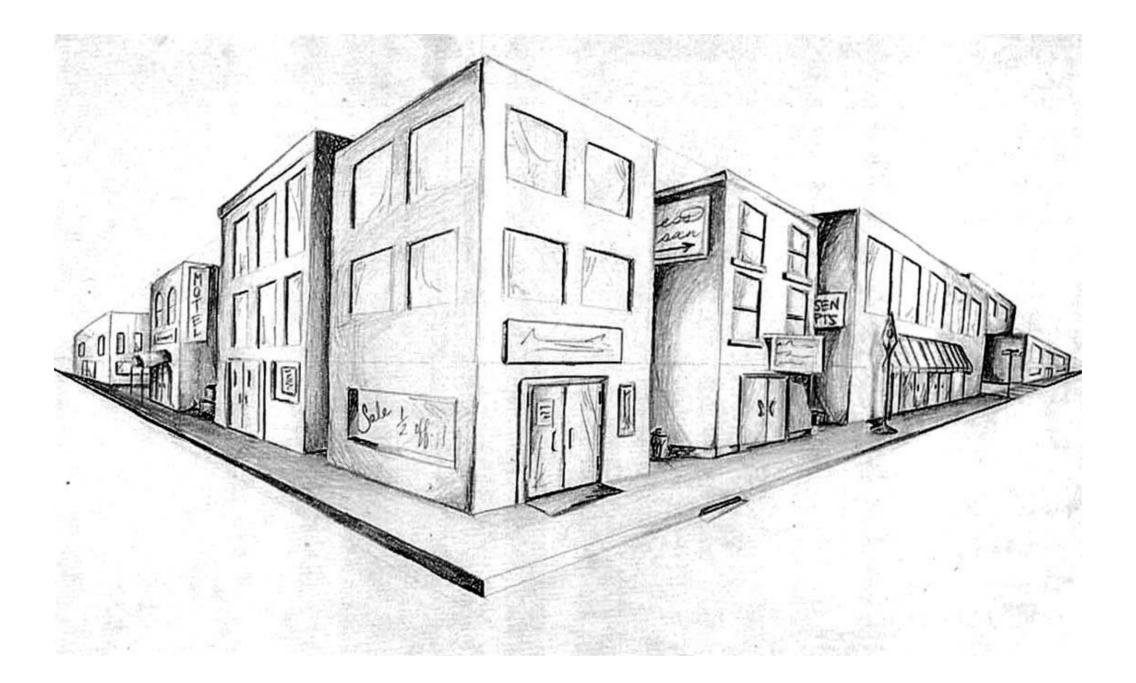
Detecting corners



http://16385.courses.cs.cmu.edu/

16-385 Computer Vision Fall 2022, Lecture 5

Overview of today's lecture

- Why detect corners?
- Visualizing quadratics.
- Harris corner detector.
- Multi-scale detection.
- Multi-scale blob detection.

Slide credits

Most of these slides were adapted from:

• Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

- Fredo Durand (MIT).
- James Hays (Georgia Tech).

Why detect corners?

Why detect corners?

Image alignment (homography, fundamental matrix)

3D reconstruction

Motion tracking

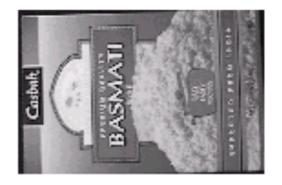
Object recognition

Indexing and database retrieval

Robot navigation

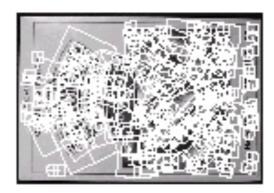
Planar object instance recognition

Database of planar objects

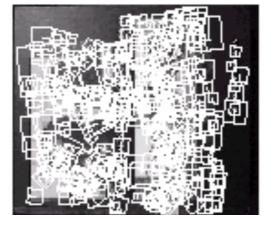












Instance recognition





3D object recognition

Database of 3D objects









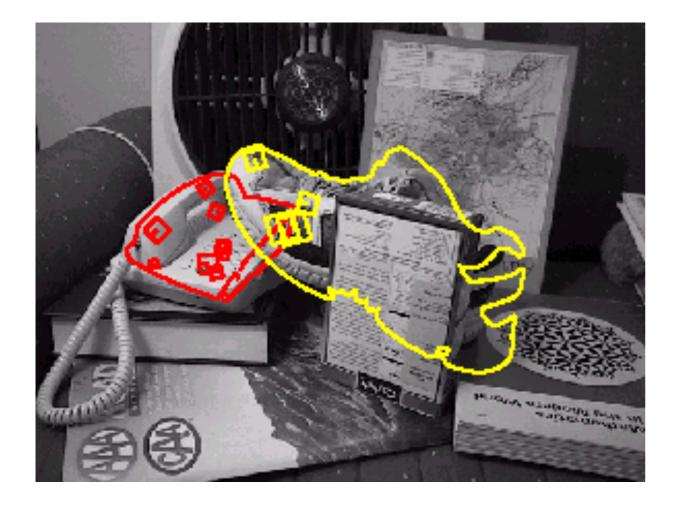


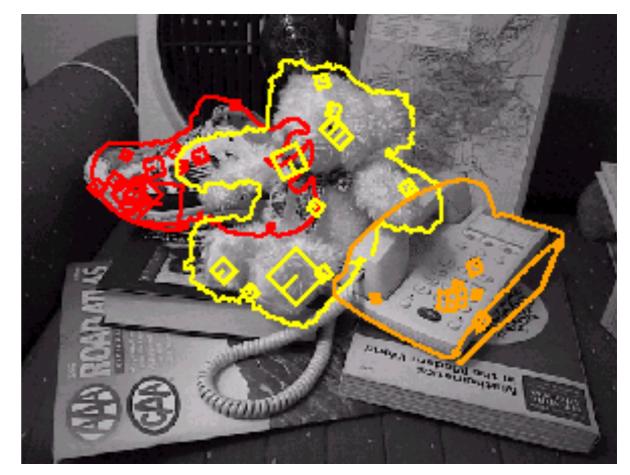


3D objects recognition









Recognition under occlusion

Location Recognition





Robot Localization

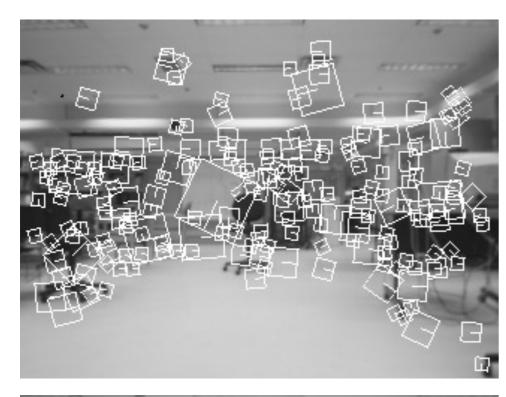
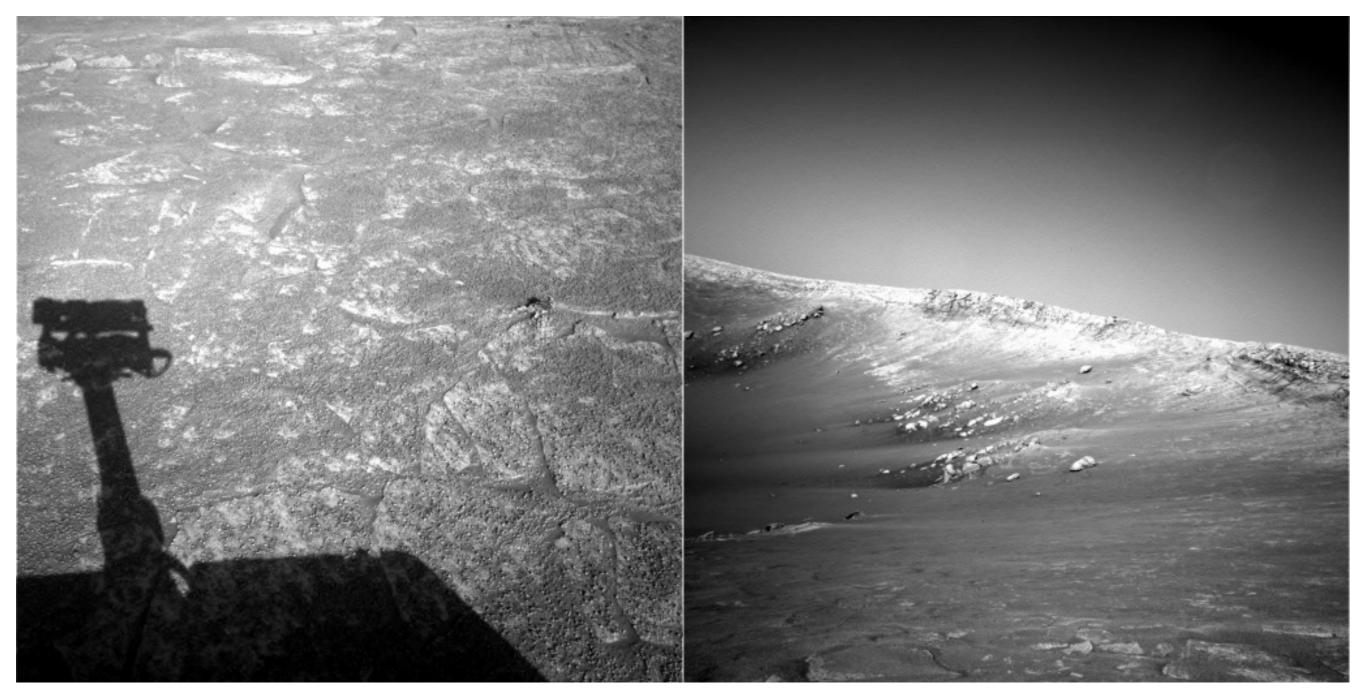






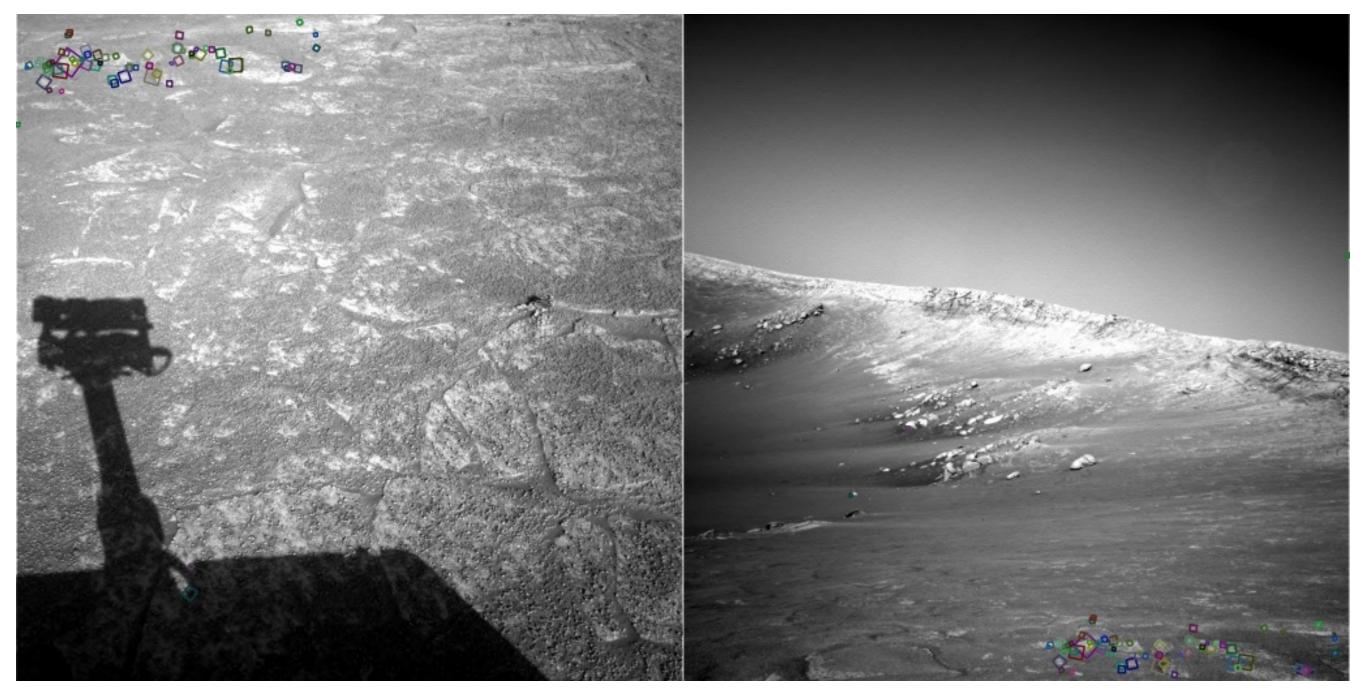
Image matching



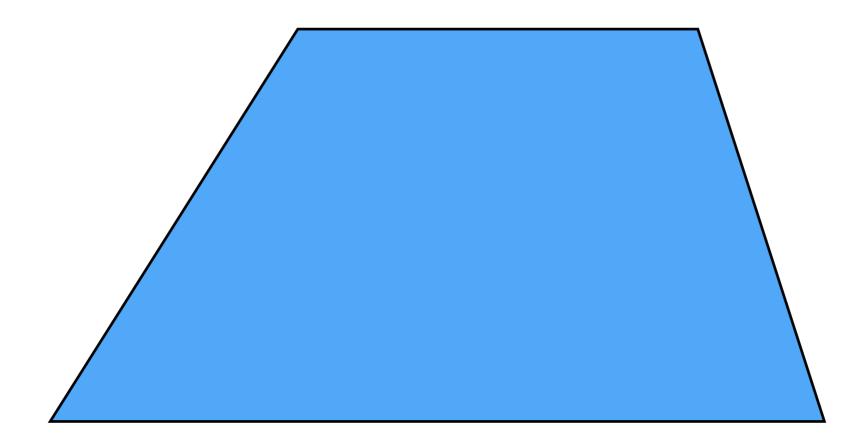


NASA Mars Rover images

Where are the corresponding points?

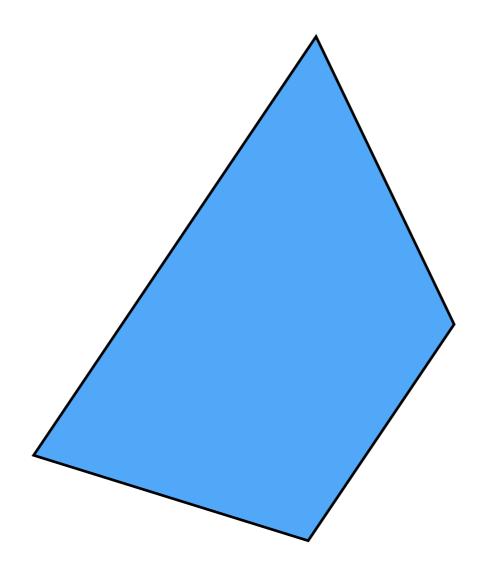


NASA Mars Rover images



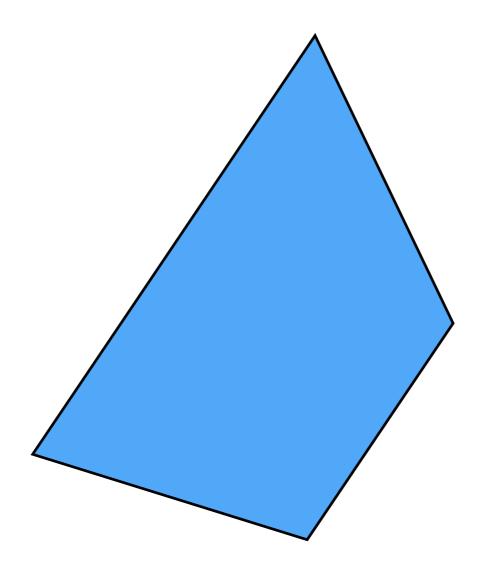
Pick a point in the image. Find it again in the next image.

What type of feature would you select?



Pick a point in the image. Find it again in the next image.

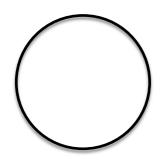
What type of feature would you select?



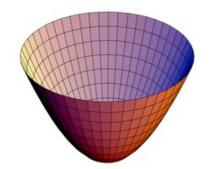
Pick a point in the image. Find it again in the next image.

What type of feature would you select? a corner

Visualizing quadratics



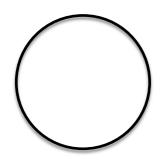
Equation of a circle $1 = x^2 + y^2$



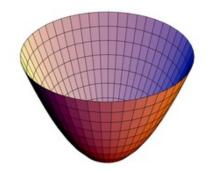
Equation of a 'bowl' (paraboloid)

$$f(x,y) = x^2 + y^2$$

If you slice the bowl at f(x, y) = 1what do you get?



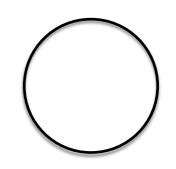
Equation of a circle $1 = x^2 + y^2$



Equation of a 'bowl' (paraboloid)

$$f(x,y) = x^2 + y^2$$

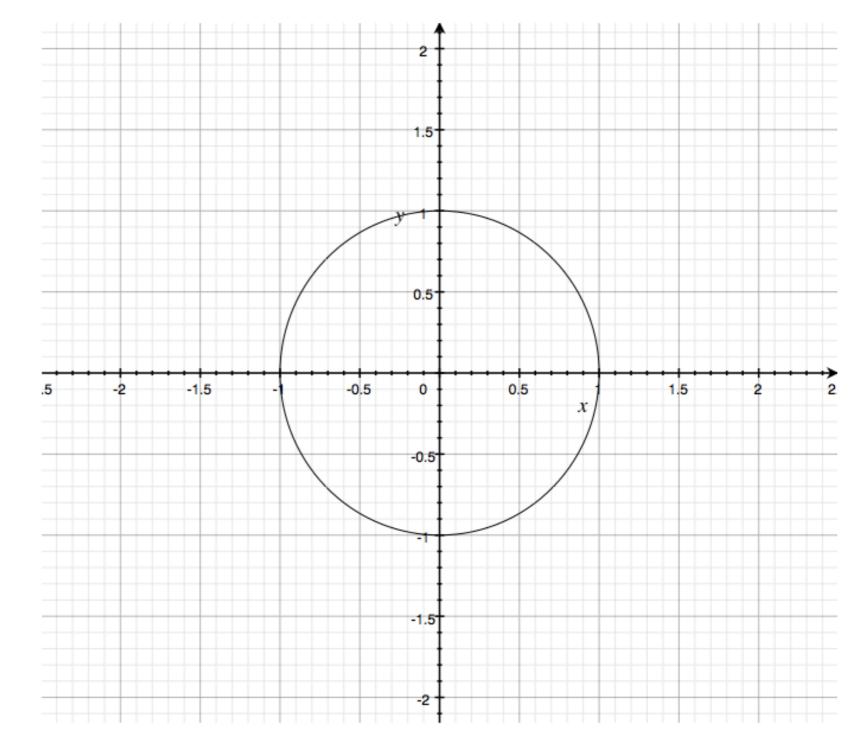
If you slice the bowl at f(x, y) = 1what do you get?



 $f(x, y) = x^2 + y^2$

can be written in matrix form like this...

