

16-385 Computer Vision Fall 2022, Lecture 12

http://16385.courses.cs.cmu.edu/

Overview of today's lecture

- Revisiting triangulation.
- Disparity.
- Stereo rectification.
- Stereo matching.
- Improving stereo matching.
- Structured light.

Slide credits

Some of these slides were adapted directly from:

- Kris Kitani (16-385, Spring 2017).
- Srinivasa Narasimhan (16-823, Spring 2017).

Revisiting triangulation



Left image





Left image



Right image

1. Select point in one image (how?)



Left image

- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)



Left image

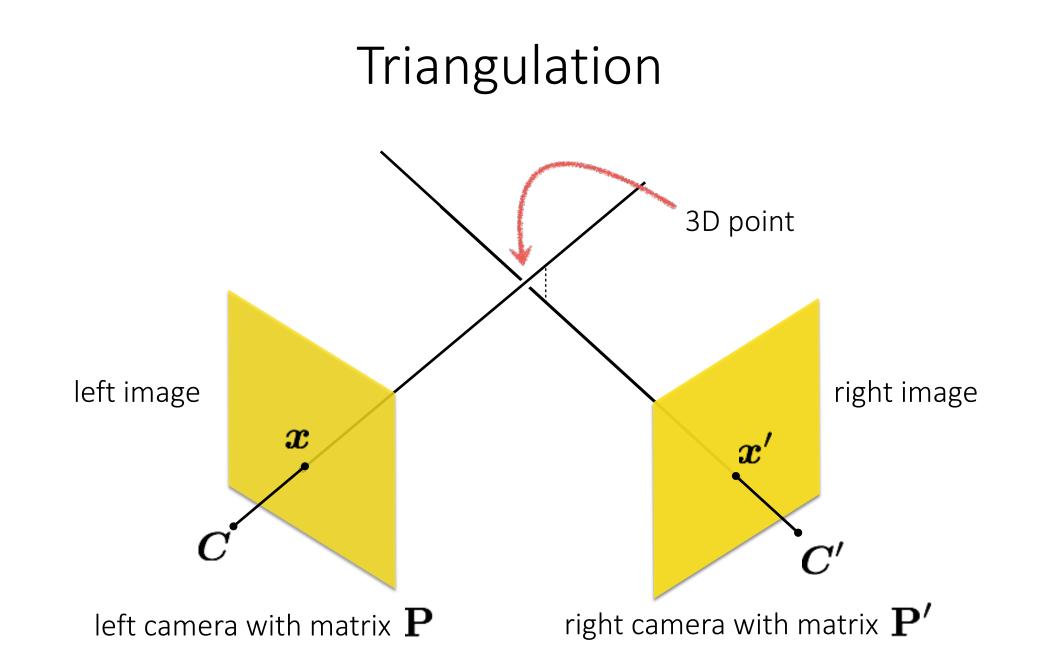
- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)
- 3. Find matching point along line (how?)



Left image



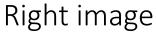
- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)
- 3. Find matching point along line (how?)
- 4. Perform triangulation (how?)





Left image





- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)
- 3. Find matching point along line (how?)
- 4. Perform triangulation (how?)

What are the disadvantages of this procedure?

Stereo rectification





What's different between these two images?







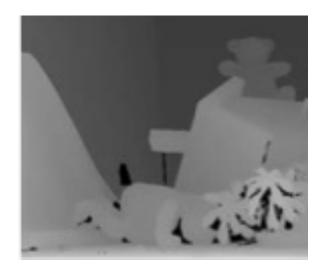


Objects that are close move more or less?

The amount of horizontal movement is inversely proportional to ...





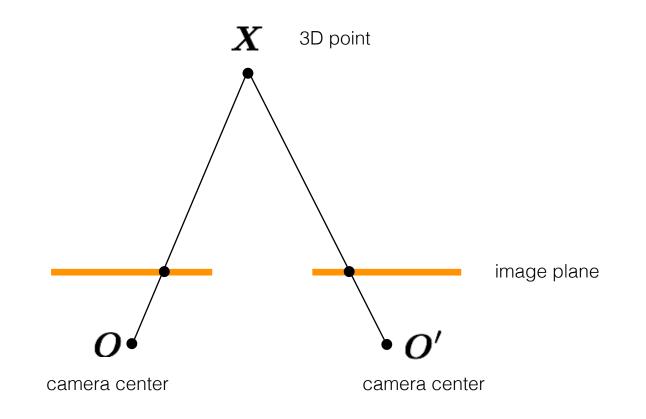


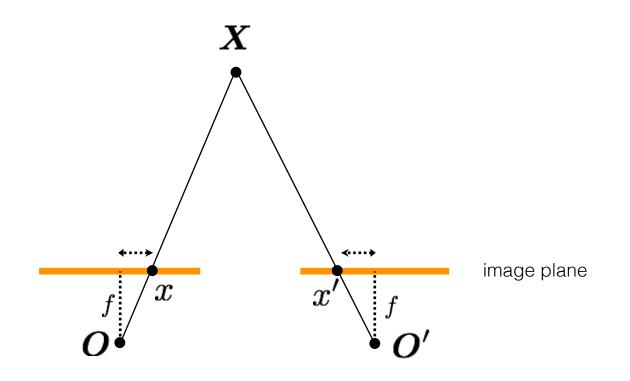
The amount of horizontal movement is inversely proportional to ...

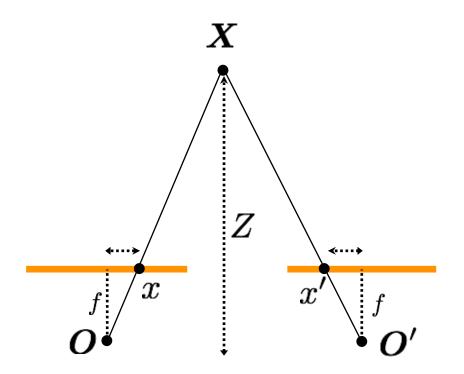


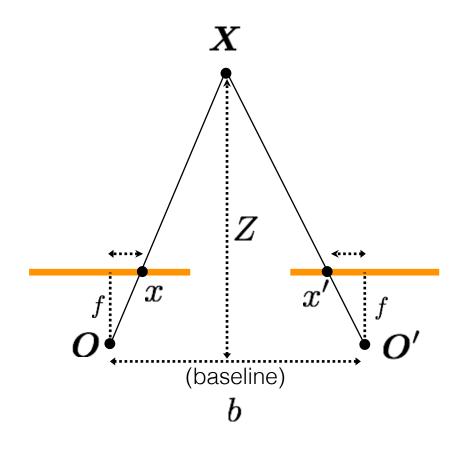
... the distance from the camera.

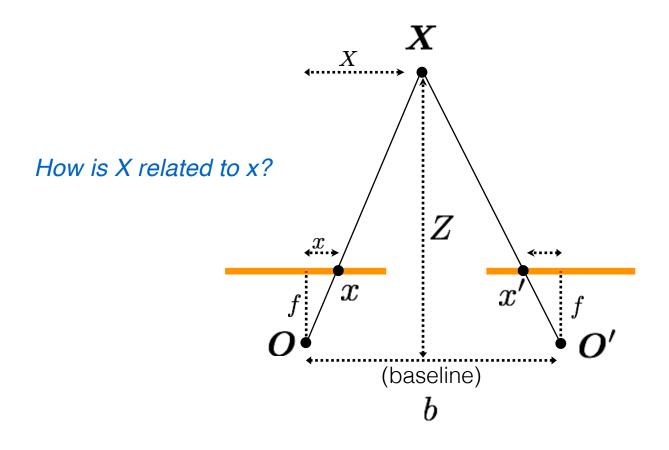
More formally...

