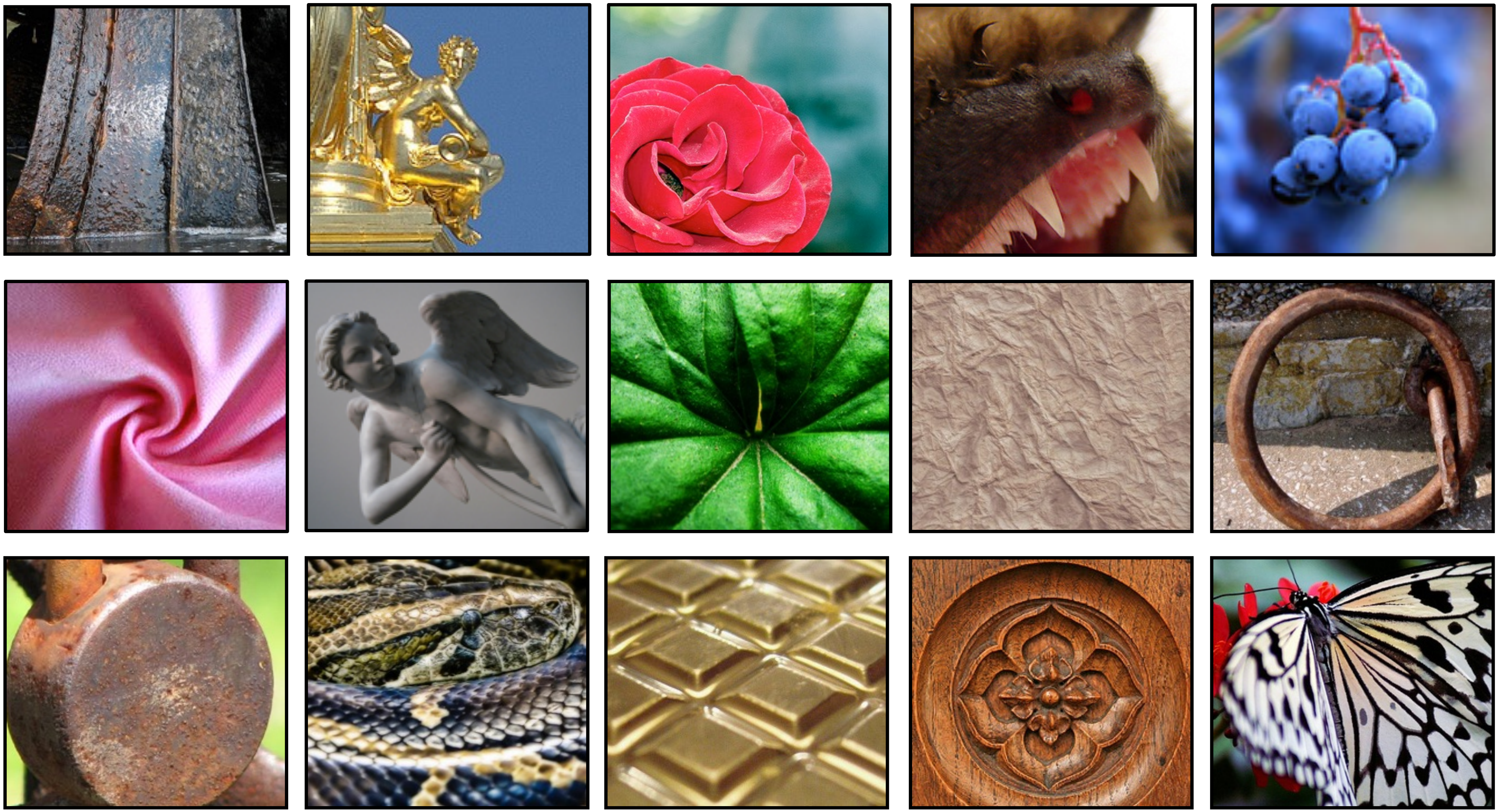
Radiometry and reflectance



16-385 Computer Vision
Spring 2021, Lectures 13 & 14

http://16385.courses.cs.cmu.edu/

Overview of today’s lecture

- Appearance phenomena.
- Measuring light and radiometry.
- Reflectance and BRDF.