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



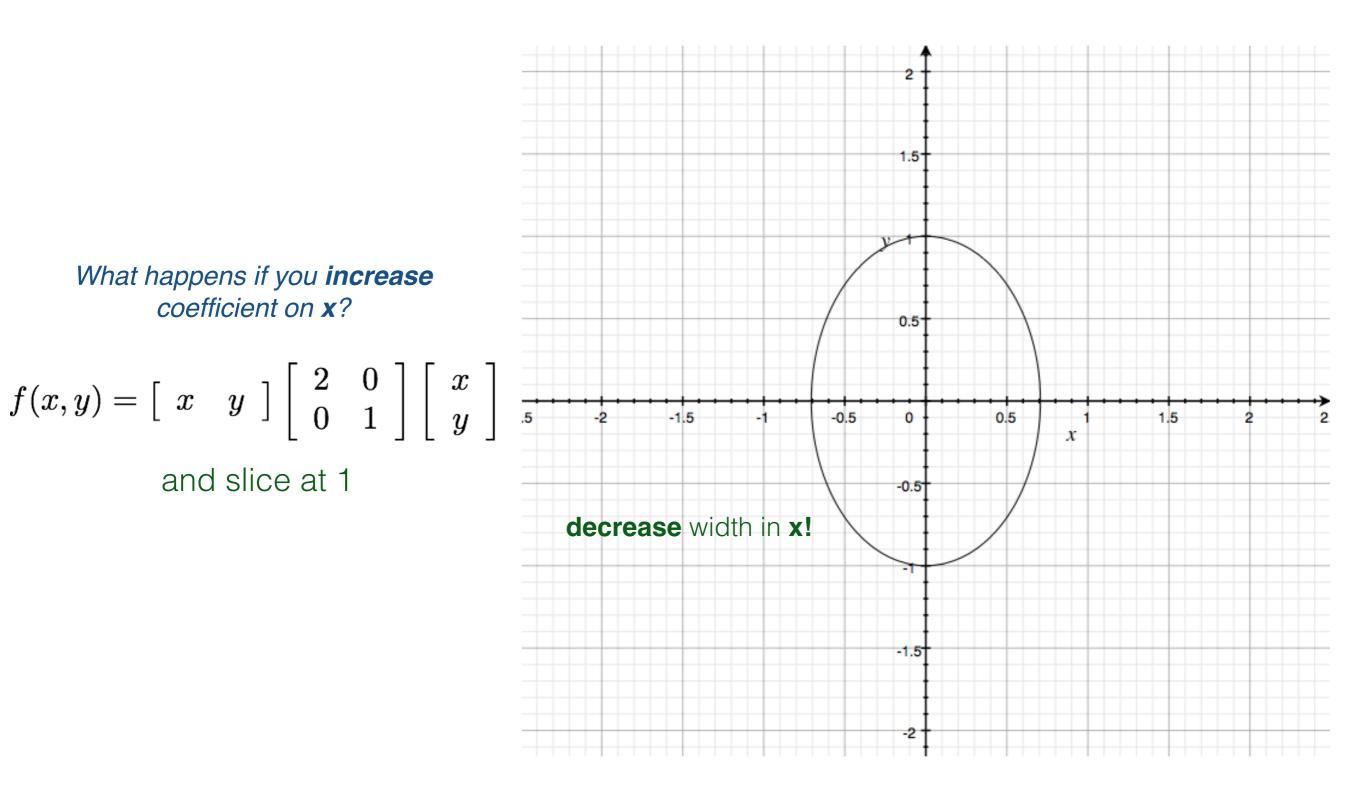
$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

'sliced at 1'

What happens if you **increase** coefficient on **x**?

$$f(x,y) = \left[\begin{array}{cc} x & y \end{array} \right] \left[\begin{array}{cc} 2 & 0 \\ 0 & 1 \end{array} \right] \left[\begin{array}{cc} x \\ y \end{array} \right]$$

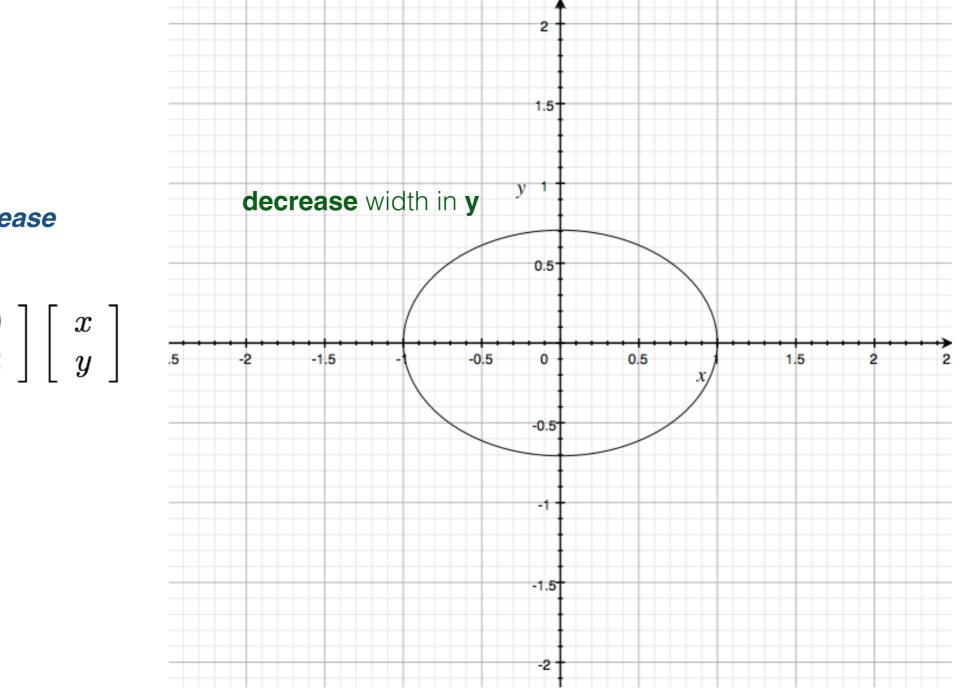
and slice at 1



What happens if you **increase** coefficient on **y**?

$$f(x,y) = \left[\begin{array}{cc} x & y \end{array} \right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 2 \end{array} \right] \left[\begin{array}{cc} x \\ y \end{array} \right]$$

and slice at 1



What happens if you **increase** coefficient on **y**?

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

and slice at 1

$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

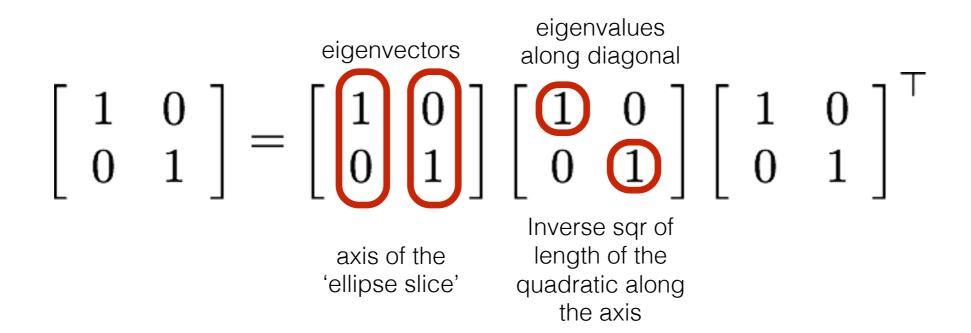
What's the shape? What are the eigenvectors? What are the eigenvalues?

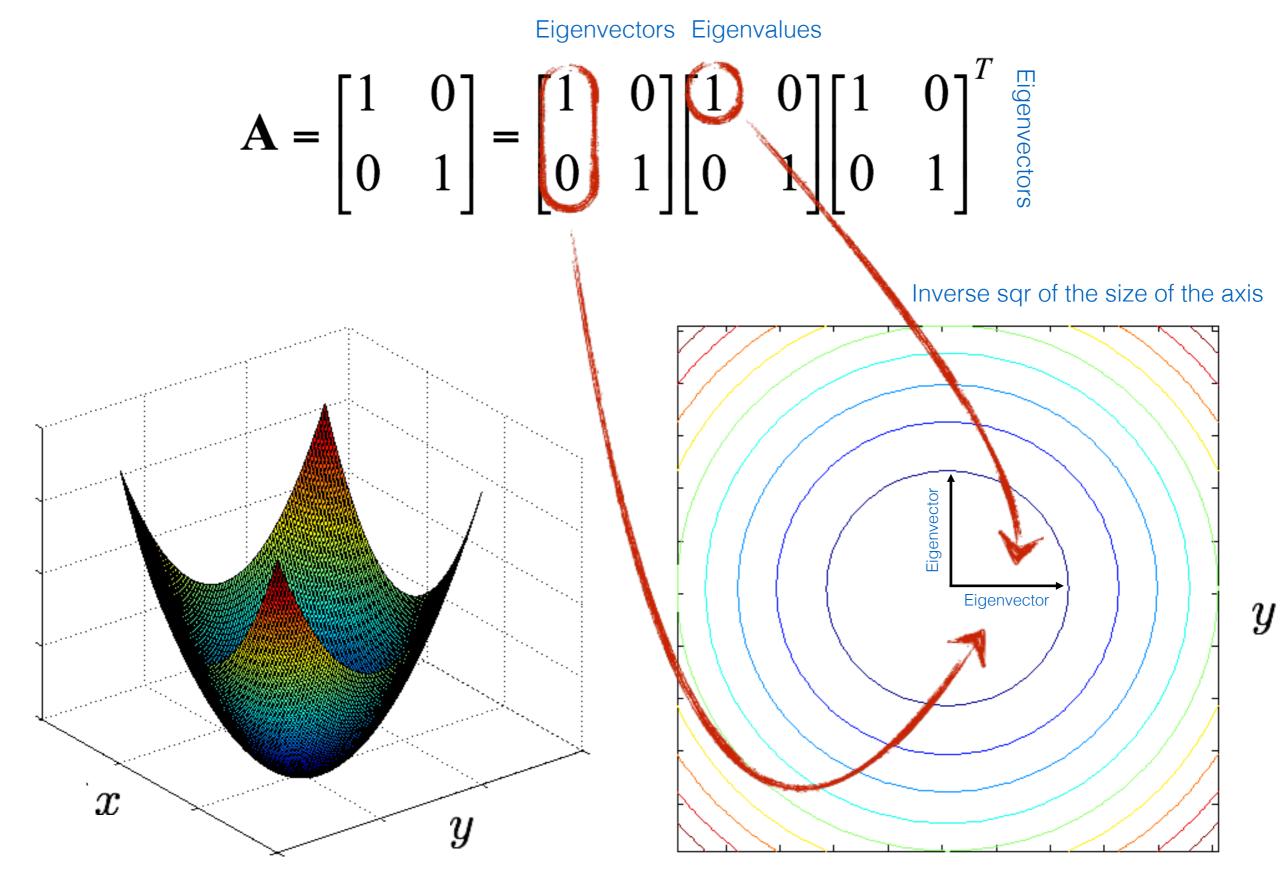
$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x,y) = \left[egin{array}{ccc} x & y \end{array}
ight] \left[egin{array}{ccc} 1 & 0 \ 0 & 1 \end{array}
ight] \left[egin{array}{ccc} x \ y \end{array}
ight]$$

Result of Singular Value Decomposition (SVD)





Recall:

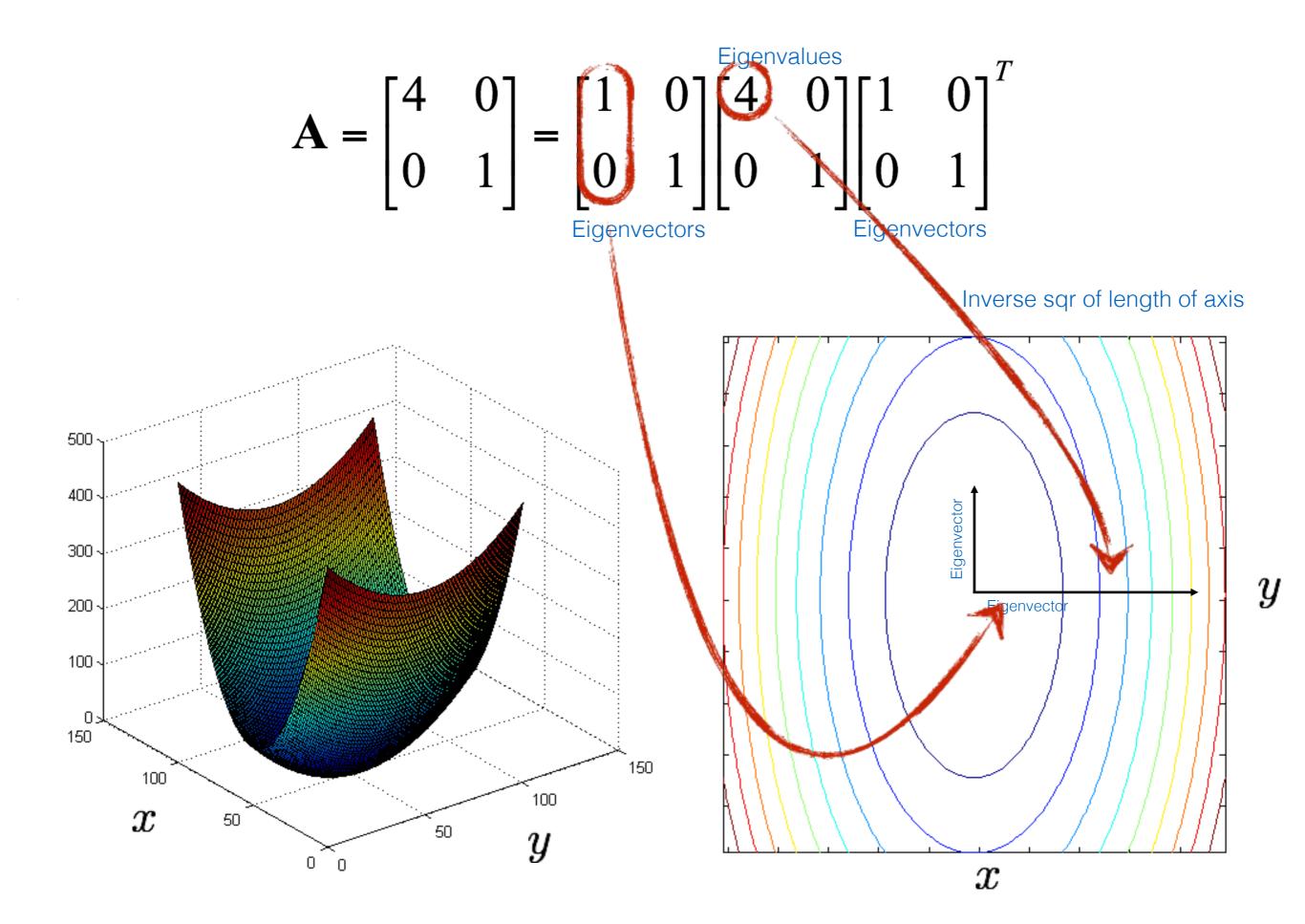
$$\left(\begin{array}{c} \end{array} \right) \quad f(x,y) = \left[\begin{array}{cc} x & y \end{array} \right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \left[\begin{array}{c} x \\ y \end{array} \right]$$

you can smash this bowl in the y direction

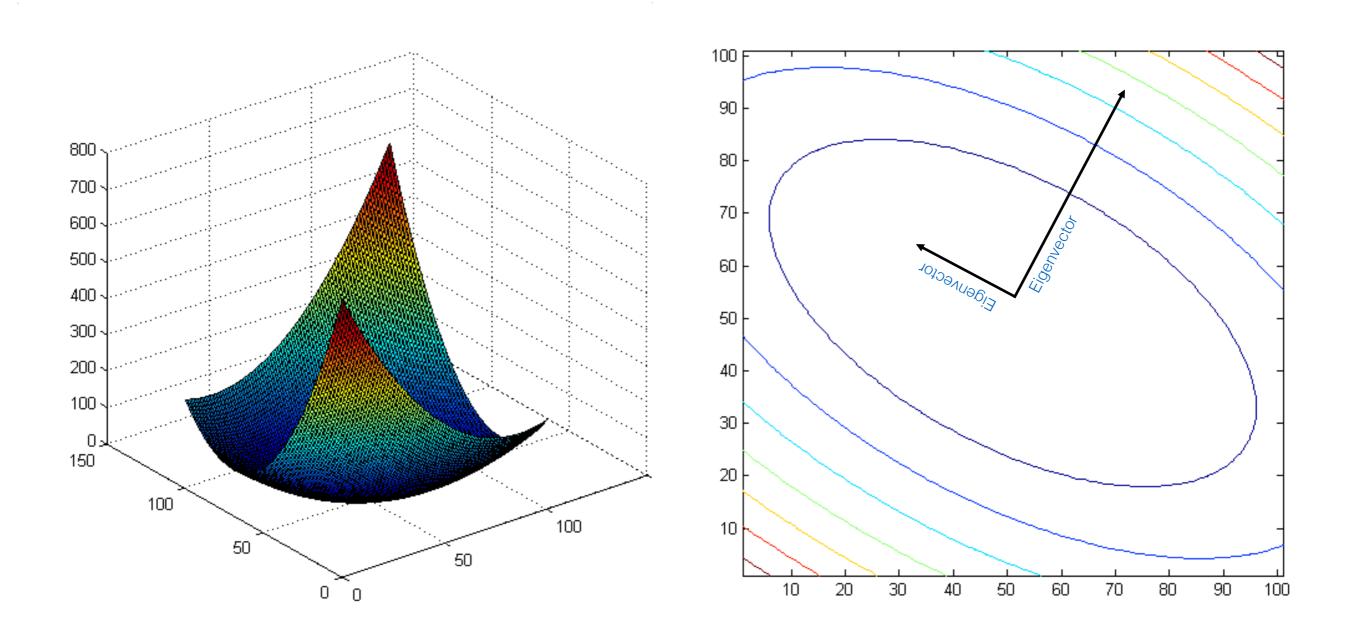
$$\bigcirc f(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

you can smash this bowl in the x direction

$$\left(\begin{array}{ccc} & f(x,y) = \left[\begin{array}{ccc} x & y\end{array}\right] \left[\begin{array}{ccc} 4 & 0\\ 0 & 1\end{array}\right] \left[\begin{array}{ccc} x\\ y\end{array}\right]$$



$$\mathbf{A} = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}_{\text{Eigenvectors}}^{T}$$



$$\mathbf{A} = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^{T}$$

Eigenvectors

We will need this to understand the...

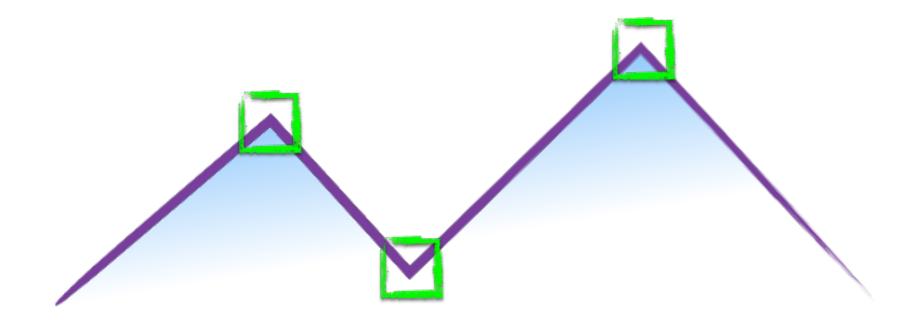
Error function for Harris Corners

The surface E(u,v) is locally approximated by a quadratic form

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$
$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

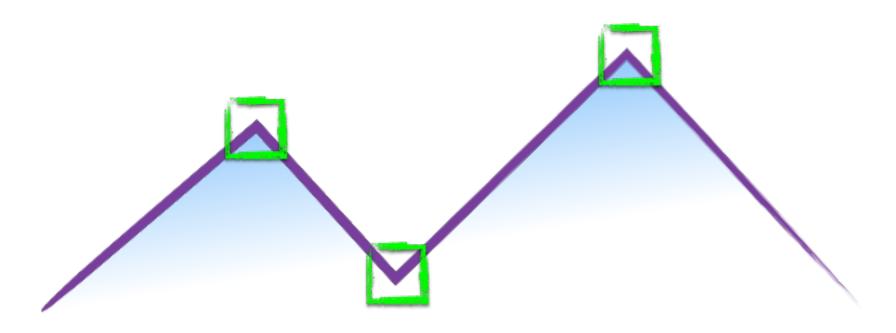
Harris corner detector

How do you find a corner?



How do you find a corner?