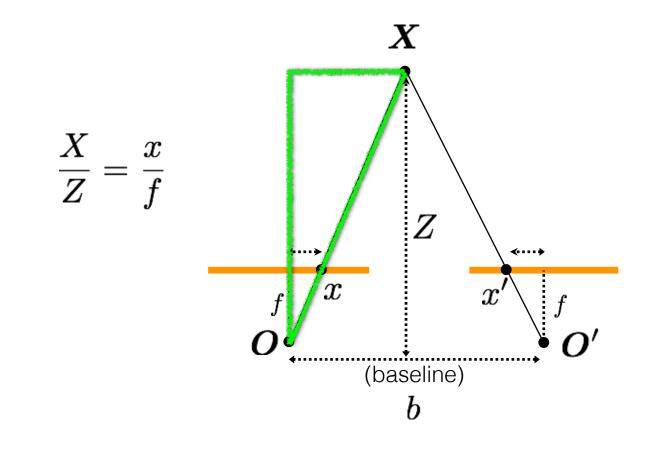


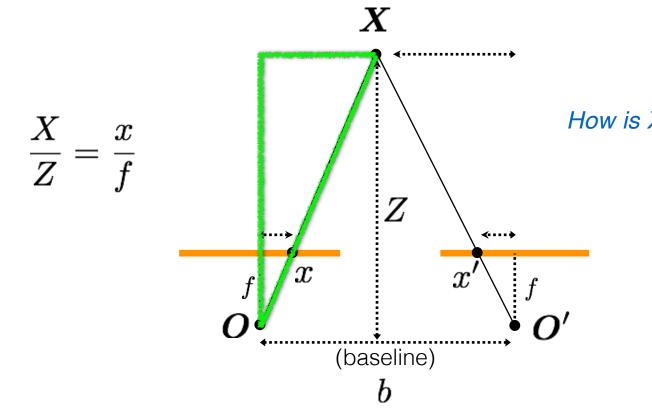




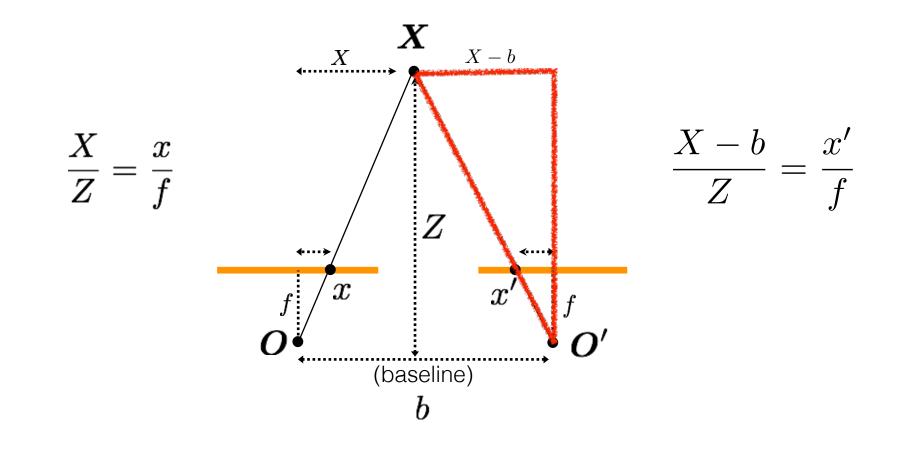


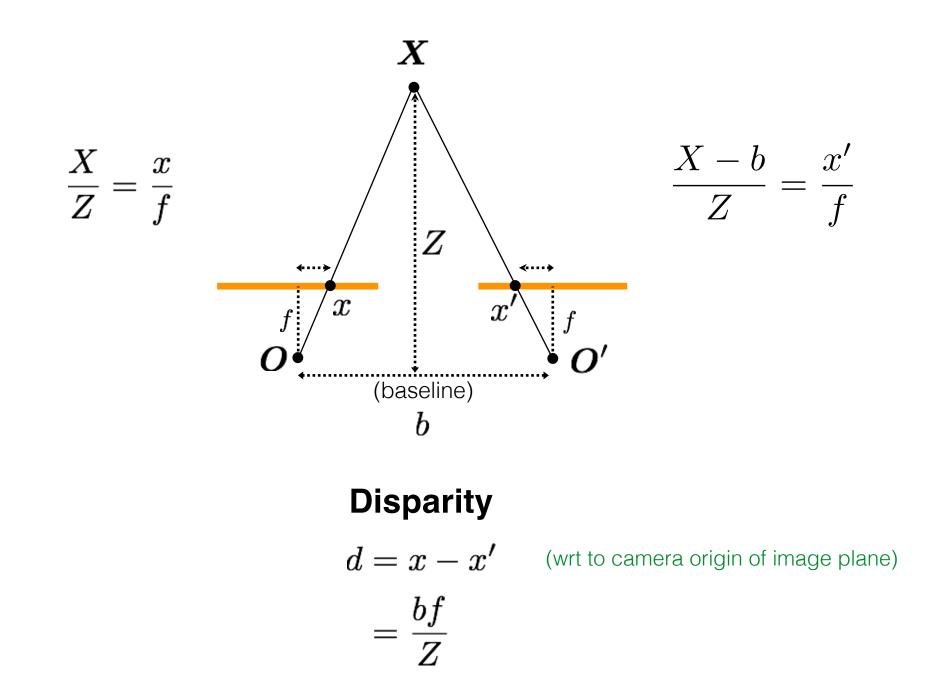


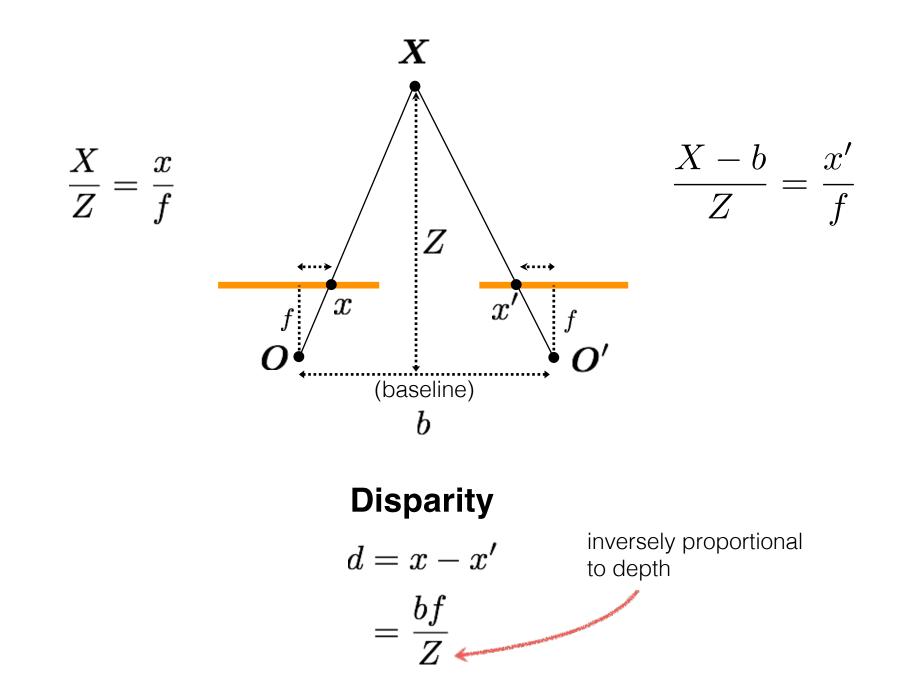












Real-time stereo sensing



Nomad robot searches for meteorites in Antartica http://www.cs.cmu.edu/~meteorite/



Subaru Eyesight system

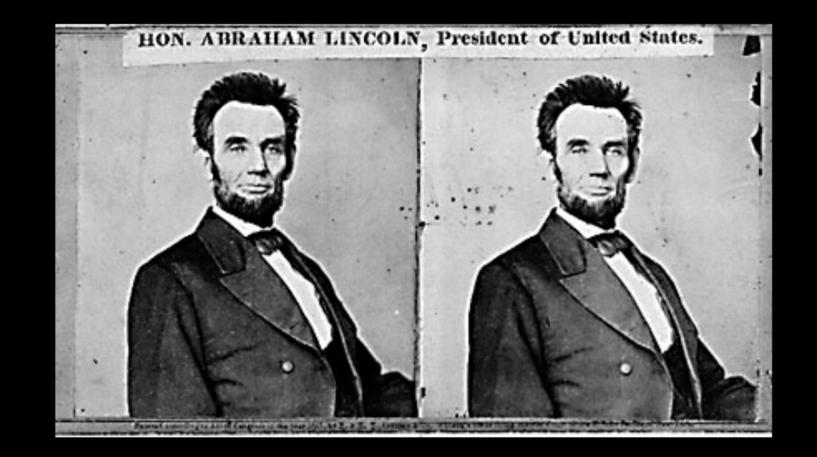




What other vision system uses disparity for depth sensing?

Stereoscopes: A 19th Century Pastime



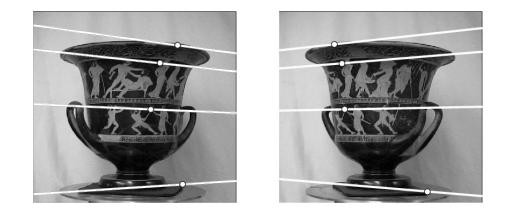




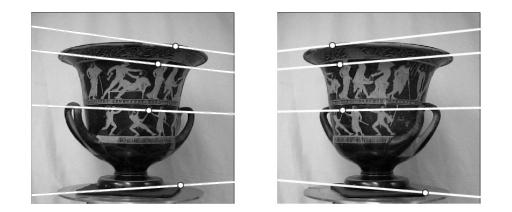
Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923



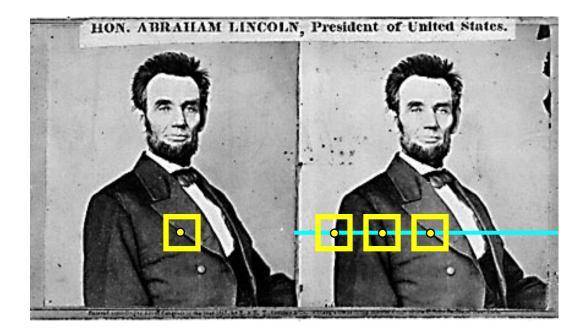
Is disparity the only depth cue the human visual system uses? So can I compute depth from any two images of the same object?



So can I compute depth from any two images of the same object?



- 1. Need sufficient baseline
- 2. Images need to be 'rectified' first (make epipolar lines horizontal)

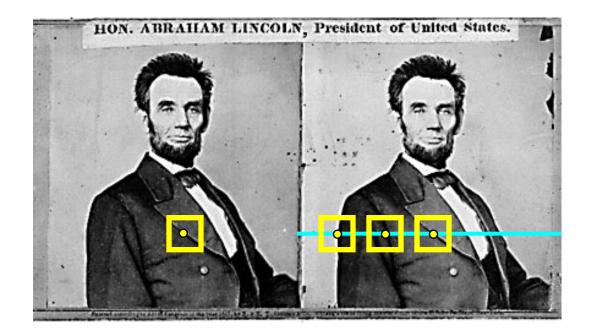


1. Rectify images

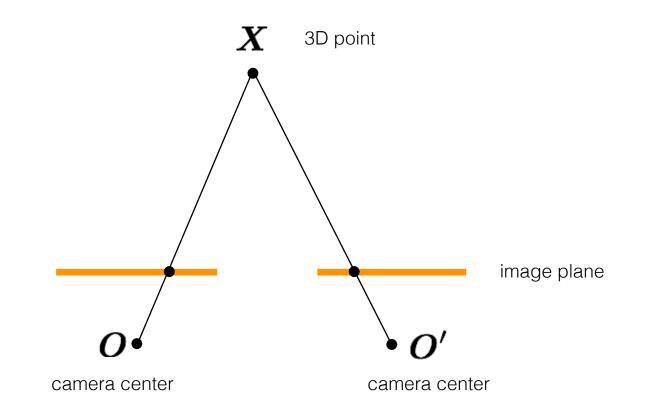
(make epipolar lines horizontal)

- 2. For each pixel
 - a. Find epipolar line
 - b.Scan line for best match
 - c.Compute depth from disparity

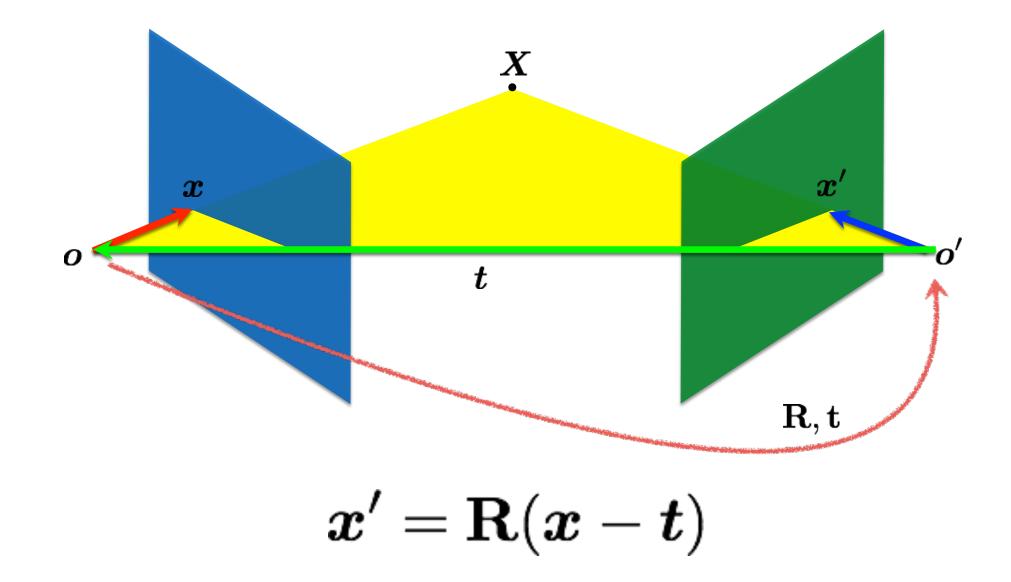
$$Z = \frac{bf}{d}$$



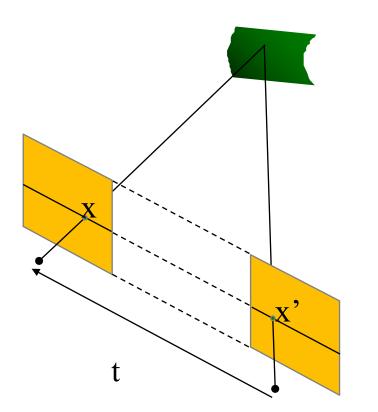
How can you make the epipolar lines horizontal?



What's special about these two cameras?

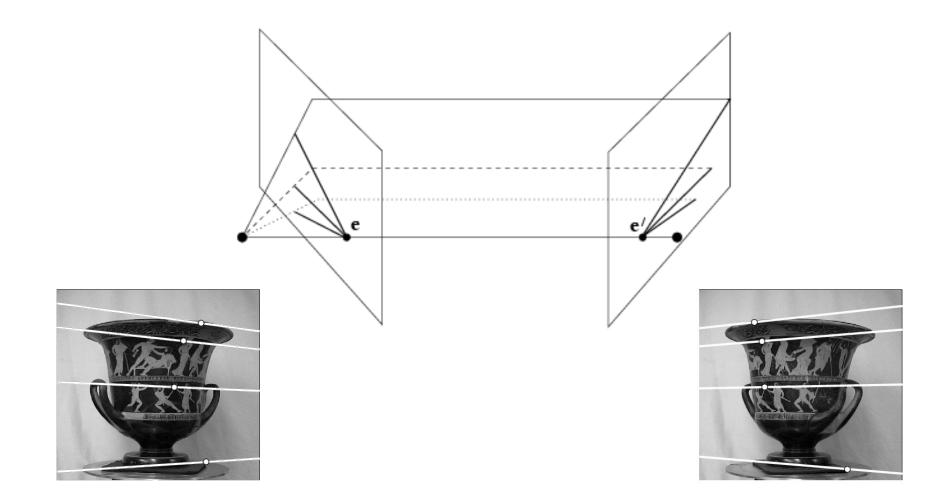


When are epipolar lines horizontal?



When this relationship holds:

$$R = I \qquad t = (T, 0, 0)$$



It's hard to make the image planes exactly parallel



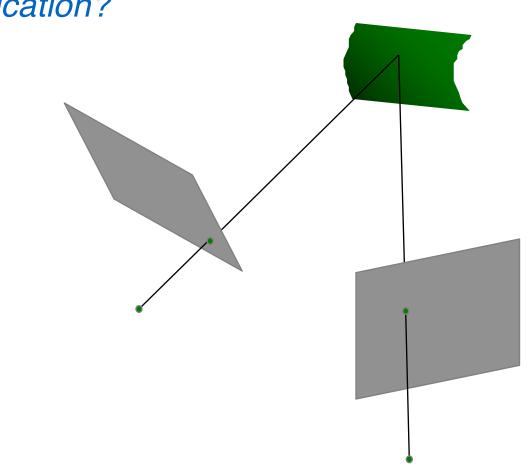
How can you make the epipolar lines horizontal?





Use stereo rectification?

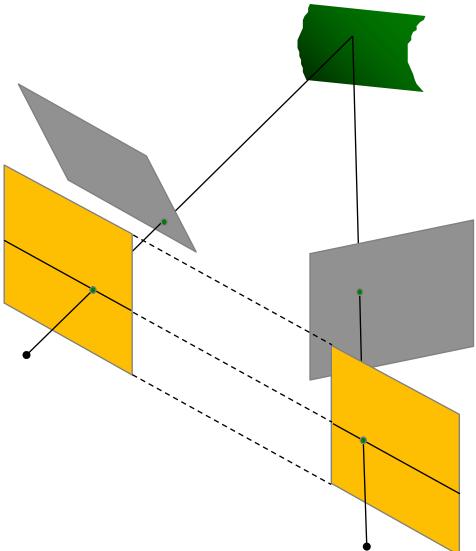




What is stereo rectification?

