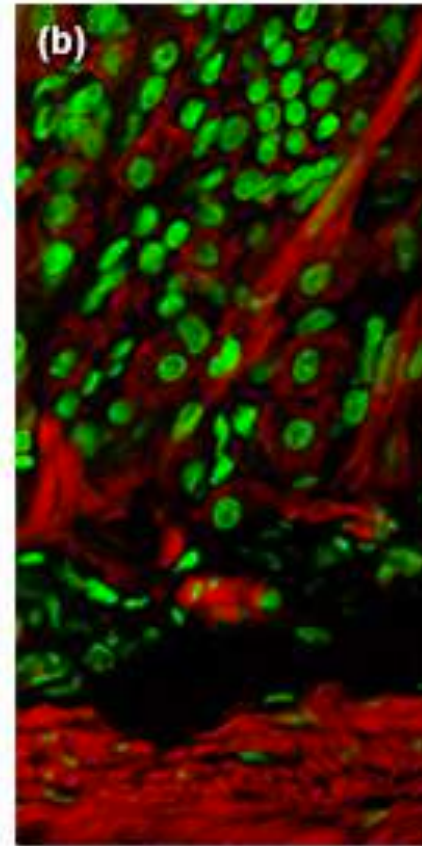
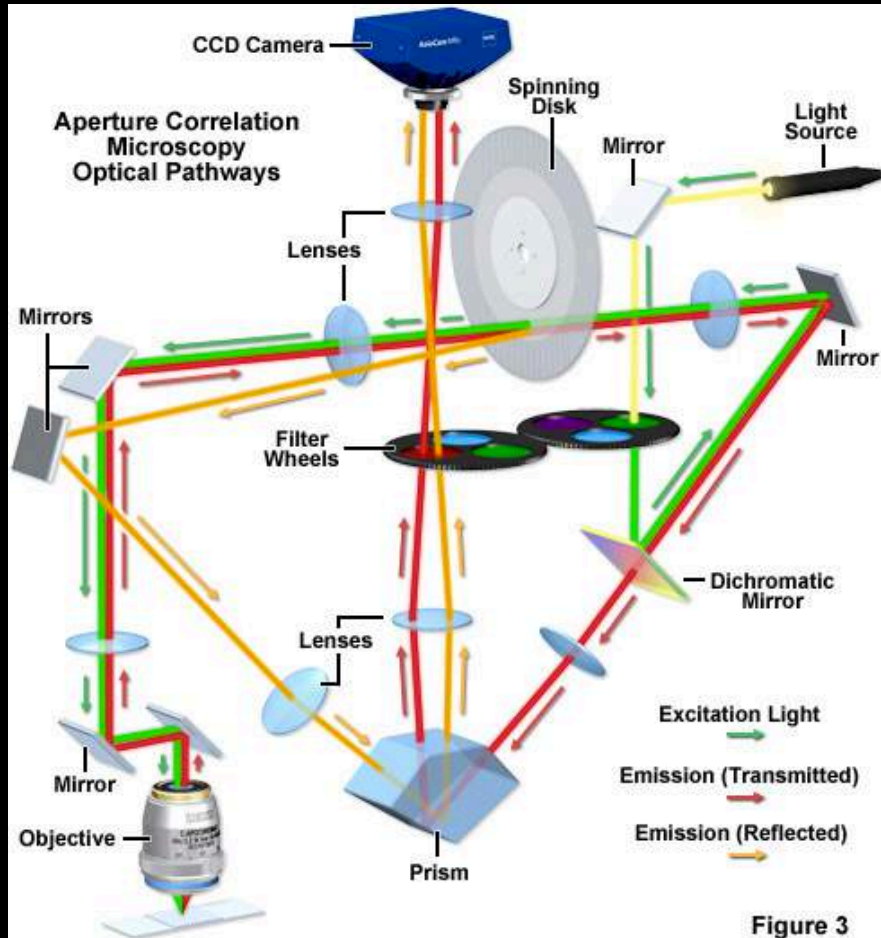


https://www.fxguide.com/fxfeatured/light_stage_6/

mobile light stage



aperture correlation microscope (source: Zeiss)



overview

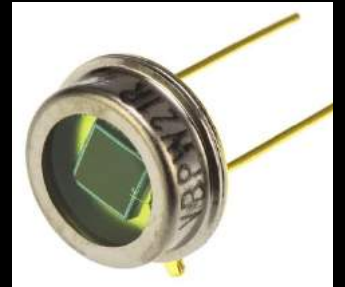
1. **the light transport matrix:** a general model for the transfer of radiant energy
2. example transport matrices for real scenes
3. challenges associated with analyzing transport matrices
4. optical algorithms to analyze light transport

overview

1. **the light transport matrix:** a general model for the transfer of radiant energy
2. example transport matrices for real scenes
3. challenges associated with analyzing transport matrices
4. optical algorithms to analyze light transport

modeling light transport

how do we model light transport between one light source and one sensor?

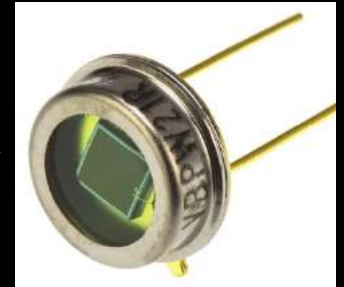
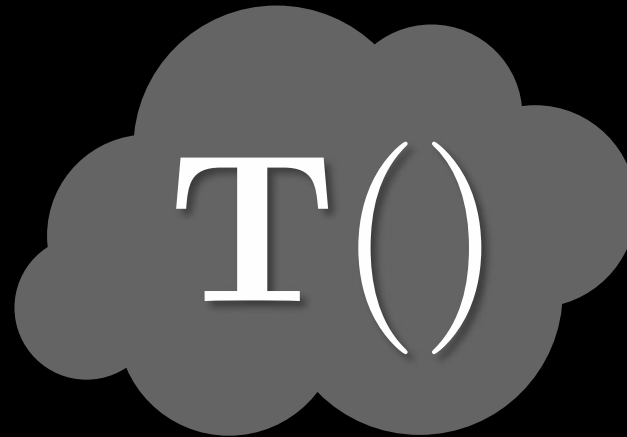


light source emitting
radiant energy x
(measured in Joules)

sensor detecting
radiant energy y
(measured in Joules)

modeling light transport

how do we model light transport between one light source and one sensor?



light source emitting
radiant energy x
(measured in Joules)

sensor detecting
radiant energy y
(measured in Joules)

observations:

- radiant energy is always non-negative, i.e., can't detect "negative" energy"
- the transport function $T()$ is homogeneous of degree 1, i.e., the function satisfies $T(s*x) = s T(x)$

property 1: homogeneity of degree 1

weight

1

x



=



scene light by light source at
100% intensity

observations:

- radiant energy is always non-negative, i.e., can't detect "negative" energy"
- the transport function $T()$ is homogeneous of degree 1, i.e., the function satisfies $T(s*x) = s T(x)$

property 1: homogeneity of degree 1

weight

2

x



=



scene light by light source at
200% intensity

observations:

- radiant energy is always non-negative, i.e., can't detect "negative" energy"
- the transport function $T()$ is homogeneous of degree 1, i.e., the function satisfies $T(s*x) = s T(x)$

property 1: homogeneity of degree 1

weight

0.5

x



=

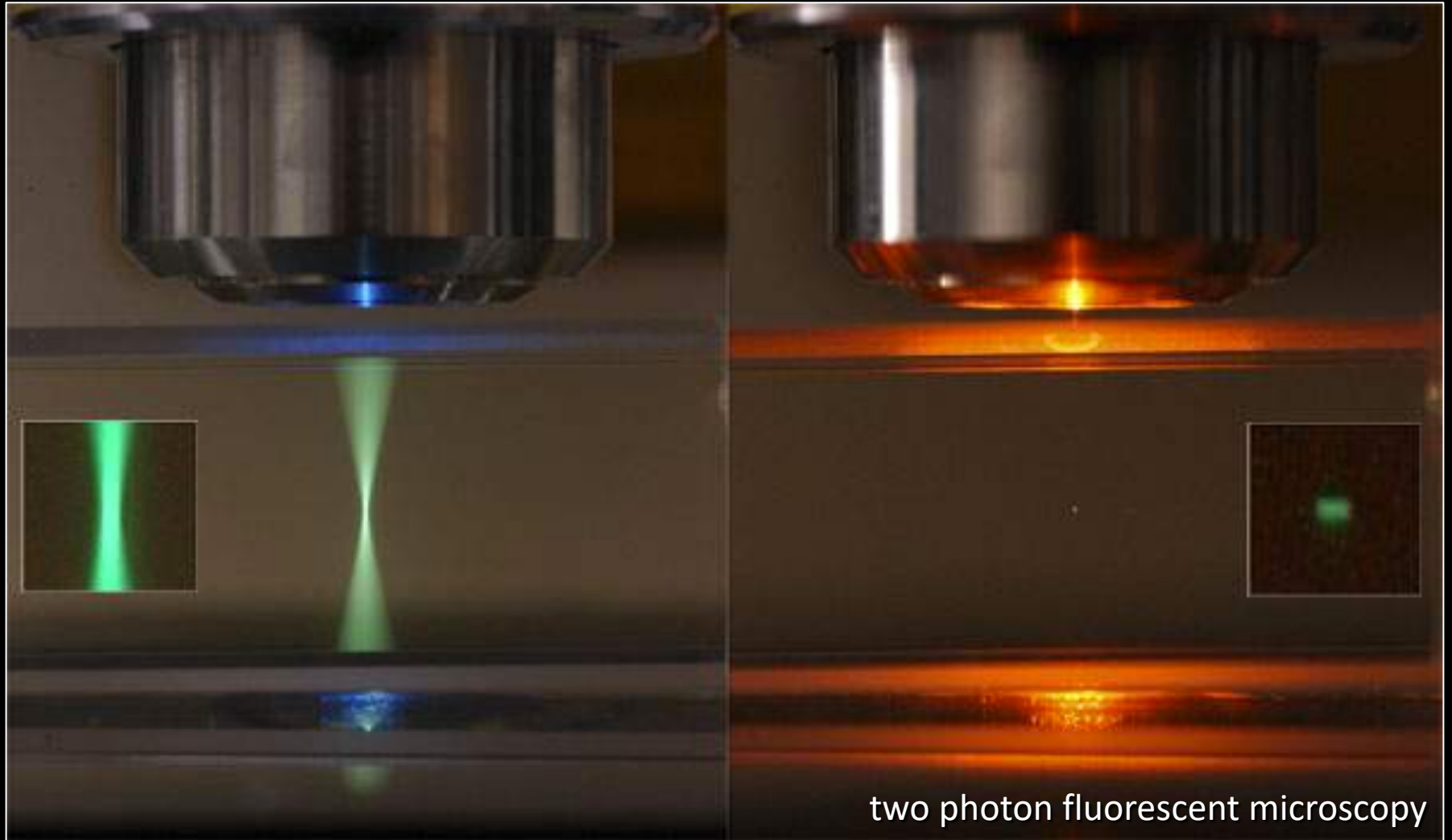


scene light by light source at
50% intensity
(fails for saturated pixels)

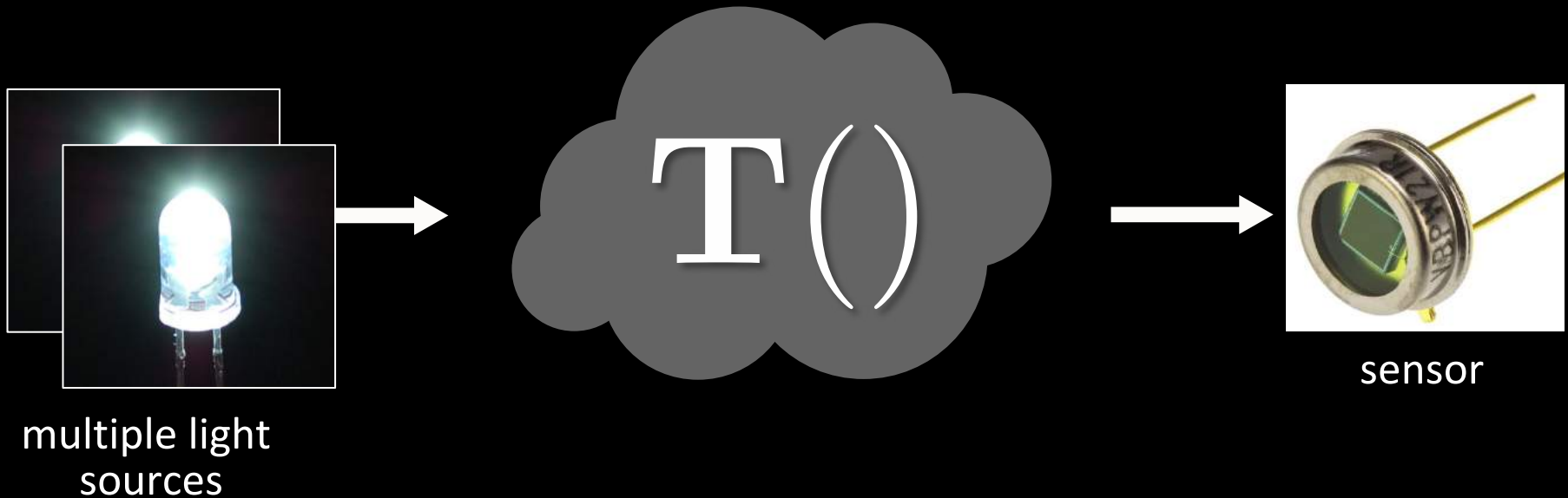
observations:

- radiant energy is always non-negative, i.e., can't detect "negative" energy"
- the transport function $T()$ is homogeneous of degree 1, i.e., the function satisfies $T(s*x) = s T(x)$

example when homogeneity of degree 1 condition does **not** hold true



computational light transport



observation:

- measurement under two light sources equals the sum of measurements taken under each source individually, i.e., $T(x_1, x_2) = T(x_1, 0) + T(0, x_2)$

property 2: additivity



=



+



observation:

- measurement under two light sources equals the sum of measurements taken under each source individually, i.e., $T(x_1, x_2) = T(x_1, 0) + T(0, x_2)$

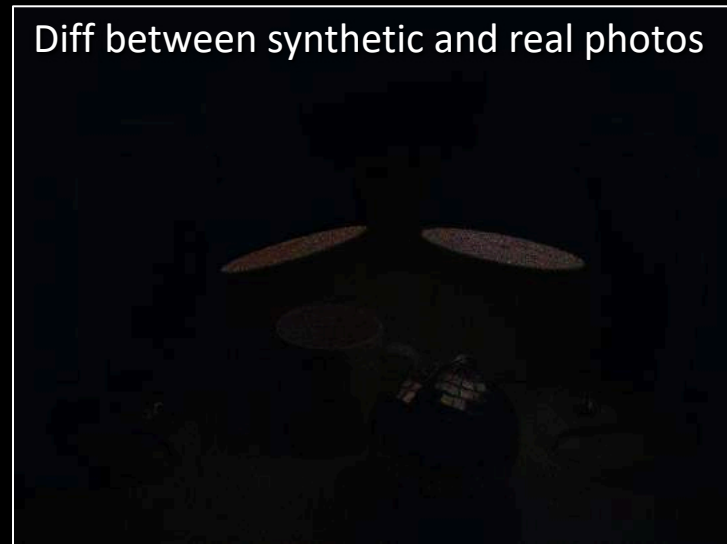
property 2: additivity



observation:

- measurement under two light sources equals the sum of measurements taken under each source individually, i.e., $T(x_1, x_2) = T(x_1, 0) + T(0, x_2)$

property 2: additivity



observation:

- measurement under two light sources equals the sum of measurements taken under each source individually, i.e., $T(x_1, x_2) = T(x_1, 0) + T(0, x_2)$

image-based relighting



=



image-based relighting



=



+



Weight 1

x

1

Weight 2

x

1

image-based relighting



=



+



Weight 1

x 1

Weight 2

x 0

image-based relighting



=



+



Weight 1



Weight 2





=



Weight 1

x \mathbf{l}_1 +



Weight 2

x \mathbf{l}_2



=



Weight 1

x \mathbf{l}_1 +



Weight 2

x \mathbf{l}_2

\mathbf{p}

=

$\sum_{i=1}^2$

\mathbf{T}_i

x

\mathbf{l}_i



=



Weight 1

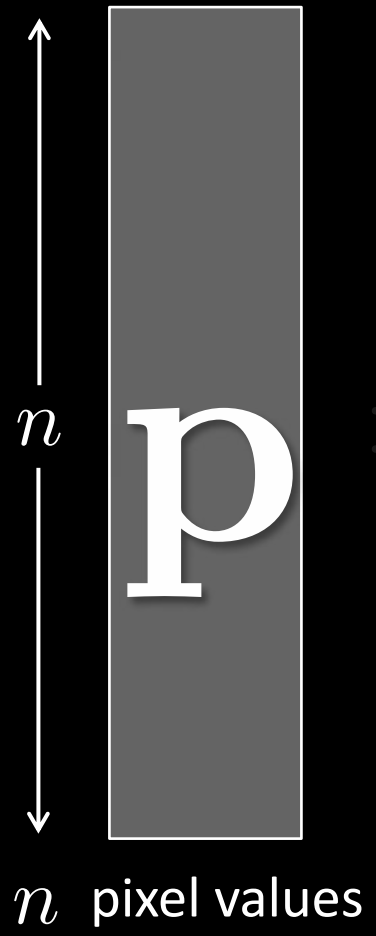
$\times \mathbf{l}_1$

+



Weight 2

$\times \mathbf{l}_2$



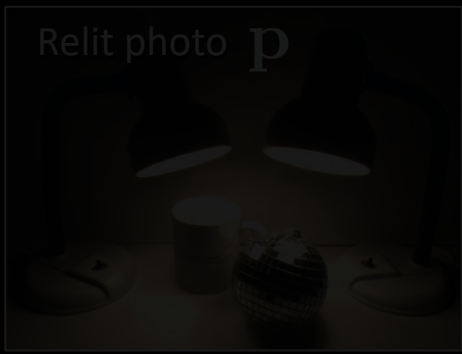
=

$$\sum_{i=1}^2$$



\times





=



Weight 1

$\times \mathbf{l}_1 +$



Weight 2

$\times \mathbf{l}_2$

n

\mathbf{p}

n pixel values

=

$\sum_{i=1}^2$

\mathbf{T}_i

\times

\mathbf{l}_i



=



Weight 1

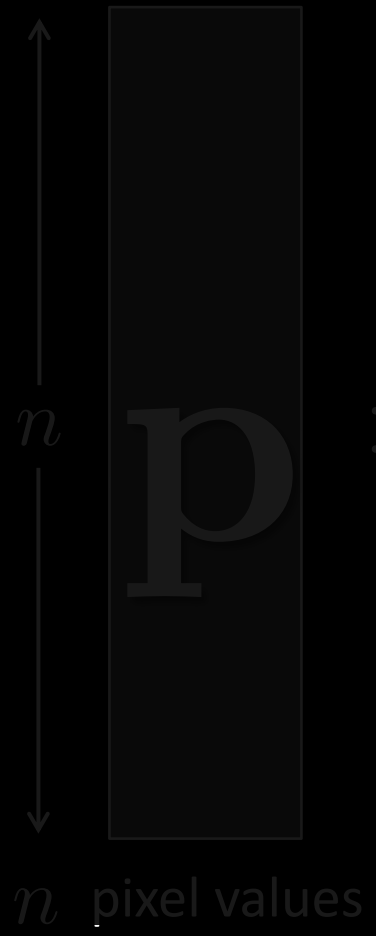
x \mathbf{l}_1

+



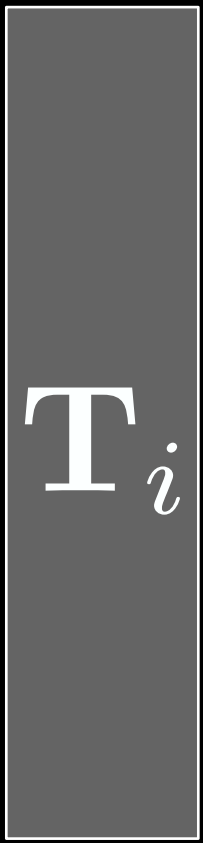
Weight 2

x \mathbf{l}_2



=

$$\sum_{i=1}^2$$



\mathbf{T}_i

x



\mathbf{l}_i

Contribution of the source





Weight 1
 $\times \mathbf{l}_1$

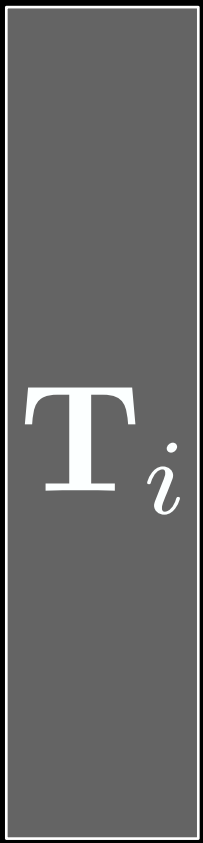


Weight 2
 $\times \mathbf{l}_2$

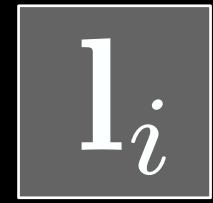
Number of
controllable
sources \downarrow