[Moravec 1980]

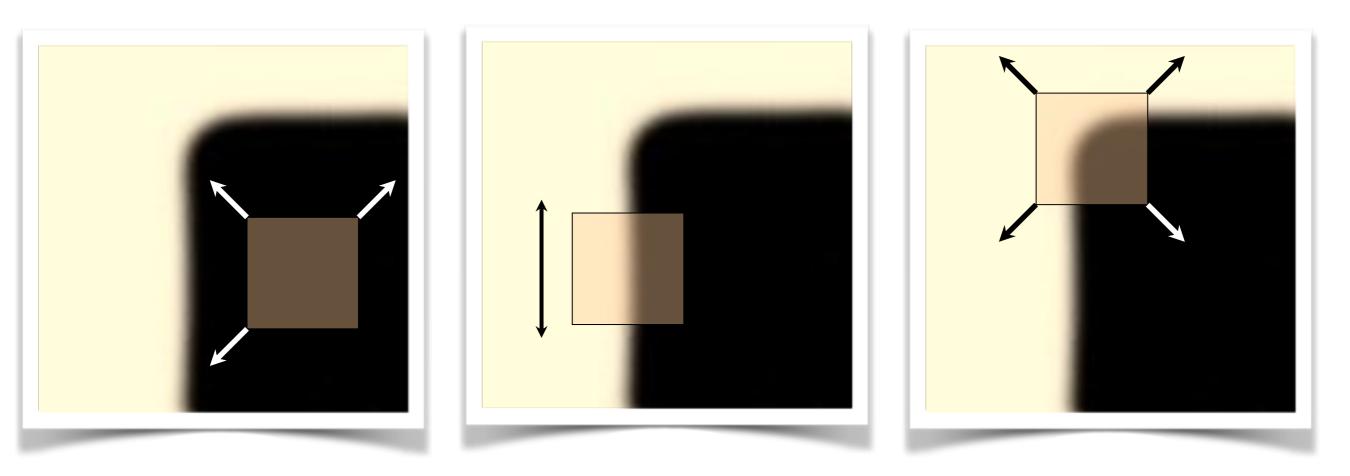


Easily recognized by looking through a small window

Shifting the window should give large change in intensity

Easily recognized by looking through a small window

Shifting the window should give large change in intensity



"flat" region: no change in all directions "edge": no change along the edge direction

"corner": significant change in all directions

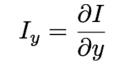
[Moravec 1980]

Design a program to detect corners (hint: use image gradients)

Finding corners (a.k.a. PCA)

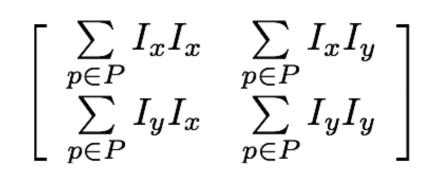
- 1.Compute image gradients over small region
- 2.Subtract mean from each image gradient
- 3.Compute the covariance matrix
- 4.Compute eigenvectors and eigenvalues
- 5.Use threshold on eigenvalues to detect corners

$$I_x = \frac{\partial I}{\partial x}$$



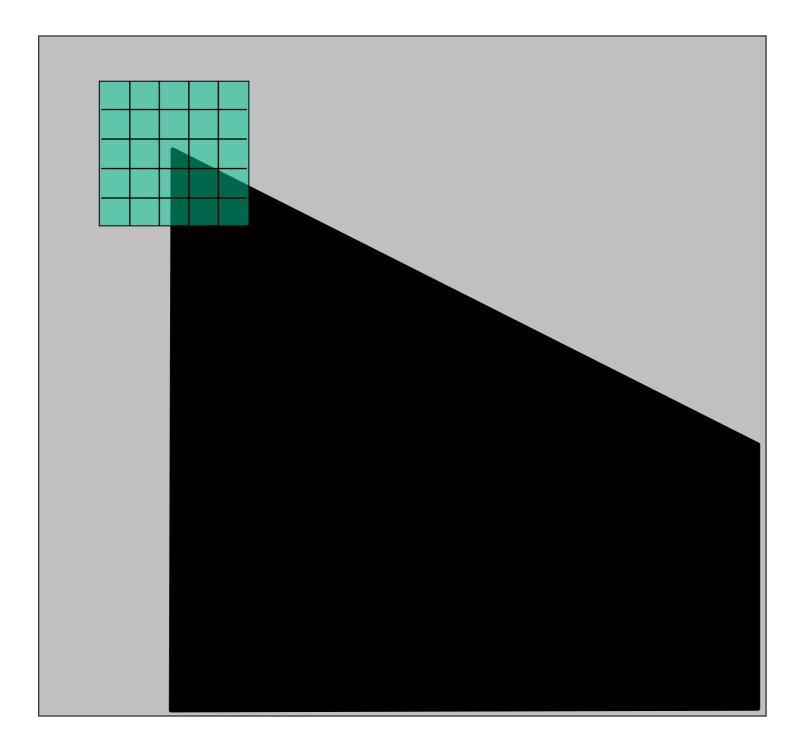






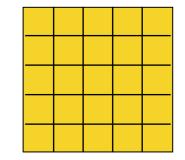
1. Compute image gradients over a small region (not just a single pixel)

1. Compute image gradients over a small region (not just a single pixel)



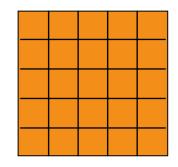
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$

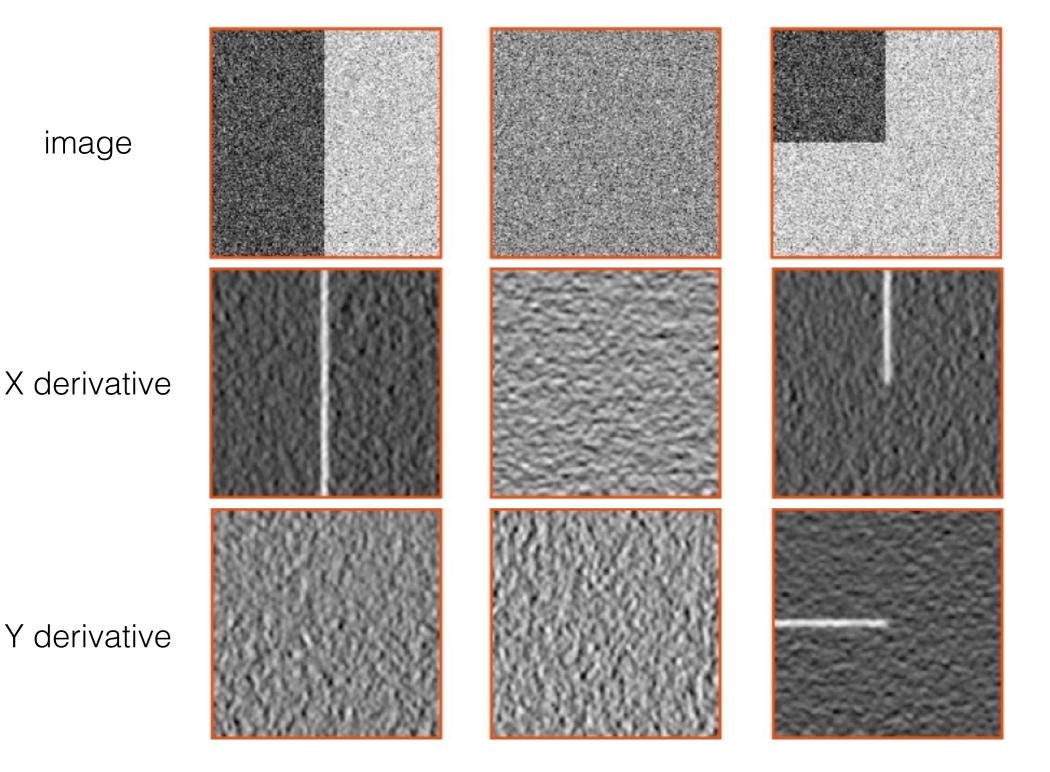


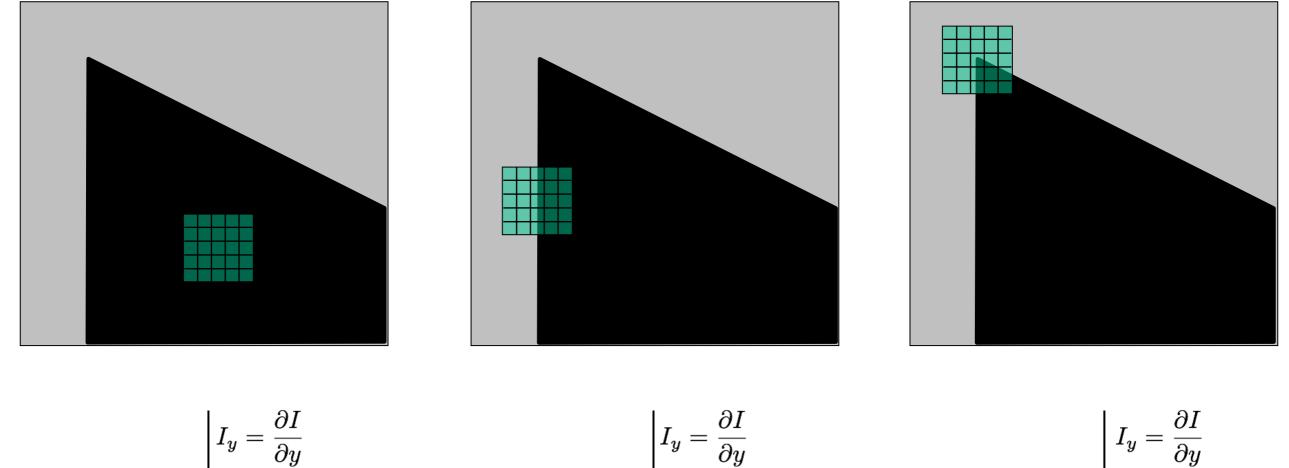
array of y gradients

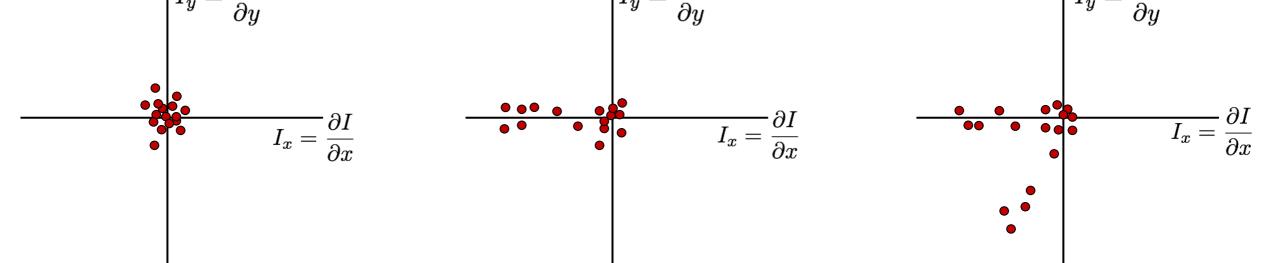
$$I_y = \frac{\partial I}{\partial y}$$



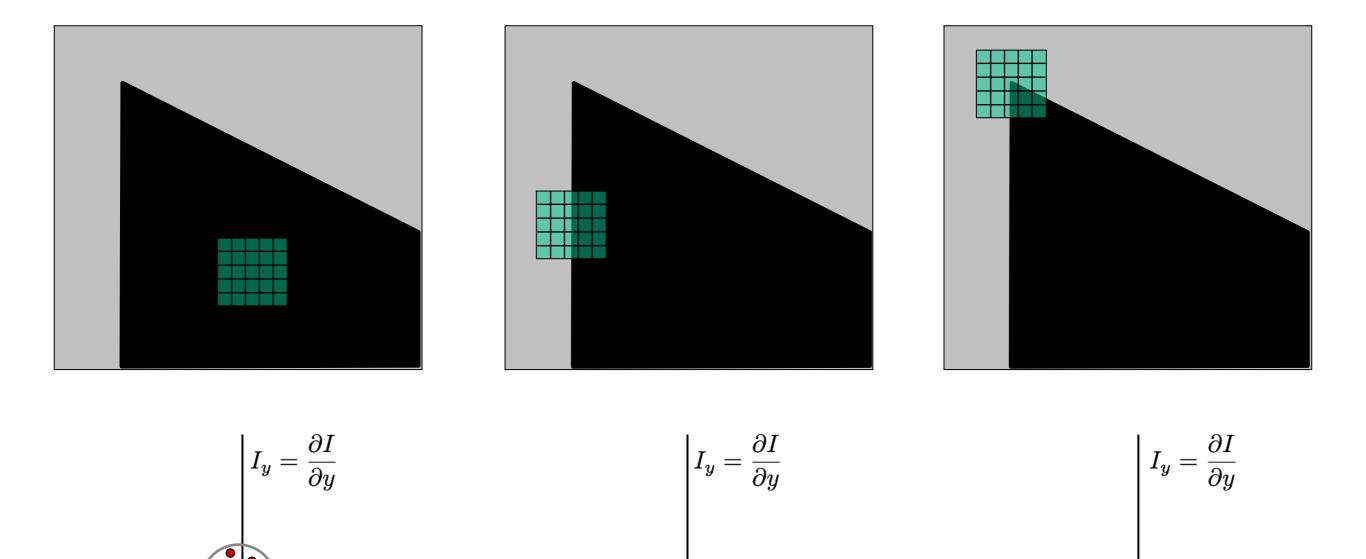
visualization of gradients







What does the distribution tell you about the region?

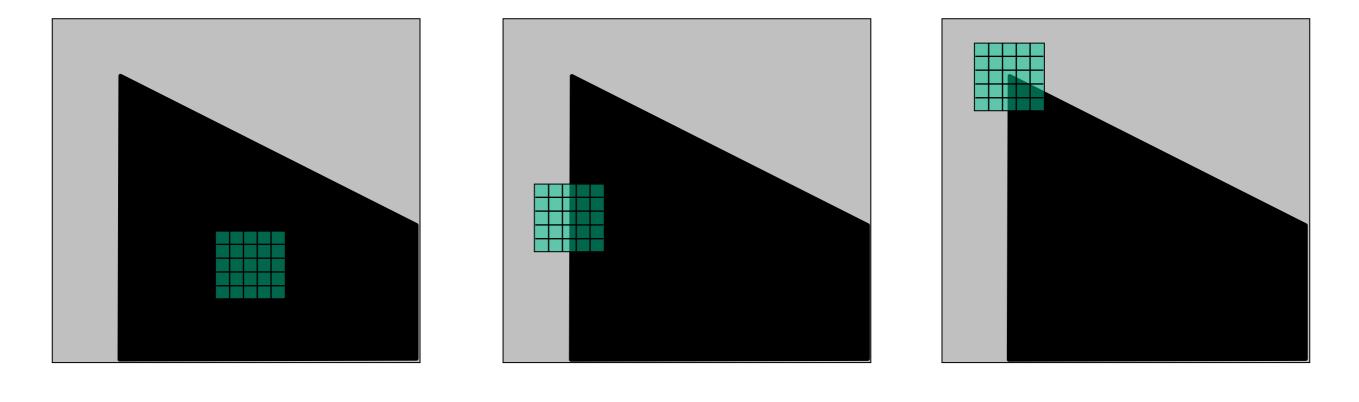


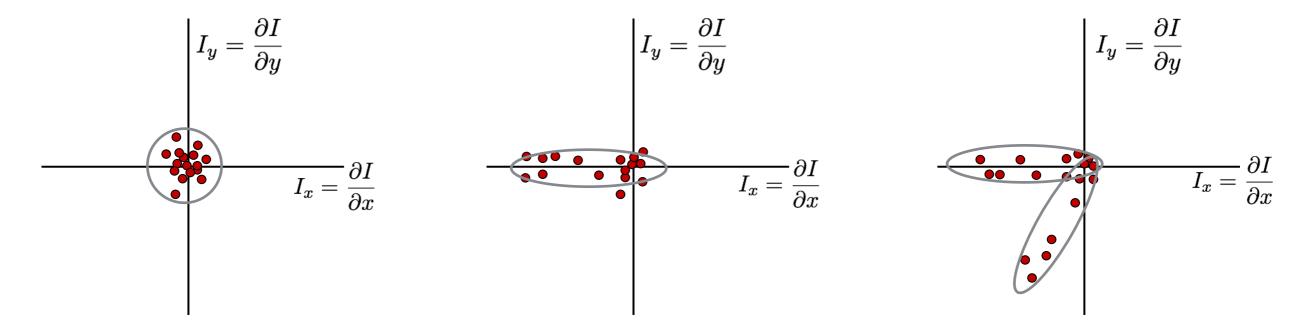
distribution reveals edge orientation and magnitude