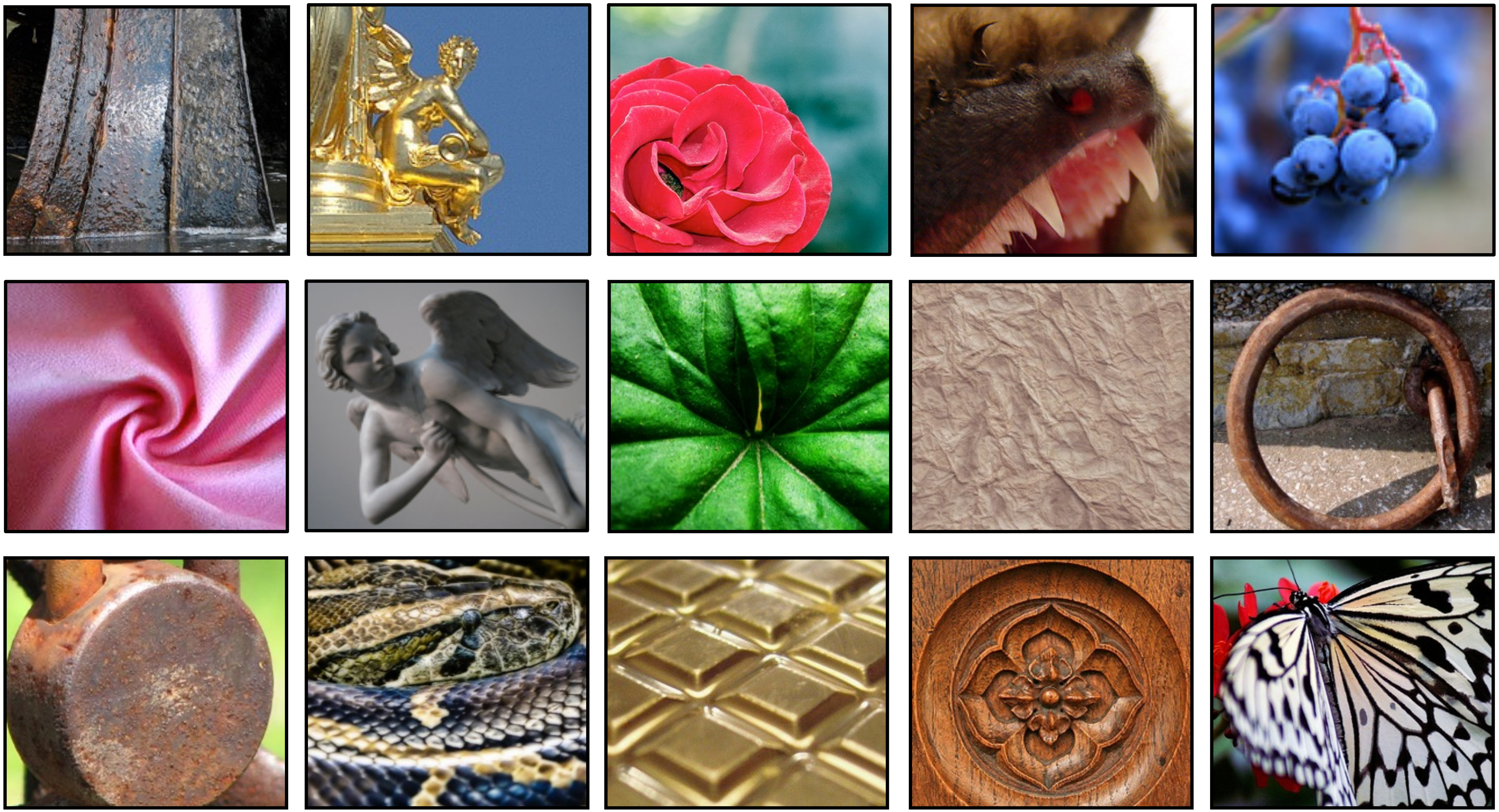
Slide credits

Most of these slides were adapted from:

- Srinivasa Narasimhan (16-385, Spring 2014).
- Todd Zickler (Harvard University).
- Steven Gortler (Harvard University).