2

$$\sum_{i=1}^2$$



Contribution of
each source \downarrow



n

n pixel values



=



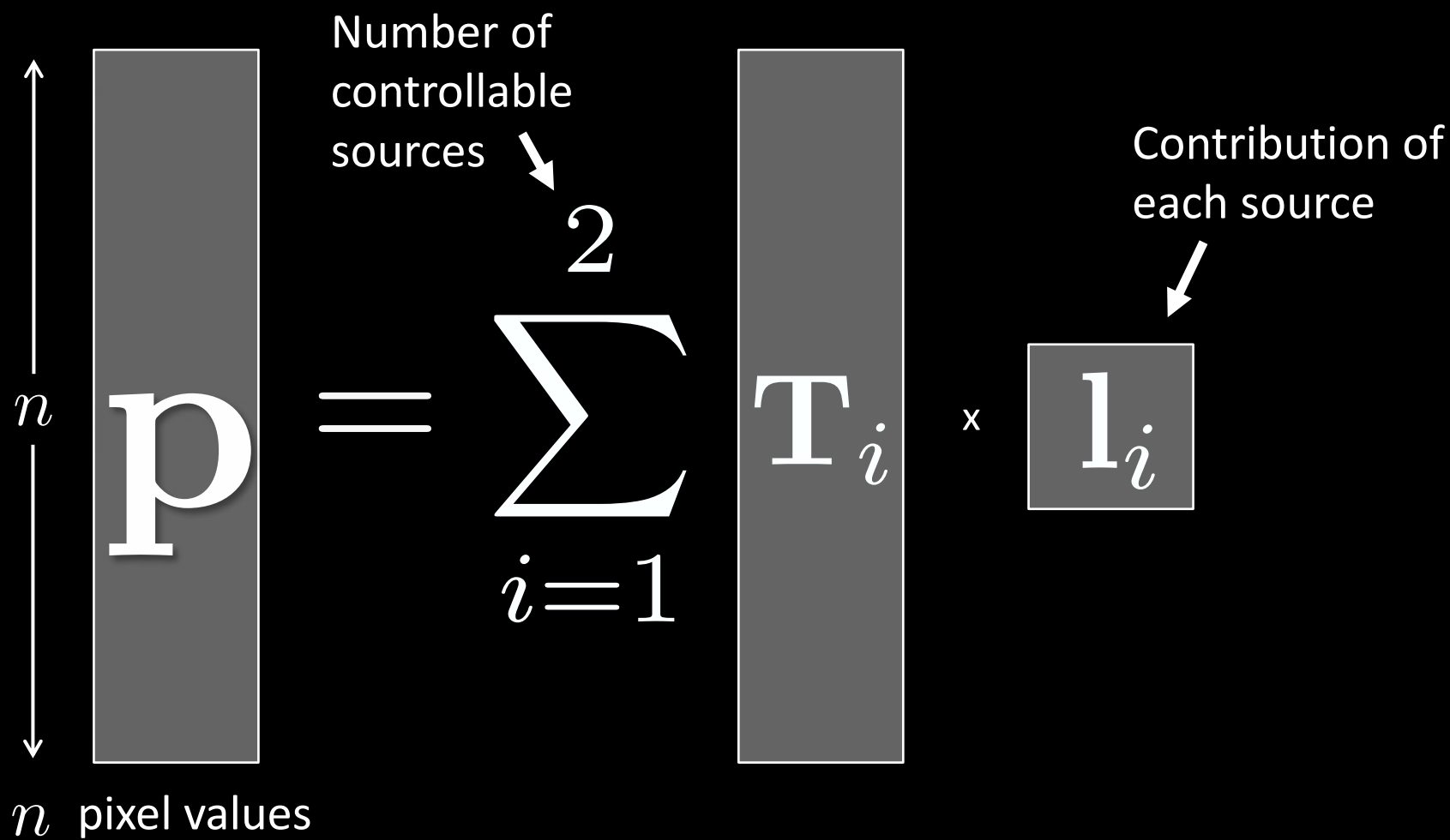
Weight 1

x \mathbf{l}_1 +



Weight 2

x \mathbf{l}_2





=



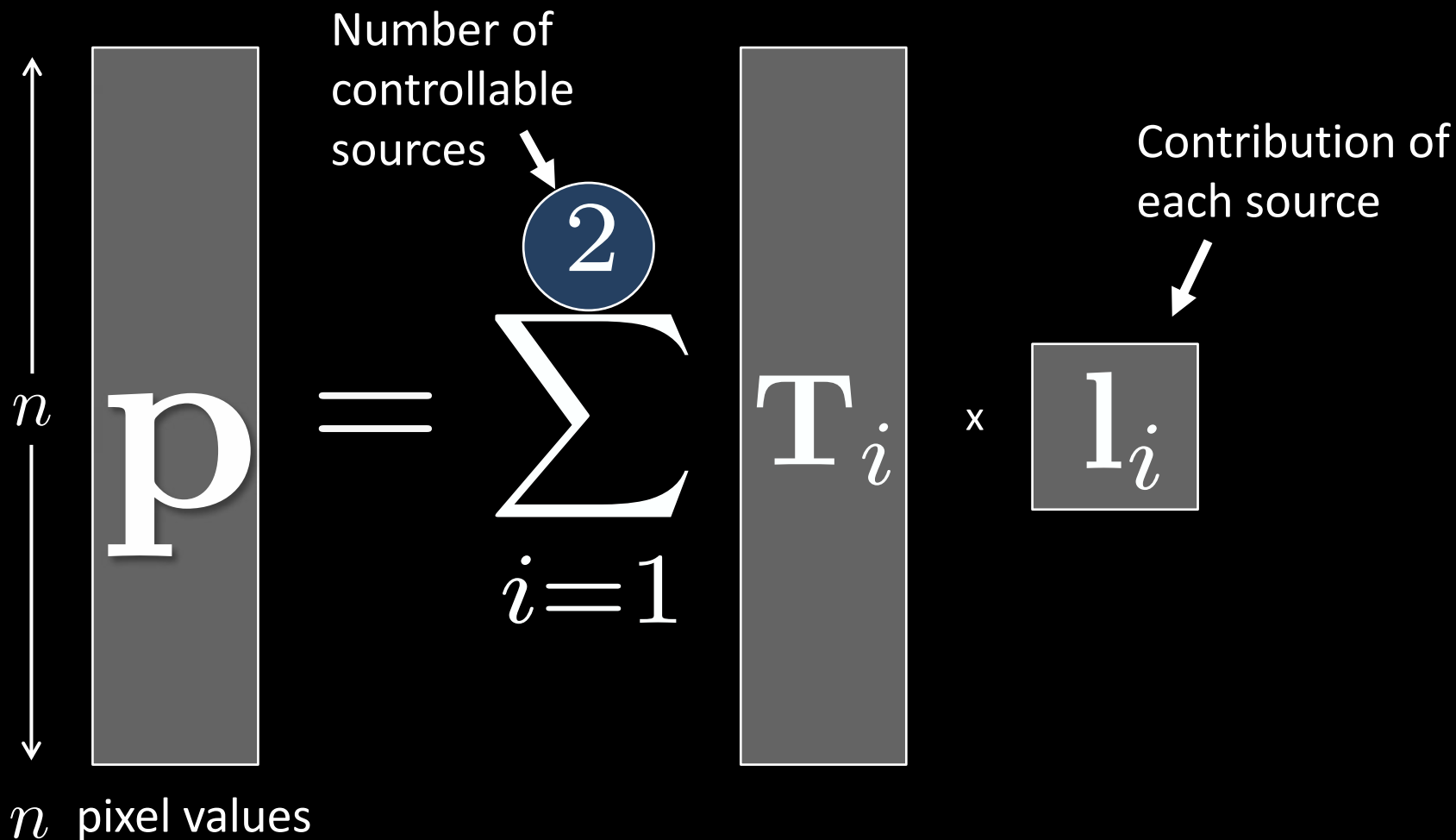
Weight 1

x \mathbf{l}_1 +



Weight 2

x \mathbf{l}_2





=



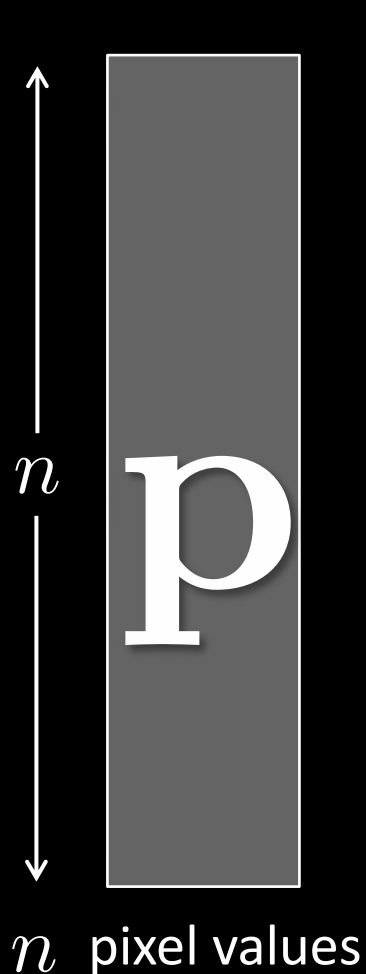
Weight 1

x \mathbf{l}_1 +



Weight 2

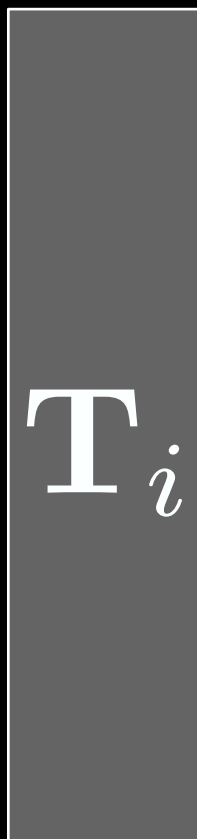
x \mathbf{l}_2



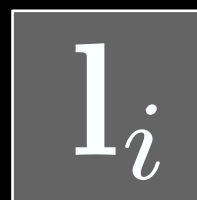
Number of controllable sources

m

$$= \sum_{i=1}^m$$



x





=



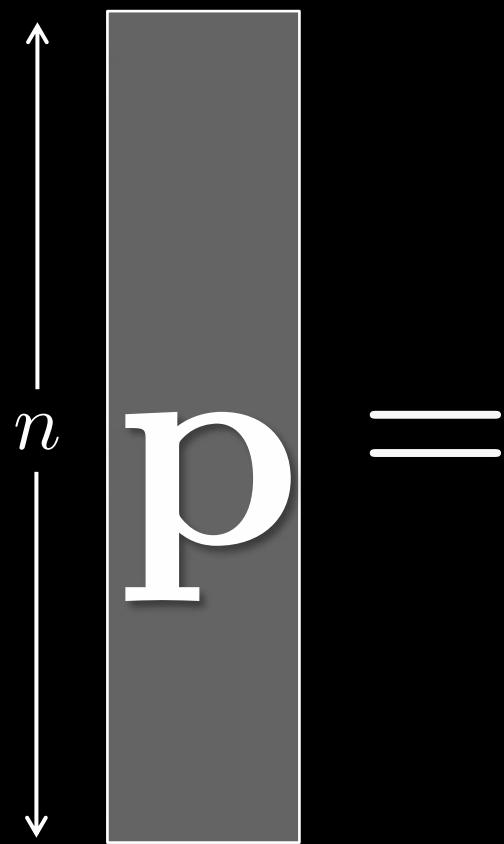
Weight 1

$\times \mathbf{l}_1 +$

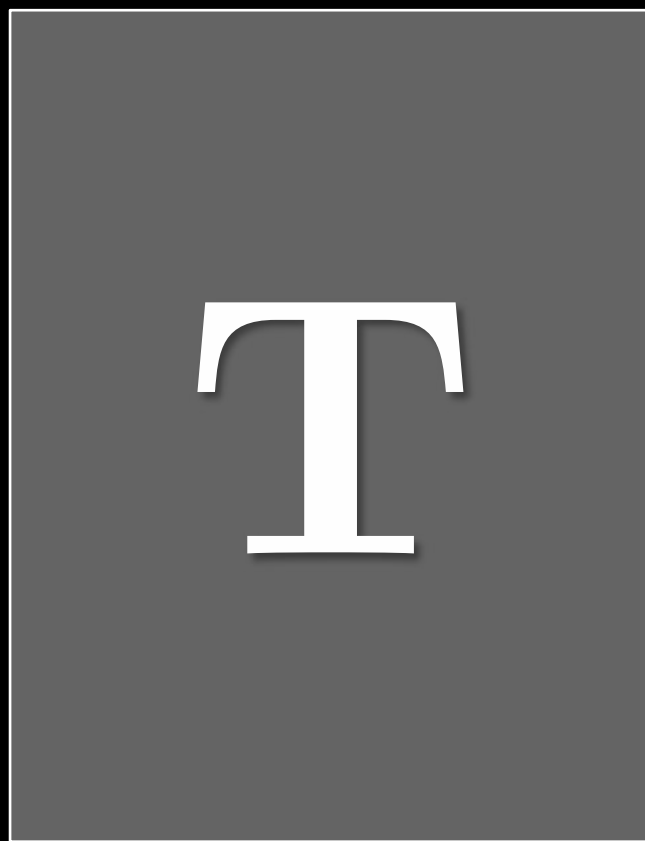


Weight 2

$\times \mathbf{l}_2$



n pixel values

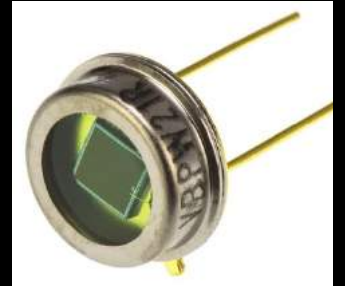
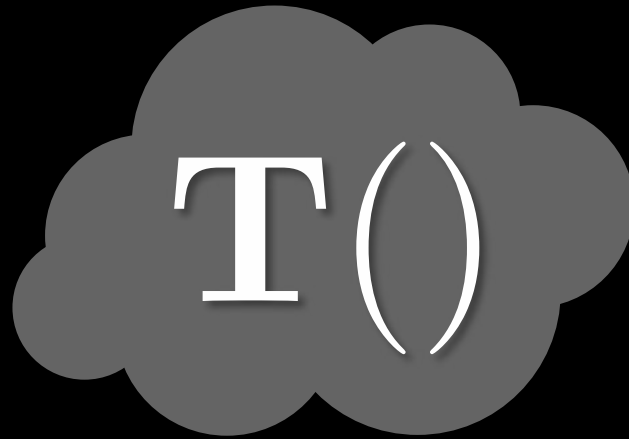


$n \times m$



m independent illumination degrees of freedom

modeling light transport with color



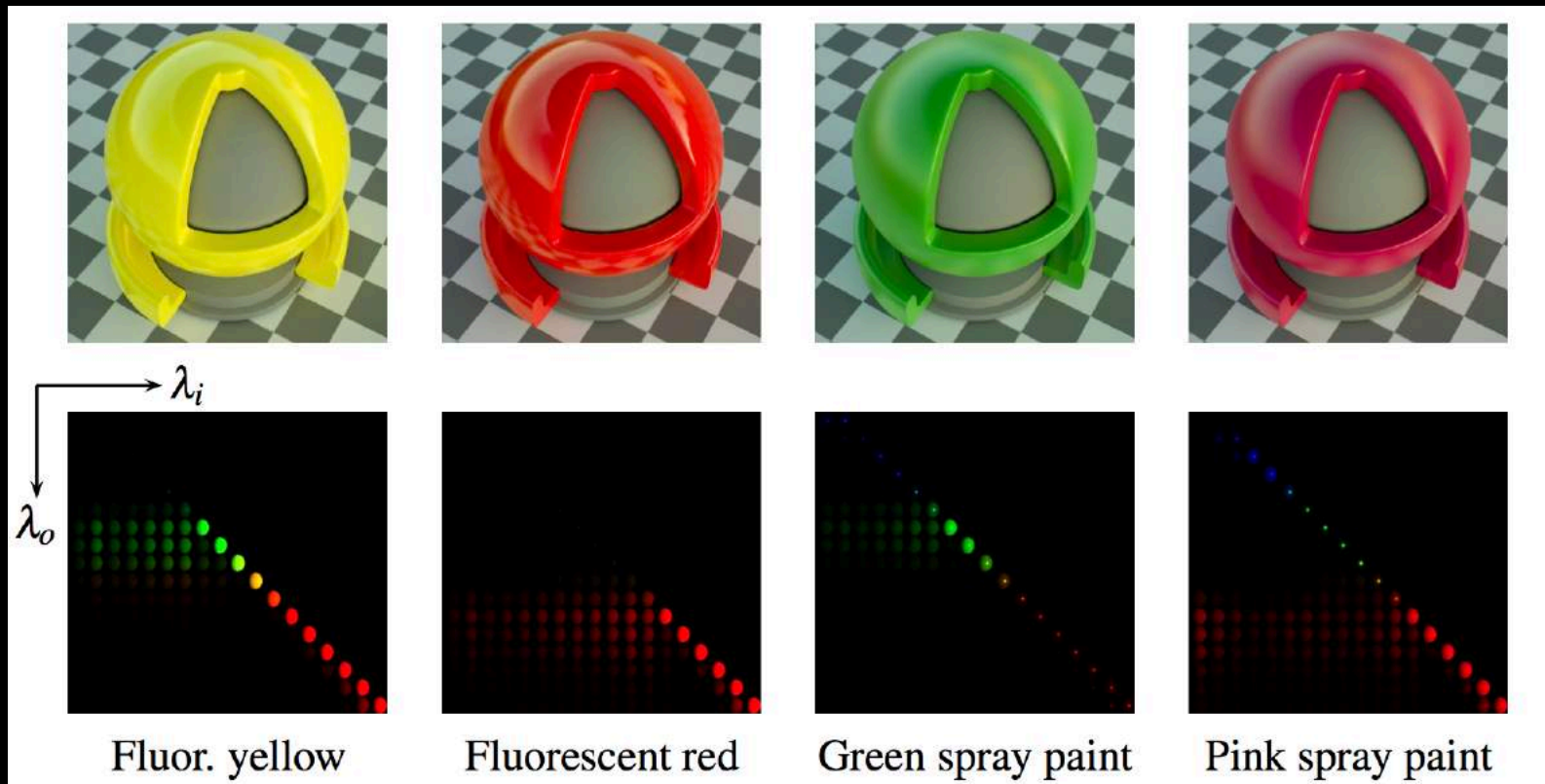
light source emitting
radiant energy x
(measured in Joules)

**with specific
wavelength w**

sensor detecting
radiant energy y
(measured in Joules)

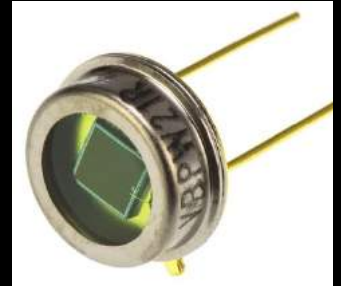
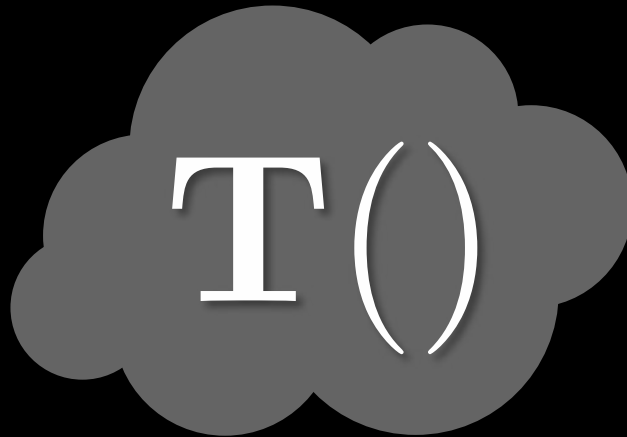
**with specific
wavelength w'**

bispectral BRDF / transport matrix



“Acquisition and Analysis of Bispectral Bidirectional Reflectance and Reradiation Distribution Functions”, Hullin et al. 2010

modeling light transport with polarization



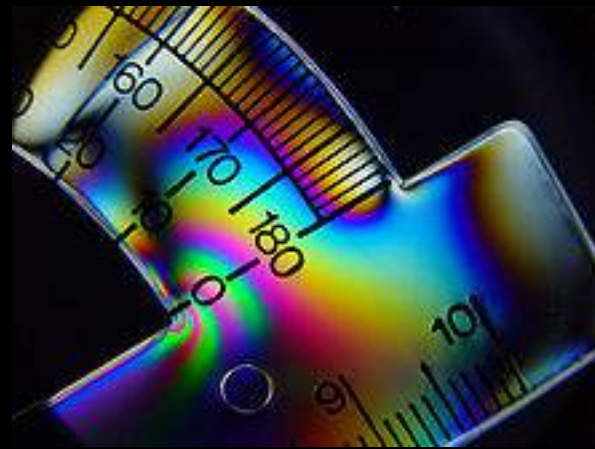
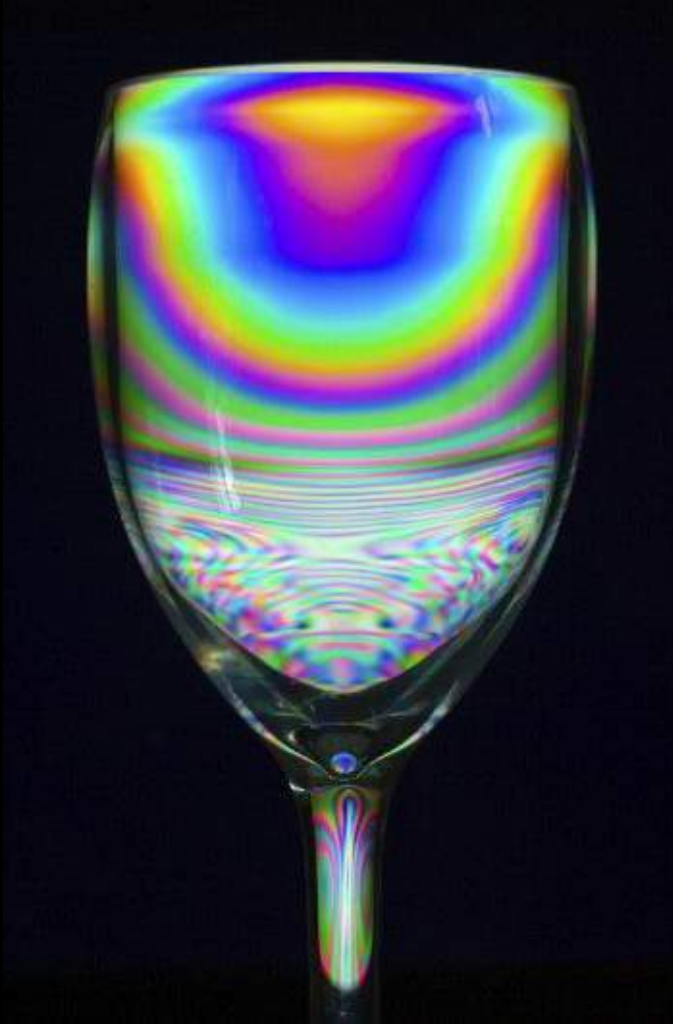
light source emitting
radiant energy x
(measured in Joules)

**with specific
polarization state**

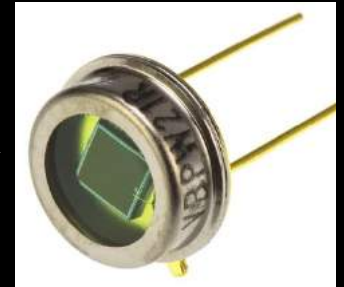
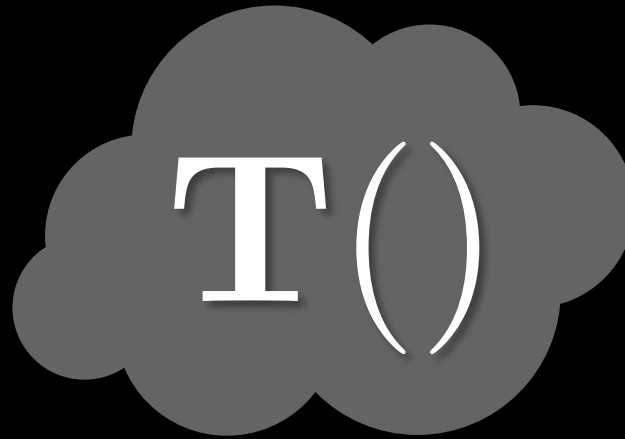
sensor detecting
radiant energy y
(measured in Joules)

**with specific
polarization state**

modeling light transport with polarization



modeling light transport with time



light source emitting
radiant energy x
(measured in Joules)

sensor detecting
radiant energy y
(measured in Joules)

at time t

at time t'

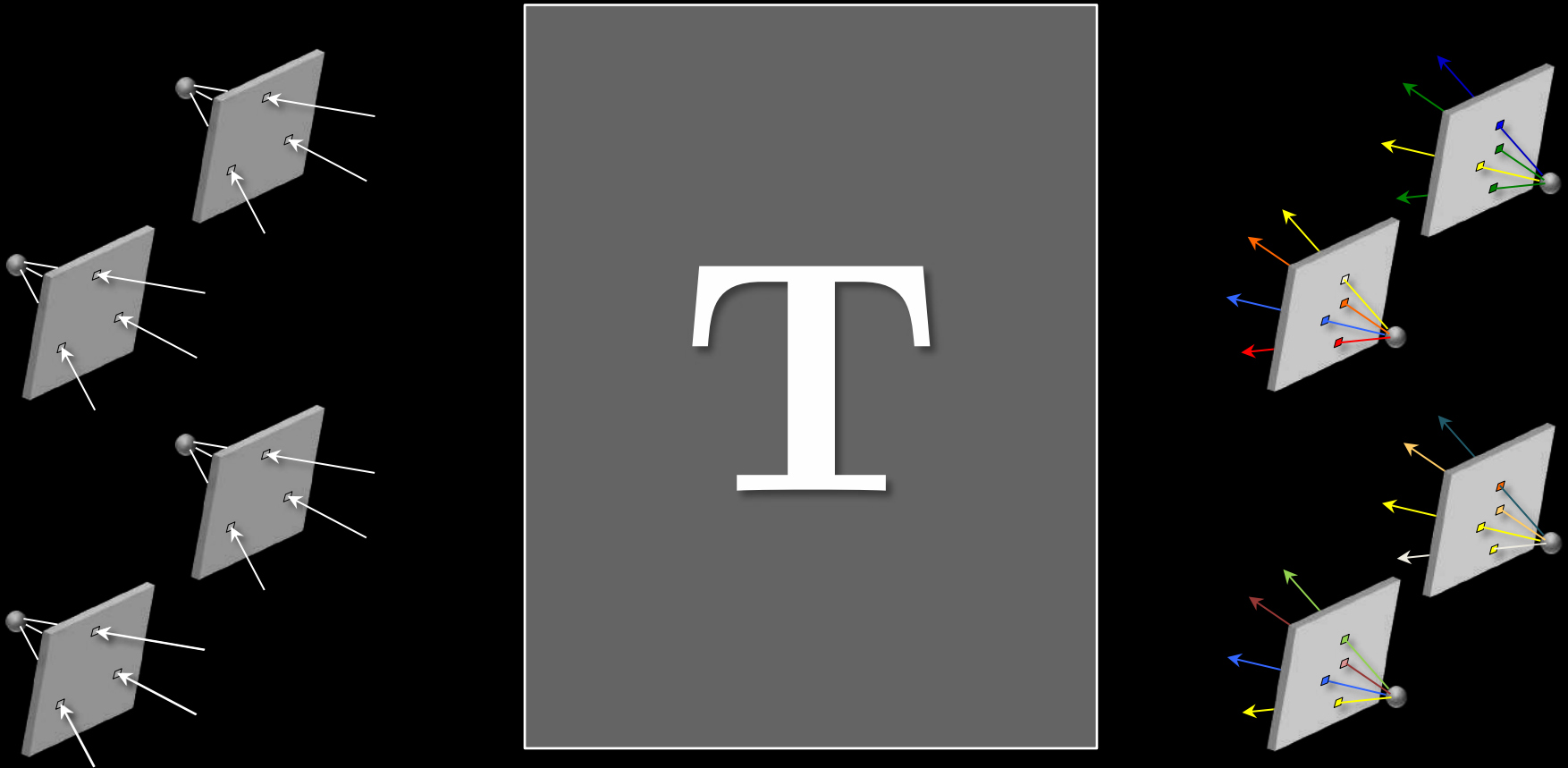
modeling light transport **with time**



[Raskar et al. 2011]

the light transport matrix

Sloan et al 02, Ng et al 03, Seitz et al 05, Sen et al 05, ...