 $I_x = \frac{\partial I}{\partial x}$

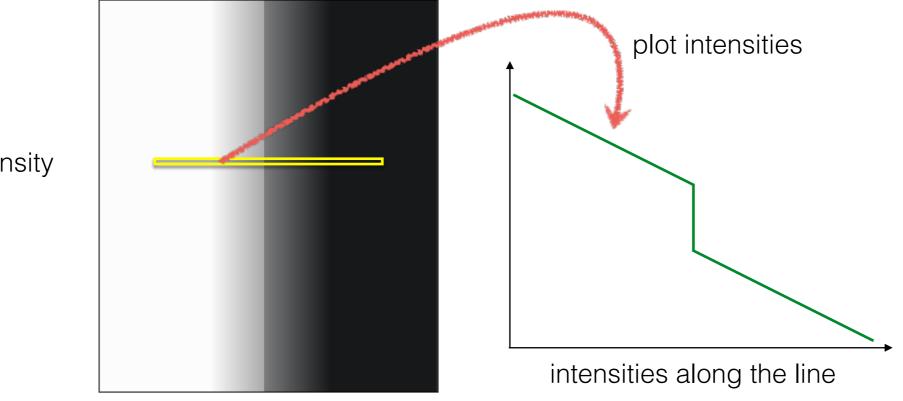
 $I_x = \frac{\partial I}{\partial x}$

 $\overline{I_x} = \frac{\partial I}{\partial x}$

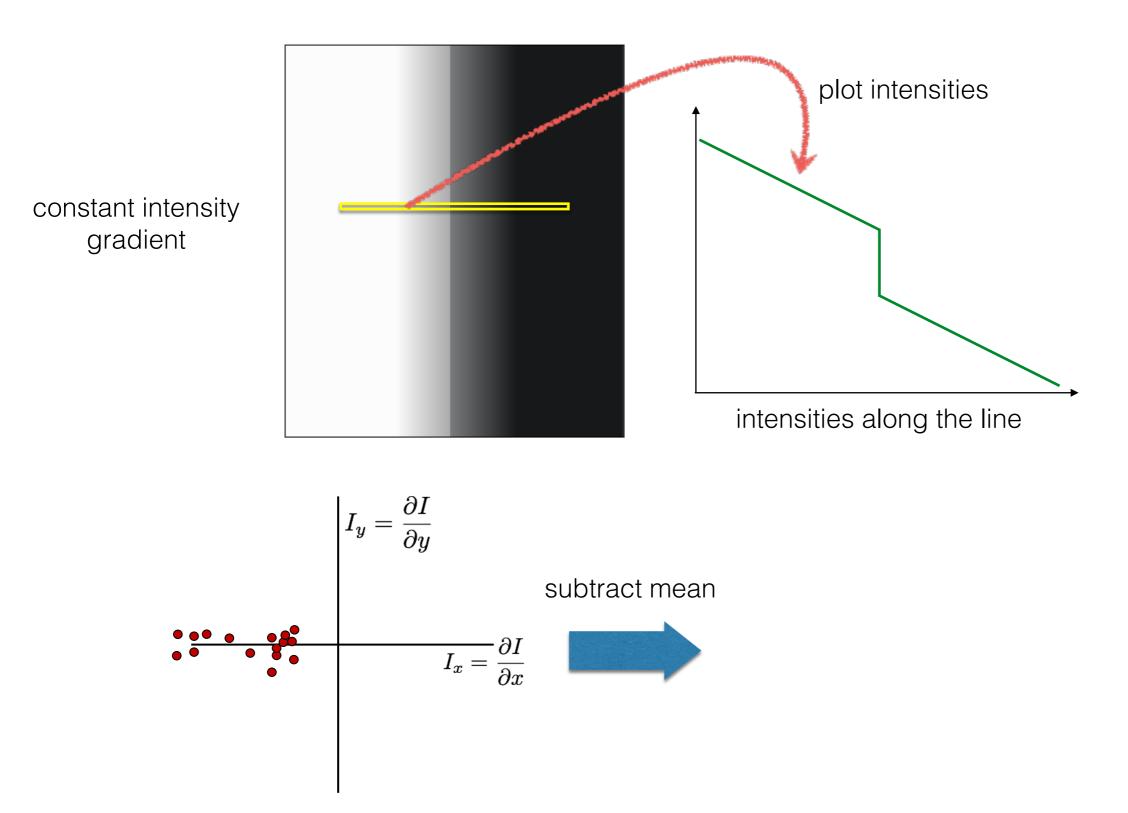




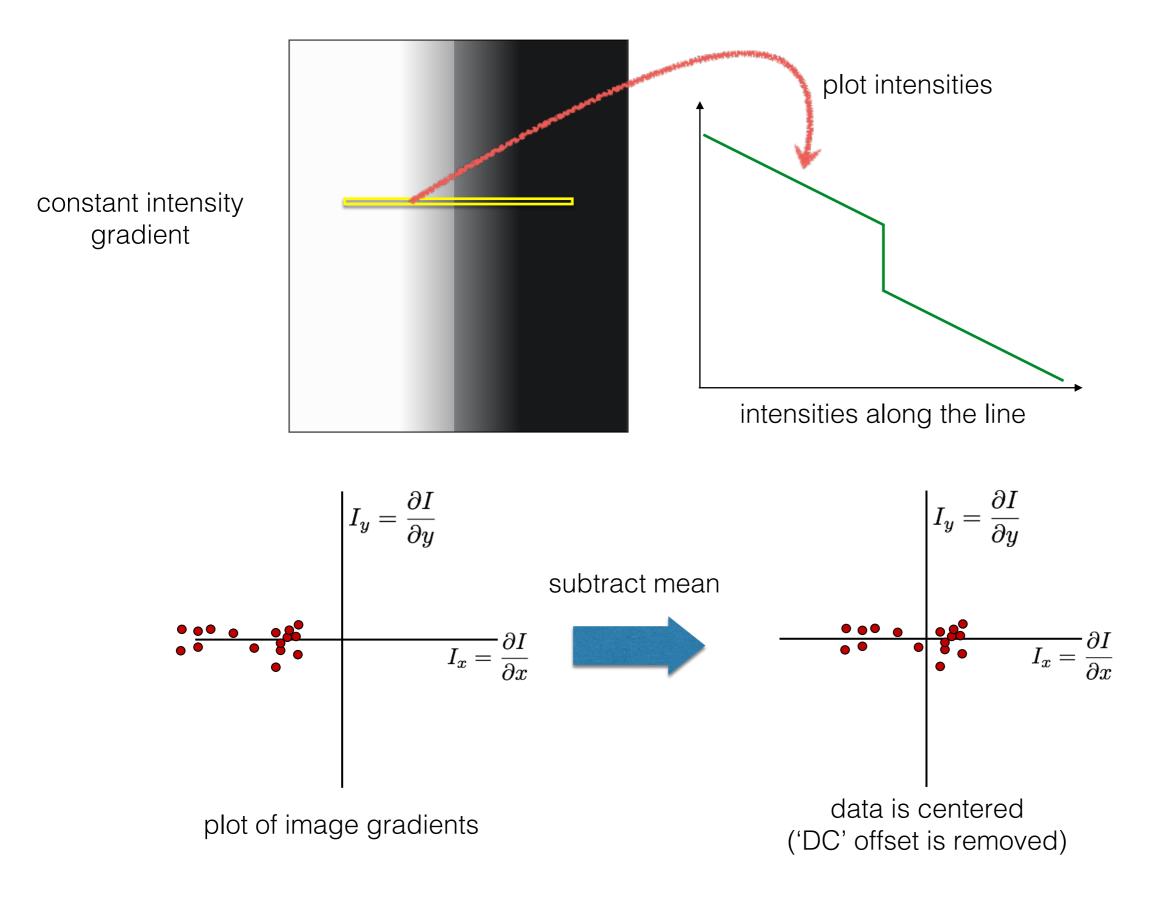
How do you quantify orientation and magnitude?



constant intensity gradient



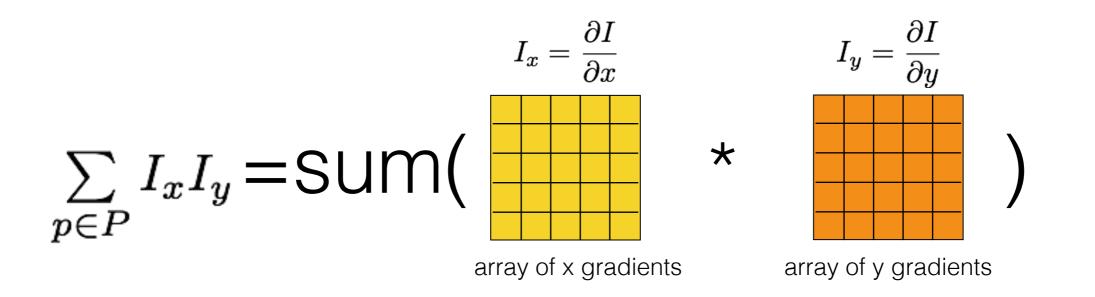
plot of image gradients



3. Compute the covariance matrix

3. Compute the covariance matrix

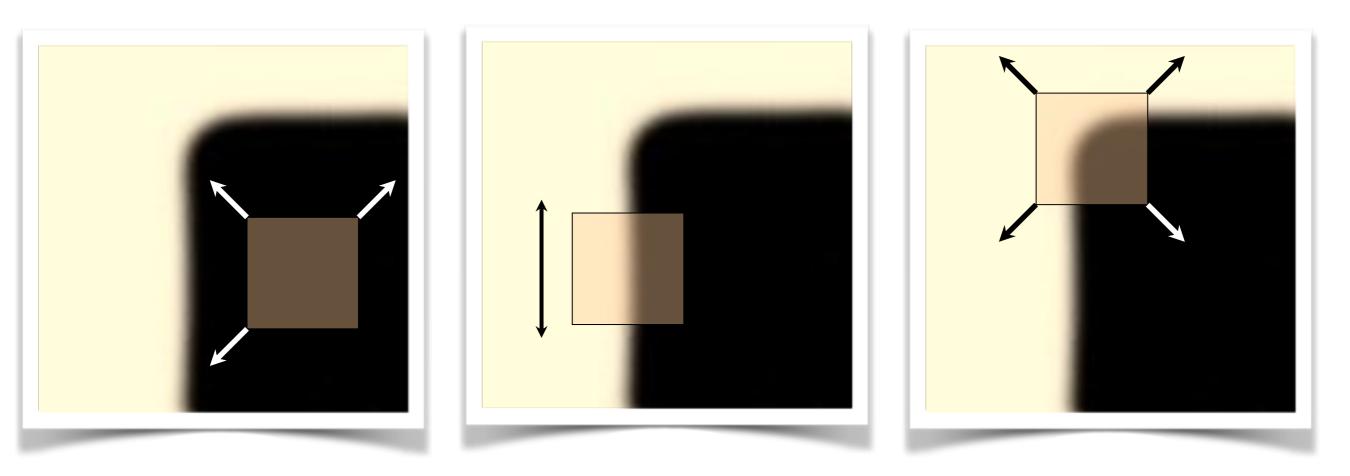
$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$



Where does this covariance matrix come from?

Easily recognized by looking through a small window

Shifting the window should give large change in intensity



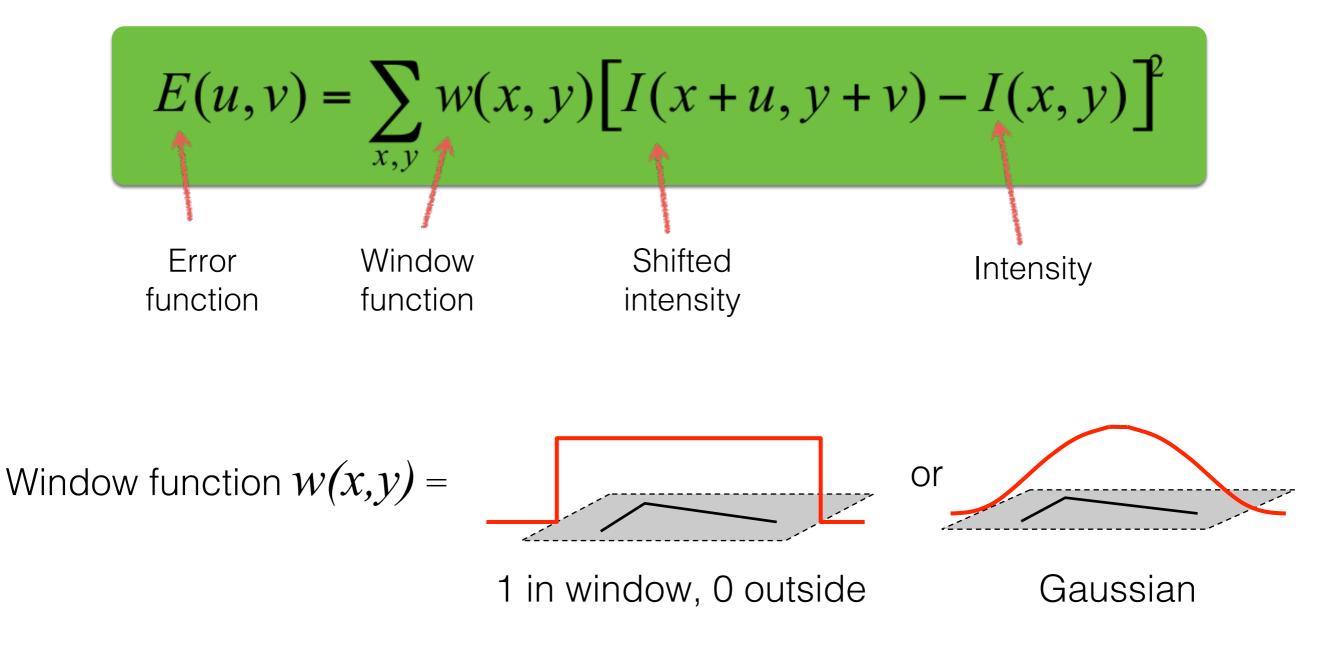
"flat" region: no change in all directions "edge": no change along the edge direction

"corner": significant change in all directions

[Moravec 1980]

Error function

Change of intensity for the shift [u, v]:



Error function approximation

Change of intensity for the shift [u, v]:

$$E(u,v) = \sum_{x,y} w(x,y) \Big[I(x+u,y+v) - I(x,y) \Big]^2$$

First-order Taylor expansion of I(x,y) about (0,0) (bilinear approximation for small shifts)

Bilinear approximation

For small shifts [u, v] we have a 'bilinear approximation':

Change in appearance for a shift [u,v]

$$E(u,v) \cong \begin{bmatrix} u,v \end{bmatrix} M \begin{bmatrix} u\\v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

'second moment' matrix 'structure tensor'

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

By computing the gradient covariance matrix...

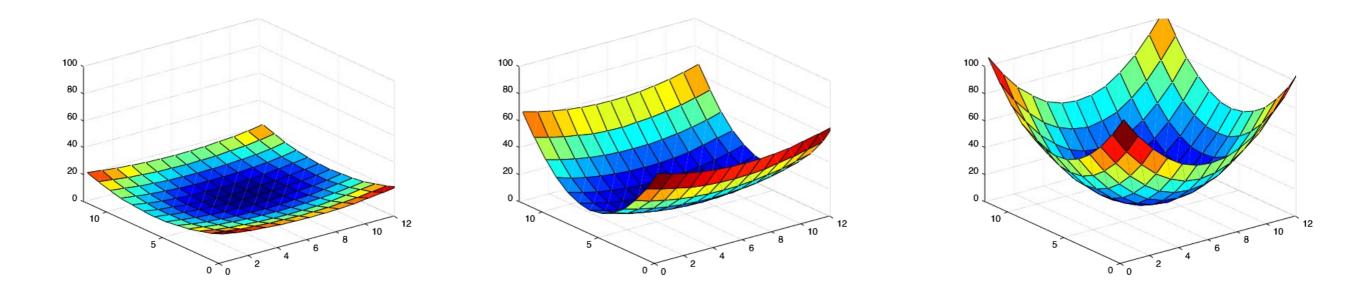
$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

we are fitting a quadratic to the gradients over a small image region

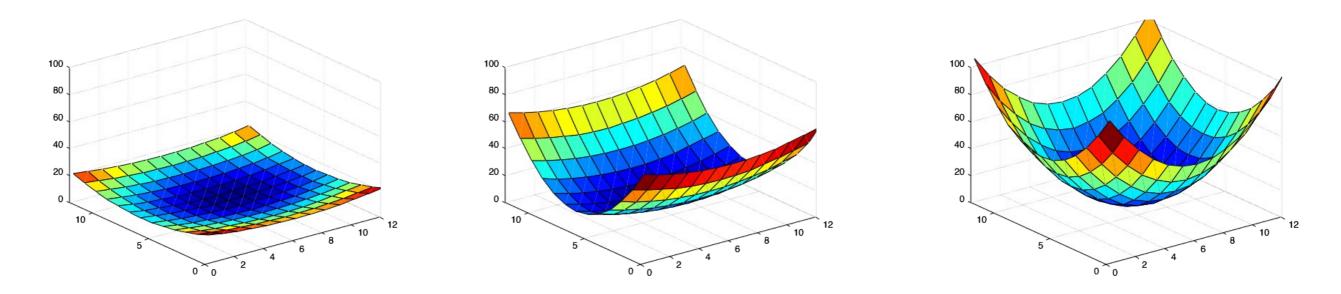
Visualization of a quadratic

The surface E(u,v) is locally approximated by a quadratic form

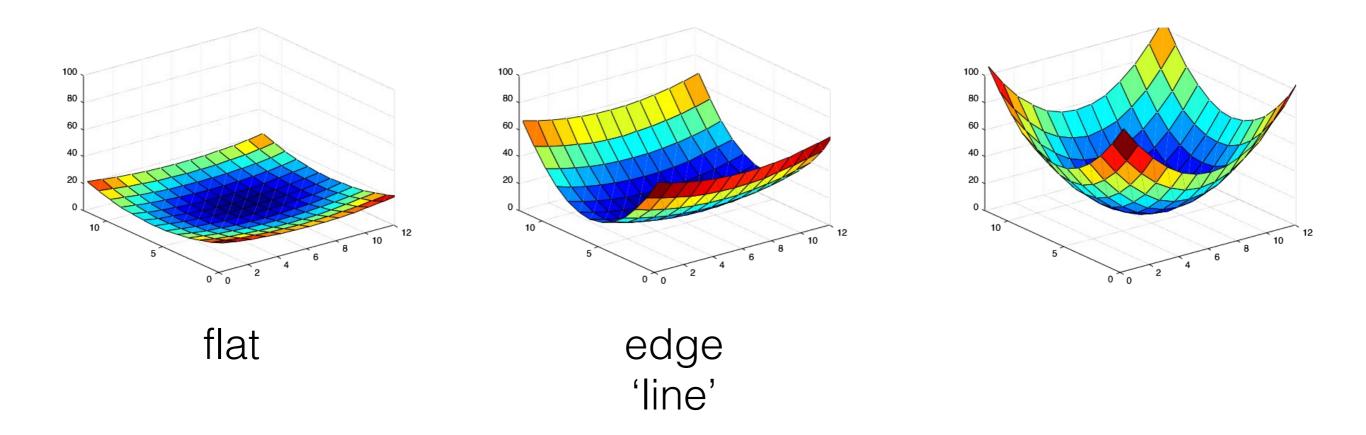
$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$
$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

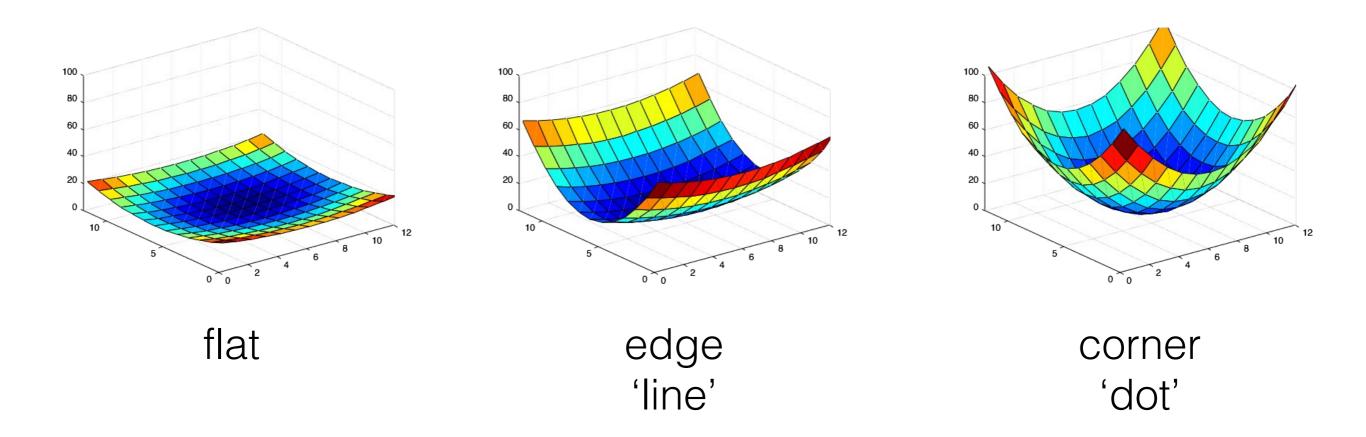


What kind of image patch do these surfaces represent?



flat





eigenvalue

$$Me = \lambda e$$

 \sim \nearrow
eigenvector

 $(M - \lambda I)\boldsymbol{e} = 0$

eigenvalue

 $Me = \lambda e$ \swarrow eigenvector

 $(M - \lambda I)\boldsymbol{e} = 0$

 $M - \lambda I$

1. Compute the determinant of (returns a polynomial)

eigenvalue

 $Me = \lambda e$ igenvector

 $(M - \lambda I)\boldsymbol{e} = 0$

1. Compute the determinant of $M - \lambda I$ (returns a polynomial)

2. Find the roots of polynomial

(returns eigenvalues)

 $\det(M - \lambda I) = 0$

eigenvalue

 $Me = \lambda e$ \swarrow eigenvector

 $(M - \lambda I)\boldsymbol{e} = 0$

1. Compute the determinant of $M - \lambda I$ (returns a polynomial)

2. Find the roots of polynomial (returns eigenvalues)

 $\det(M - \lambda I) = 0$

3. For each eigenvalue, solve (returns eigenvectors)

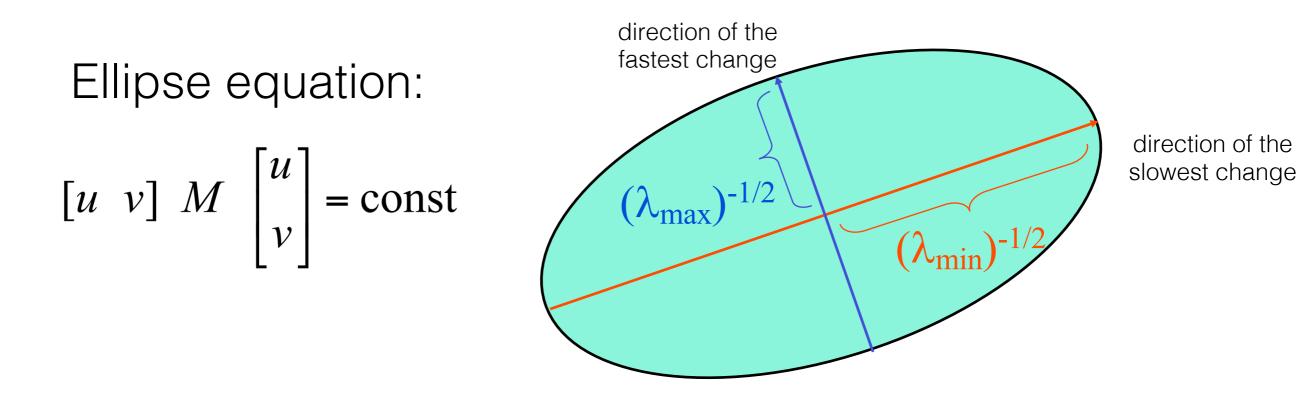
 $(M - \lambda I)\boldsymbol{e} = 0$

eig(M)