What is stereo rectification?

Reproject image planes onto a common plane parallel to the line between camera centers

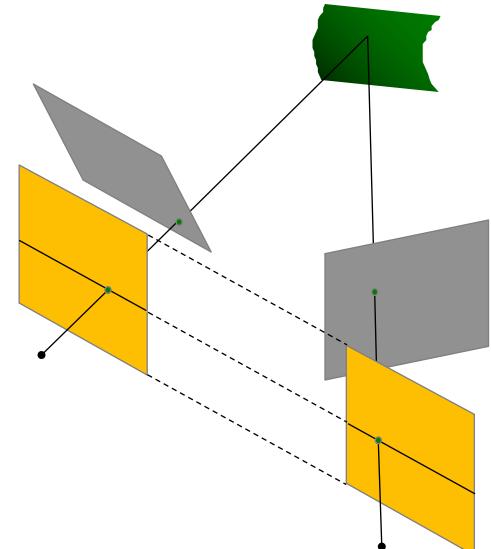


How can you do this?

What is stereo rectification?

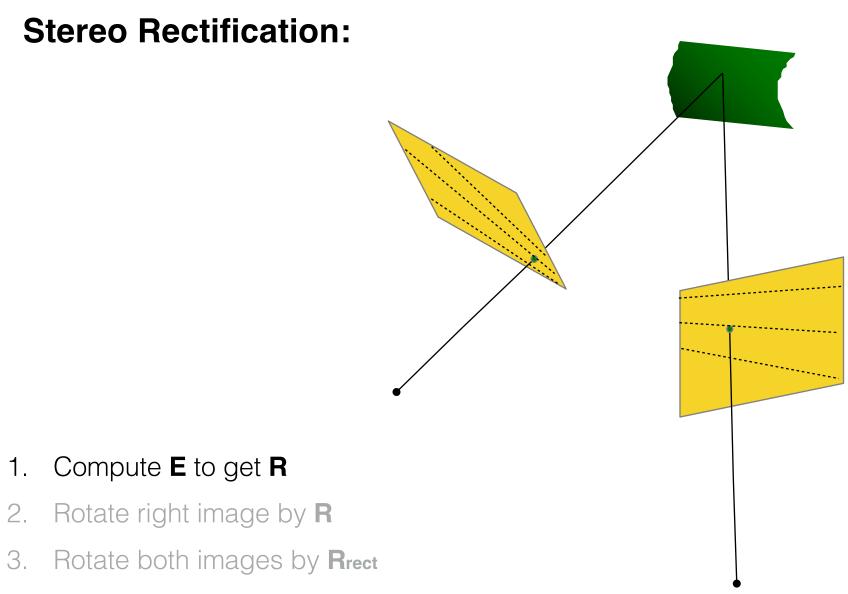
Reproject image planes onto a common plane parallel to the line between camera centers

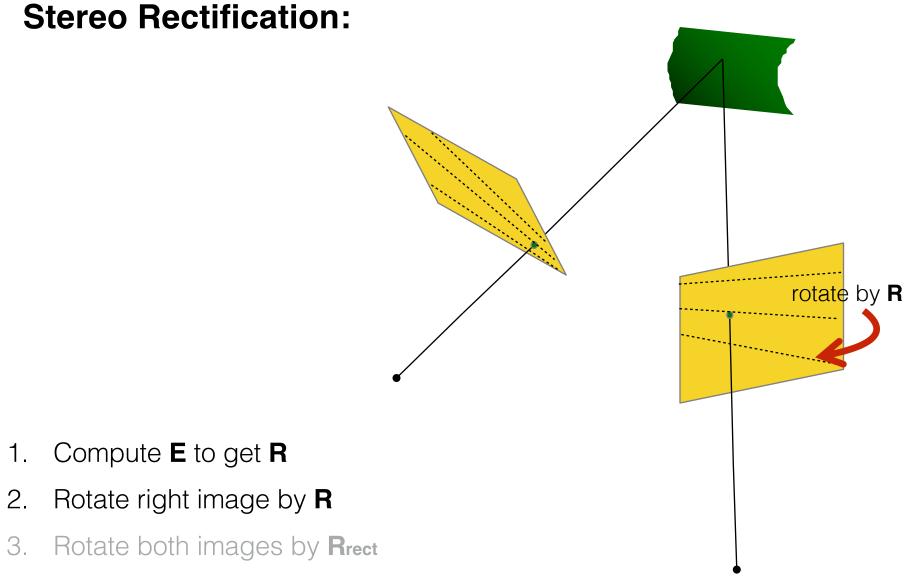
Need two homographies (3x3 transform), one for each input image reprojection

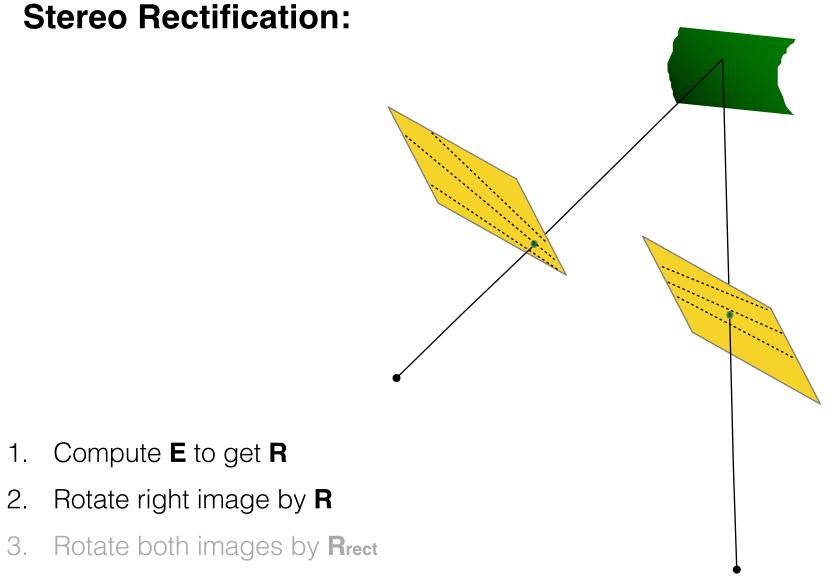


Stereo Rectification

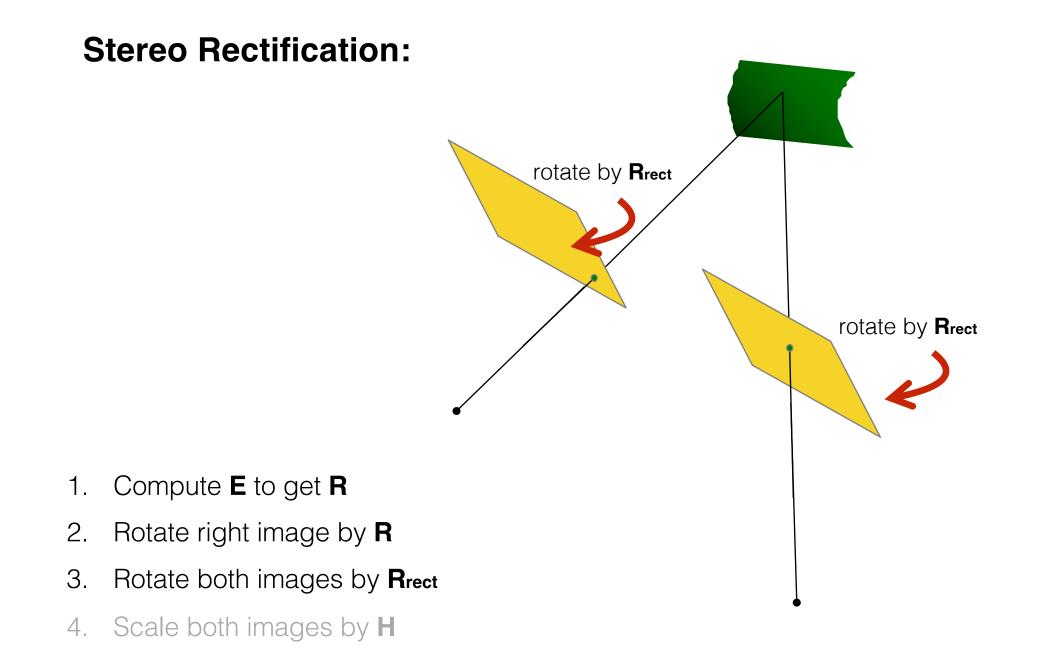
- Rotate the right camera by R (aligns camera coordinate system orientation only)
- 2. Rotate (**rectify**) the left camera so that the epipole is at infinity
- 3. Rotate (**rectify**) the right camera so that the epipole is at infinity
- 4. Adjust the **scale**



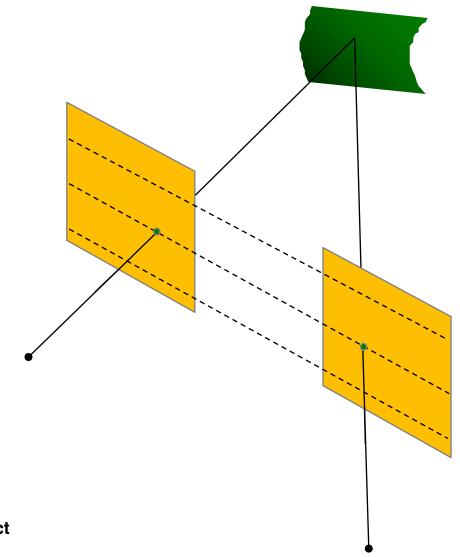




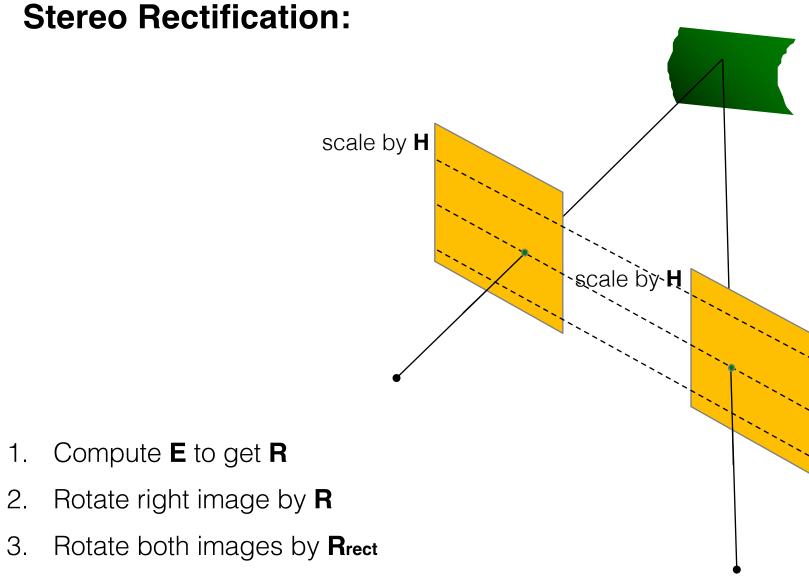
1.