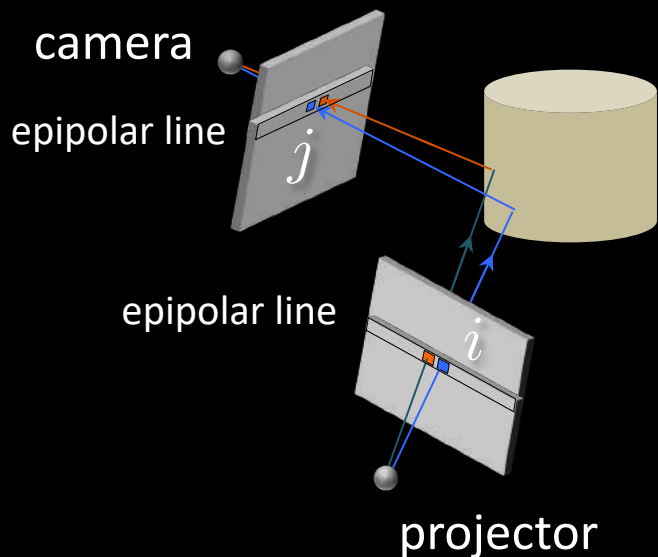


transport matrix represents the set of photos under
all possible (controllable) lighting conditions

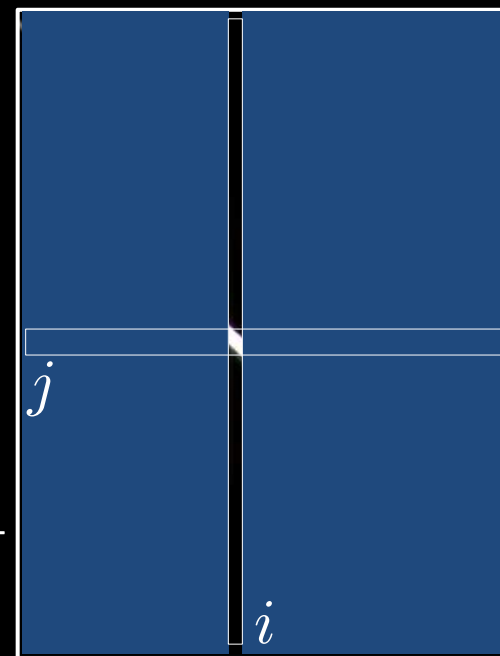
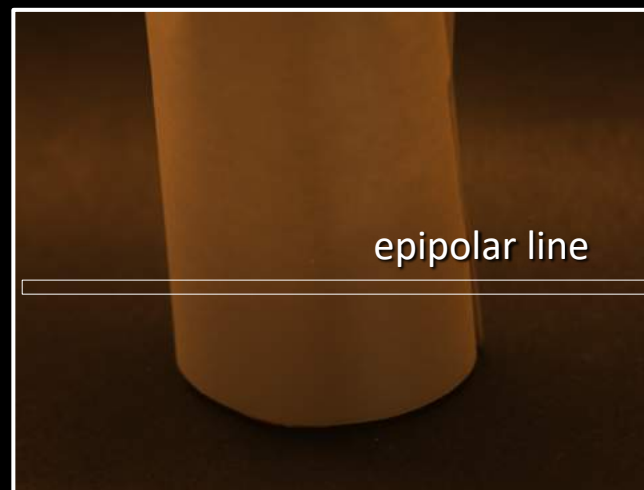
overview

1. **the light transport matrix:** a general model for the transfer of radiant energy
2. **example transport matrices for real scenes**
3. challenges associated with analyzing transport matrices
4. optical algorithms to analyze light transport

convex scene, diffuse reflectance, projector



camera view

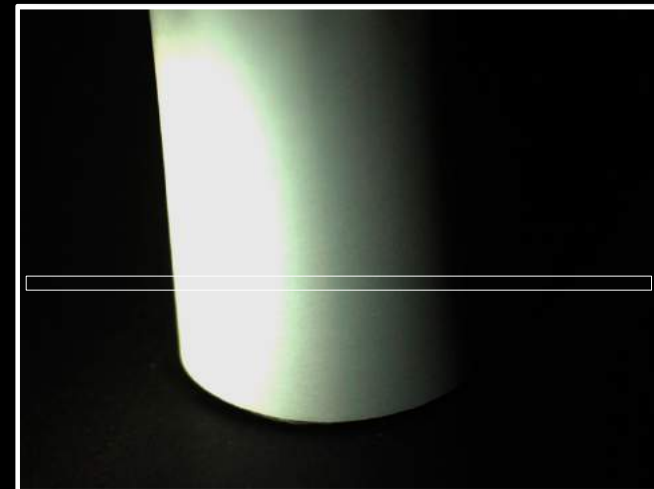
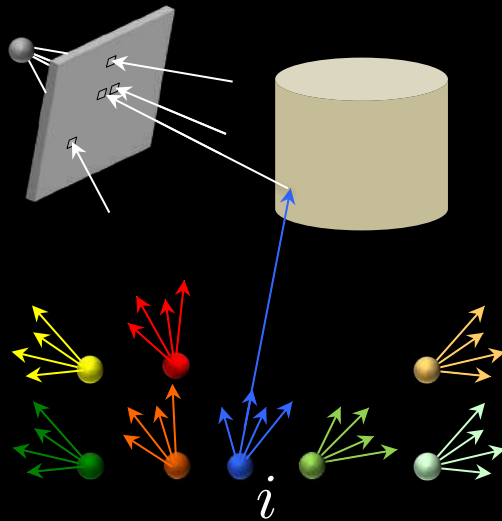


$T \Leftrightarrow$ stereo disparity map

acquiring $T \Leftrightarrow$
structured-light 3D scanning

T sparse, high rank

convex scene, diffuse reflectance, point sources



ambient source illumination

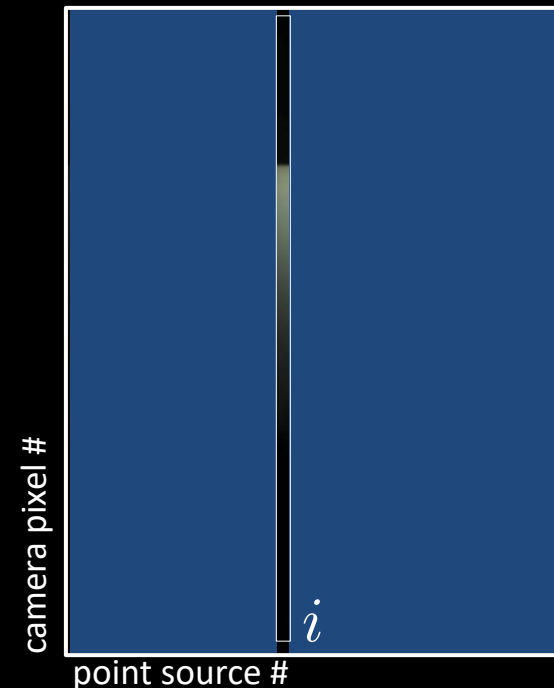
no shadows \Rightarrow

$$\text{rank}(\mathbf{T}) = 3 \text{ [Shashua, PhD 92]}$$

attached shadows \Rightarrow

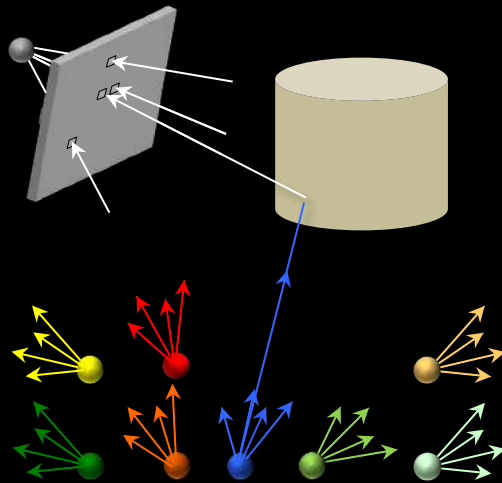
$$\text{rank}(\mathbf{T}) \approx 9 \text{ [Basri \& Jacobs, PAMI 01]}$$

analyzing $\mathbf{T} \Leftrightarrow$ photometric stereo



\mathbf{T} (for 1 image row)

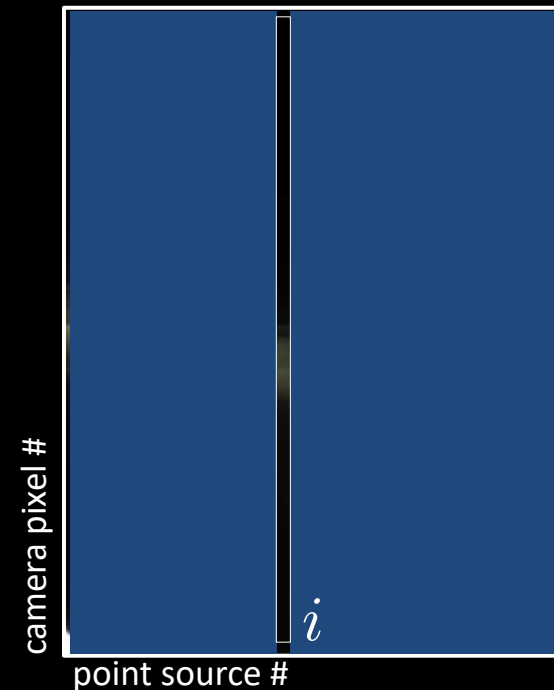
convex scene, specular reflectance, point sources



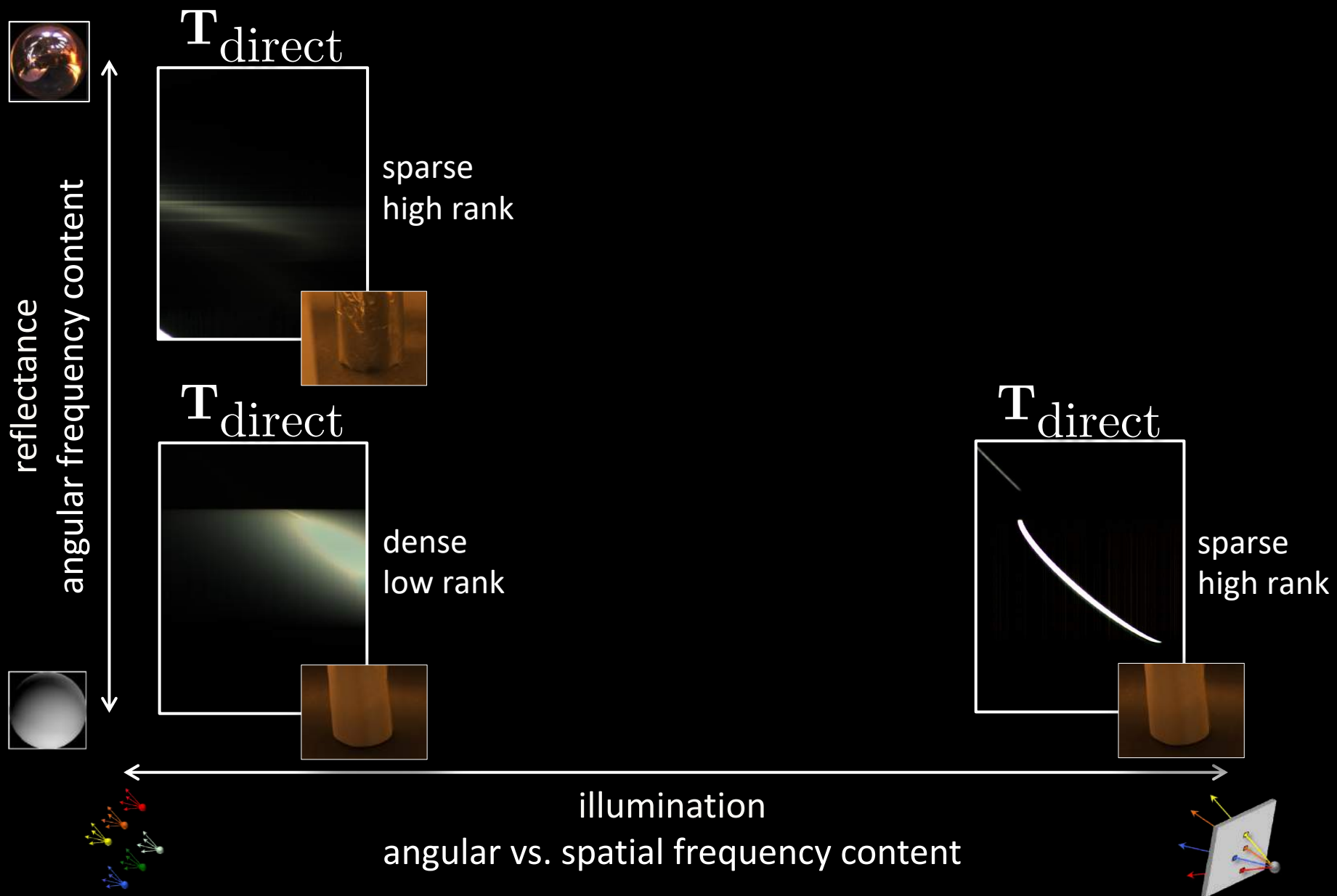
ambient source illumination

specular reflectance \Rightarrow
 \mathbf{T} can become full rank
[Ramamoorthi & Hanrahan, SIG 01]