Visualization as an ellipse

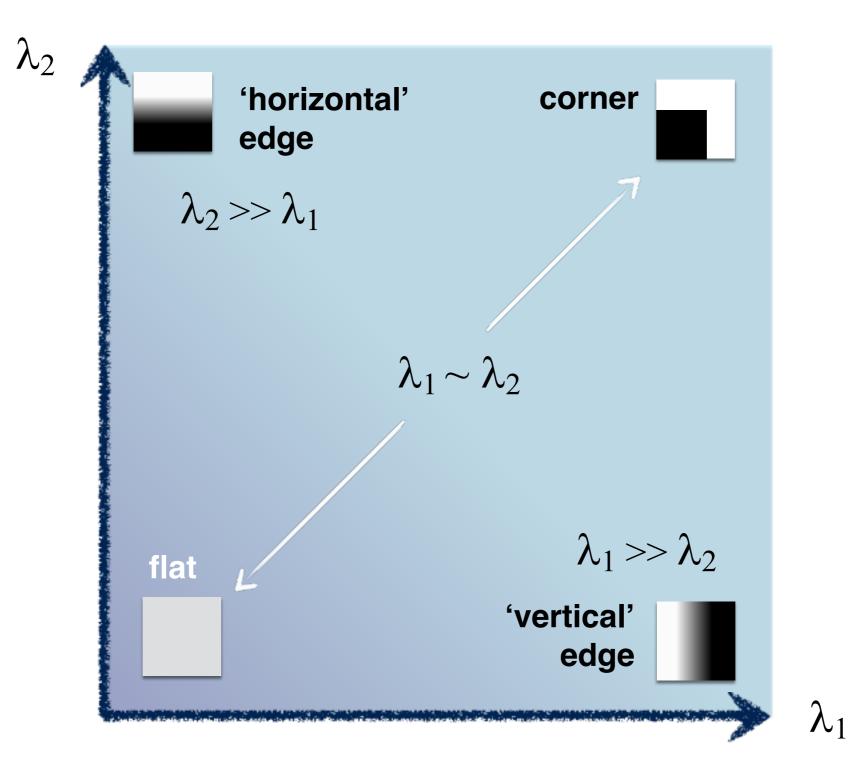
Since M is symmetric, we have $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

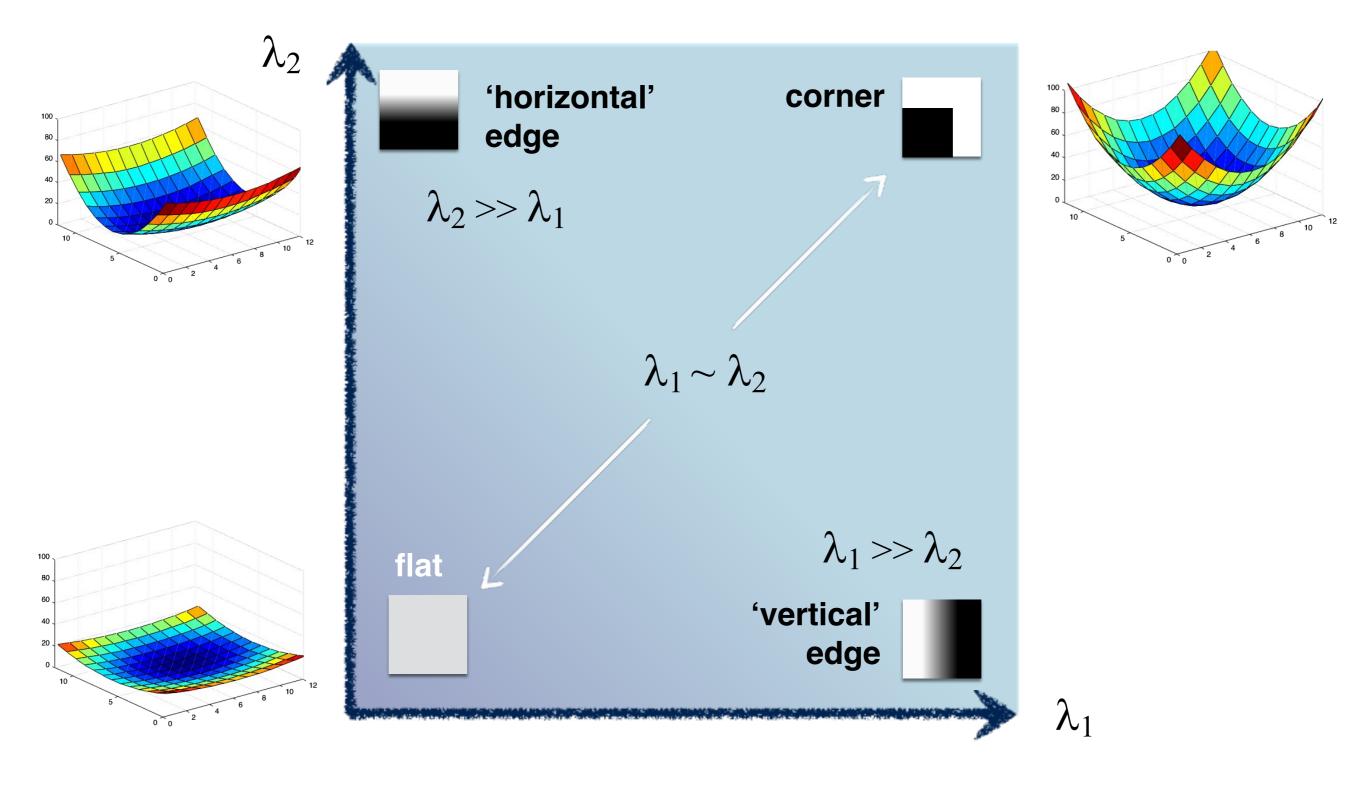
We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R

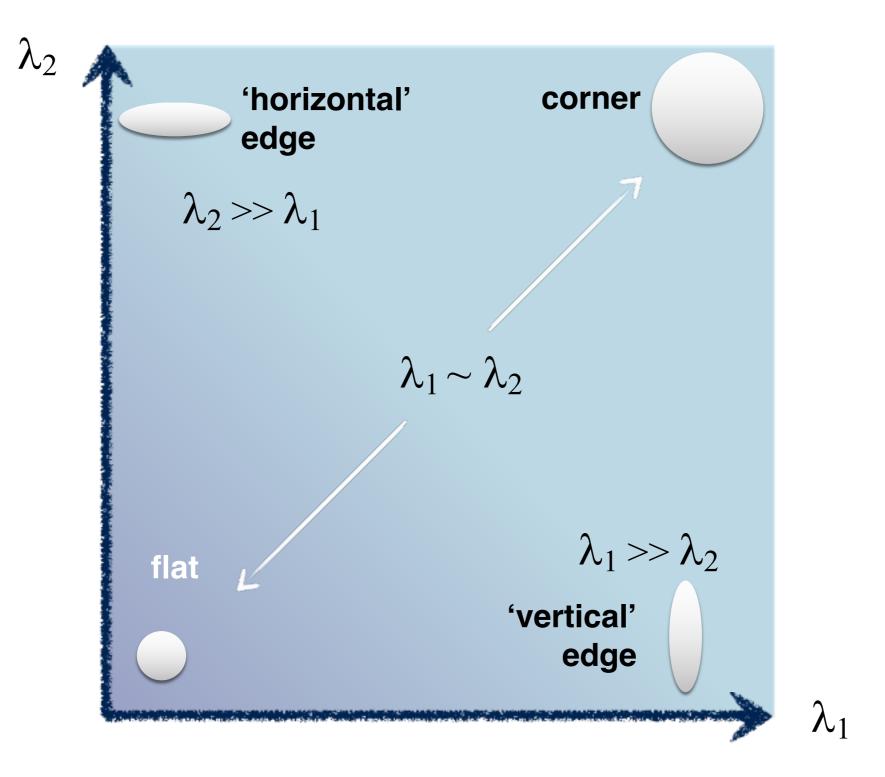


 λ_2 $\lambda_2 >> \lambda_1$ What kind of image patch does each region represent? $\lambda_1 \sim 0$ $\lambda_2 \sim 0$ $\lambda_1 >> \lambda_2$



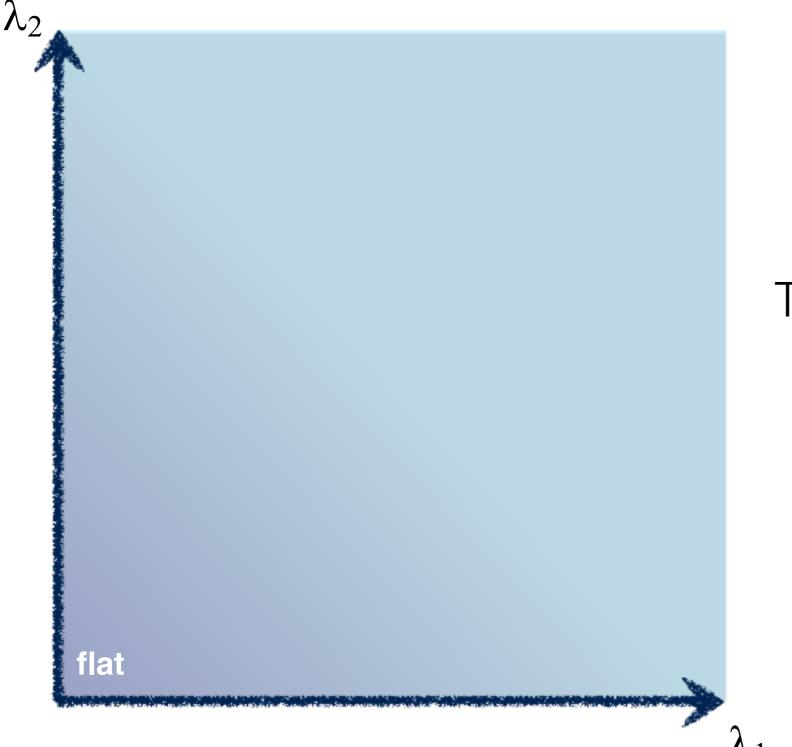






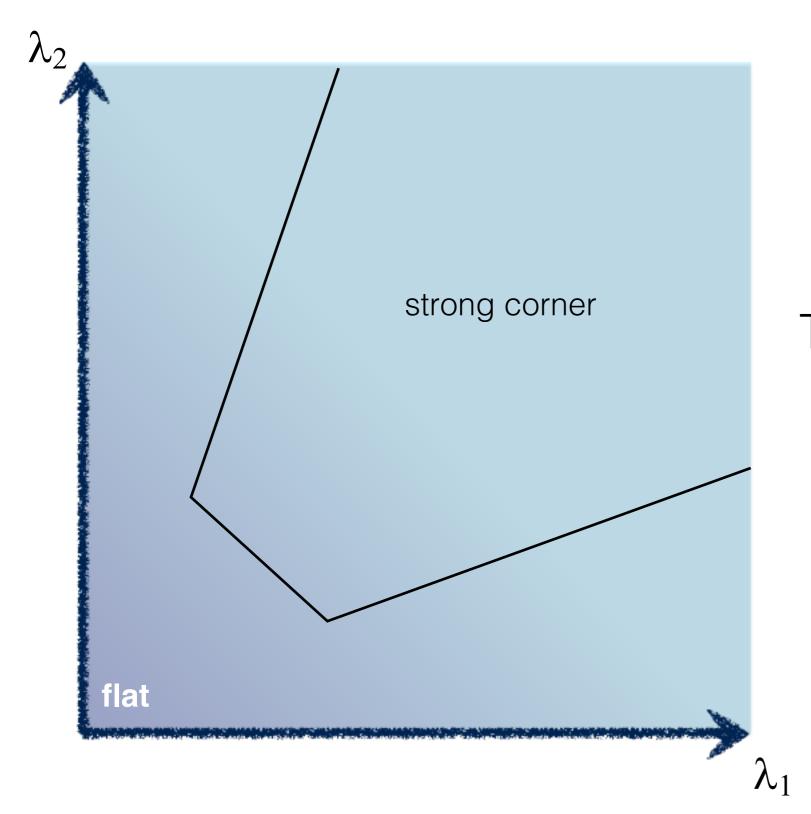
5. Use threshold on eigenvalues to detect corners

5. Use threshold on eigenvalues to detect corners



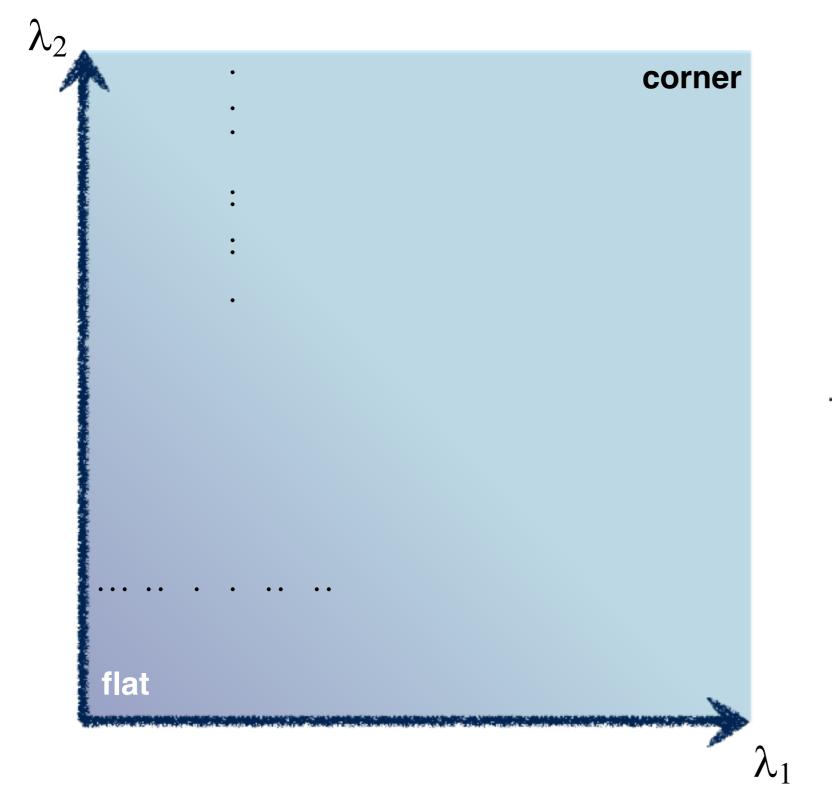
Think of a function to score 'cornerness'

5. Use threshold on eigenvalues to detect corners



Think of a function to score 'cornerness'

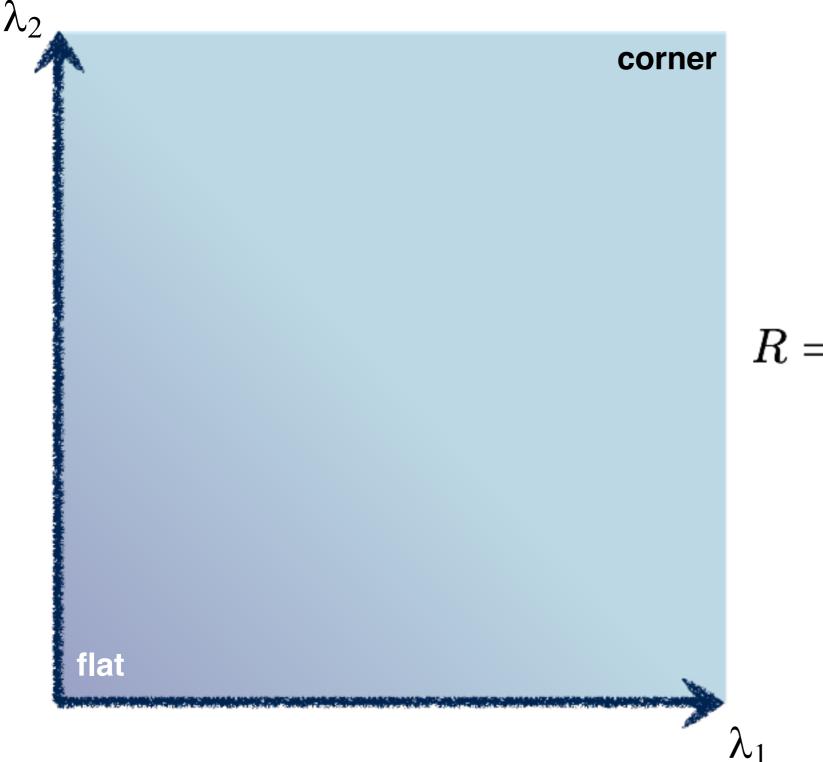
5. Use threshold on eigenvalues to detect corners (a function of)



Use the smallest eigenvalue as the response function

 $R = \min(\lambda_1, \lambda_2)$

5. Use threshold on eigenvalues to detect corners (a function of)

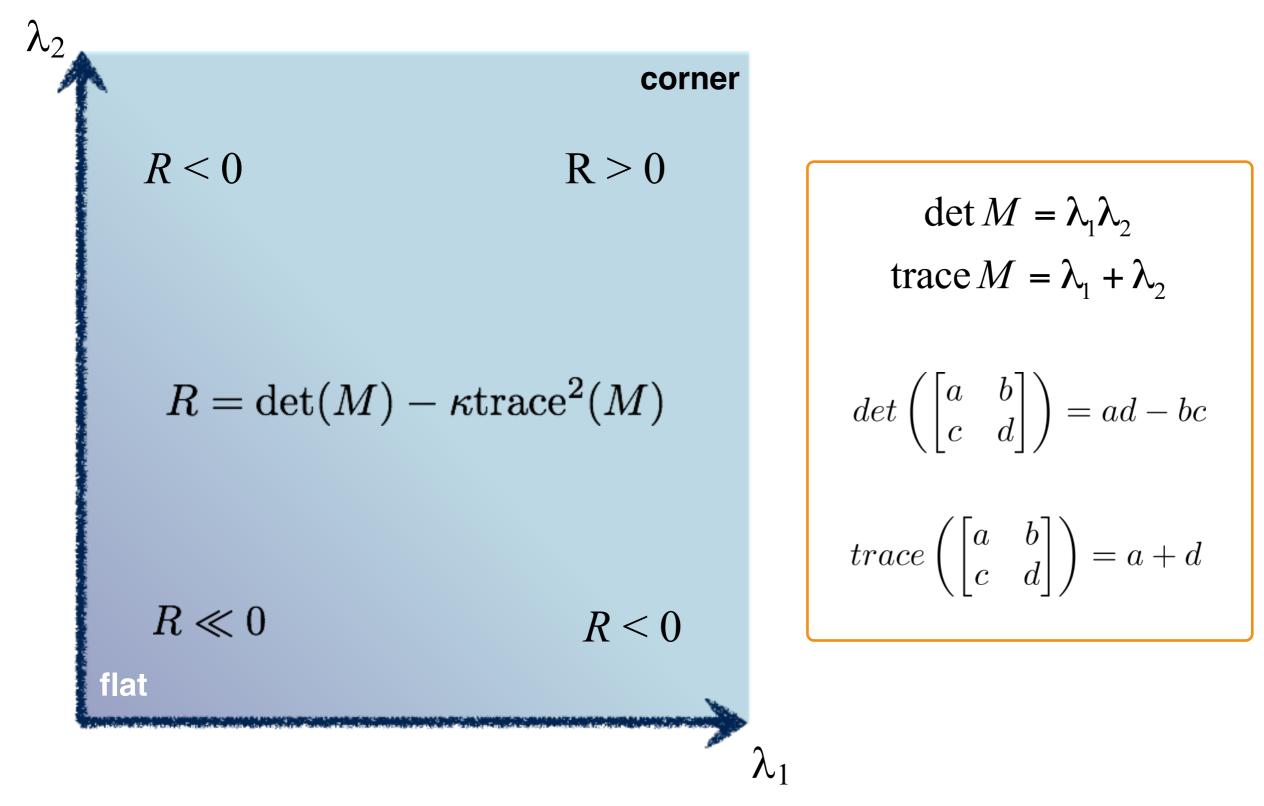


Eigenvalues need to be bigger than one.

$$R = \lambda_1 \lambda_2 - \kappa (\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently...