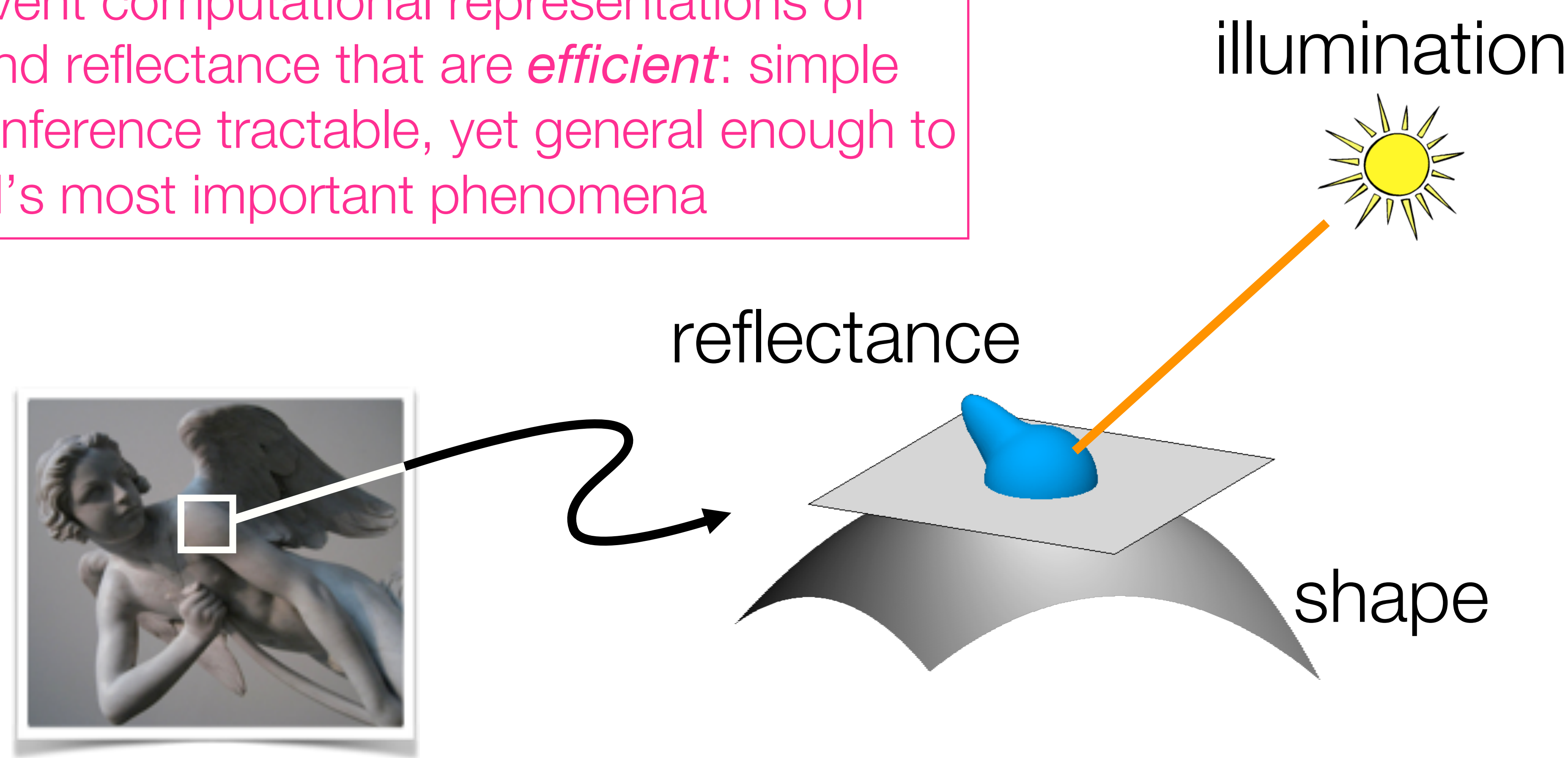
Appearance

Appearance



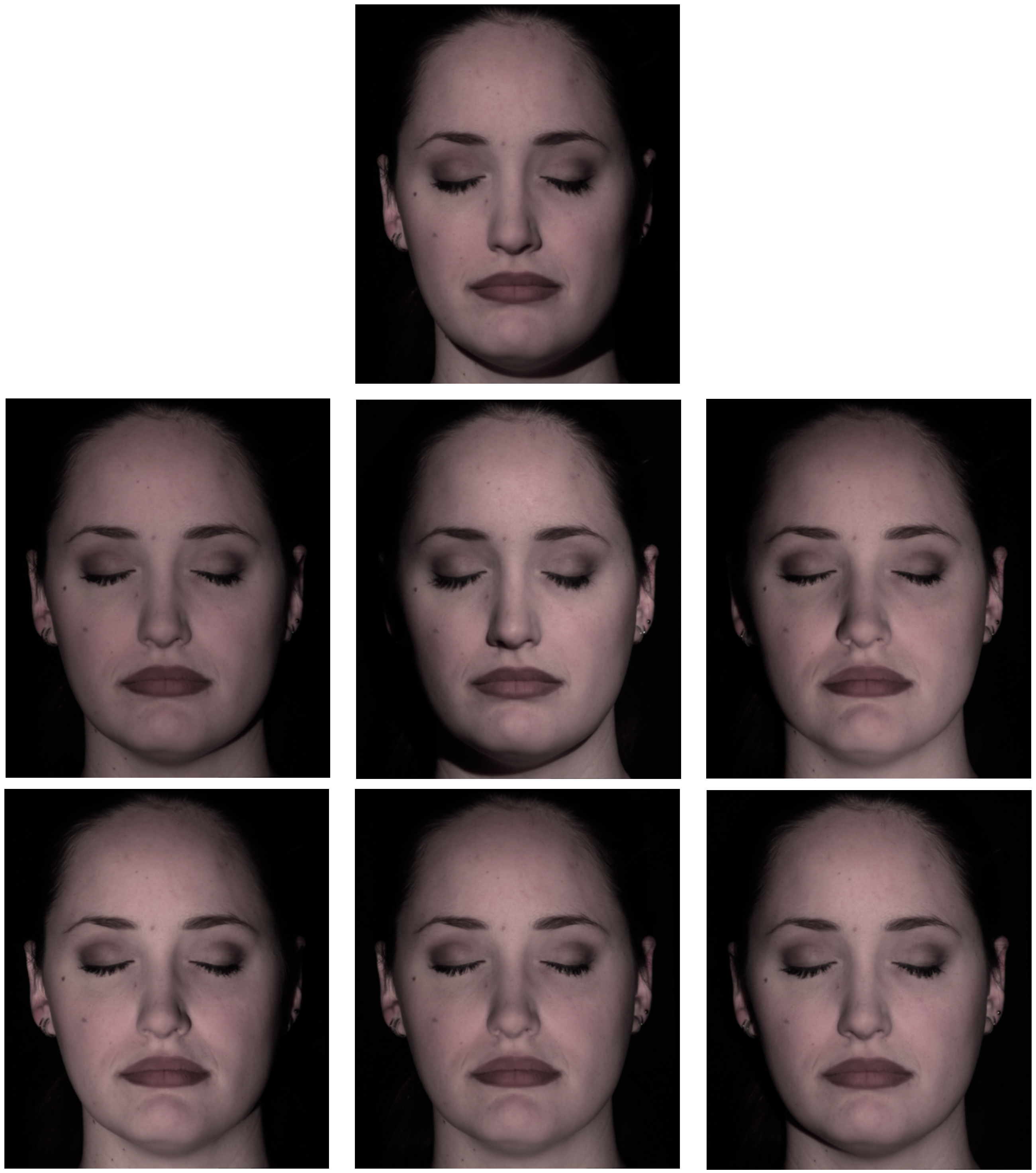
“Physics-based” computer vision
(a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are ***efficient***: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena



$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Example application: Photometric Stereo

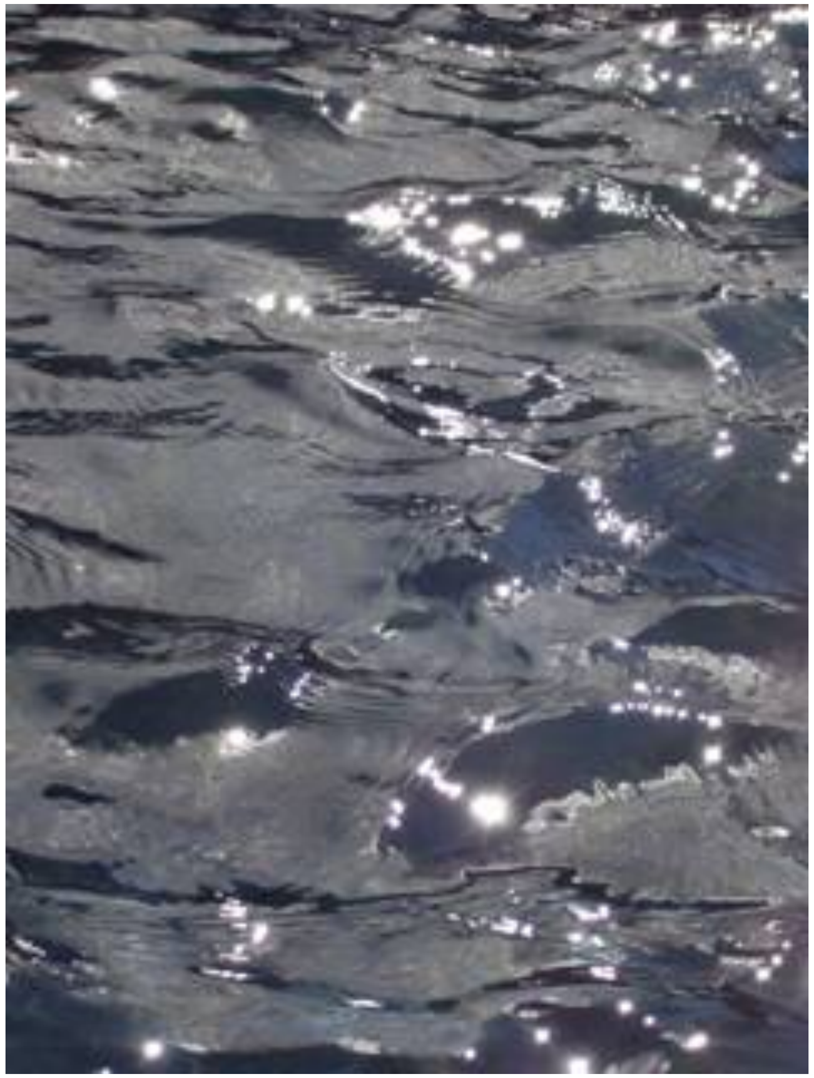


Why study the physics (optics) of the world?