Stereo Rectification:



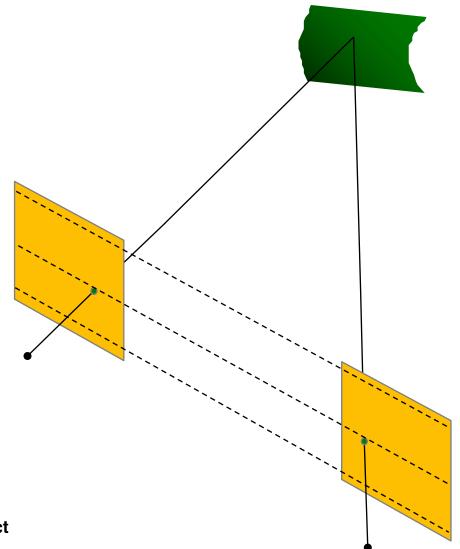
- 1. Compute **E** to get **R**
- 2. Rotate right image by **R**
- 3. Rotate both images by **R**rect
- 4. Scale both images by **H**



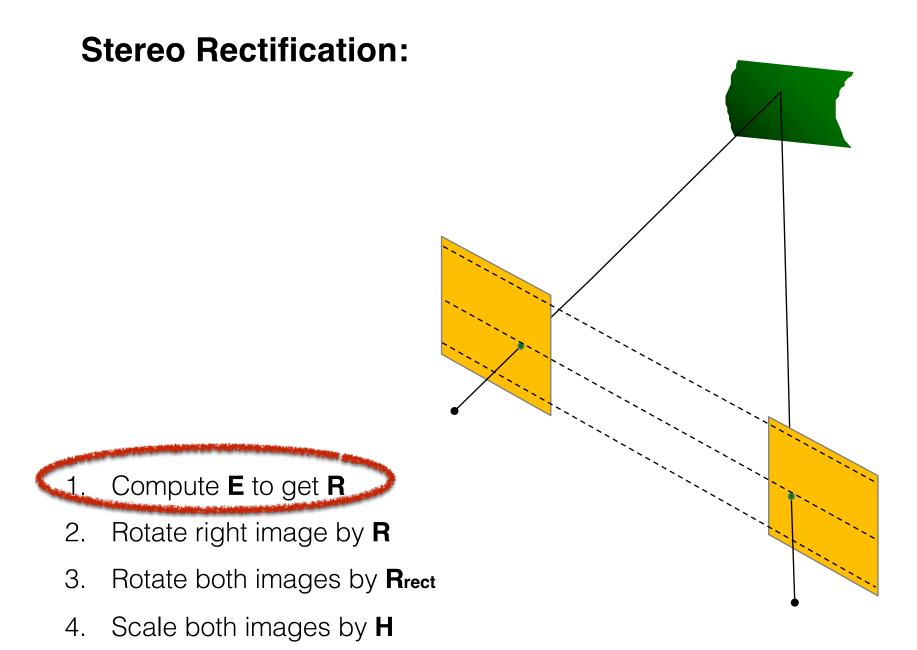
1.

З.

Stereo Rectification:



- 1. Compute **E** to get **R**
- 2. Rotate right image by **R**
- 3. Rotate both images by **R**rect
- 4. Scale both images by **H**



Suppose
$$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$$

Step 1: Use \mathbf{E} to get \mathbf{R}
SVD: $\mathbf{E} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^{\top}$ Let $\mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

We get FOUR solutions:

 $\begin{aligned} \mathbf{R}_1 &= \mathbf{U} \mathbf{W} \mathbf{V}^\top \quad \mathbf{R}_2 &= \mathbf{U} \mathbf{W}^\top \mathbf{V}^\top \qquad \begin{bmatrix} \mathbf{t} \end{bmatrix}_\times &= \pm \mathbf{U} \mathbf{W} \mathbf{\Sigma} \mathbf{U}^T \\ \text{two possible rotations} \qquad & \text{two possible translations} \end{aligned}$

We get FOUR solutions:

$$\begin{aligned} \mathbf{R}_1 &= \mathbf{U} \mathbf{W} \mathbf{V}^\top & \mathbf{R}_1 &= \mathbf{U} \mathbf{W} \mathbf{V}^\top \\ \left[\mathbf{t} \right]_{\times} &= \mathbf{U} \mathbf{W} \mathbf{\Sigma} \mathbf{U}^T & \left[\mathbf{t} \right]_{\times} &= -\mathbf{U} \mathbf{W} \mathbf{\Sigma} \mathbf{U}^T \end{aligned}$$

$$\begin{aligned} \mathbf{R}_2 &= \mathbf{U} \mathbf{W}^\top \mathbf{V}^\top & \mathbf{R}_2 &= \mathbf{U} \mathbf{W}^\top \mathbf{V}^\top \\ \left[\mathbf{t} \right]_{\times} &= \mathbf{U} \mathbf{W} \boldsymbol{\Sigma} \mathbf{U}^T & \left[\mathbf{t} \right]_{\times} &= -\mathbf{U} \mathbf{W} \boldsymbol{\Sigma} \mathbf{U}^T \end{aligned}$$

Which one do we choose?

Compute determinant of R, valid solution must be equal to 1 (note: det(R) = -1 means rotation and reflection)

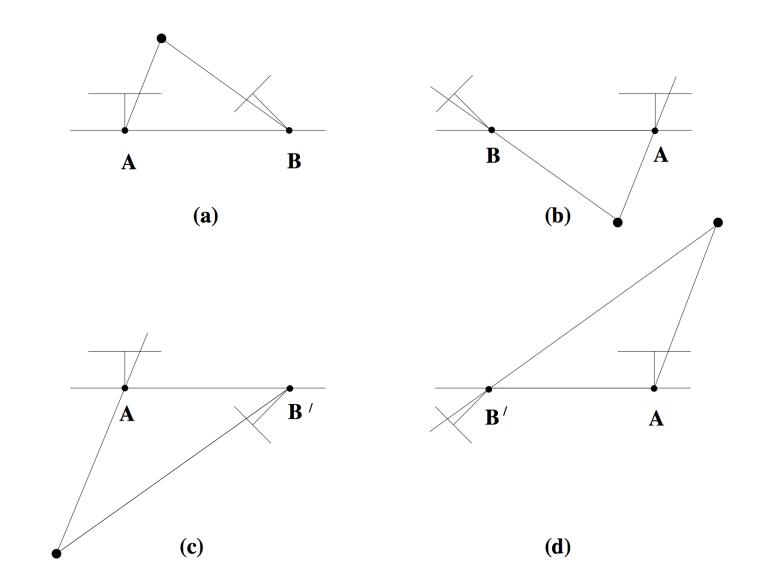
Compute 3D point using triangulation, valid solution has positive Z value (Note: negative Z means point is behind the camera)

Let's visualize the four configurations...

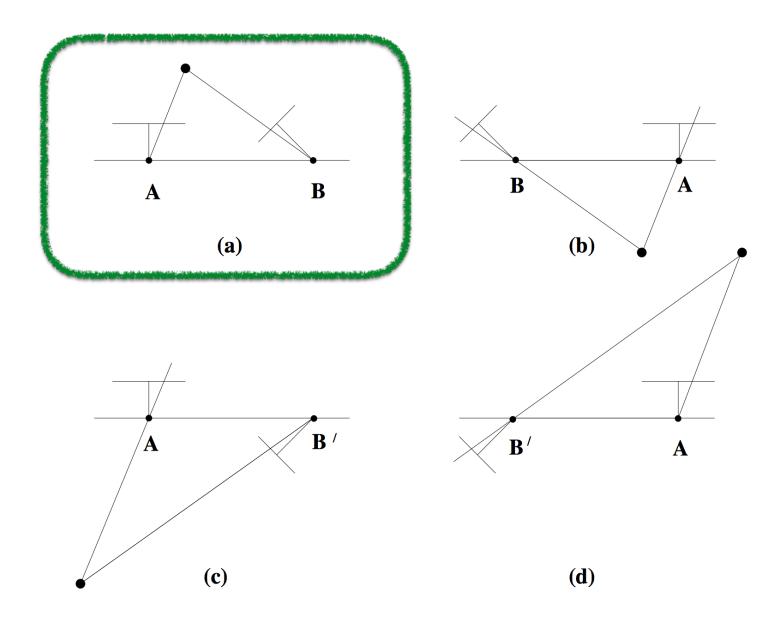


Find the configuration where the point is in front of both cameras

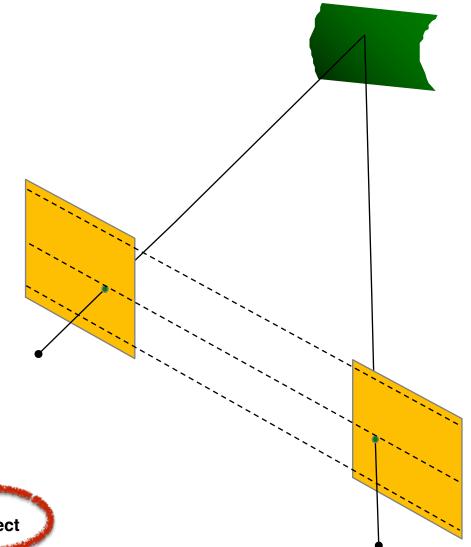
Find the configuration where the points is in front of both cameras



Find the configuration where the points is in front of both cameras



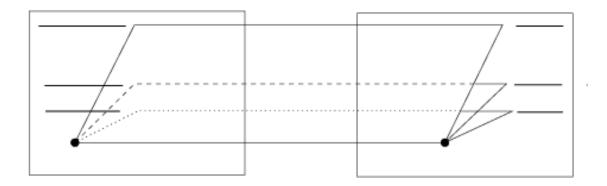
Stereo Rectification:

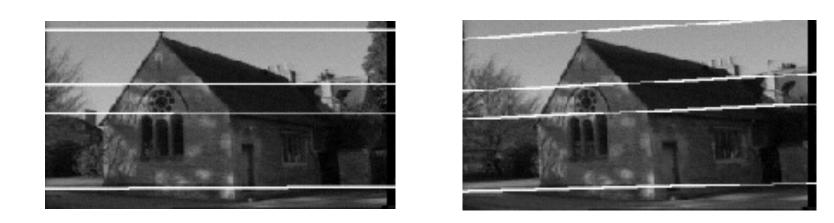


- 1. Compute **E** to get **R**
- 2. Rotate right image by R
- 3. Rotate both images by **R**rect
- 4. Scale both images by H

When do epipolar lines become horizontal?

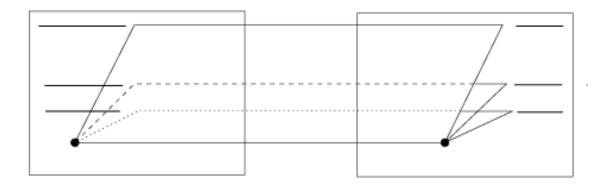
Parallel cameras

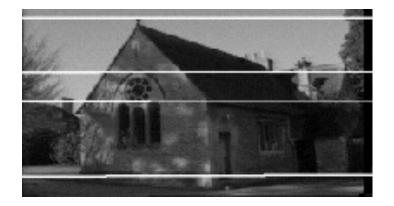


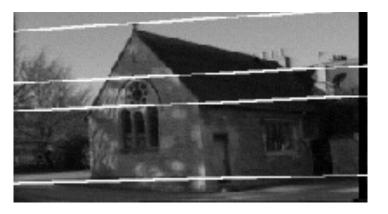


Where is the epipole?

Parallel cameras







epipole at infinity

Setting the epipole to infinity

(Building **R**_{rect} from **e**)

Let
$$R_{\text{rect}} = \begin{bmatrix} \boldsymbol{r}_1^\top \\ \boldsymbol{r}_2^\top \\ \boldsymbol{r}_3^\top \end{bmatrix}$$
 Given: (using SVD on E)
(translation from E)

$$m{r}_1 = m{e}_1 = rac{T}{||T||}$$
 epipole coincides with translation vector
 $m{r}_2 = rac{1}{\sqrt{T_x^2 + T_y^2}} \begin{bmatrix} -T_y & T_x & 0 \end{bmatrix}$ cross product of e and the direction vector of the optical axis

If
$$\mathbf{r}_1 = \mathbf{e}_1 = \frac{T}{||T||}$$
 and \mathbf{r}_2 \mathbf{r}_3 orthogona
then $R_{\text{rect}}\mathbf{e}_1 = \begin{bmatrix} \mathbf{r}_1^{\top}\mathbf{e}_1 \\ \mathbf{r}_2^{\top}\mathbf{e}_1 \\ \mathbf{r}_3^{\top}\mathbf{e}_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$

If
$$r_1 = e_1 = \frac{T}{||T||}$$
 and r_2 r_3 orthogonal
then $R_{\text{rect}}e_1 = \begin{bmatrix} r_1^\top e_1 \\ r_2^\top e_1 \\ r_3^\top e_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Where is this point located on the image plane?

If
$$r_1 = e_1 = \frac{T}{||T||}$$
 and r_2 r_3 orthogonal
then $R_{\text{rect}}e_1 = \begin{bmatrix} r_1^\top e_1 \\ r_2^\top e_1 \\ r_3^\top e_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Where is this point located on the image plane?

At x-infinity

Stereo Rectification Algorithm

- 1. Estimate E using the 8 point algorithm (SVD)
- 2. Estimate the epipole e (SVD of E)
- 3. Build **R**rect from **e**
- 4. Decompose E into R and T
- 5. Set $\mathbf{R}_1 = \mathbf{R}_{\text{rect}}$ and $\mathbf{R}_2 = \mathbf{R}\mathbf{R}_{\text{rect}}$
- 6. Rotate each left camera point (warp image)
 [x' y' z'] = R1 [x y z]
- 7. Rectified points as $\mathbf{p} = f/z'[x' y' z']$
- 8. Repeat 6 and 7 for right camera points using \mathbf{R}_2



What can we do after

rectification?