5. Use threshold on eigenvalues to detect corners (a function of)



Harris & Stephens (1988)

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)
$$R = \frac{\det(M)}{\operatorname{trace}(M) + \epsilon}$$

Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." 1988.

1. Compute x and y derivatives of image

$$I_x = G_{\sigma}^x * I \qquad I_y = G_{\sigma}^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad \qquad I_{y^2} = I_y \cdot I_y \qquad \qquad I_{xy} = I_x \cdot I_y$$

 Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2}$$
 $S_{y^2} = G_{\sigma'} * I_{y^2}$ $S_{xy} = G_{\sigma'} * I_{xy}$

Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." 1988.

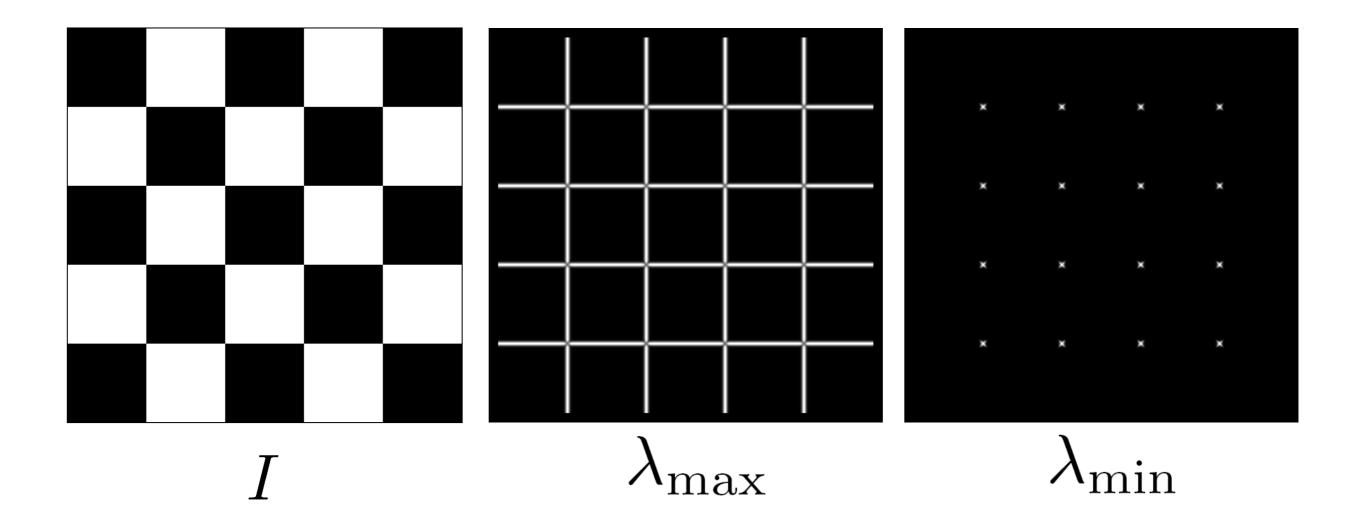
4. Define the matrix at each pixel

$$M(x, y) = \begin{bmatrix} S_{x^2}(x, y) & S_{xy}(x, y) \\ S_{xy}(x, y) & S_{y^2}(x, y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = \det M - k (\operatorname{trace} M)^2$$

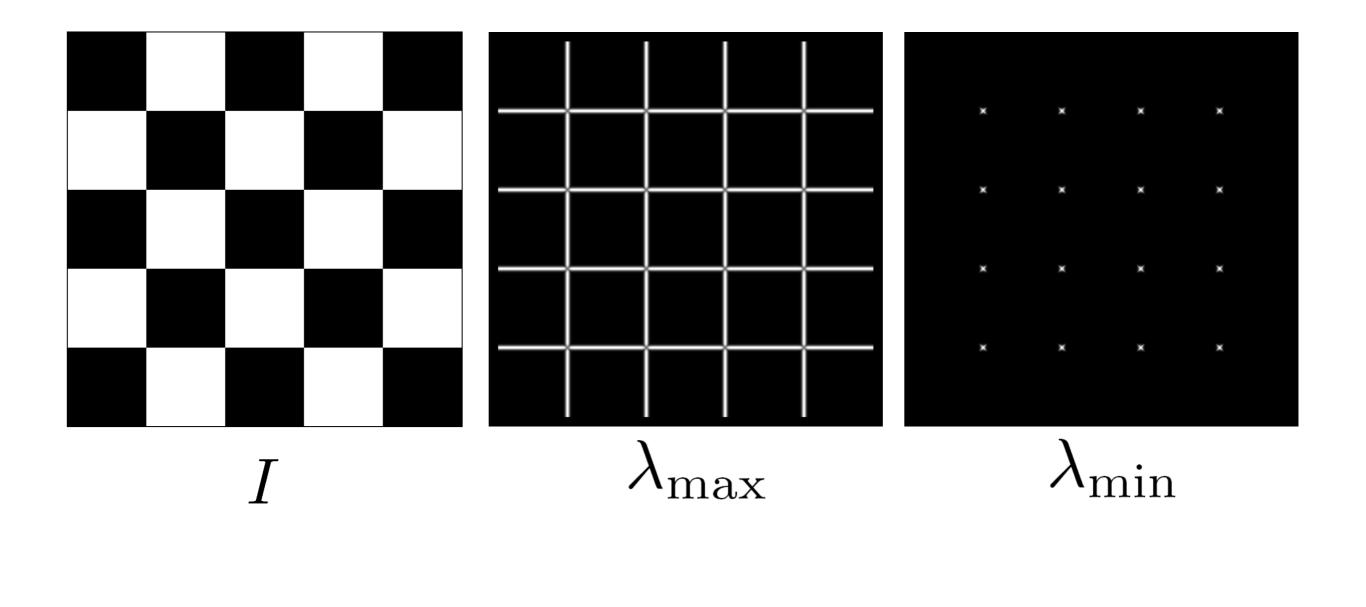
 Threshold on value of R; compute non-max suppression.



Yet another option:

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

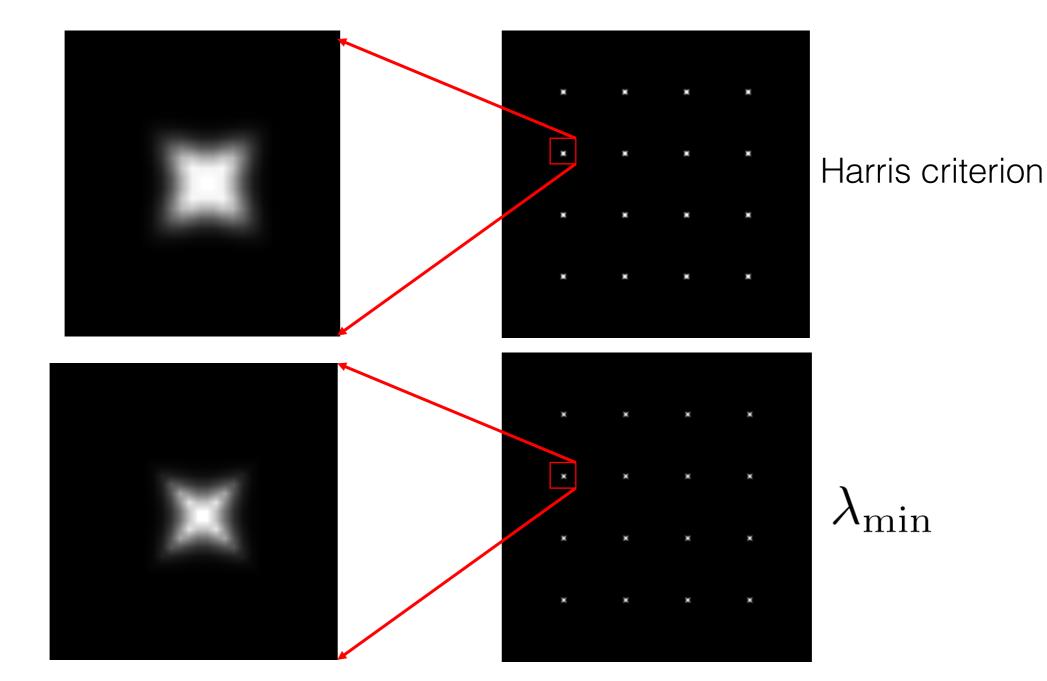
How do you write this equivalently using determinant and trace?



Yet another option:

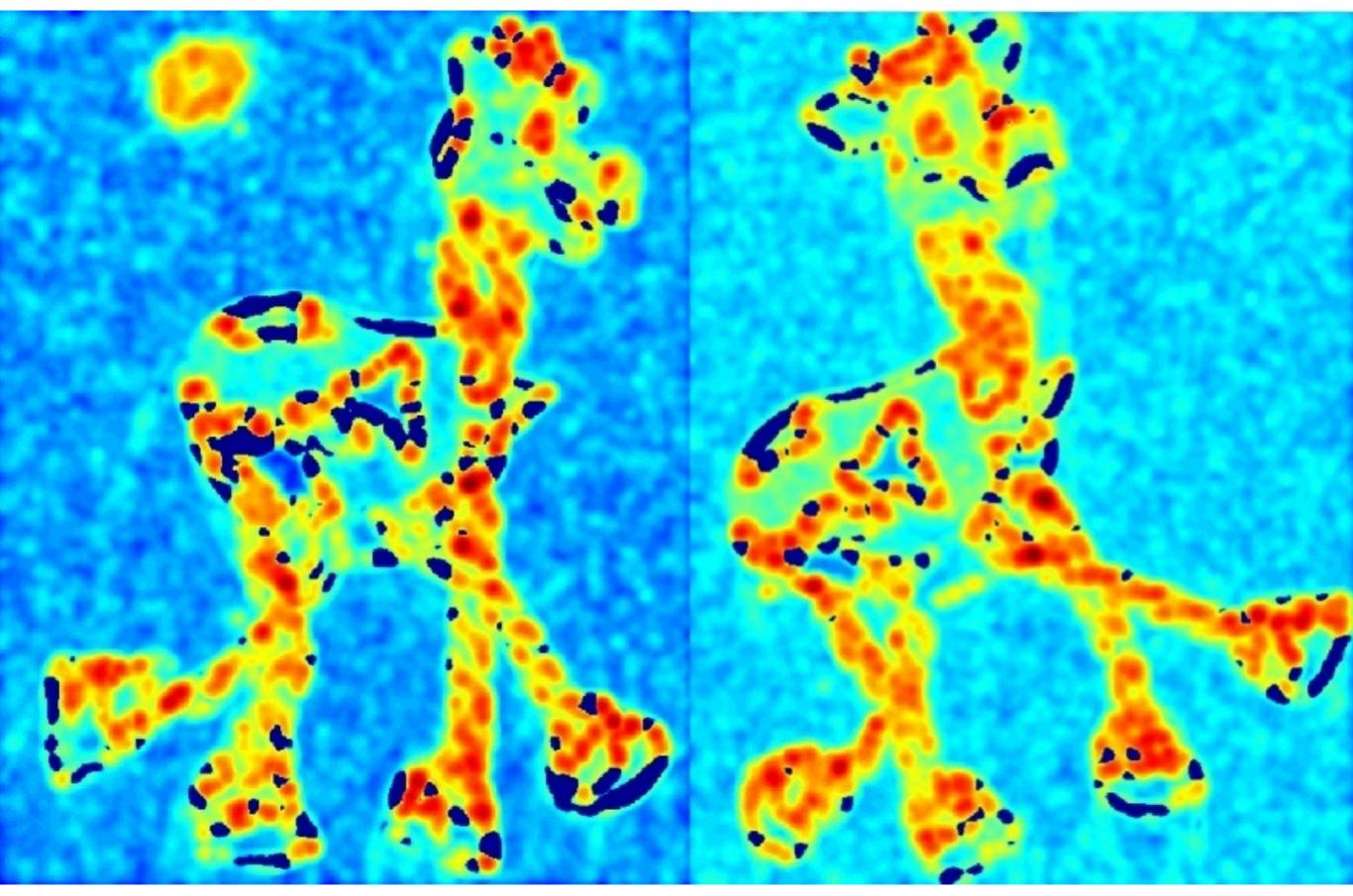
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{determinant(H)}{trace(H)}$$

Different criteria

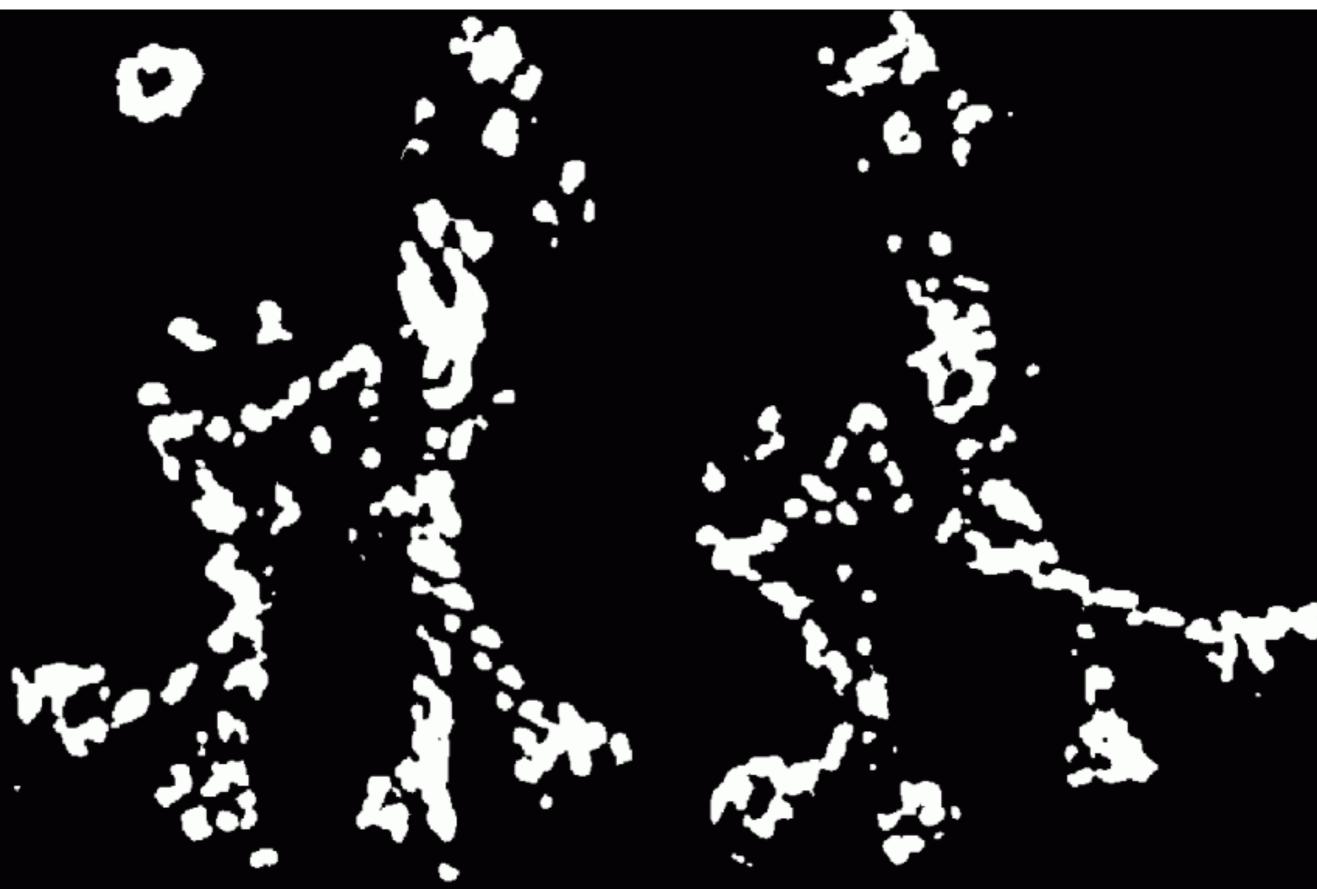




Corner response



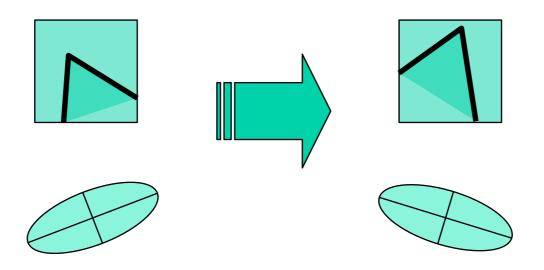
Thresholded corner response



Non-maximal suppression



Harris corner response is invariant to rotation



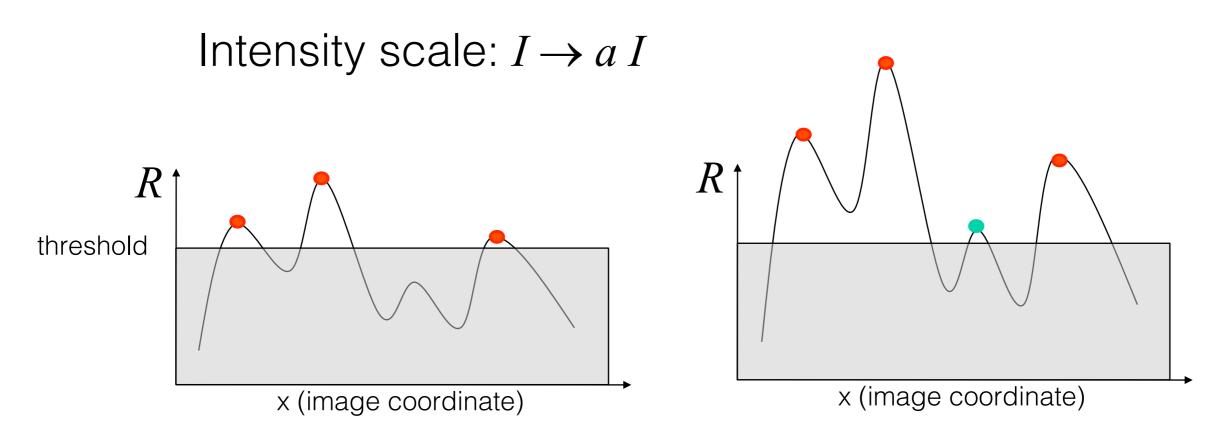
Ellipse rotates but its shape (**eigenvalues**) remains the same

Corner response R is invariant to image rotation

Harris corner response is invariant to intensity changes

Partial invariance to *affine intensity* change

Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$



The Harris detector is not invariant to changes in ...

The Harris corner detector is not invariant to scale



Multi-scale detection

How can we make a feature detector scale-invariant?