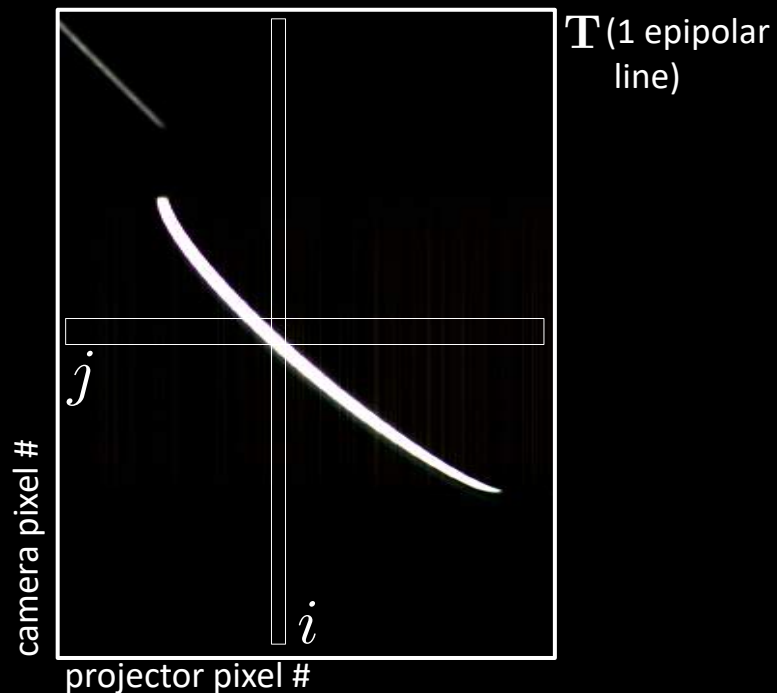
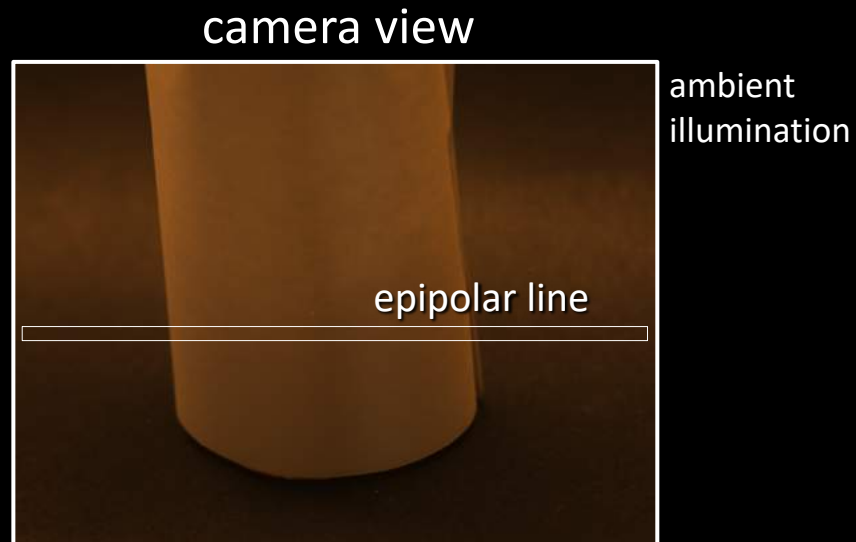
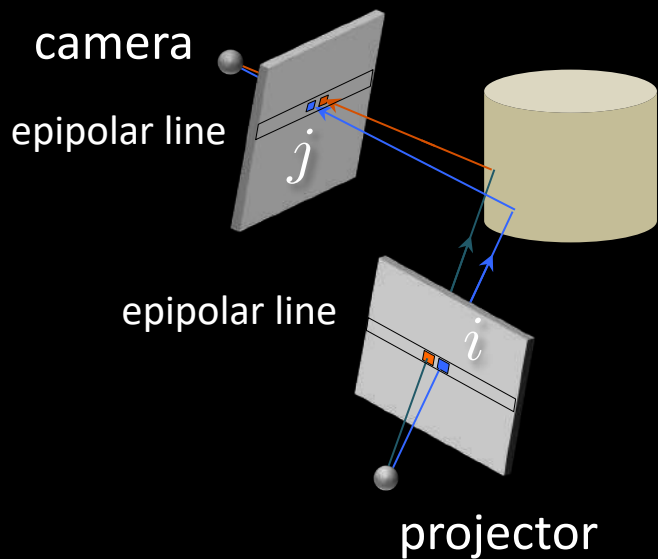
analyzing $\mathbf{T} \Leftrightarrow$
shape-from-specularities
[Sanderson et al, PAMI 89]



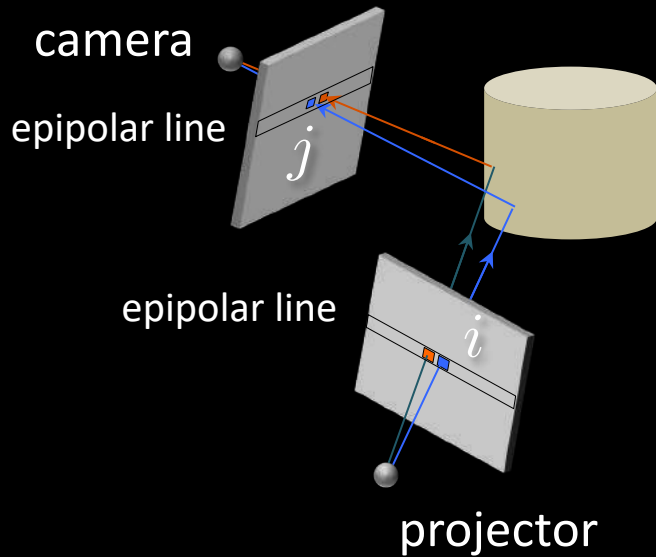
\mathbf{T} (for 1 image row)



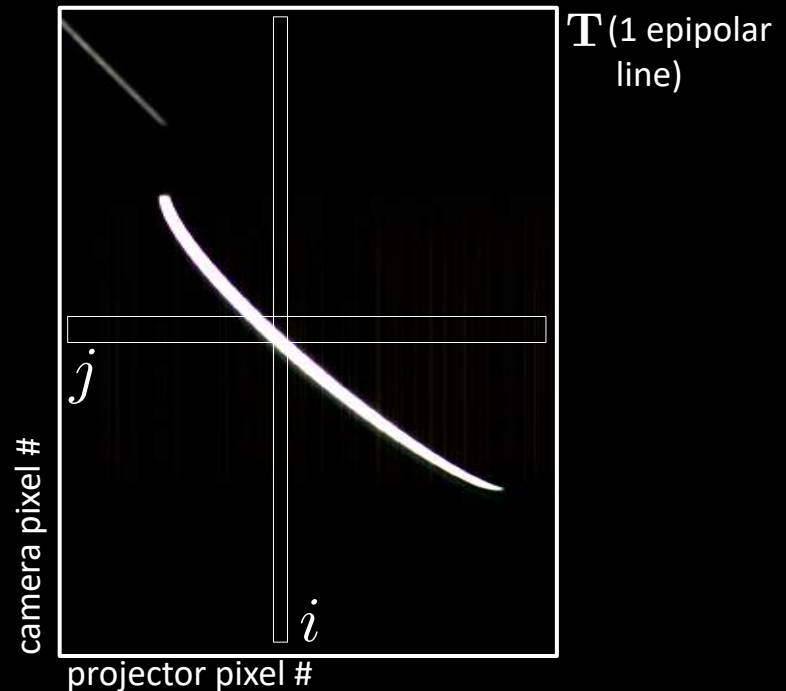
convex scene, diffuse reflectance, projector



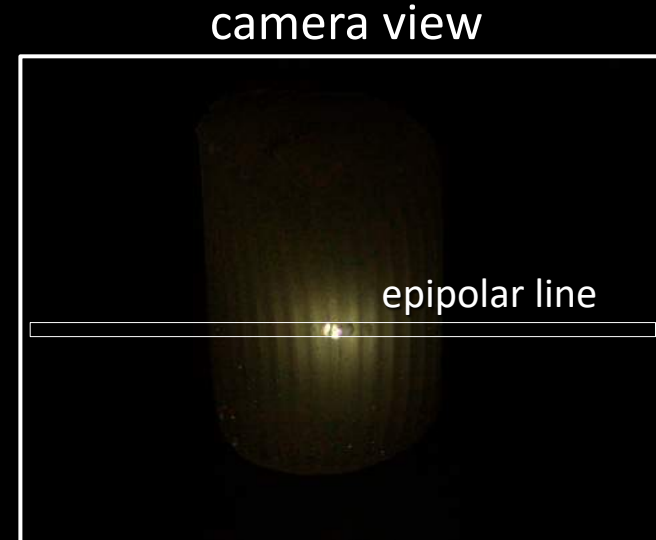
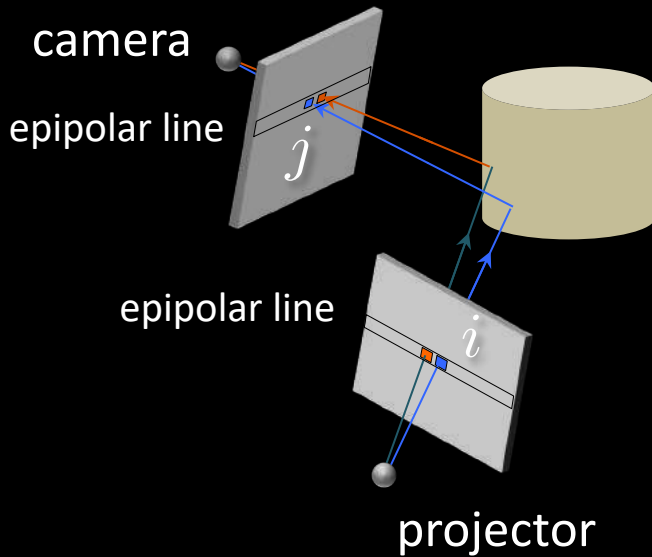
convex scene, translucency, projector



camera view



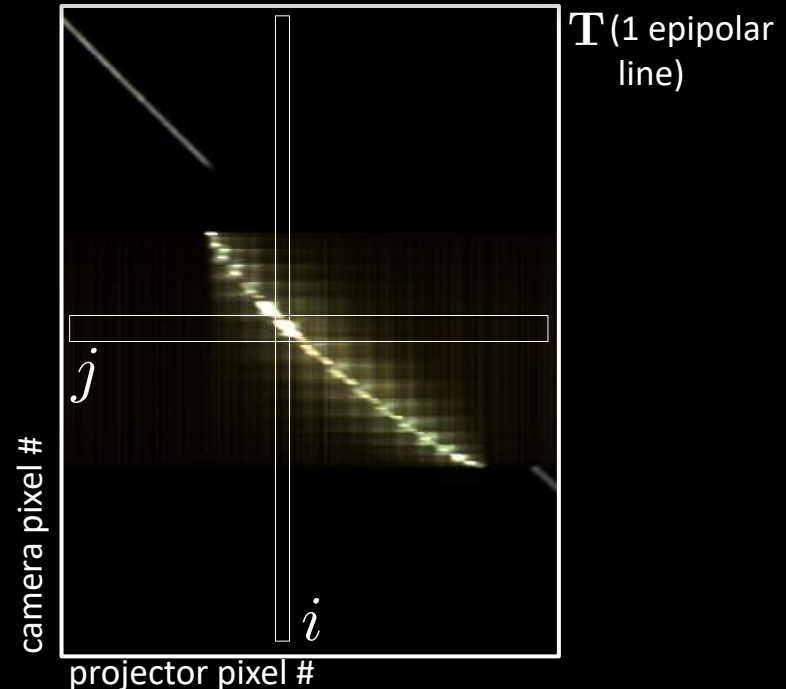
convex scene, translucency, projector



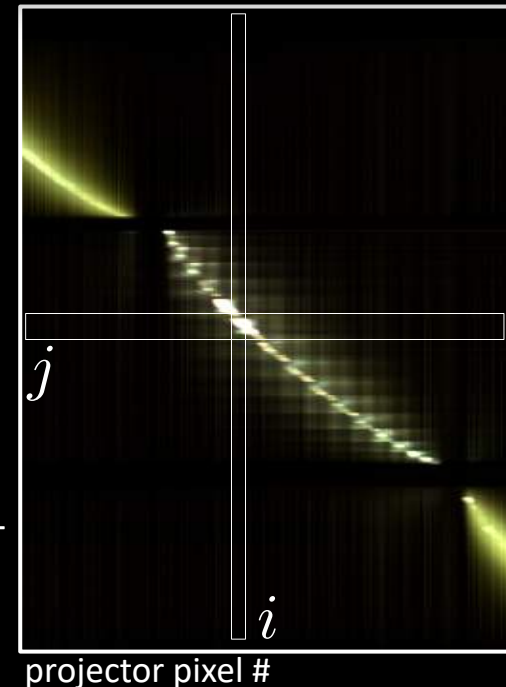
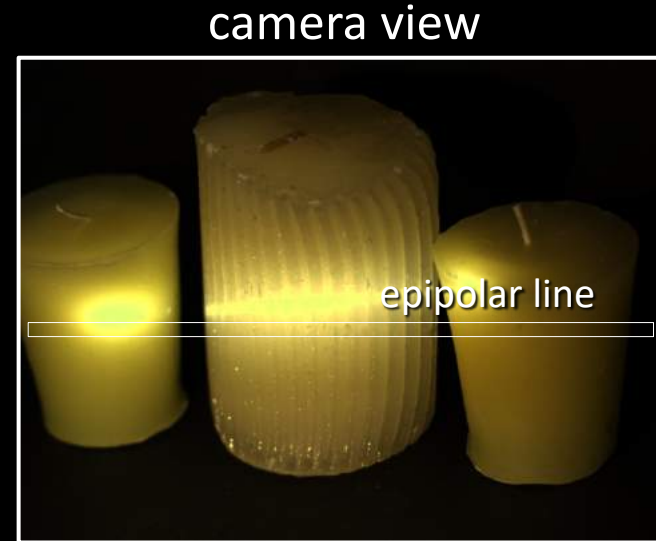
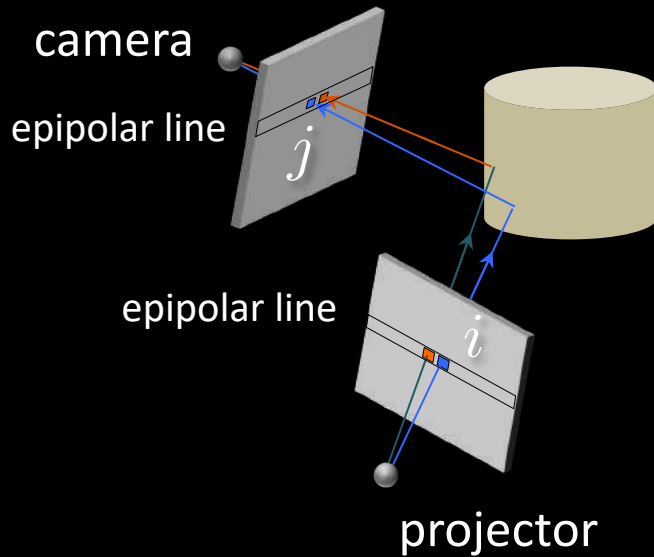
$$\mathbf{T} = \mathbf{T}_{\text{direct}} + \mathbf{T}_{\text{indirect}}$$

recovering $\mathbf{T}_{\text{direct}} \Leftrightarrow$
structured-light 3D scanning