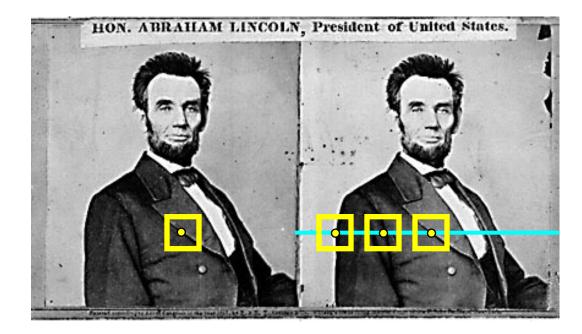


Stereo matching



Depth Estimation via Stereo Matching





1. Rectify images

c.Compute depth from disparity

$$Z = \frac{bf}{d}$$

How would you do this?

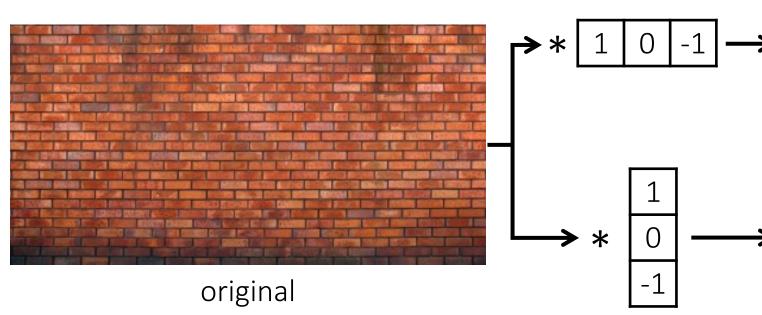
Reminder from filtering

How do we detect an edge?

Reminder from filtering

How do we detect an edge?

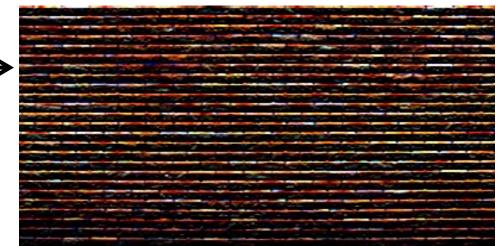
• We filter with something that looks like an edge.



We can think of linear filtering as a way to evaluate how similar an image is *locally* to some template.

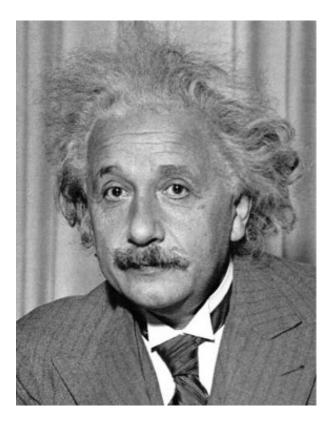


horizontal edge filter

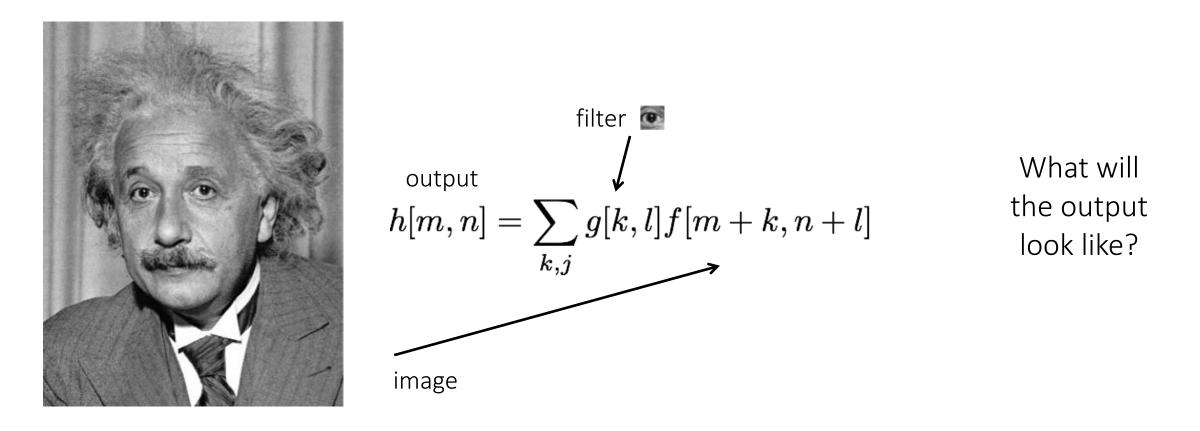


vertical edge filter

How do we detect the template **m** in he following image?

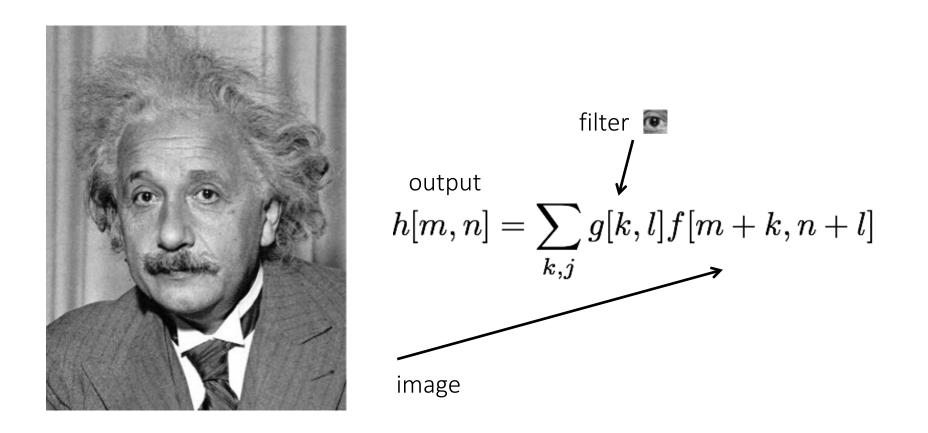


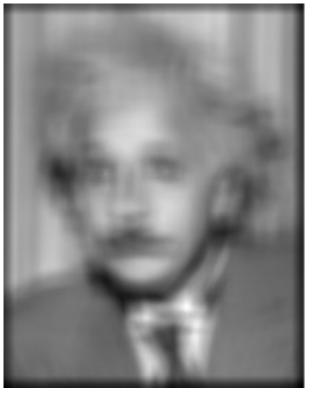
How do we detect the template **m** in he following image?



Solution 1: Filter the image using the template as filter kernel.

How do we detect the template **m** in he following image?

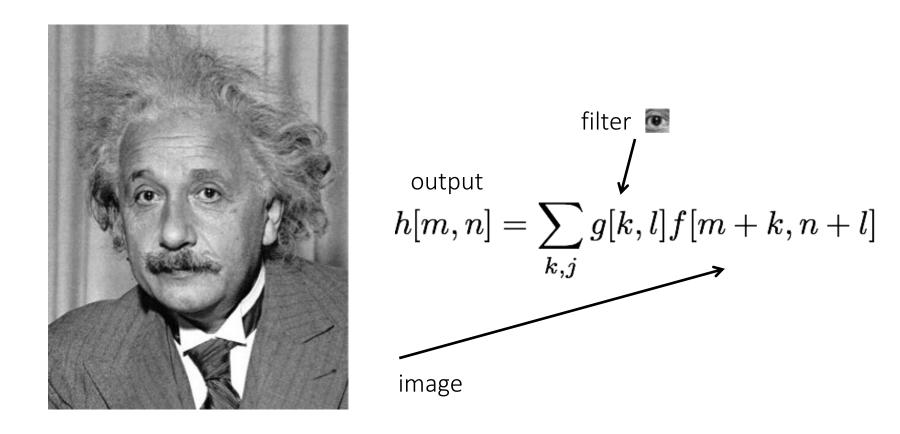


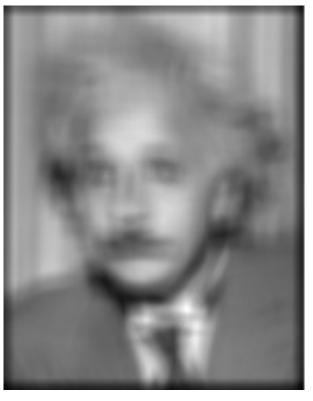


Solution 1: Filter the image using the template as filter kernel.

What went wrong?

How do we detect the template **m** in he following image?

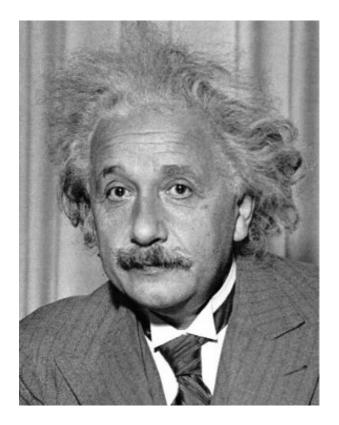


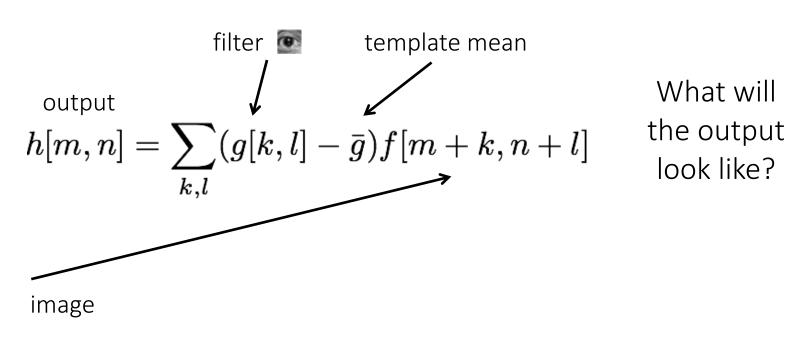


Increases for higher local intensities.

Solution 1: Filter the image using the template as filter kernel.

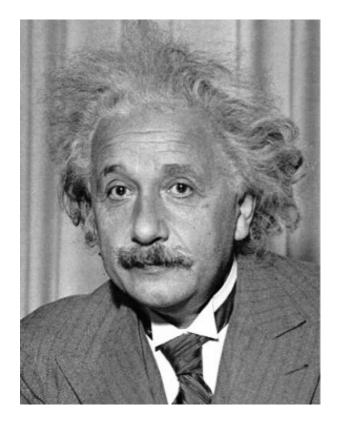
How do we detect the template **m** in he following image?

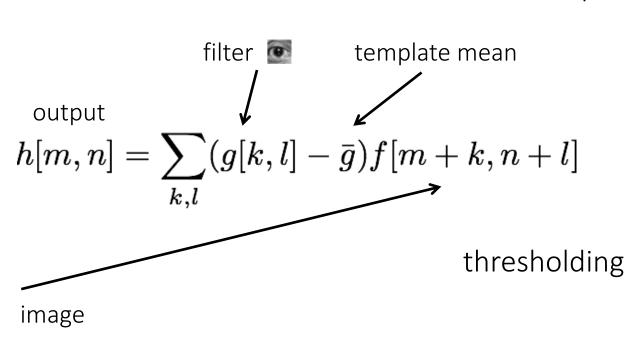




Solution 2: Filter the image using a *zero-mean* template.

How do we detect the template **m** in he following image?





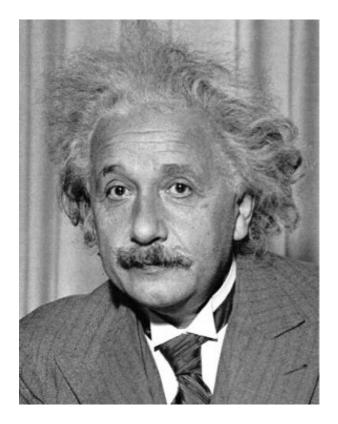
output

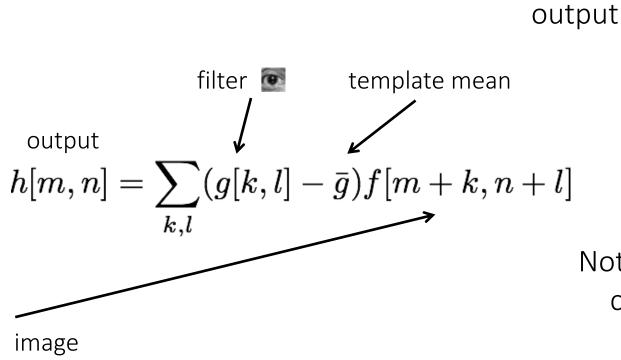
True detection False detections

Solution 2: Filter the image using a *zero-mean* template.

What went wrong?

How do we detect the template **m** in he following image?