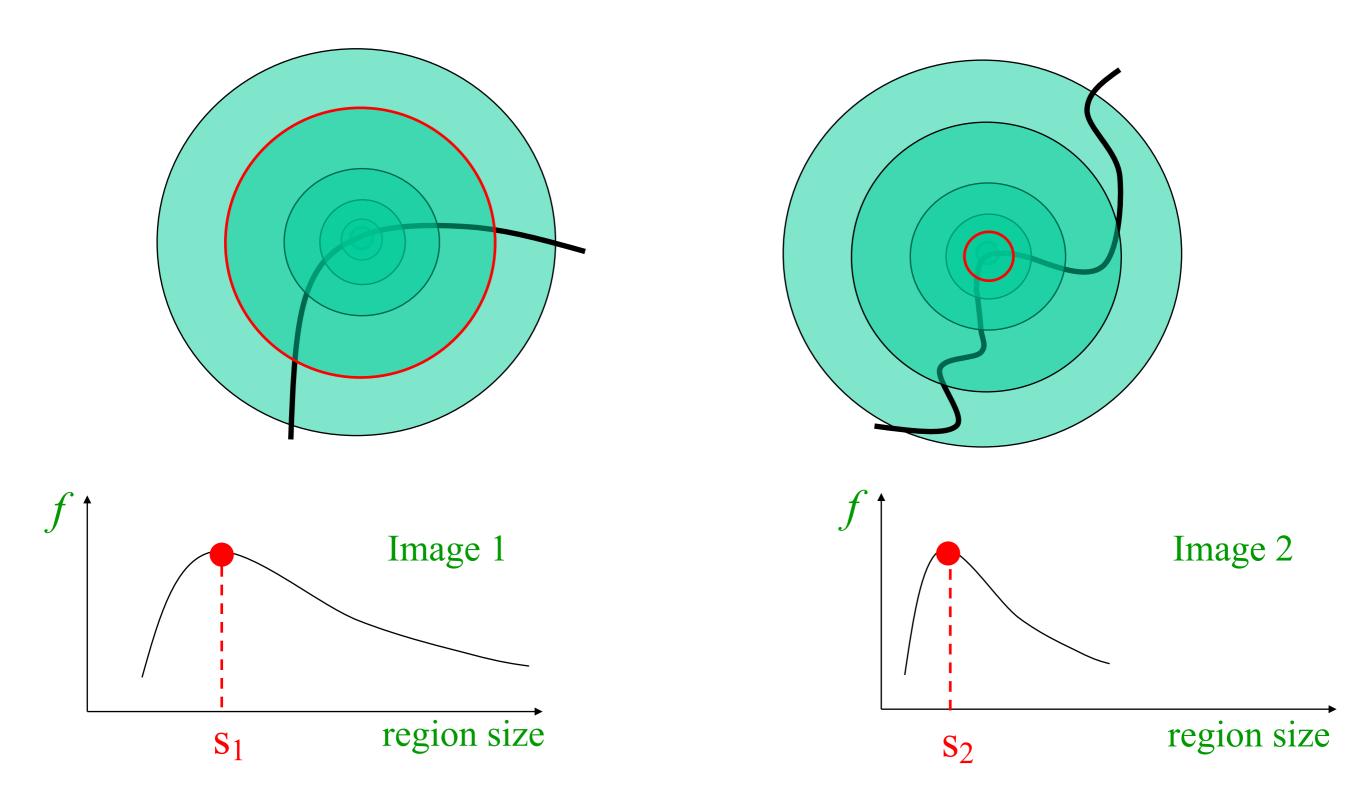
How can we automatically select the scale?

Multi-scale blob detection



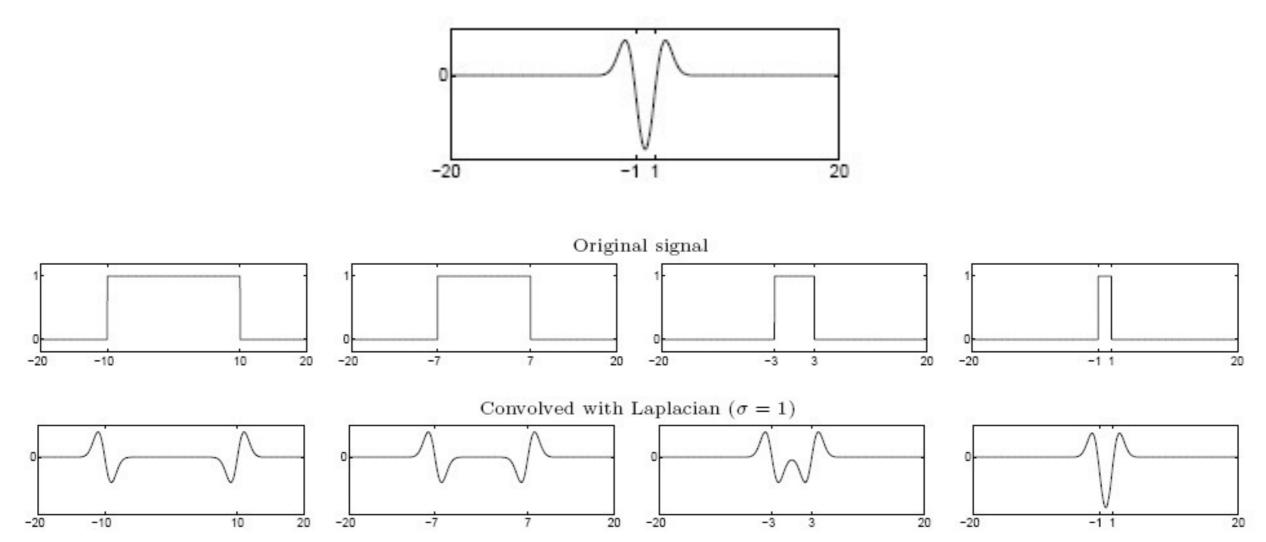
Intuitively...

Find local maxima in both **position** and **scale**



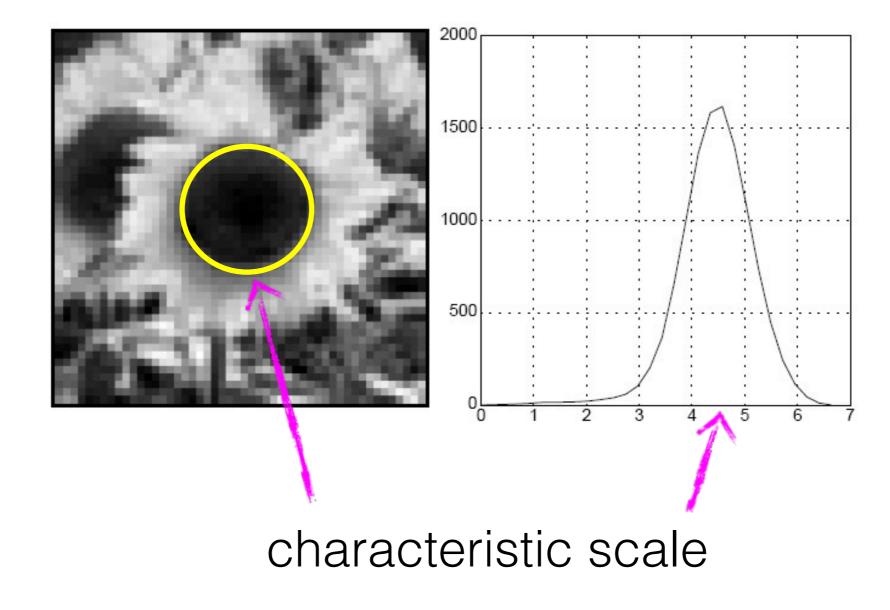
Formally...





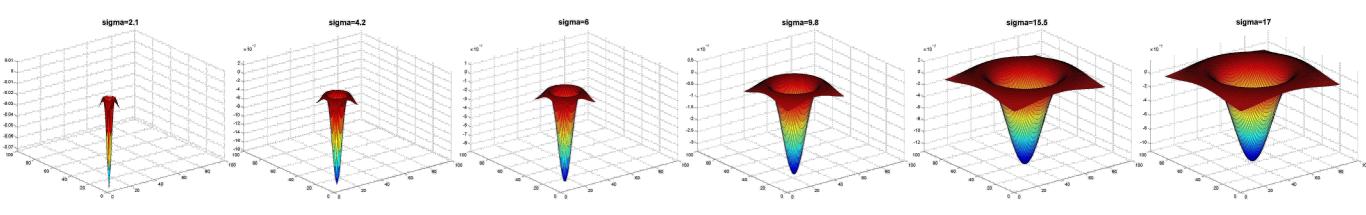
Highest response when the signal has the same **characteristic scale** as the filter

characteristic scale - the scale that produces peak filter response



we need to search over characteristic scales

What happens if you apply different Laplacian filters?

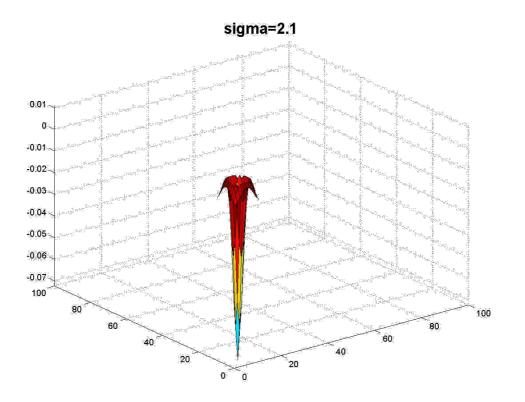


Full size

3/4 size



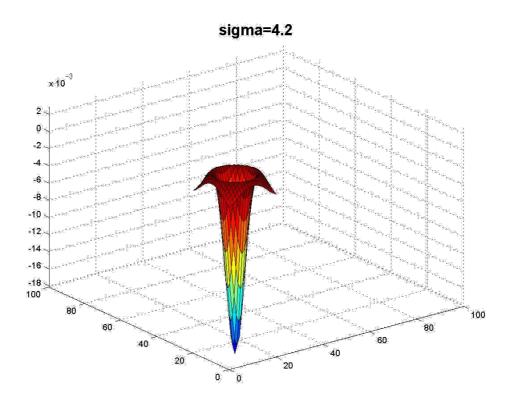


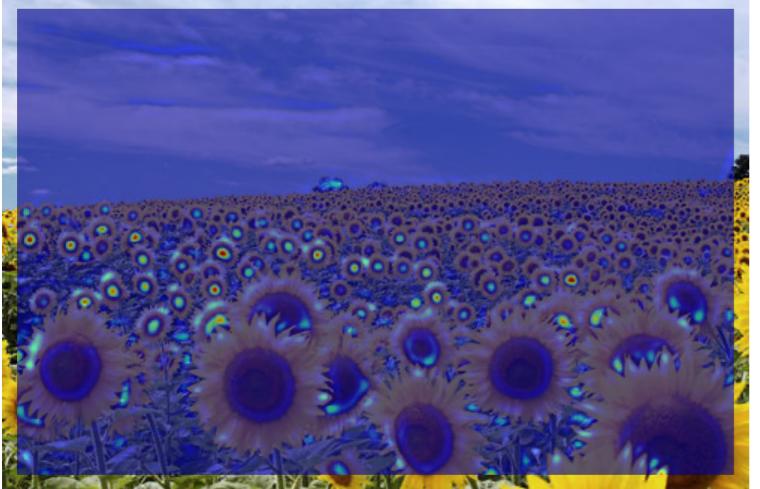




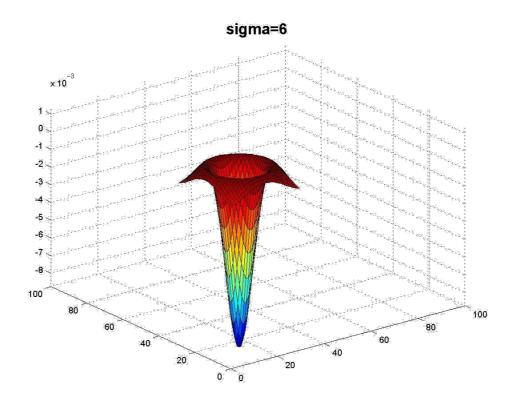


jet color scale blue: low, red: high



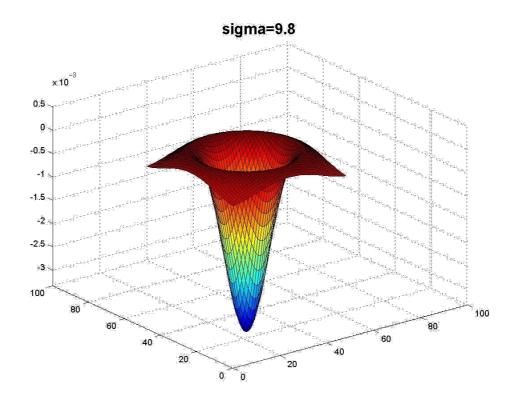




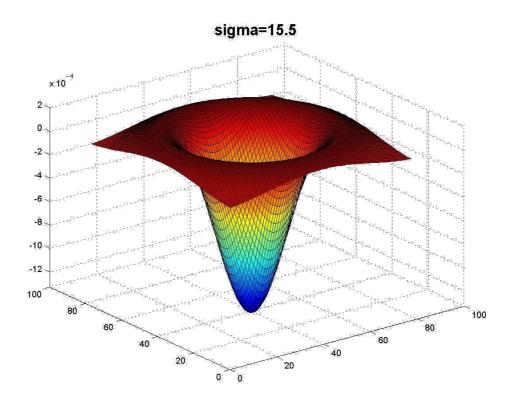






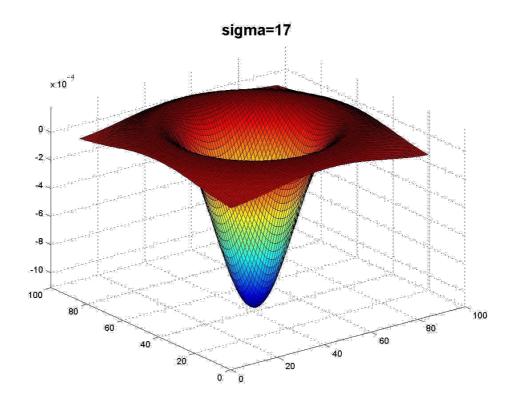
















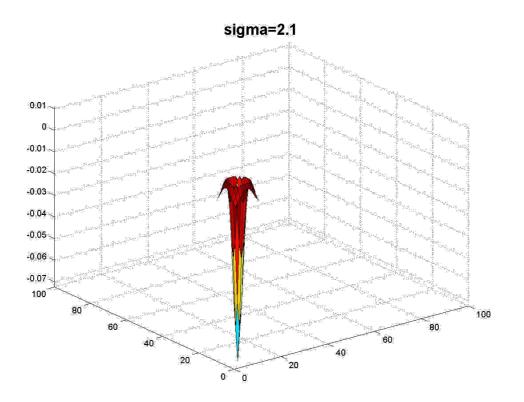
What happened when you applied different Laplacian filters?