\mathbf{T} less sparse, usually high rank



convex scene, translucency, projector

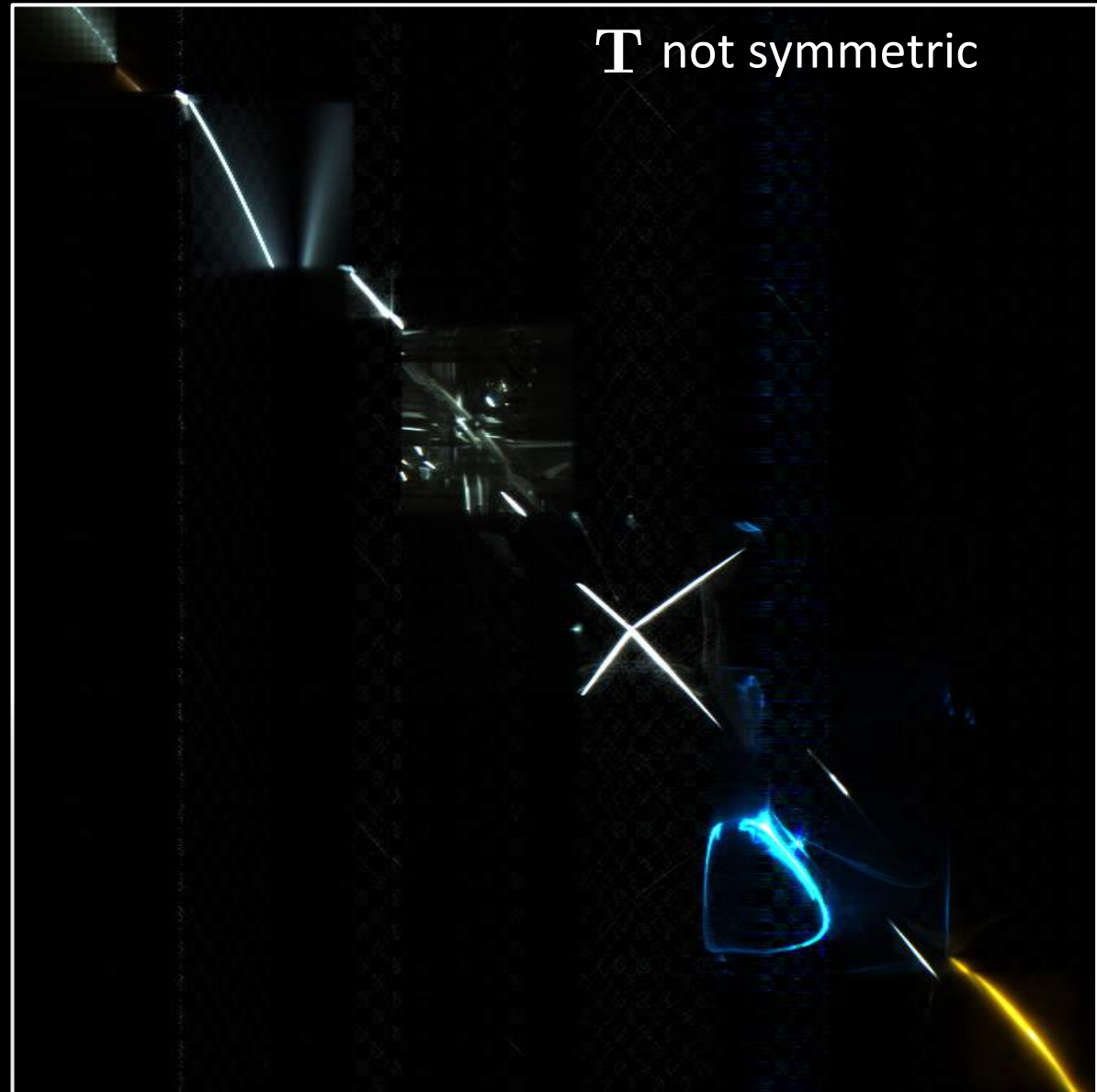
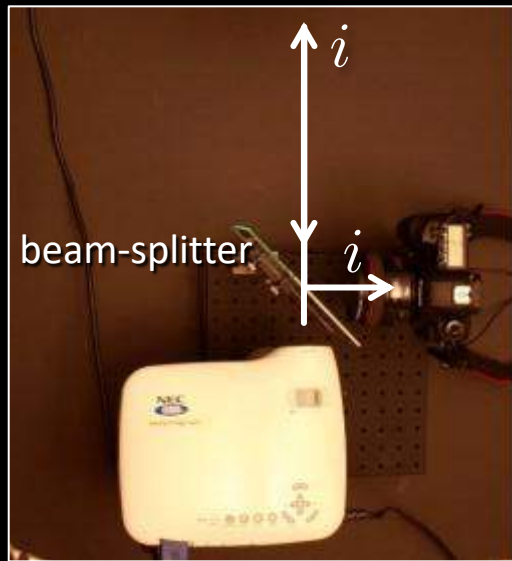


$$\mathbf{T} = \mathbf{T}_{\text{direct}} + \mathbf{T}_{\text{indirect}}$$

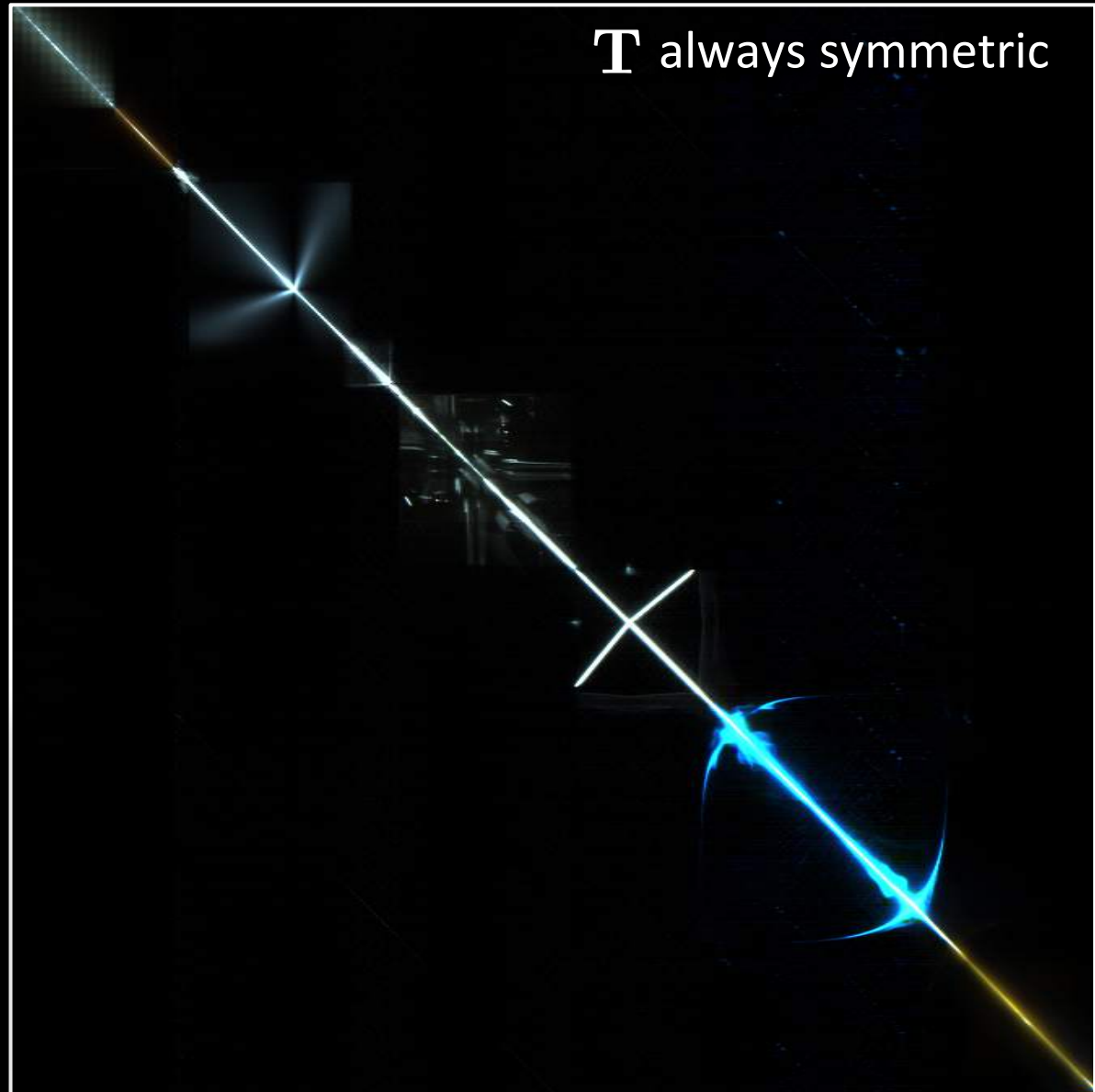
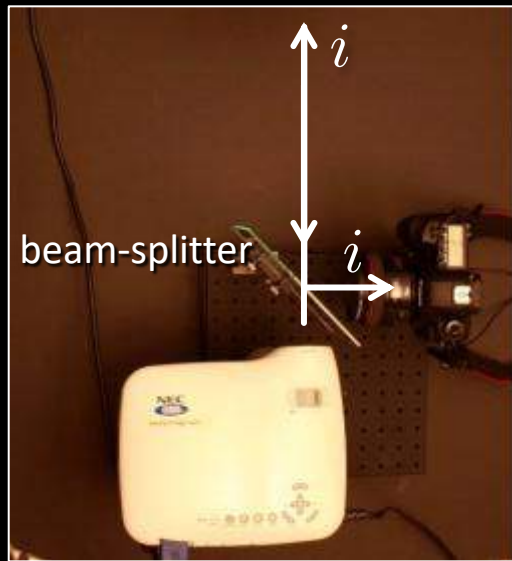
recovering $\mathbf{T}_{\text{direct}} \Leftrightarrow$
structured-light 3D scanning

\mathbf{T} less sparse, usually high rank

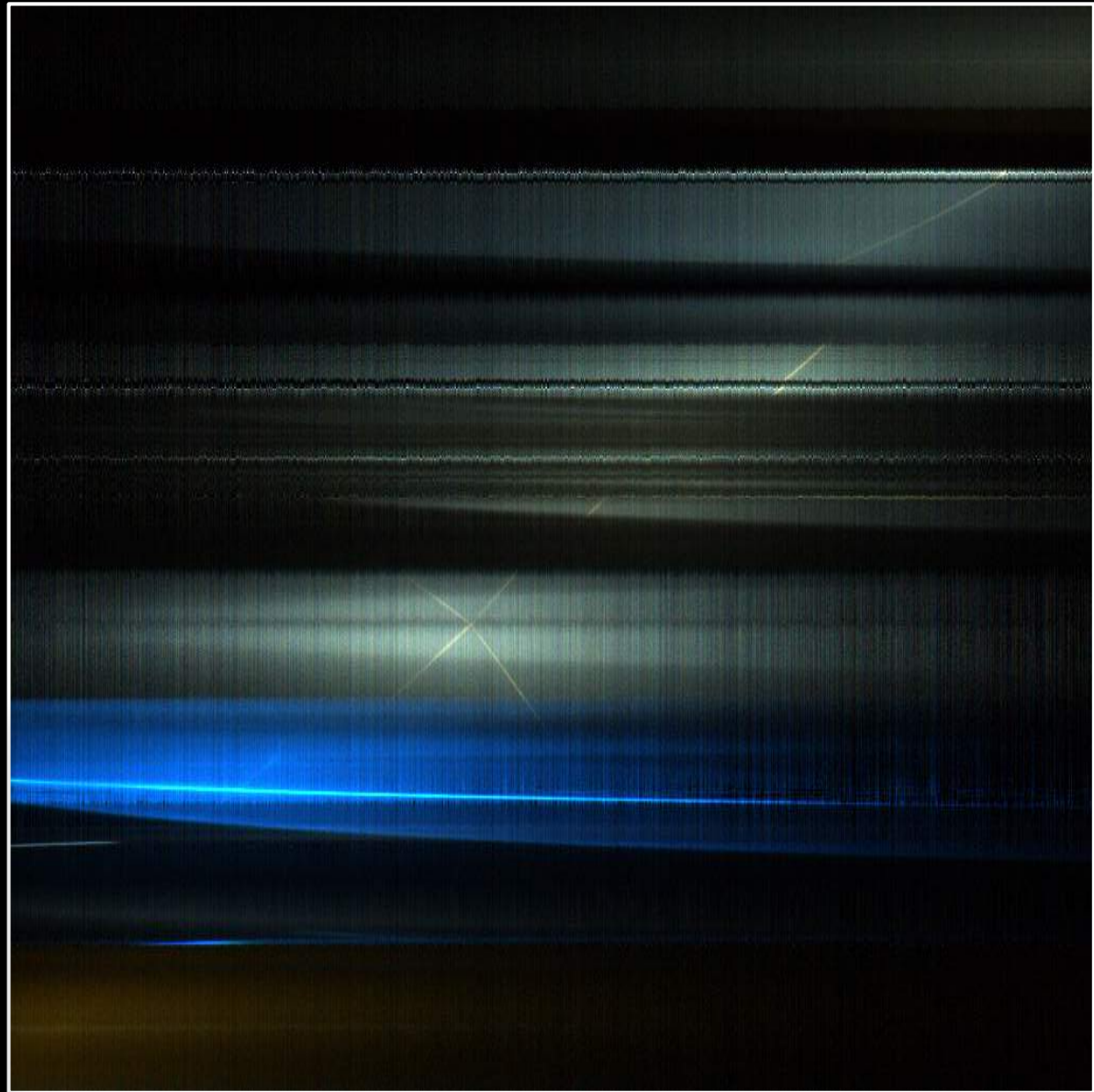
general scene, projector



general scene, coaxial projector & camera



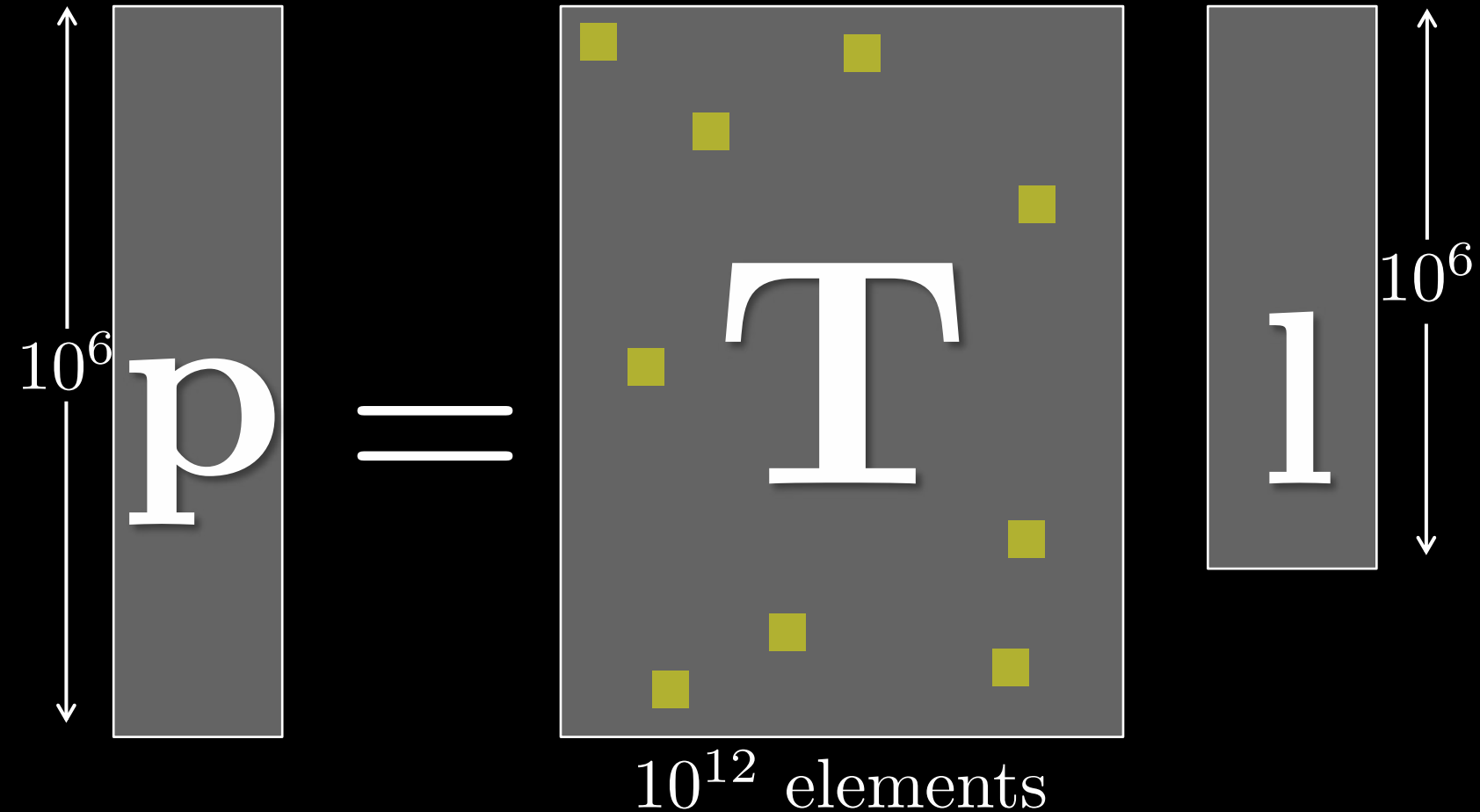
general scene, array of point sources



overview

1. **the light transport matrix:** a general model for the transfer of radiant energy
2. example transport matrices for real scenes
3. **challenges associated with analyzing transport matrices**
4. optical algorithms to analyze light transport

basic matrix properties



unknown & extremely large
no random access to its elements
relation to scene geometry & reflectance can be complex

overview

1. **the light transport matrix:** a general model for the transfer of radiant energy
2. example transport matrices for real scenes
3. challenges associated with analyzing transport matrices
4. **optical algorithms to analyze light transport**