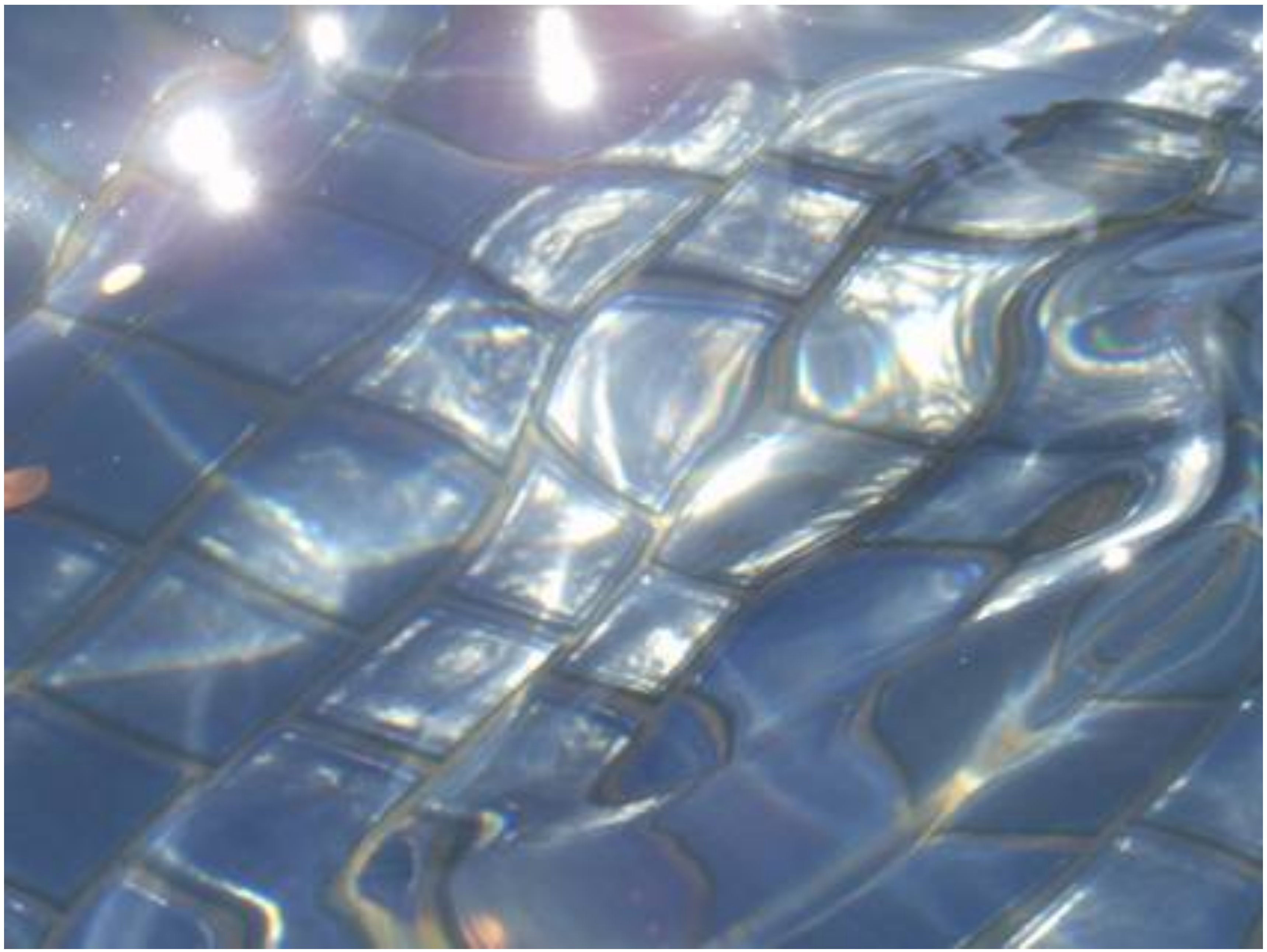
Lets see some pictures!