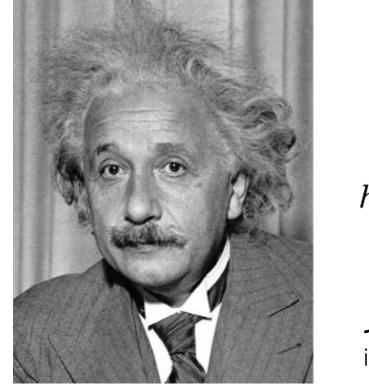


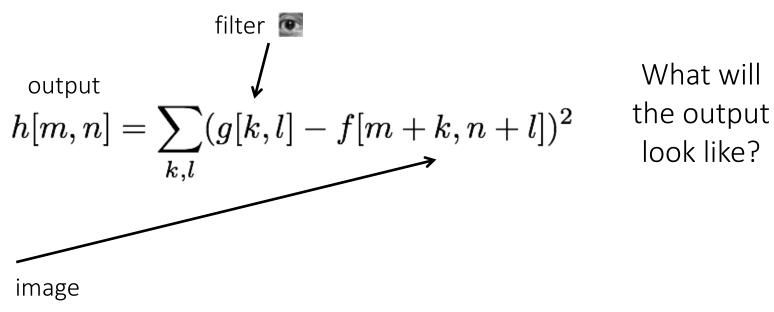


Not robust to highcontrast areas

Solution 2: Filter the image using a *zero-mean* template.

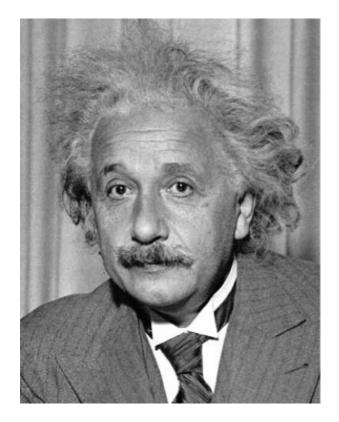
How do we detect the template **m** in he following image?

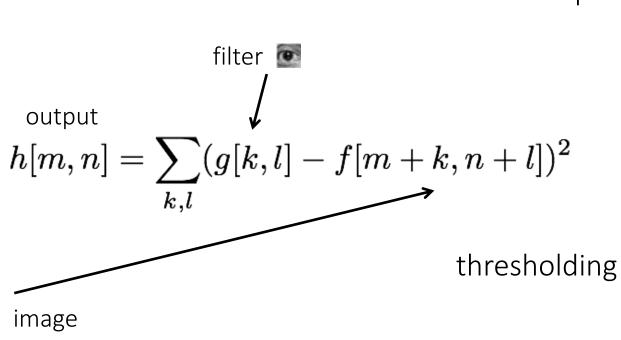




Solution 3: Use sum of squared differences (SSD).

How do we detect the template **m** in he following image?





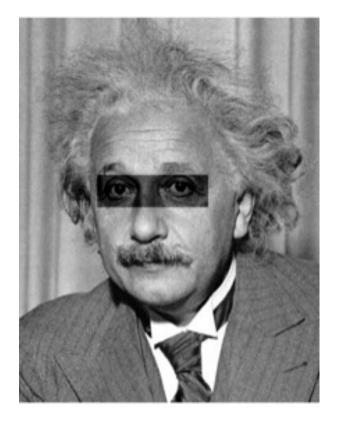
1-output

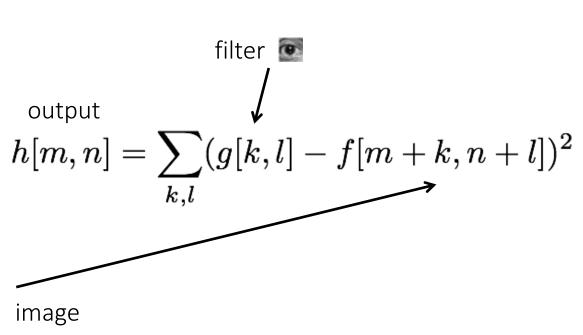
True detection

Solution 3: Use sum of squared differences (SSD).

What could go wrong?

How do we detect the template **m** in he following image?



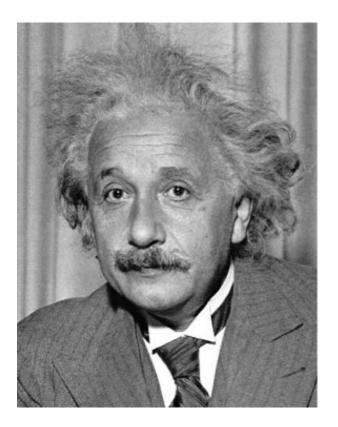


1-output

Not robust to local intensity changes

Solution 3: Use sum of squared differences (SSD).

How do we detect the template **m** in he following image?

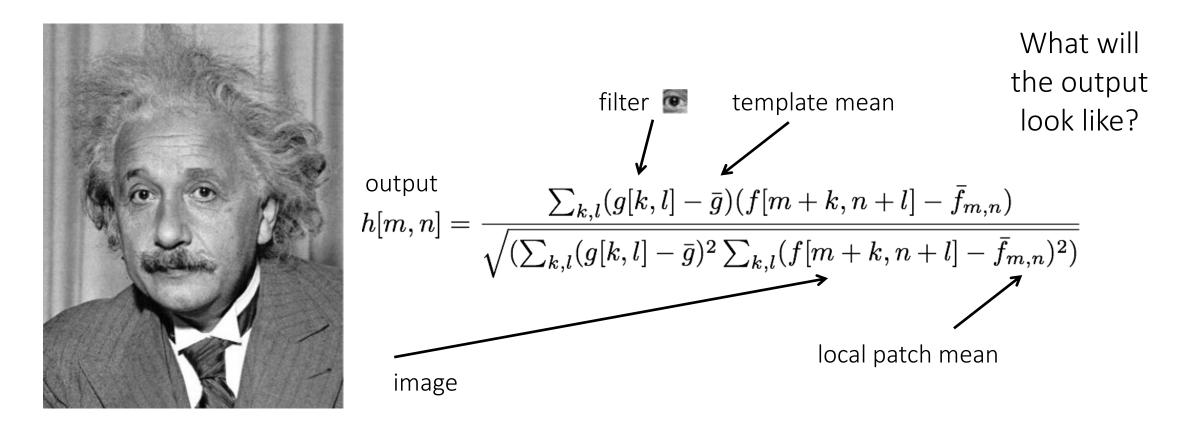


Observations so far:

- subtracting mean deals with brightness bias
- dividing by standard deviation removes contrast bias

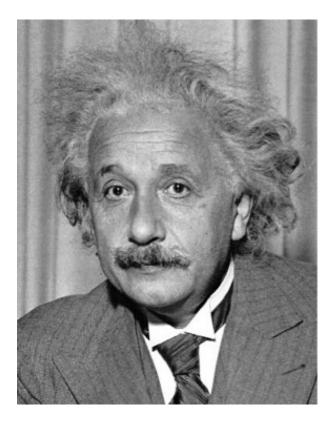
Can we combine the two effects?

How do we detect the template **m** in he following image?



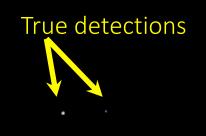
Solution 4: Normalized cross-correlation (NCC).

How do we detect the template **m** in he following image?



1-output

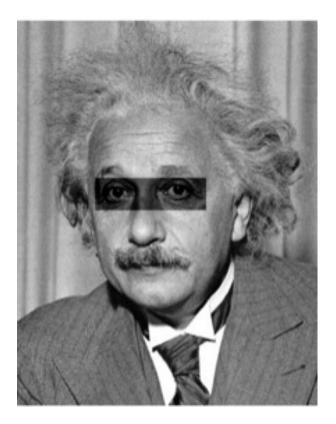




thresholding

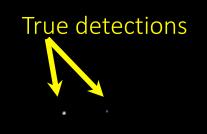
Solution 4: Normalized cross-correlation (NCC).

How do we detect the template **m** in he following image?



1-output





thresholding

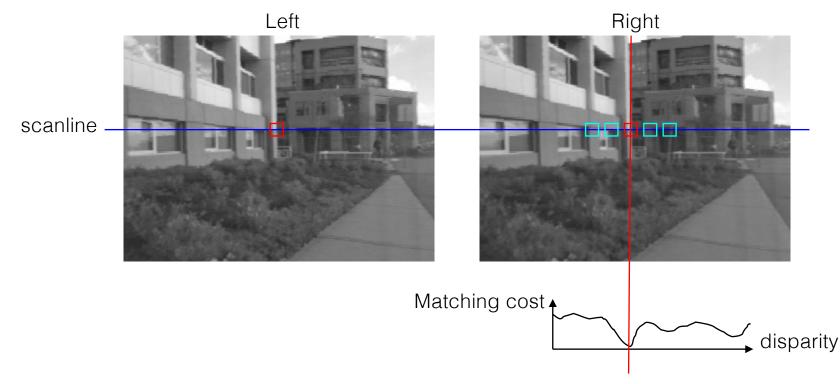
Solution 4: Normalized cross-correlation (NCC).

What is the best method?

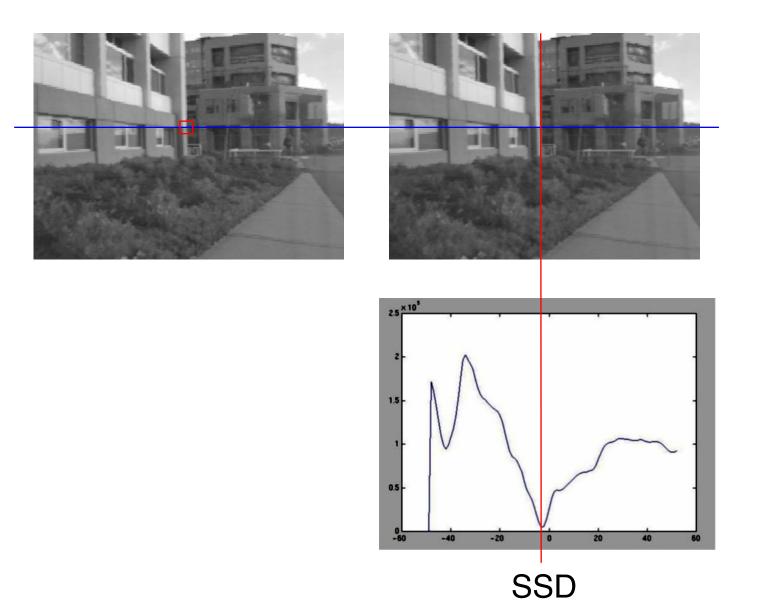
It depends on whether you care about speed or invariance.

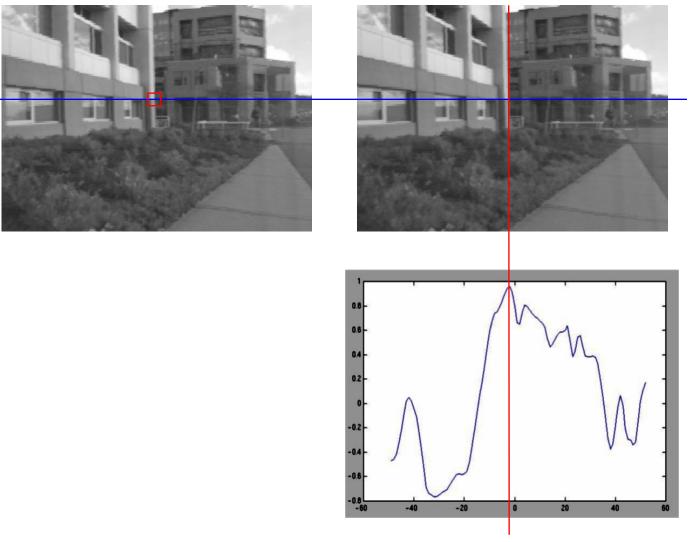
- Zero-mean: Fastest, very sensitive to local intensity.
- Sum of squared differences: Medium speed, sensitive to intensity offsets.
- Normalized cross-correlation: Slowest, invariant to contrast and brightness.