Full size

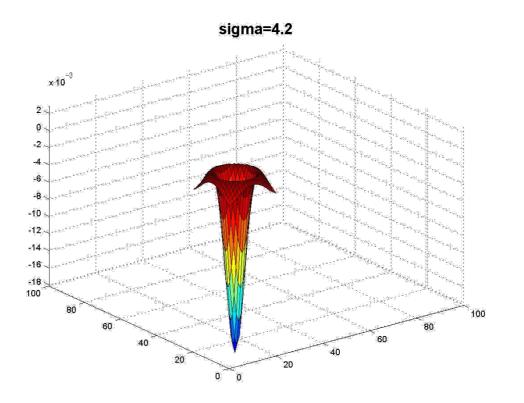
3/4 size

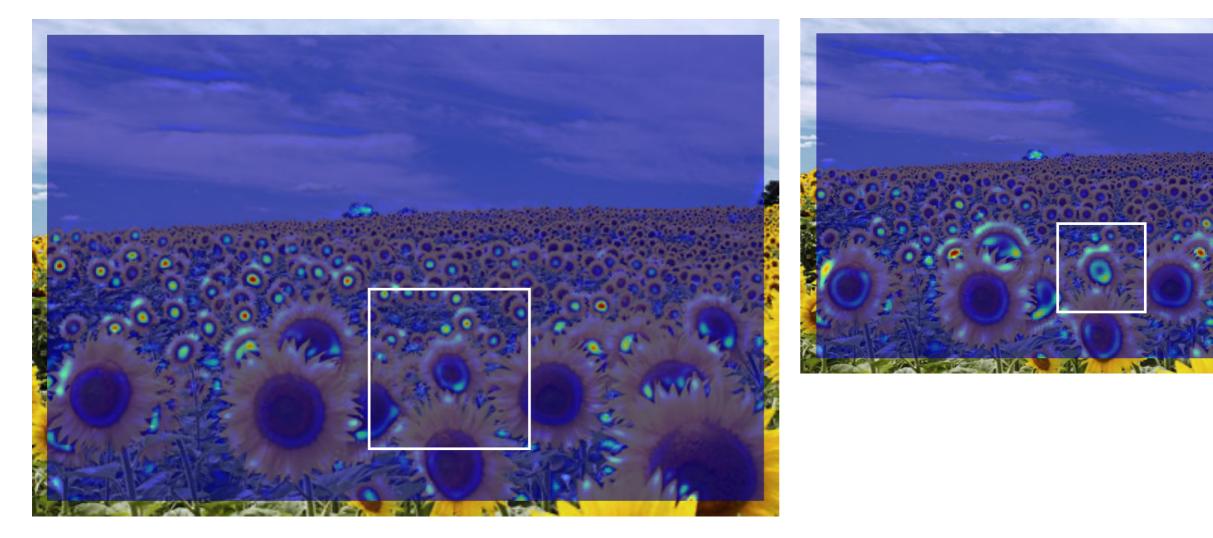


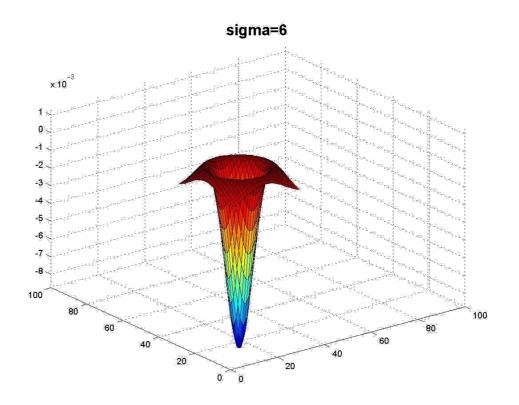


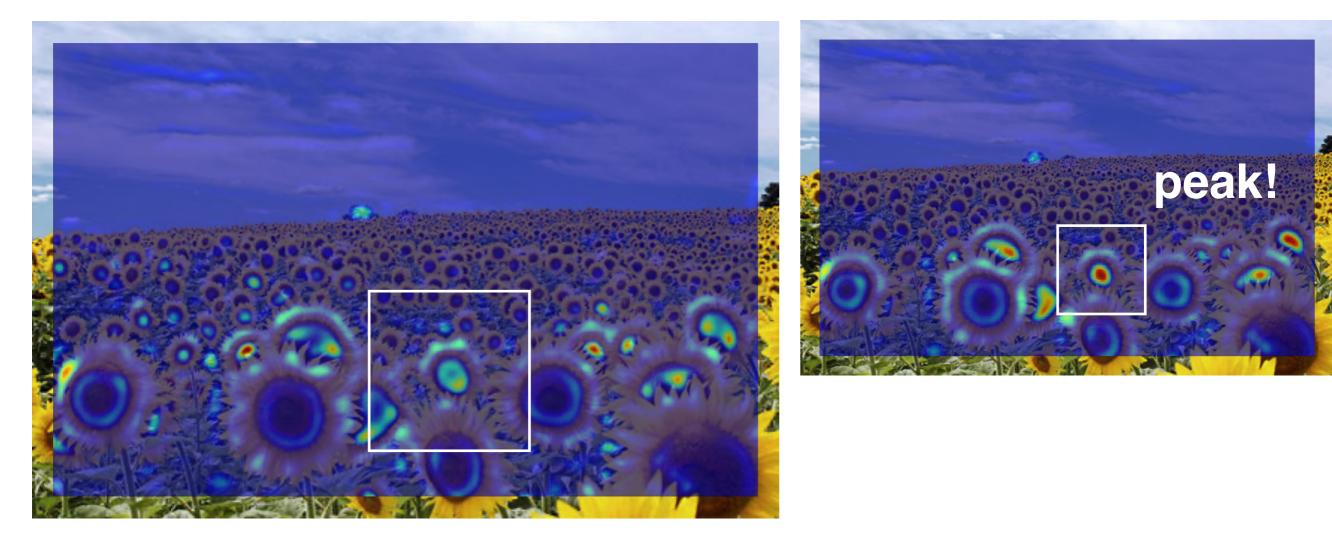


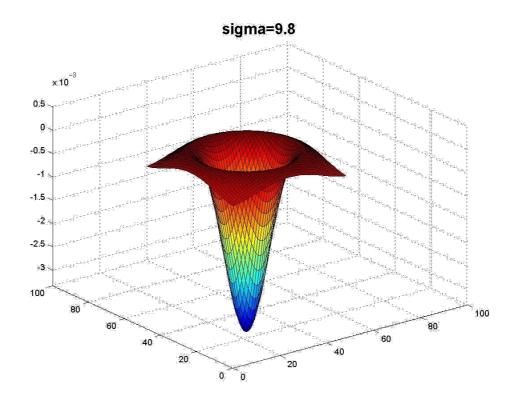


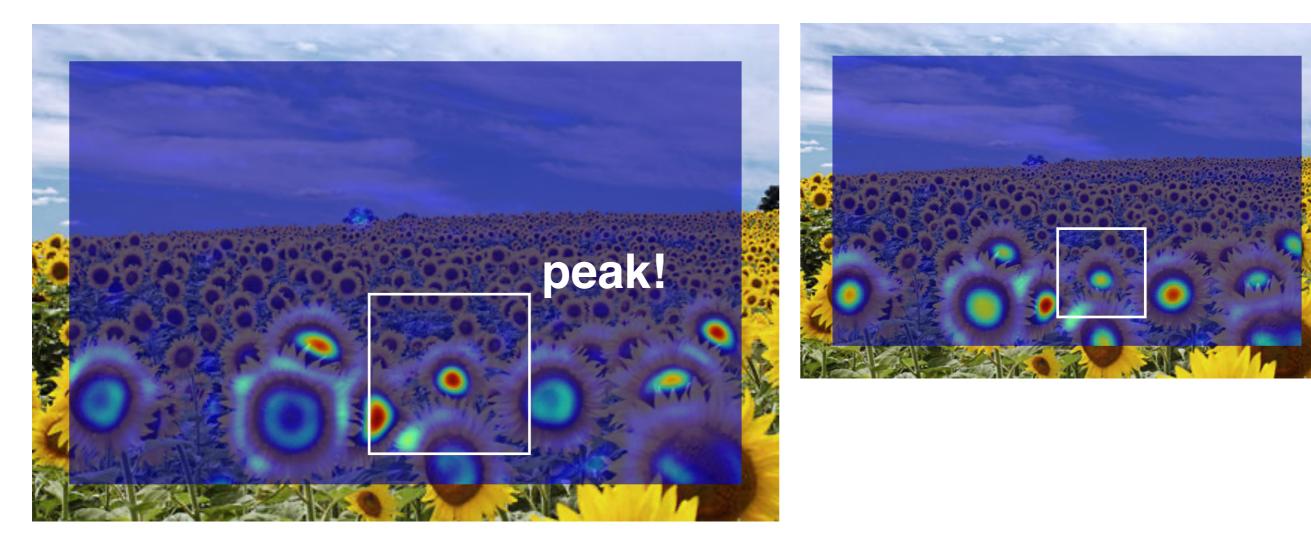


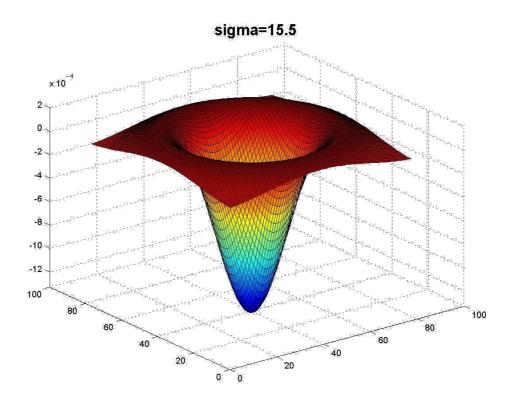


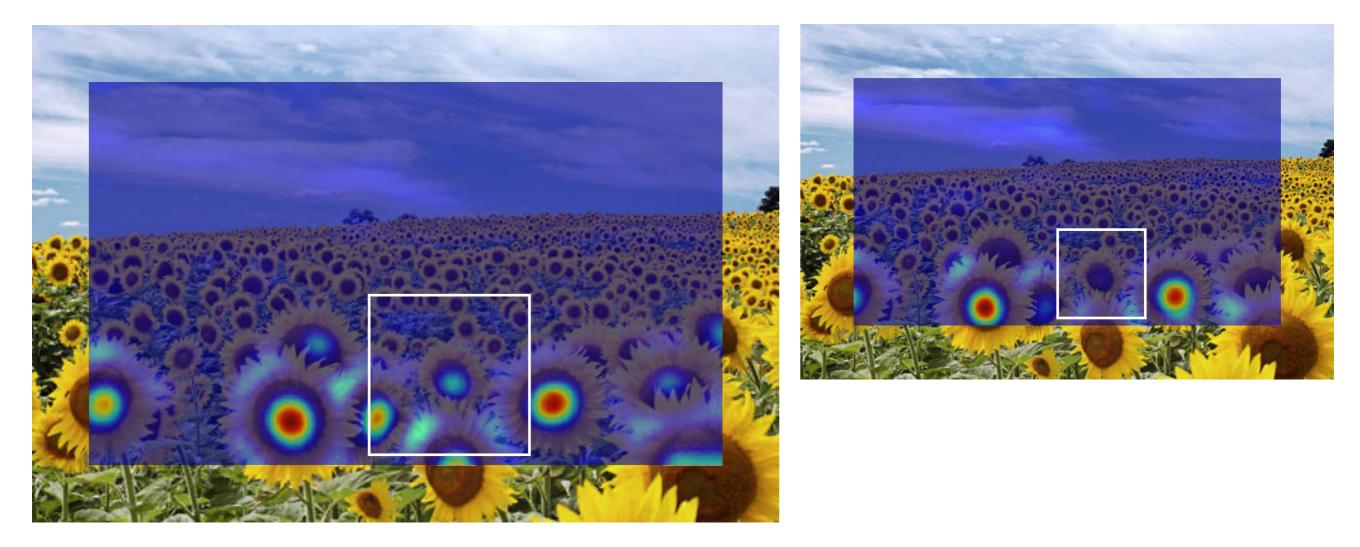


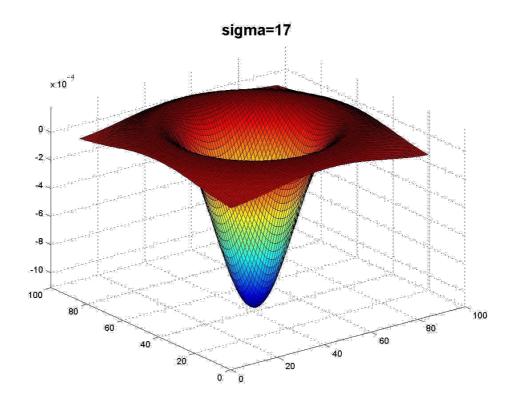


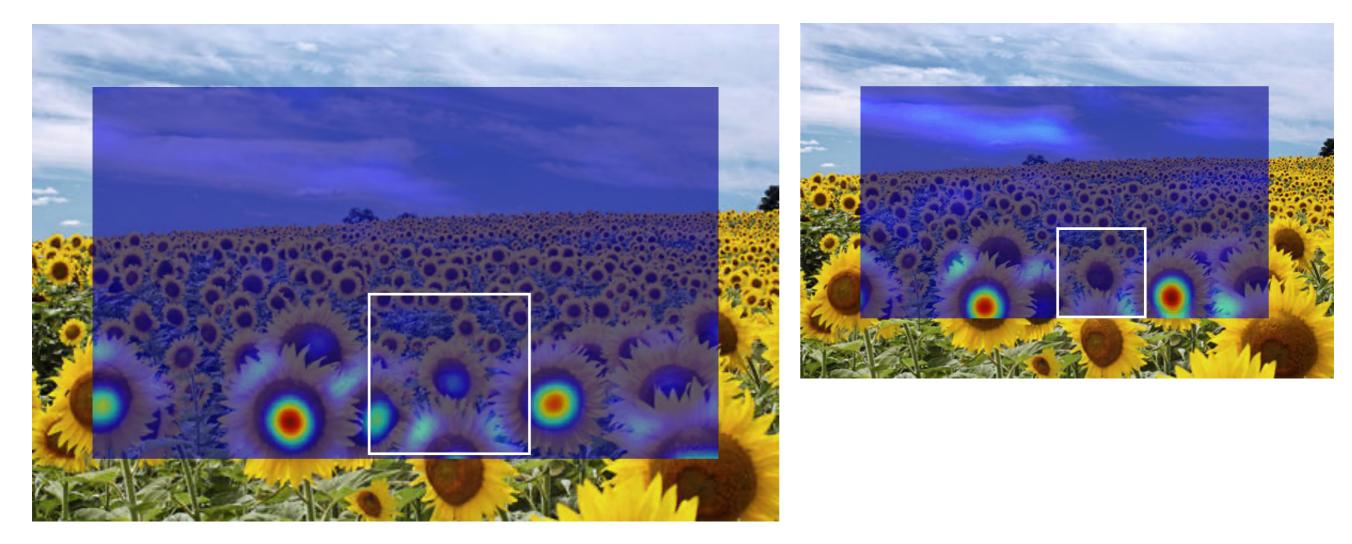


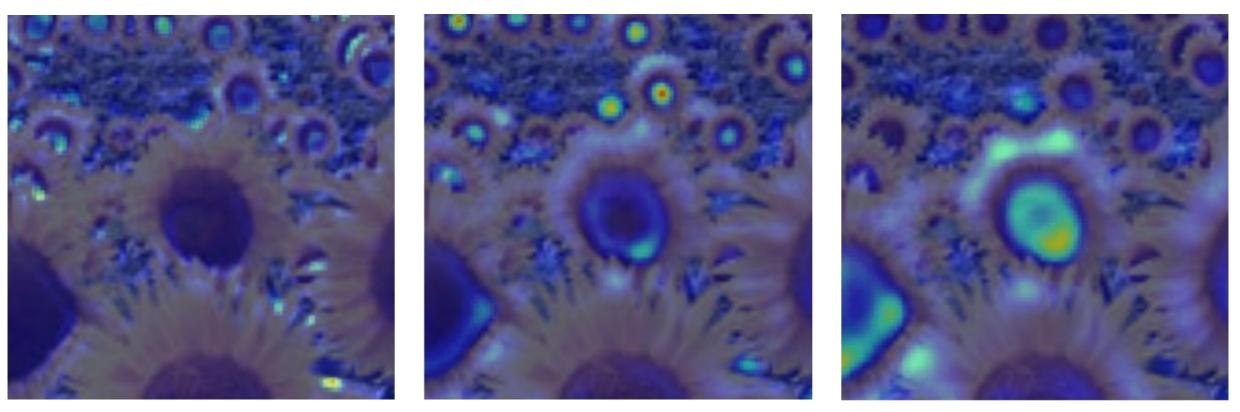






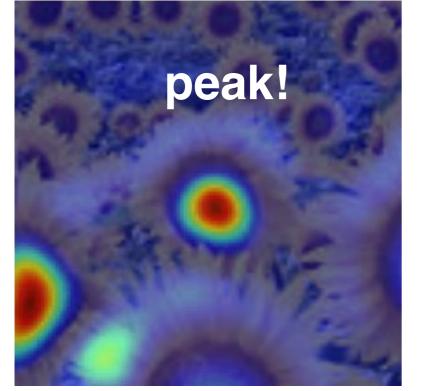


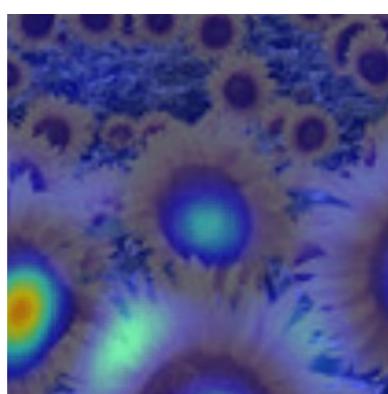


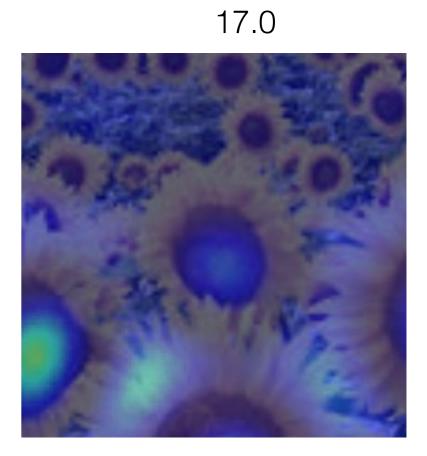


9.8

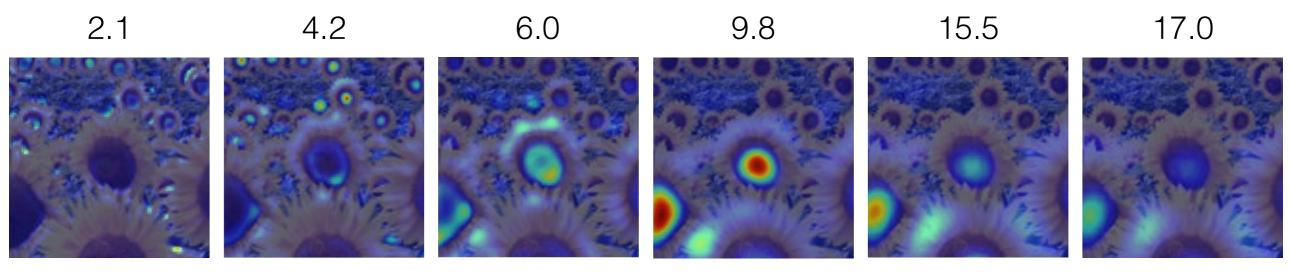
15.5



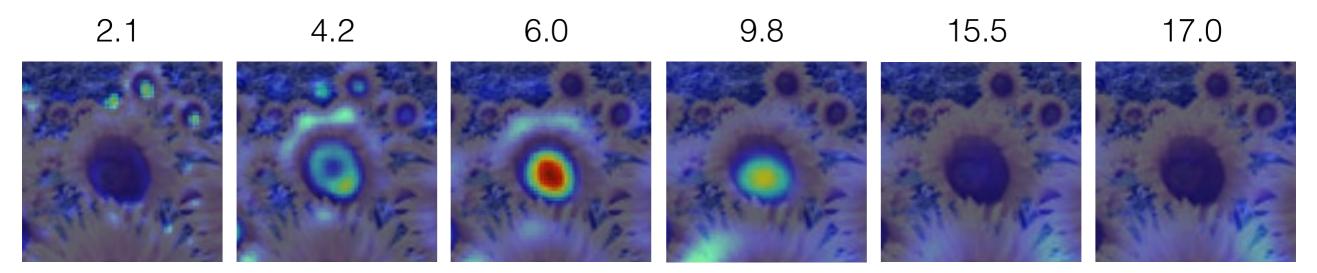




optimal scale

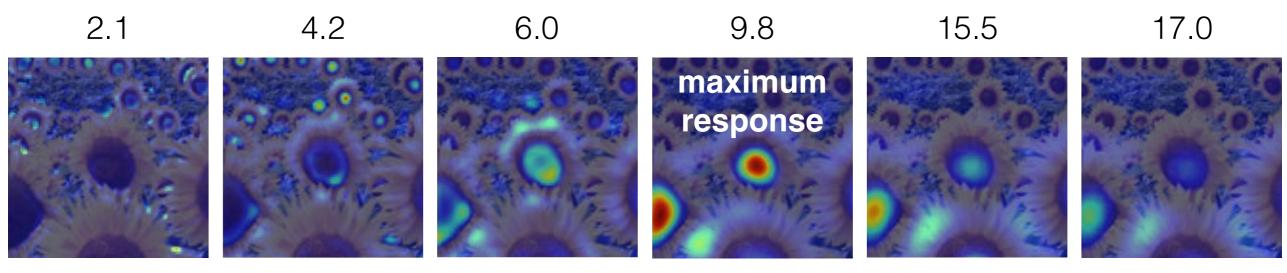


Full size image

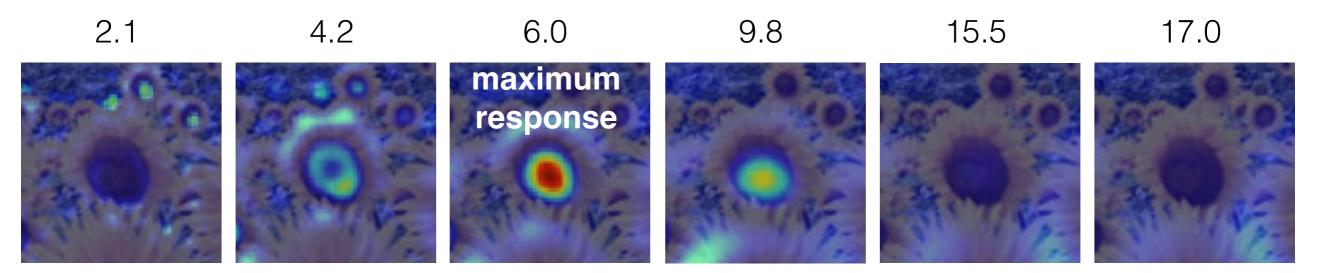


3/4 size image

optimal scale

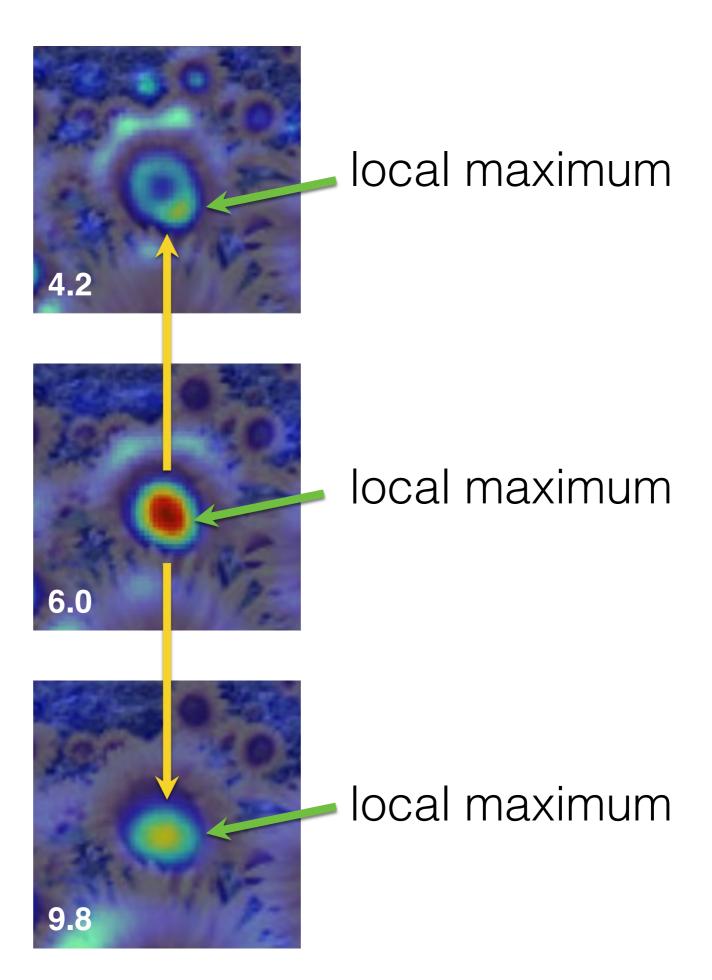


Full size image



3/4 size image

cross-scale maximum



How would you implement scale selection?

Implementation

For each level of the Gaussian pyramid

compute feature response (e.g. Harris, Laplacian)

For each level of the Gaussian pyramid

if local maximum and cross-scale

save scale and location of feature (x,y,s)