Light and Shadows

Reflections

Refractions

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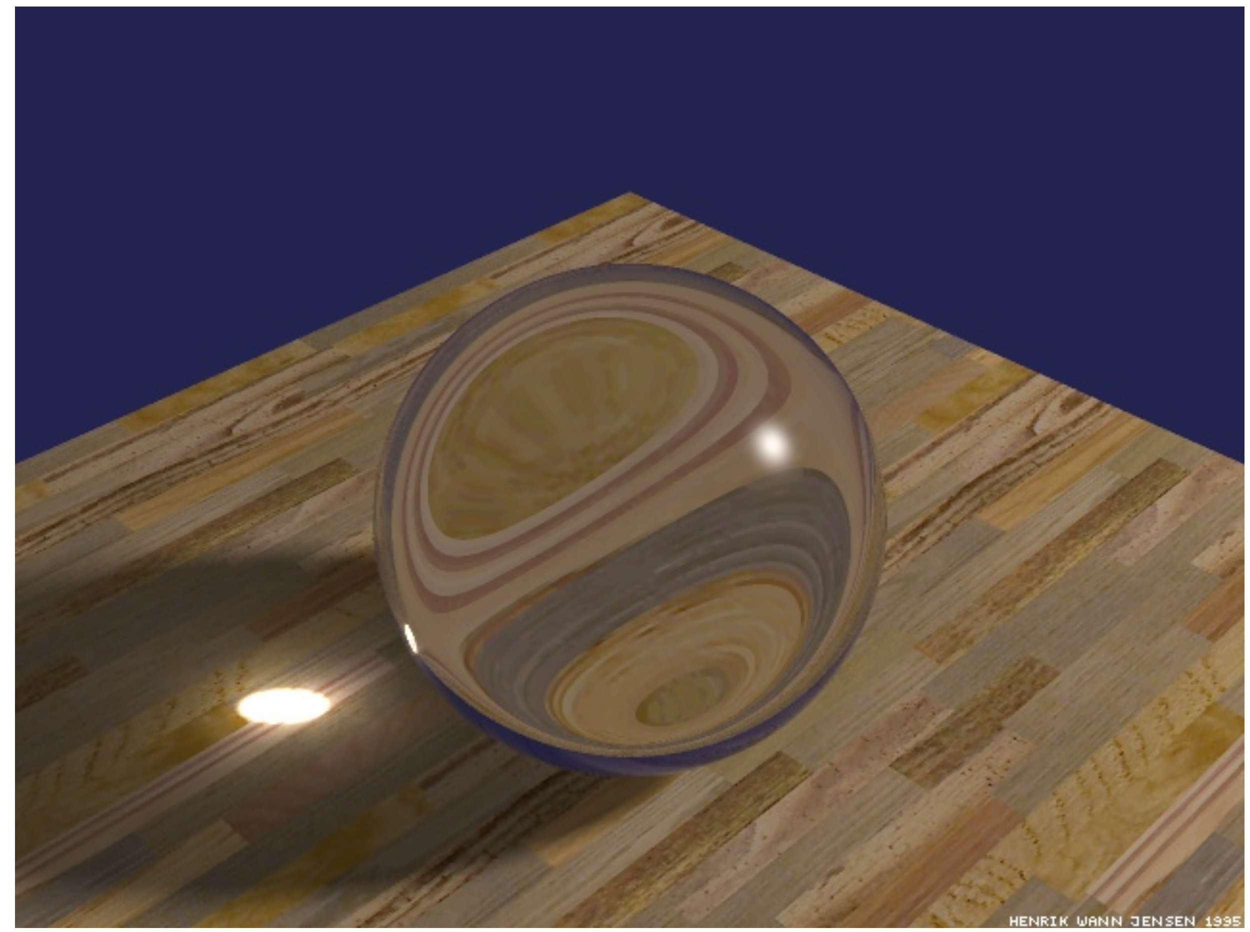
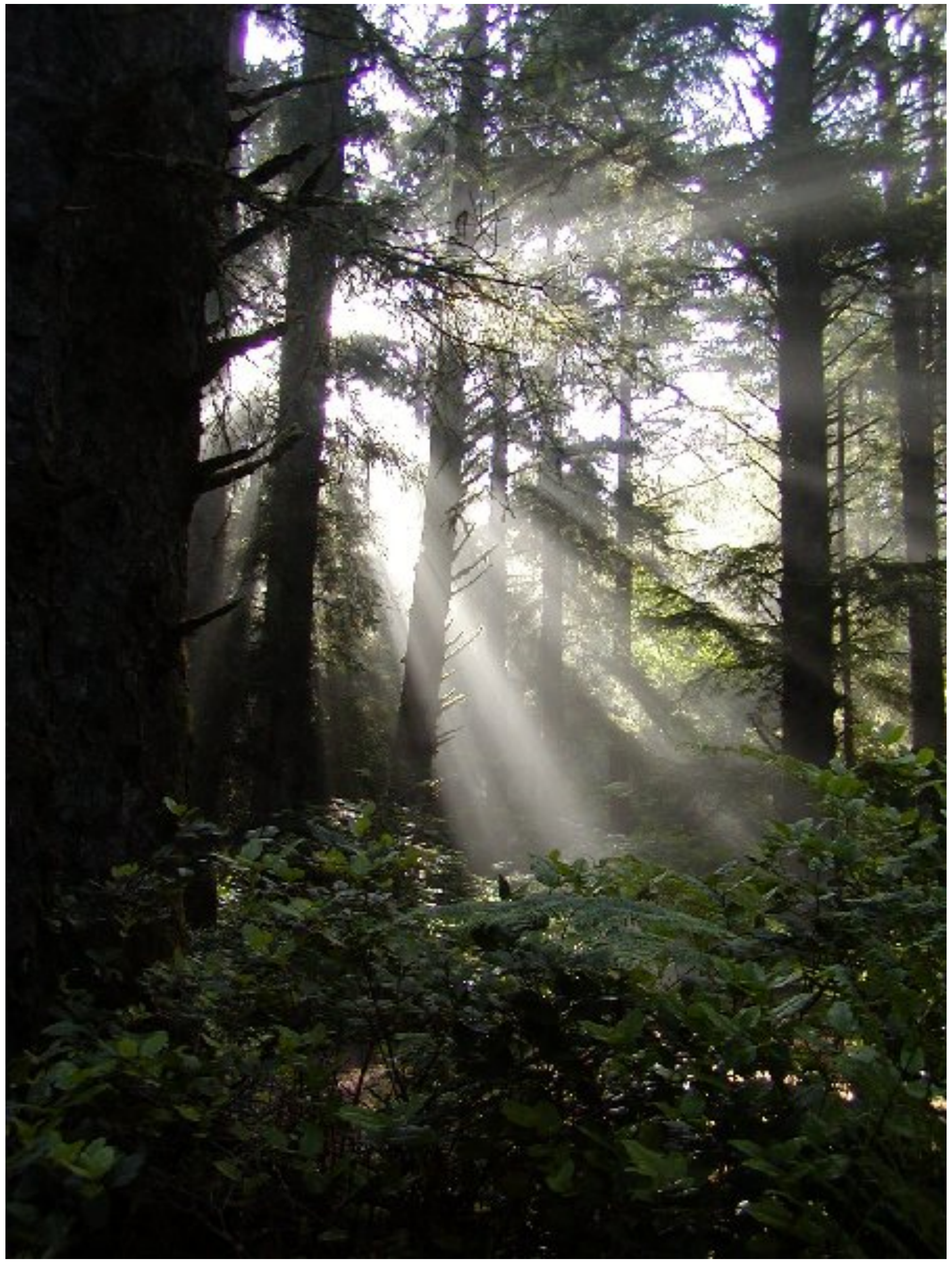