Stereo Block Matching



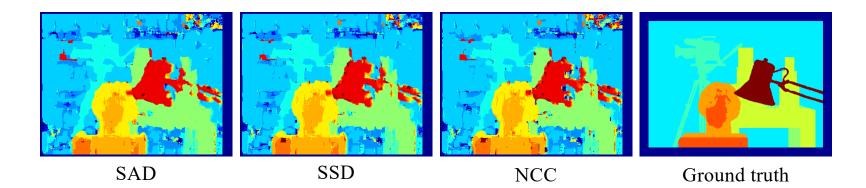
- Slide a window along the epipolar line and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation





Normalized cross-correlation

Similarity Measure	Formula
Sum of Absolute Differences (SAD)	$\sum_{(i,j)\in W} I_1(i,j) - I_2(x+i,y+j) $
Sum of Squared Differences (SSD)	$\sum_{(i,j)\in W} (I_1(i,j) - I_2(x+i,y+j))^2$
Zero-mean SAD	$\sum_{(i,j)\in W} I_1(i,j) - \overline{I}_1(i,j) - I_2(x+i,y+j) + \overline{I}_2(x+i,y+j) $
Locally scaled SAD	$\sum_{(i,j)\in W} I_1(i,j) - \frac{\bar{I}_1(i,j)}{\bar{I}_2(x+i,y+j)} I_2(x+i,y+j) $
Normalized Cross Correlation (NCC)	$\frac{\sum_{(i,j)\in W} I_1(i,j).I_2(x+i,y+j)}{\sqrt[2]{\sum_{(i,j)\in W} I_1^2(i,j).\sum_{(i,j)\in W} I_2^2(x+i,y+j)}}$



Effect of window size



W = 3

W = 20

Effect of window size



$$W = 3$$

$$W = 20$$

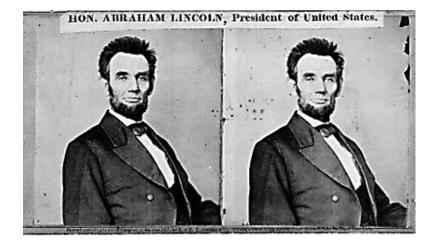
Smaller window

- + More detail
- More noise

Larger window

- + Smoother disparity maps
- Less detail
- Fails near boundaries

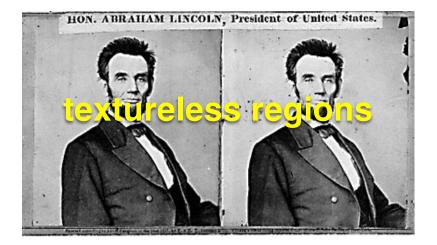
When will stereo block matching fail?

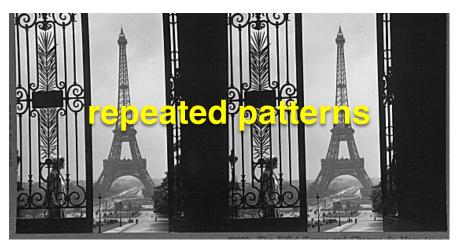






When will stereo block matching fail?





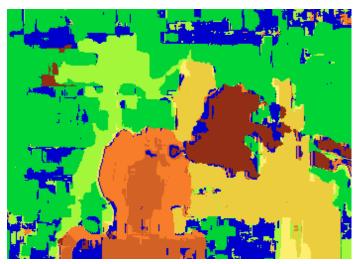


Improving stereo matching



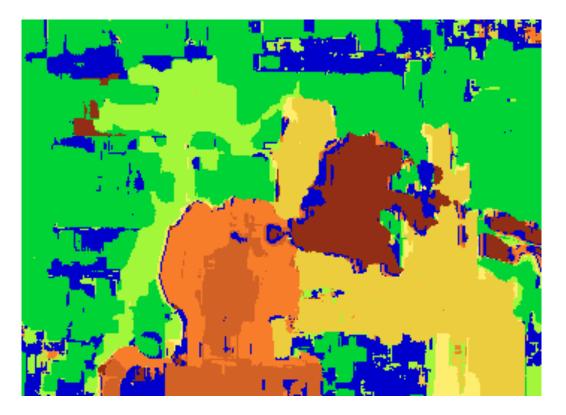
Block matching

Ground truth

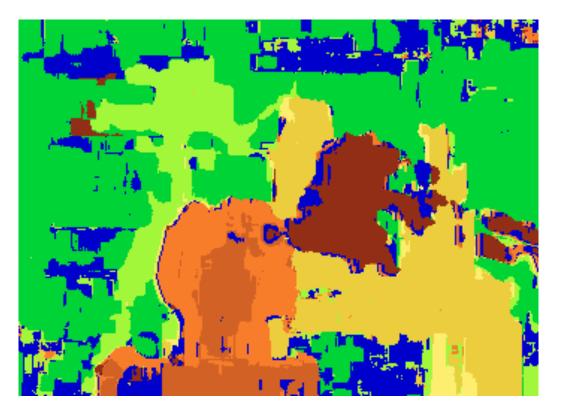




What are some problems with the result?



How can we improve depth estimation?



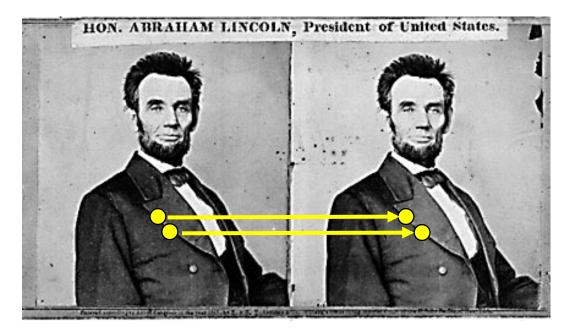
How can we improve depth estimation?

Too many discontinuities. We expect disparity values to change slowly.

Let's make an assumption: depth should change smoothly

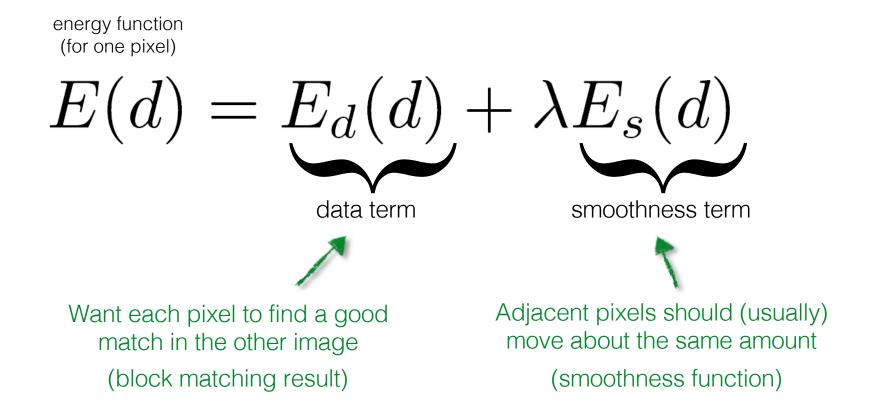
Stereo matching as ...

Energy Minimization



What defines a good stereo correspondence?

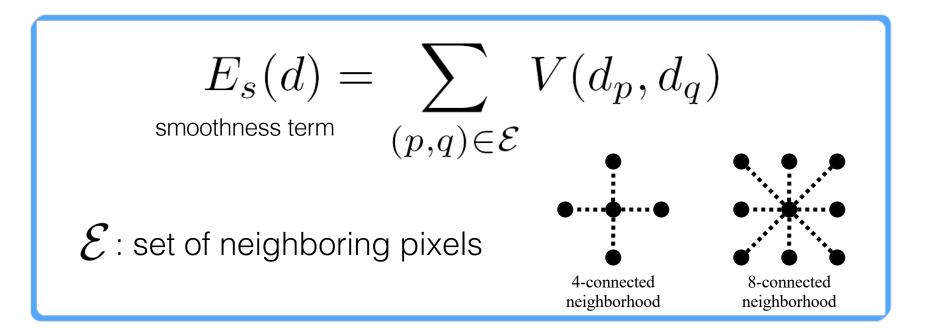
- 1. Match quality
 - Want each pixel to find a good match in the other image
- 2. Smoothness
 - If two pixels are adjacent, they should (usually) move about the same amount



$$E(d) = E_d(d) + \lambda E_s(d)$$
$$E_d(d) = \sum_{(x,y)\in I} C(x,y,d(x,y))$$
$$C(x,y,d(x,y))$$
$$C(x,y,d(x,y))$$
$$C(x,y,d(x,y))$$
$$C(x,y,d(x,y))$$

 $E(d) = E_d(d) + \lambda E_s(d)$ $E_d(d) = \sum C(x, y, d(x, y))$ $(x,y) \in I$

SSD distance between windows centered at I(x, y) and J(x+ d(x,y), y)



$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p,d_q)$$
 smoothness term

$$V(d_p, d_q) = |d_p - d_q|$$

$$L_1 \text{ distance}$$

One possible solution...

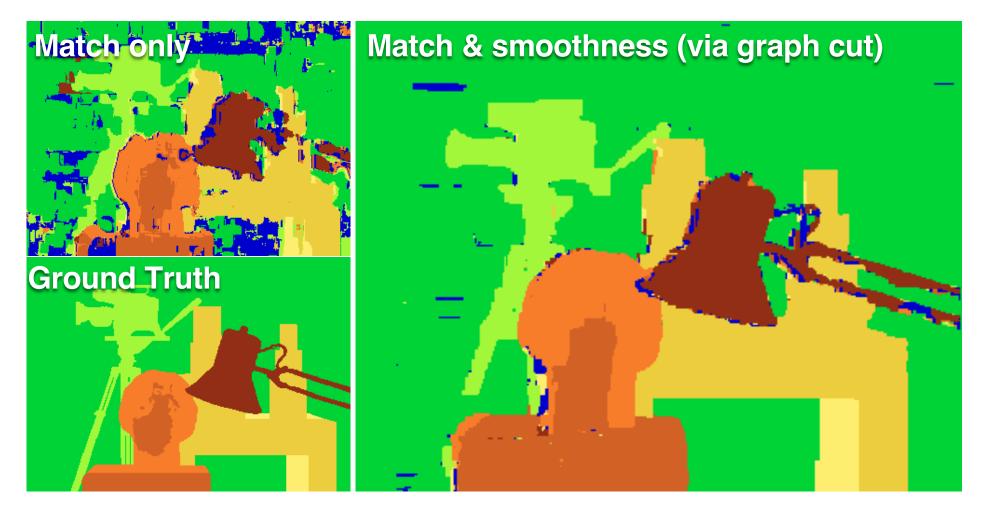
Dynamic Programming

$$E(d) = E_d(d) + \lambda E_s(d)$$

Can minimize this independently per scanline using dynamic programming (DP)

D(x, y, d) : minimum cost of solution such that d(x,y) = d

$$D(x, y, d) = C(x, y, d) + \min_{d'} \left\{ D(x - 1, y, d') + \lambda \left| d - d' \right| \right\}$$

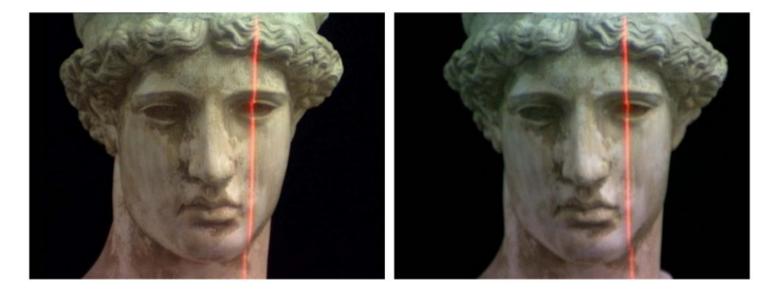


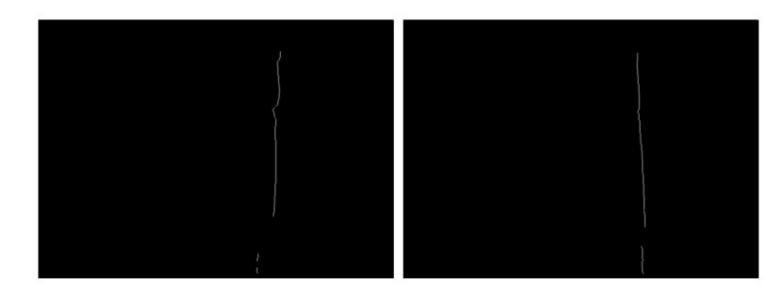
Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001

Structured light

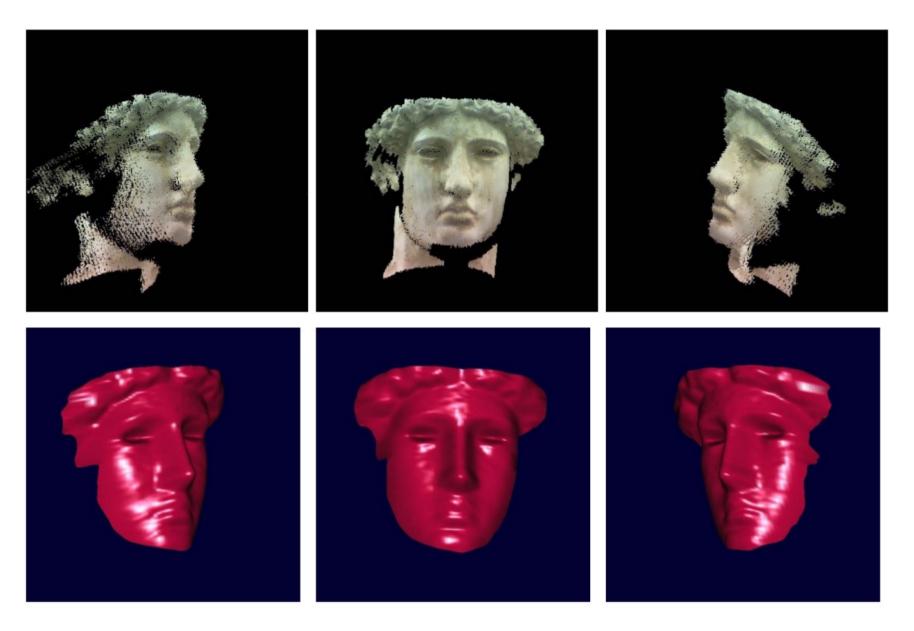
Use controlled ("structured") light to make correspondences easier