Computing with Light

Key idea: analyze the transport matrix by implementing iterative numerical algorithms directly in optics

numerical domain

$$\begin{array}{c} \text{transport} \\ \text{matrix} \\ \downarrow \\ \mathbf{p} = \mathbf{T} \mathbf{l} \\ \begin{array}{cc} \uparrow & \uparrow \\ \text{photo} & \text{illumination} \\ & \text{pattern} \end{array} \end{array}$$



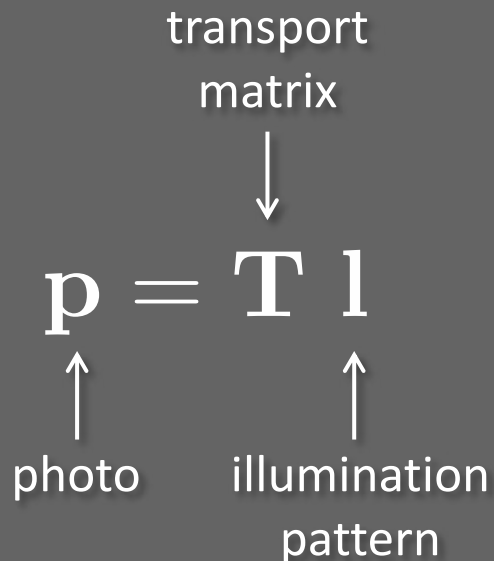
optical domain



Computing with Light

Key idea: analyze the transport matrix by implementing iterative numerical algorithms directly in optics

numerical domain



optical domain



Computing with Light

Key idea: analyze the transport matrix by implementing iterative numerical algorithms directly in optics

numerical domain

```
function analyze(T)
```

```
...
```

```
for  $i = 1$  to  $k$  {
```

```
...
```

$$\mathbf{p}_i = \mathbf{T} \mathbf{l}_i$$

```
...
```

$$\mathbf{d}_i = \mathbf{T} \mathbf{r}_i$$

```
...
```

```
}
```

```
...
```

```
return result
```



optical domain



Computing with Light

Key idea: analyze the transport matrix by implementing iterative numerical algorithms directly in optics

numerical domain

```
function analyze(T)
```

...

```
for  $i = 1$  to  $k$  {
```

...

```
p $i$  = Tl $i$ 
```

...

```
d $i$  = Tr $i$ 
```

...

```
}
```

...

```
return result
```



optical domain

```
function analyze()
```

...

```
for  $i = 1$  to  $k$  {
```

...

```
project l $i$ , capture p $i$ 
```

...

```
project r $i$ , capture d $i$ 
```

...

```
}
```

...

```
return result
```

Computing Transport Eigenvectors



Eigenvector of a square matrix T
when projected onto scene,
we get the same photo back
(multiplied by a scalar)



Numerical goal

find \mathbf{l} , λ such that

$$T\mathbf{l} = \lambda\mathbf{l}$$

and λ is maximal

Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$

numerical domain

function PowerIt(\mathbf{T})

$\mathbf{l}_1 =$ initial vector

for $i = 1$ to k {

$\mathbf{p}_i = \mathbf{T}\mathbf{l}_i$

$\mathbf{l}_{i+1} = \mathbf{p}_i / \|\mathbf{p}_i\|_2$

}

return \mathbf{l}_{i+1}

Properties

- linear convergence [Trefethen and Bau 1997]
- eigenvalues must be distinct
- \mathbf{l}_1 cannot be orthogonal to principal eigenvector

Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

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numerical domain

function PowerIt(\mathbf{T})

$\mathbf{l}_1 =$ initial vector

for $i = 1$ to k {

$\mathbf{p}_i = \mathbf{T}\mathbf{l}_i$

$\mathbf{l}_{i+1} = \mathbf{p}_i / \|\mathbf{p}_i\|_2$

}

return \mathbf{l}_{i+1}



optical domain

function PowerIt()

$\mathbf{l}_1 =$ initial vector

for $i = 1$ to k {

project \mathbf{l}_i , capture \mathbf{p}_i

$\mathbf{l}_{i+1} = \mathbf{p}_i / \|\mathbf{p}_i\|_2$

}

return \mathbf{l}_{i+1}

Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$

numerical domain

function $\text{PowerIt}(\mathbf{T})$

$\mathbf{l}_1 =$ initial vector

for $i = 1$ to k {

$$\mathbf{p}_i = \mathbf{T}\mathbf{l}_i$$

$$\mathbf{l}_{i+1} = \mathbf{p}_i / \|\mathbf{p}_i\|_2$$

}

return \mathbf{l}_{i+1}



optical domain

initialize \mathbf{l}_1

\mathbf{l}_i

project

$\mathbf{T}\mathbf{l}_i$

capture

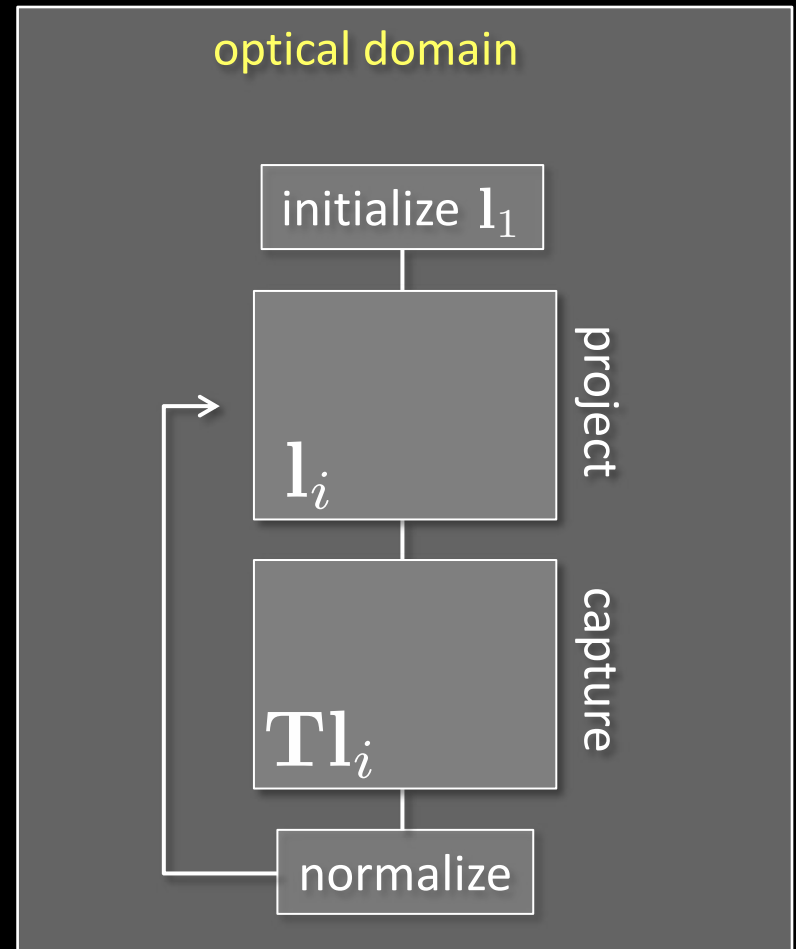
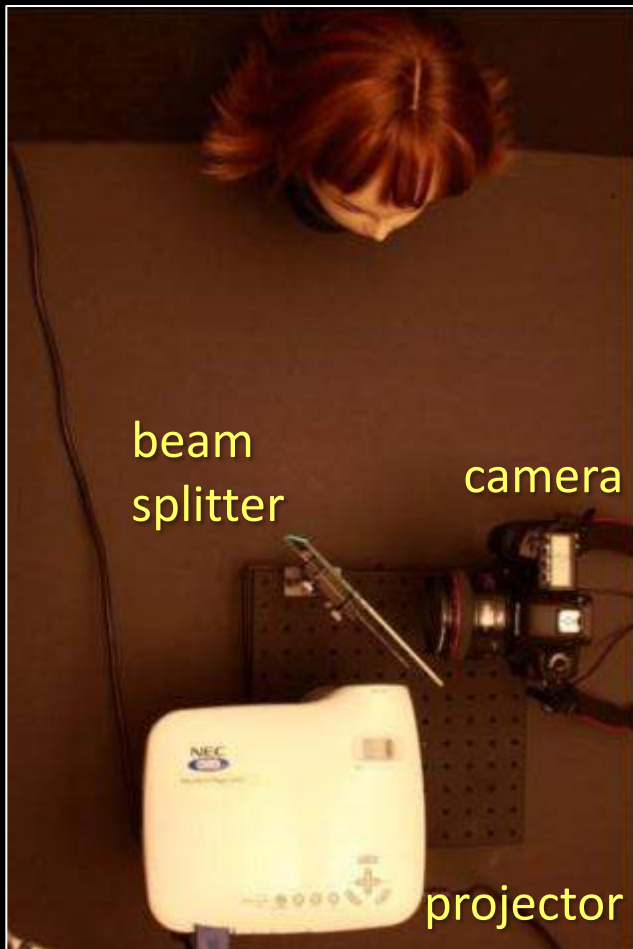
normalize



Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

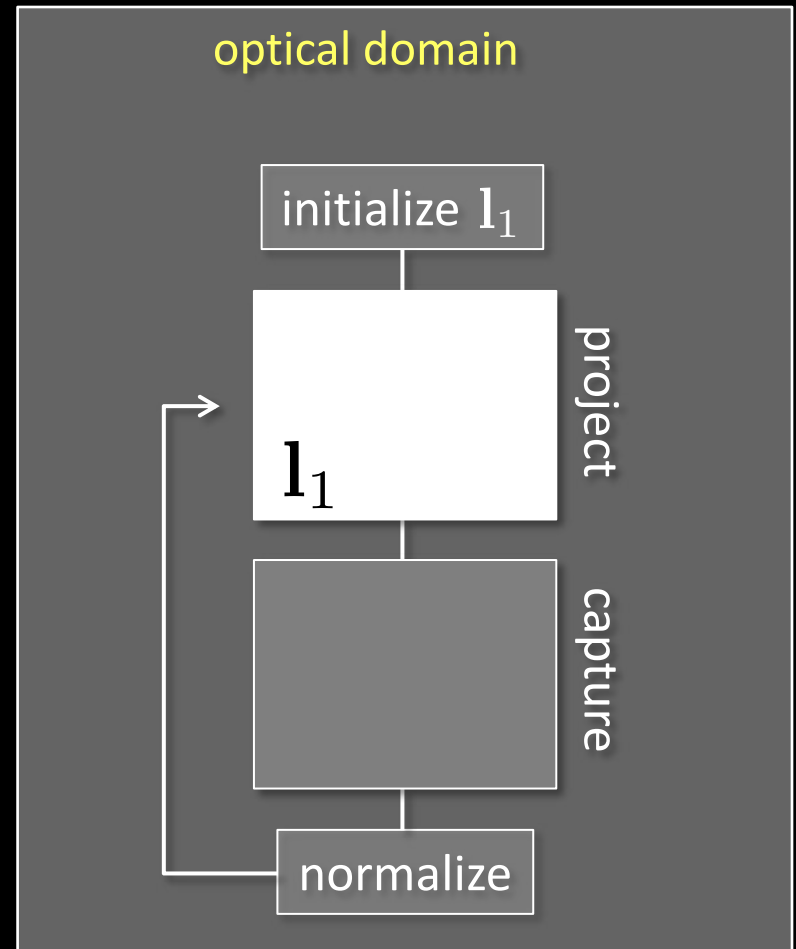
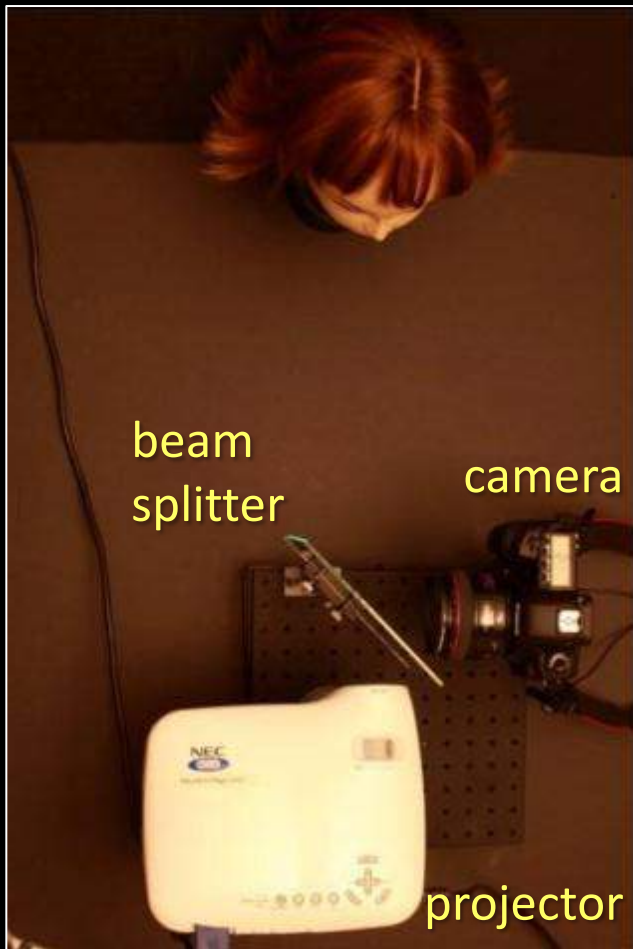
Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$



Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

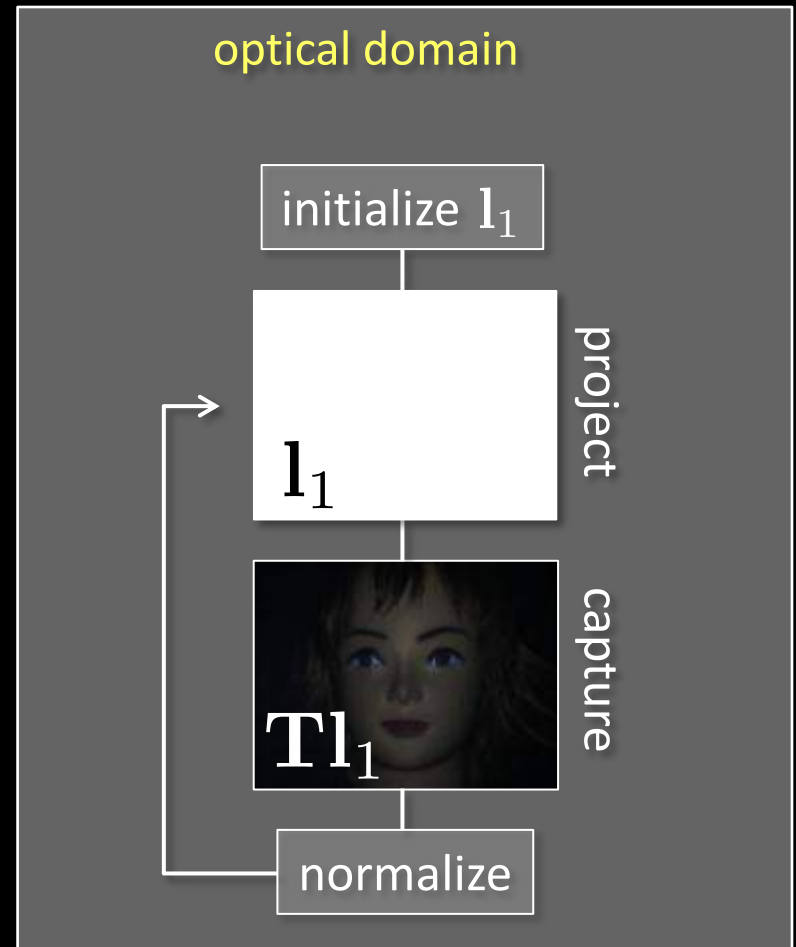
Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$



Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

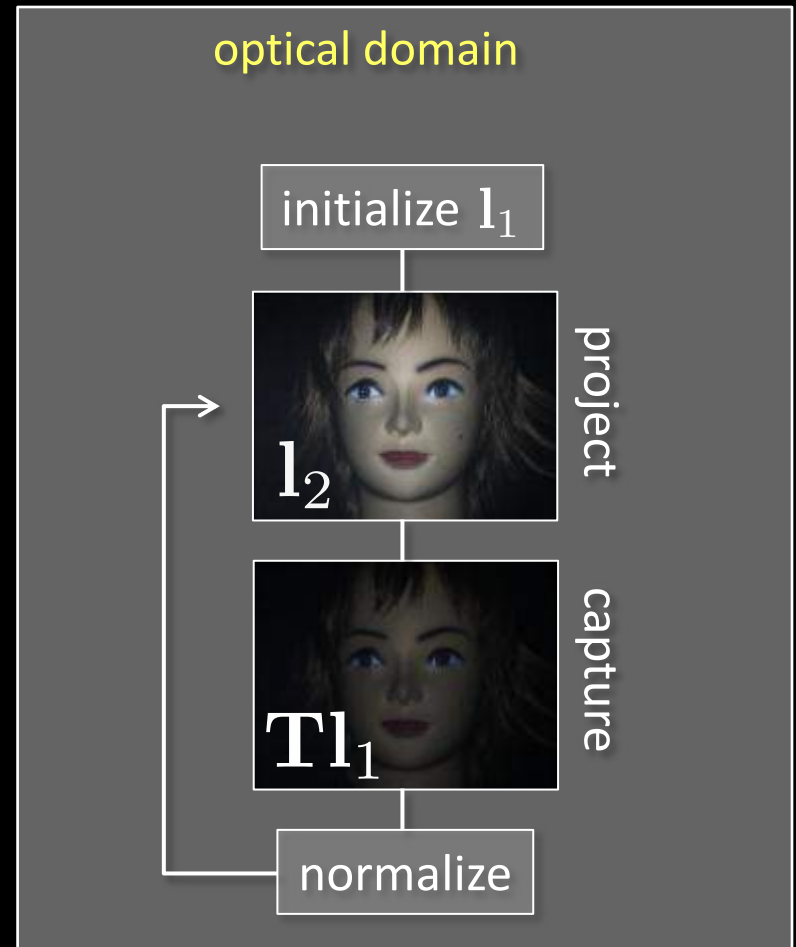
Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$



Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

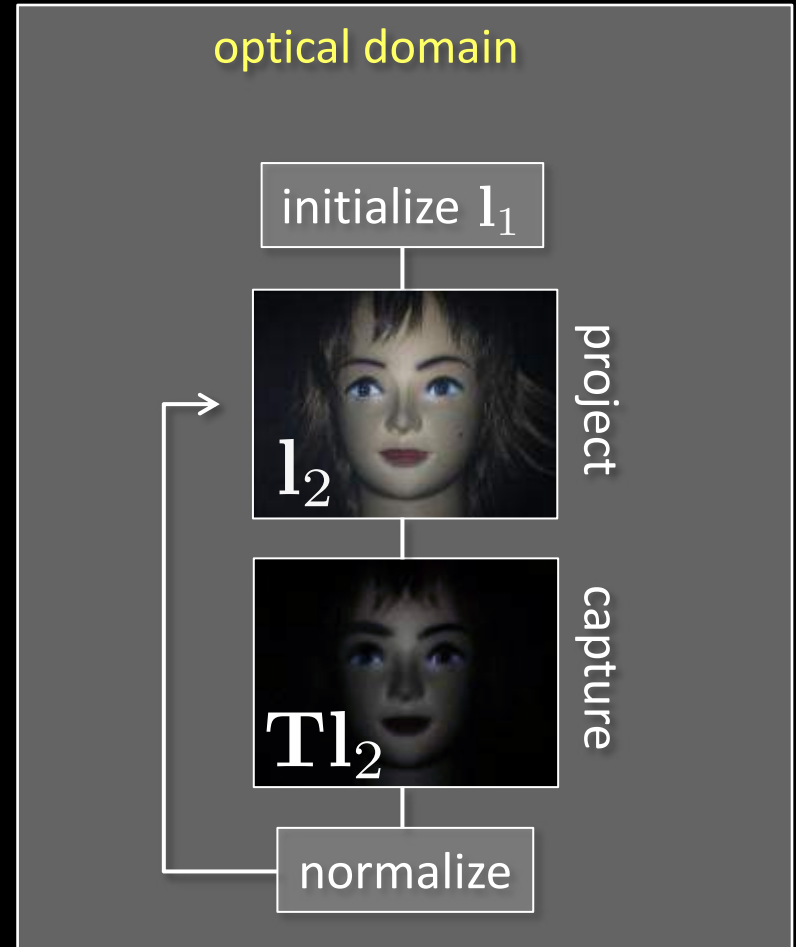
Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$



Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

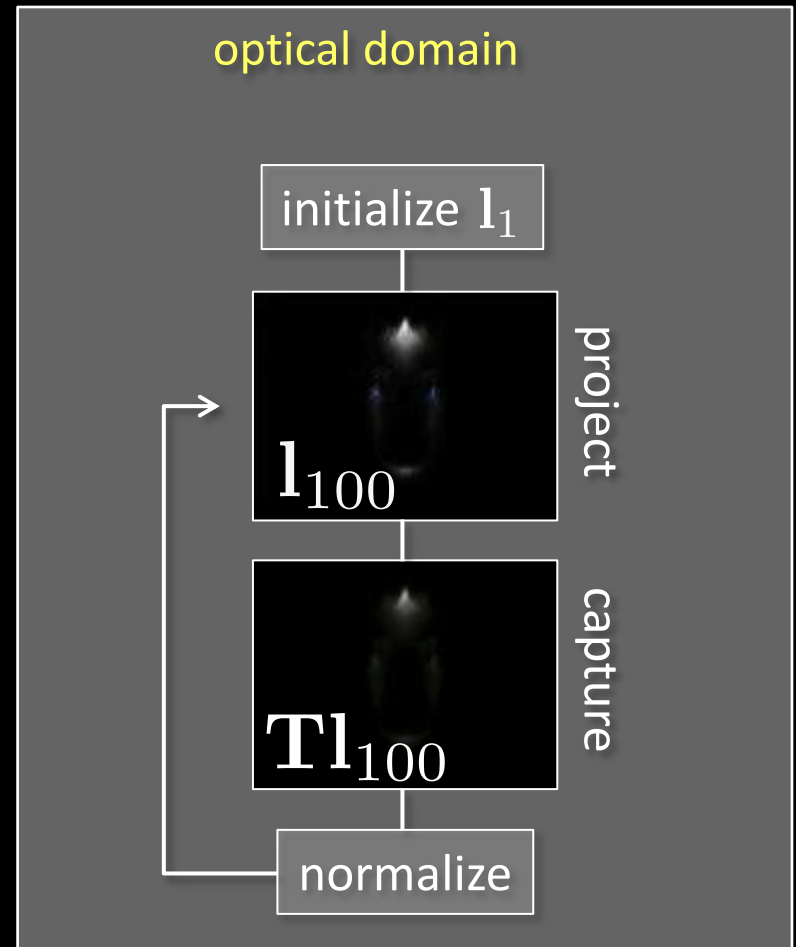
Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$



Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

Observation: it is a fixed point of the sequence $\mathbf{l}, \mathbf{T}\mathbf{l}, \mathbf{T}^2\mathbf{l}, \mathbf{T}^3\mathbf{l}, \dots$



Optical Power Iteration

Goal: find principal eigenvector of \mathbf{T}

Observation: it is a fixed point of the sequence $\mathbf{1}, \mathbf{T}\mathbf{1}, \mathbf{T}^2\mathbf{1}, \mathbf{T}^3\mathbf{1}, \dots$

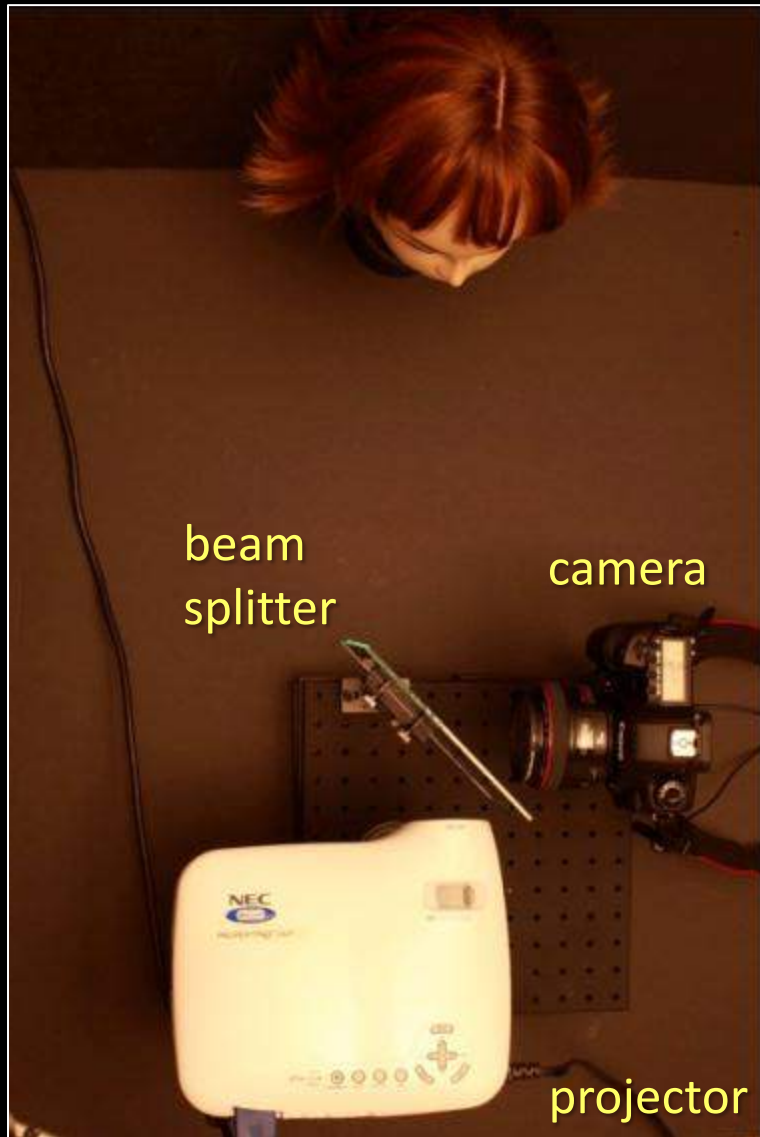


optical domain

(approximate)
principal eigenvector



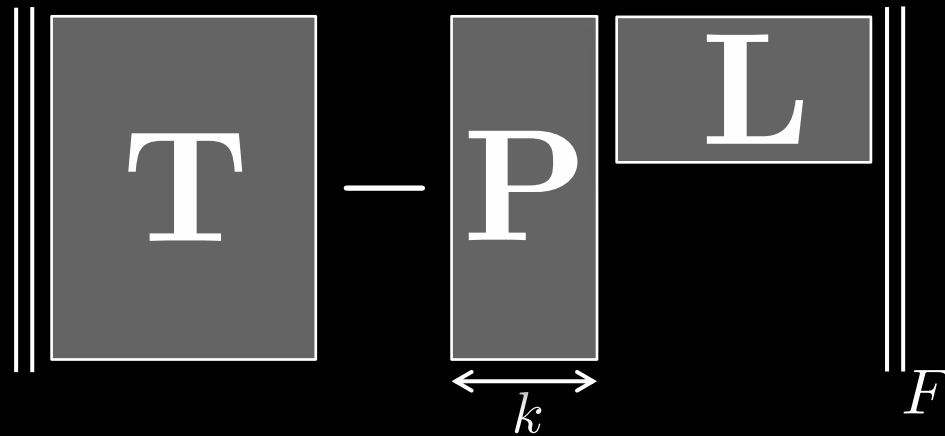
Rank-k Transport Approximation



Numerical goal [Simon and Zha 2000]

find matrices
that minimize

$$\mathbf{P}_{n \times k}, \mathbf{L}_{k \times m}$$



Symmetric \mathbf{T}

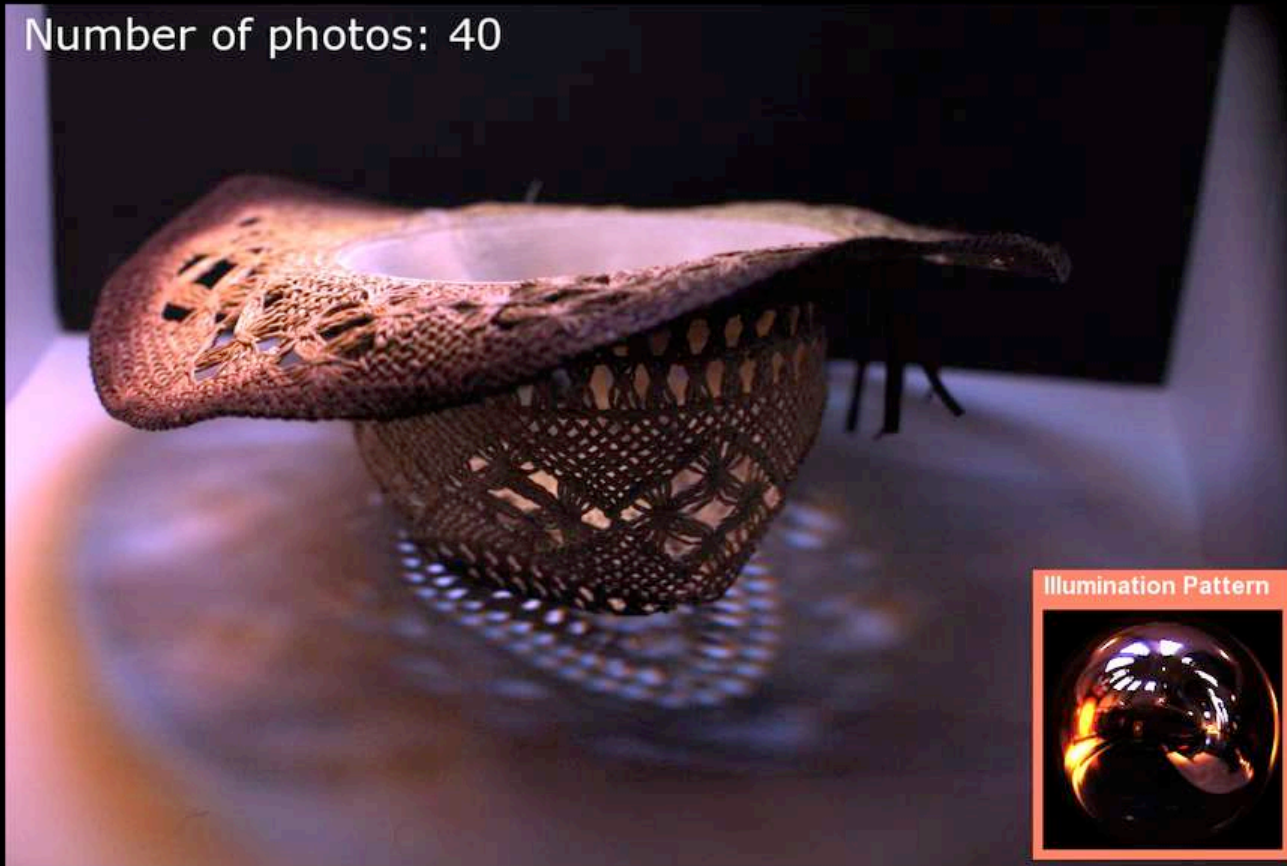
- 1 camera, 1 projector
- $2k$ photos for rank- k approx.

Nonsymmetric \mathbf{T}

- 2 cameras, 2 projectors
- $4k$ photos for rank- k approx.

Results: Optical Arnoldi

Number of photos: 40



concluding remarks

- the light transport matrix is a general model for describing the transfer or radiant energy
- the entries of a transport matrix describes all possible observations one can make of a scene
- transport matrix is often too large to measure directly in practice
- numerical algorithms can be partially or fully implemented in the optical domain