HENRIK WANN JENSEN 1995

Interreflections

Scattering



More Complex Appearances

opaque translucent





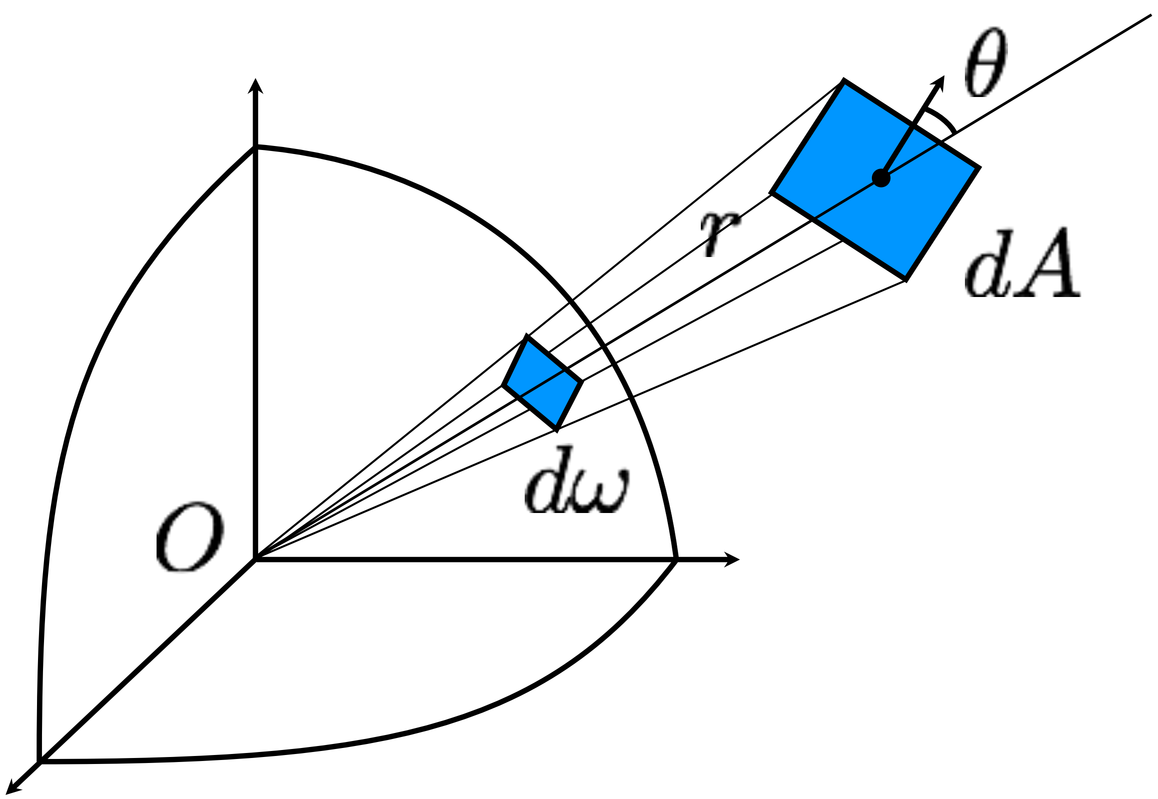




Measuring light and radiometry

Solid angle

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

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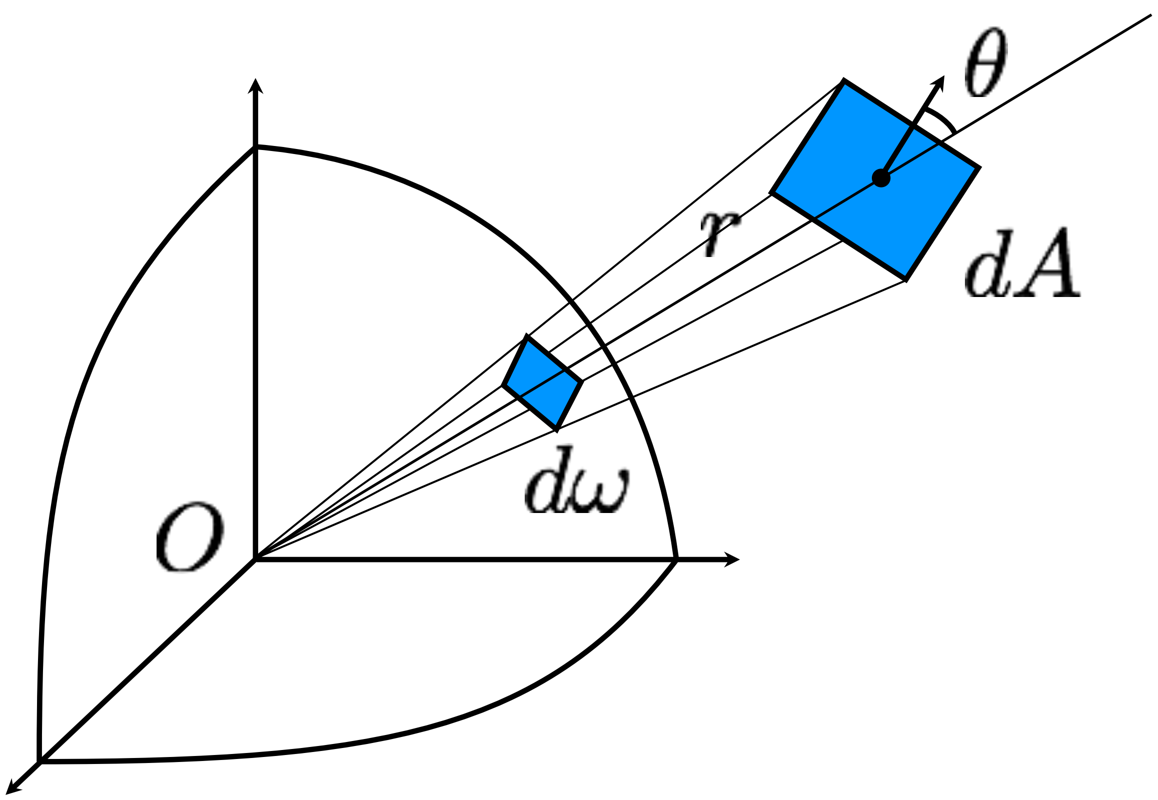

Depends on:

- orientation of patch
- distance of patch

One can show:

$$\boxed{d\omega = \frac{dA\cos\theta}{r^2}}$$

Units: steradians [sr]

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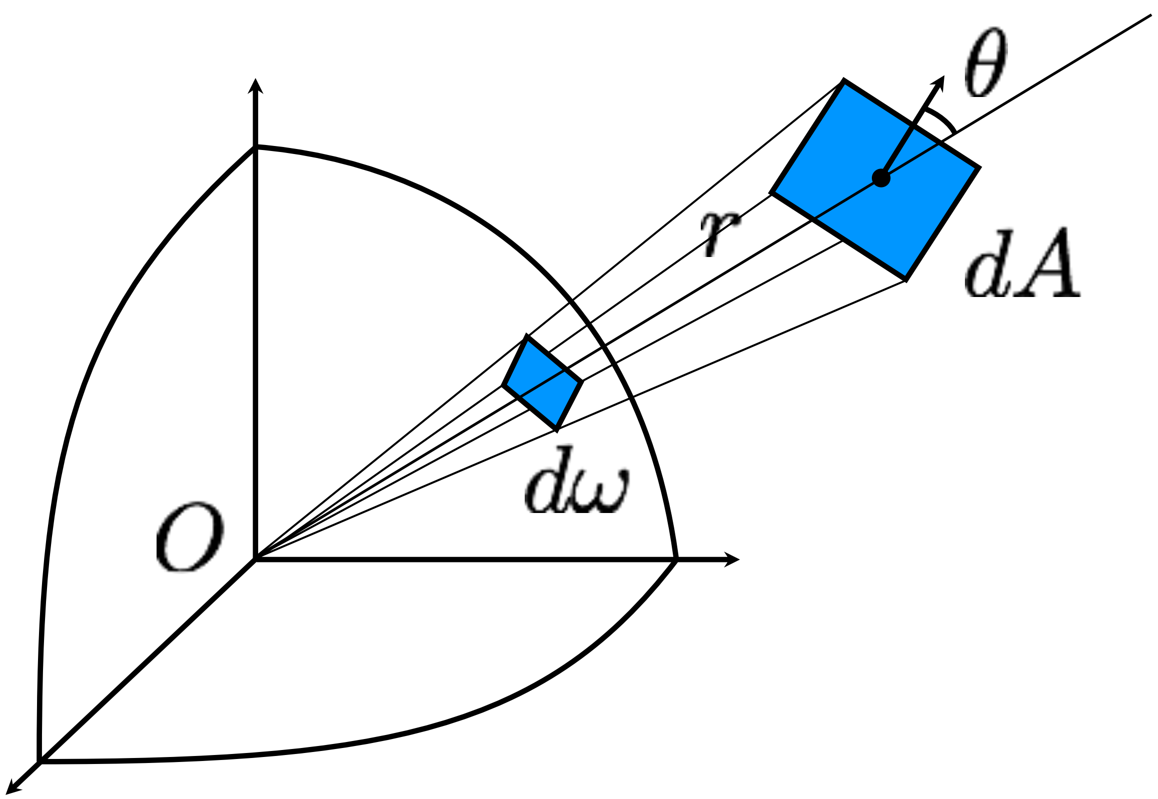