Disparity between laser points on the same scanline in the images determines the 3-D coordinates of the laser point on object

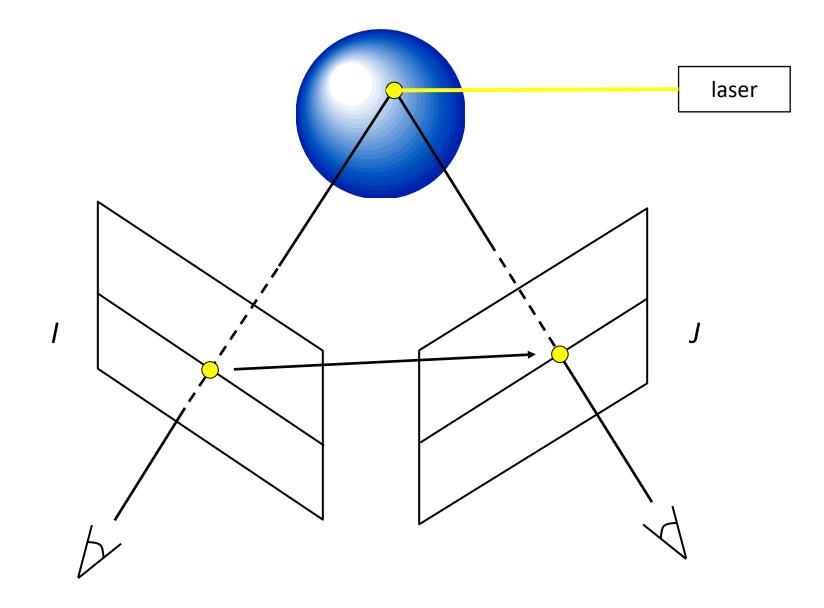




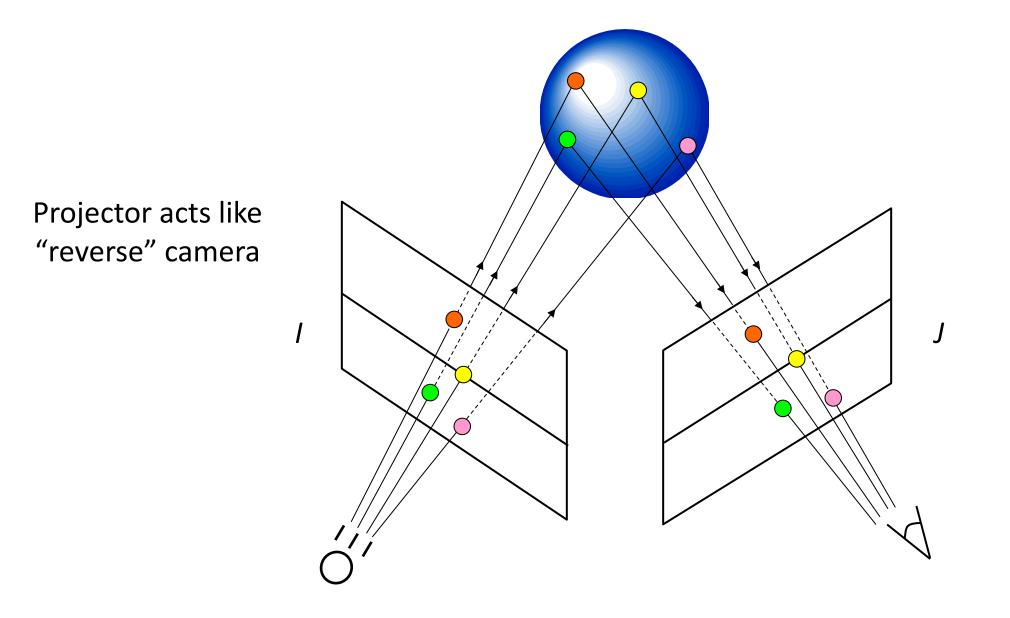
Use controlled ("structured") light to make correspondences easier



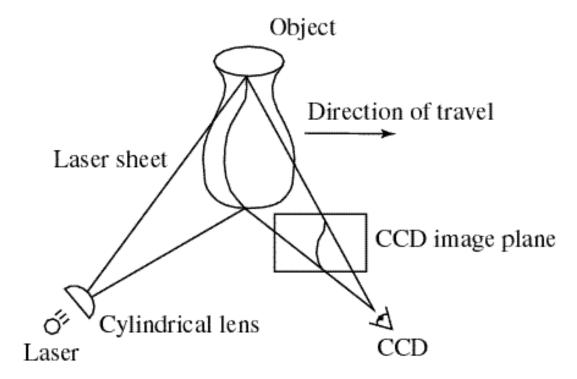
Structured light and two cameras

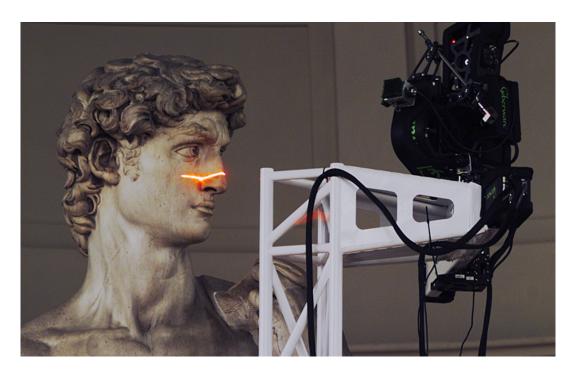


Structured light and one camera



Example: Laser scanner





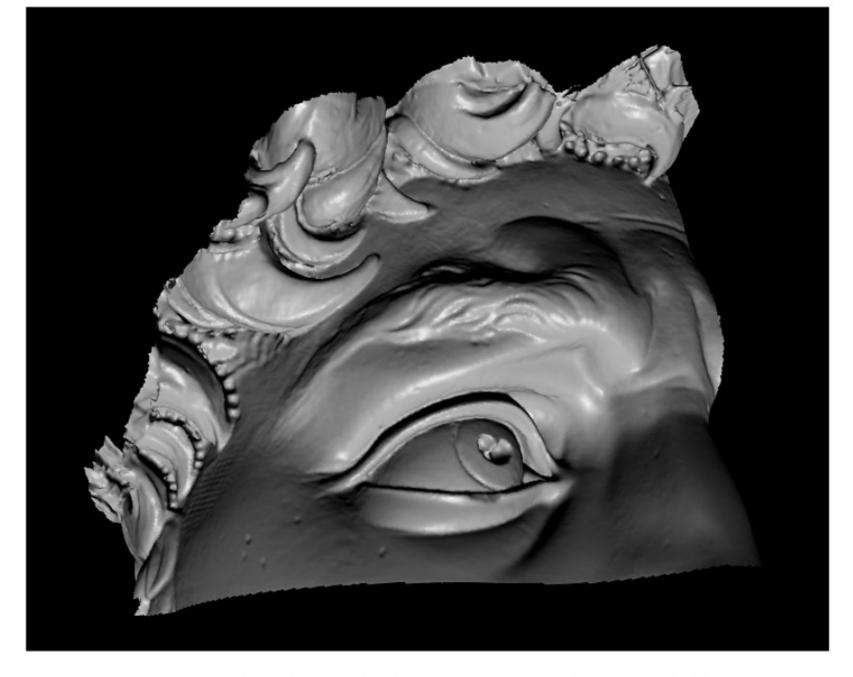
Digital Michelangelo Project http://graphics.stanford.edu/projects/mich/



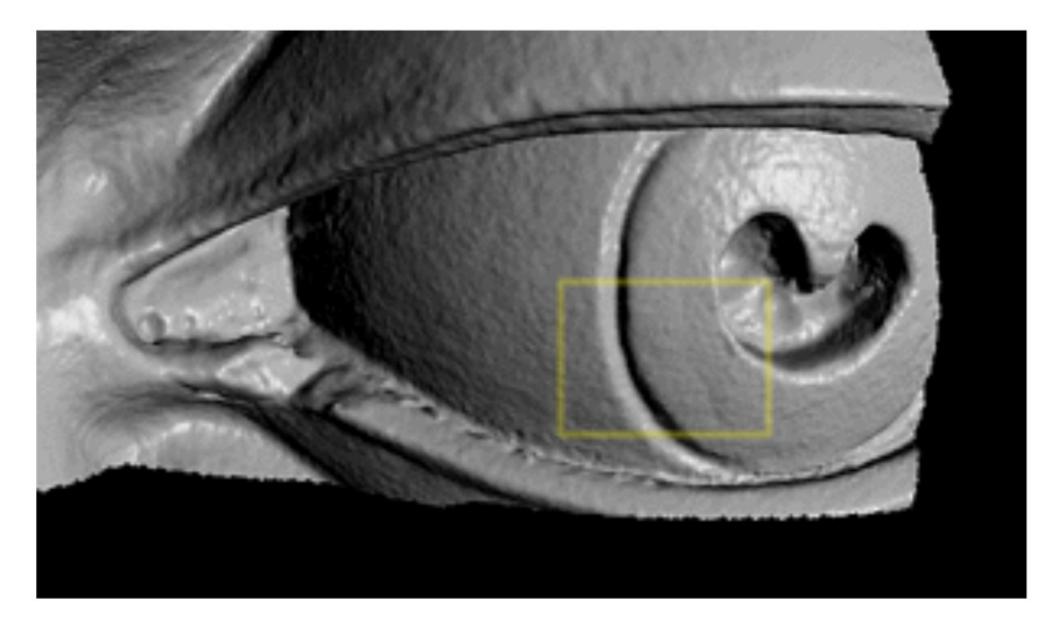
The Digital Michelangelo Project, Levoy et al.



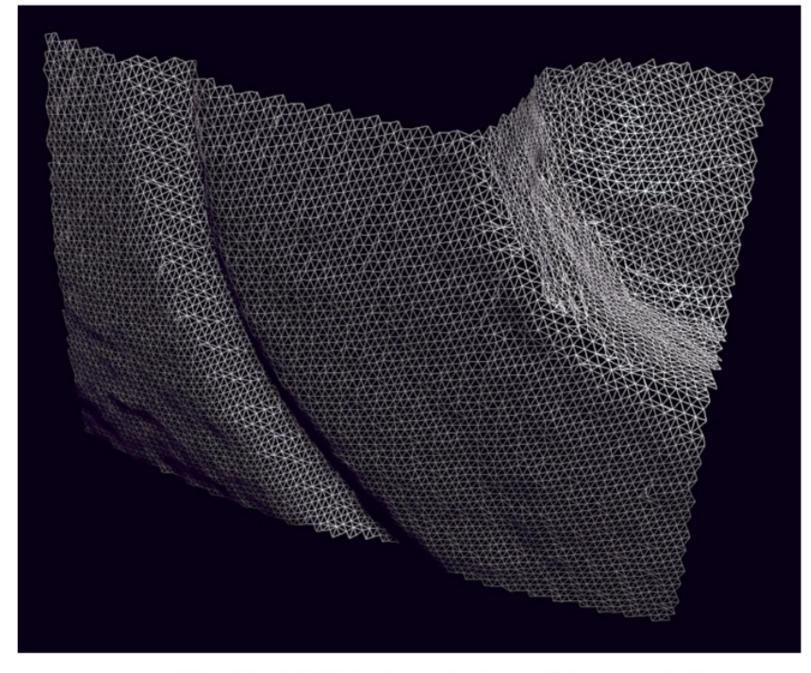
The Digital Michelangelo Project, Levoy et al.



The Digital Michelangelo Project, Levoy et al.



The Digital Michelangelo Project, Levoy et al.



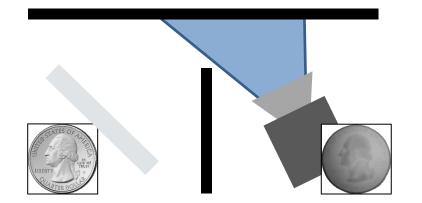
The Digital Michelangelo Project, Levoy et al.

15-463/15-663/15-862 Computational Photography

Learn about structured light and other cameras – and build some on your own!



cameras that take video at the speed of light



cameras that see around corners



cameras that measure depth in real time



cameras that capture entire focal stacks

http://graphics.cs.cmu.edu/courses/15-463/