Depends on:

- orientation of patch
- distance of patch

One can show:

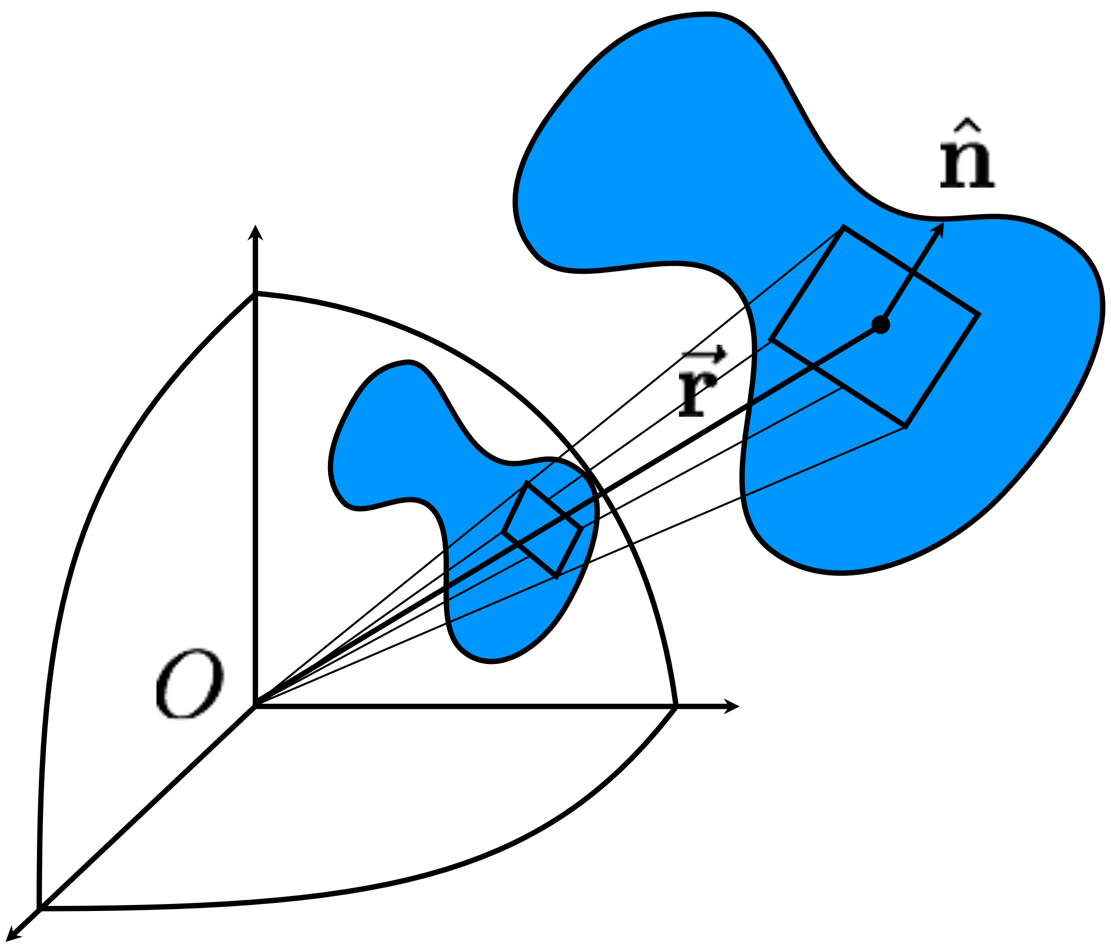
"surface foreshortening"

$$\boxed{d\omega = \frac{dA \cos\theta}{r^2}}$$

Units: steradians [sr]

Solid angle

- To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{\mathbf{r}} \cdot \hat{\mathbf{n}}\, dS}{|\vec{\mathbf{r}}|^3}$$

One can show:

“surface foreshortening”

$$\boxed{d\omega = \frac{dA \cos\theta}{r^2}}$$

Units: steradians [sr]

Question

- Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

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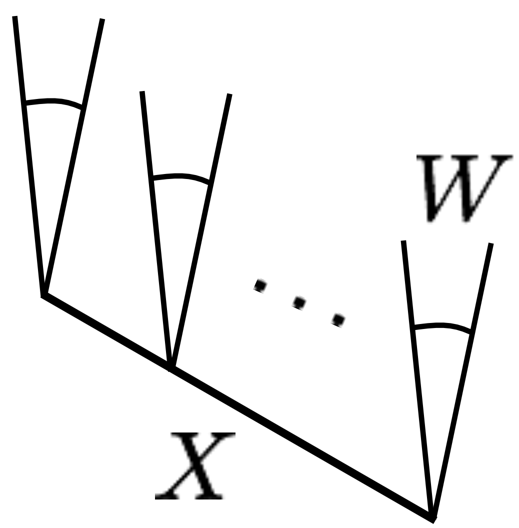
- Answer: 2\pi (area of sphere is 4\pi*r^2; area of unit sphere is 4\pi; half of that is 2\pi)

Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W
- It measures *radiant flux* [watts = joules/sec]: rate of photons hitting sensor area
- Measurement depends on sensor area |X|

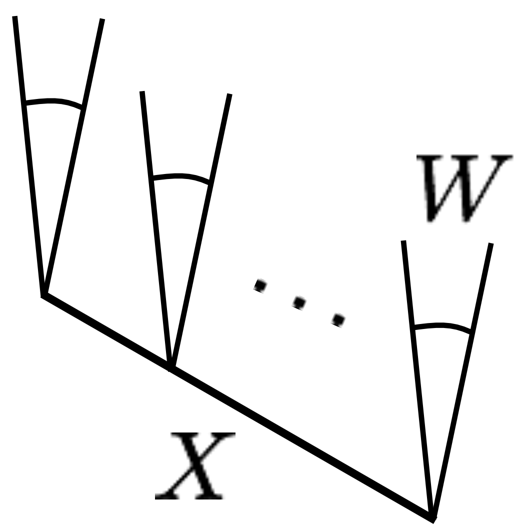


* shown in 2D for clarity; imagine three dimensions

radiant flux $\Phi(W, X)$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]



$$\frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

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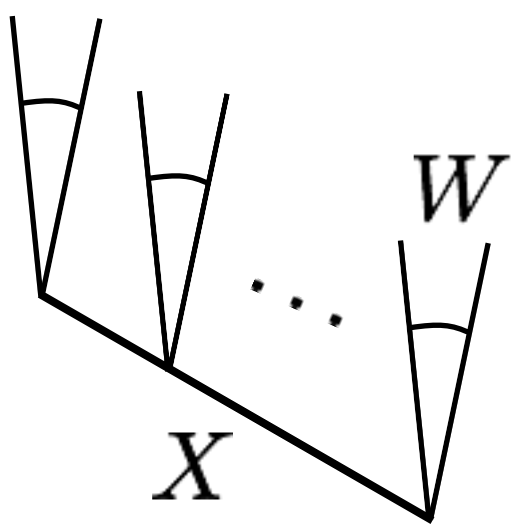
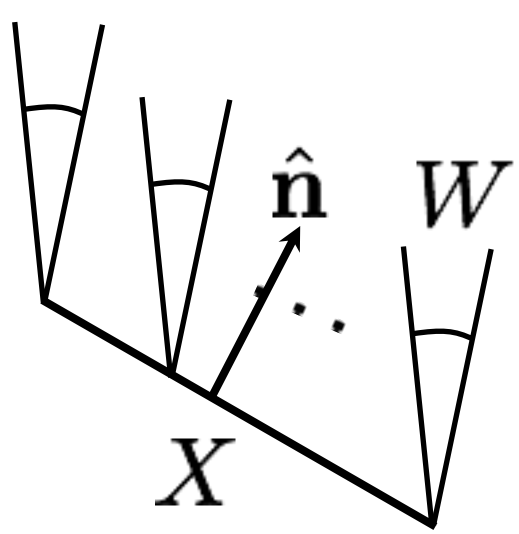


$$\lim_{X \longrightarrow x}$$

$$\frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]
- Depends on sensor direction normal.



$$\frac{\Phi(W, X)}{|X|}$$

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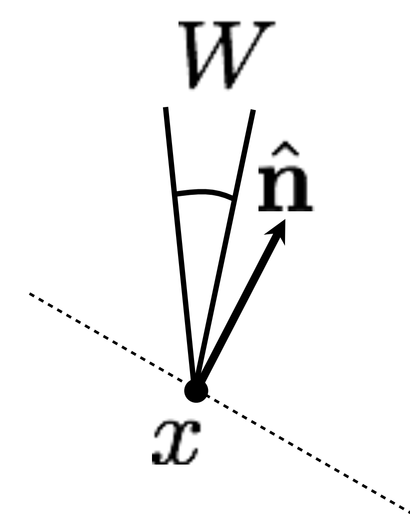
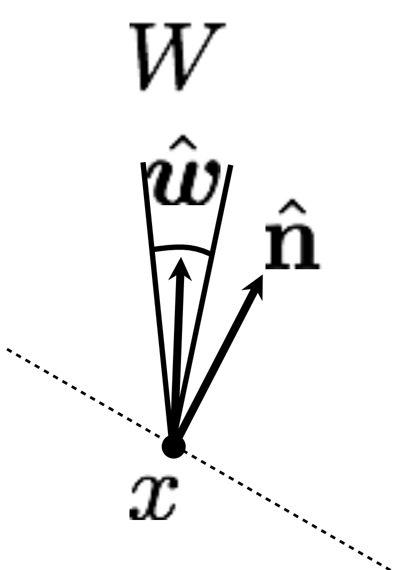


$$E_{\hat{\mathbf{n}}}(W, x)$$

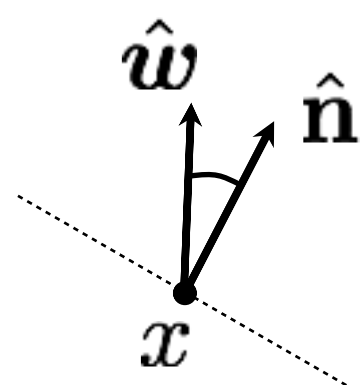
- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit
- In the literature, notations n and W are often omitted, and values are implied by context

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|
- Units: watts per steradian per square meter [W/(m^2·sr)]



$$\lim_{W \longrightarrow \hat{\boldsymbol{\omega}}}$$



$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|}$$

$$L_{\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}, x)$$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

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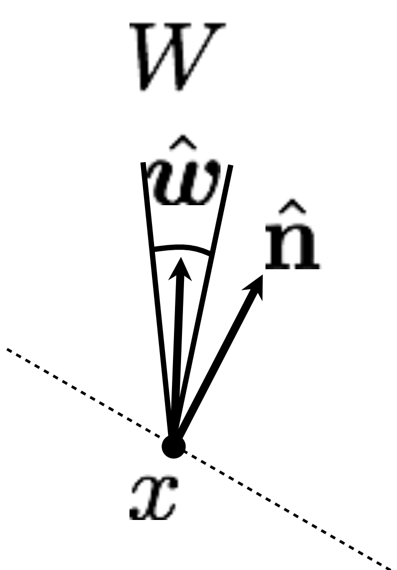
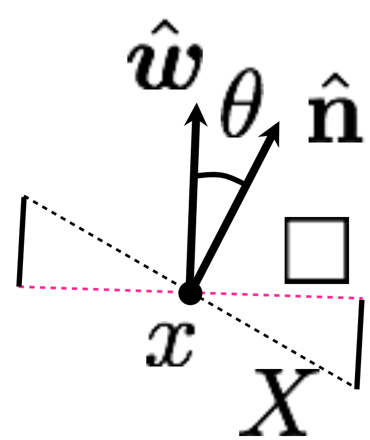


$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \quad \lim_{W \longrightarrow \hat{\boldsymbol{\omega}}} \quad L_{\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}, x)$$



$$\cos\theta = \frac{\square/2}{|X|/2}$$

$$\longrightarrow \square = |X| \cos\theta$$

"foreshortened area"

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

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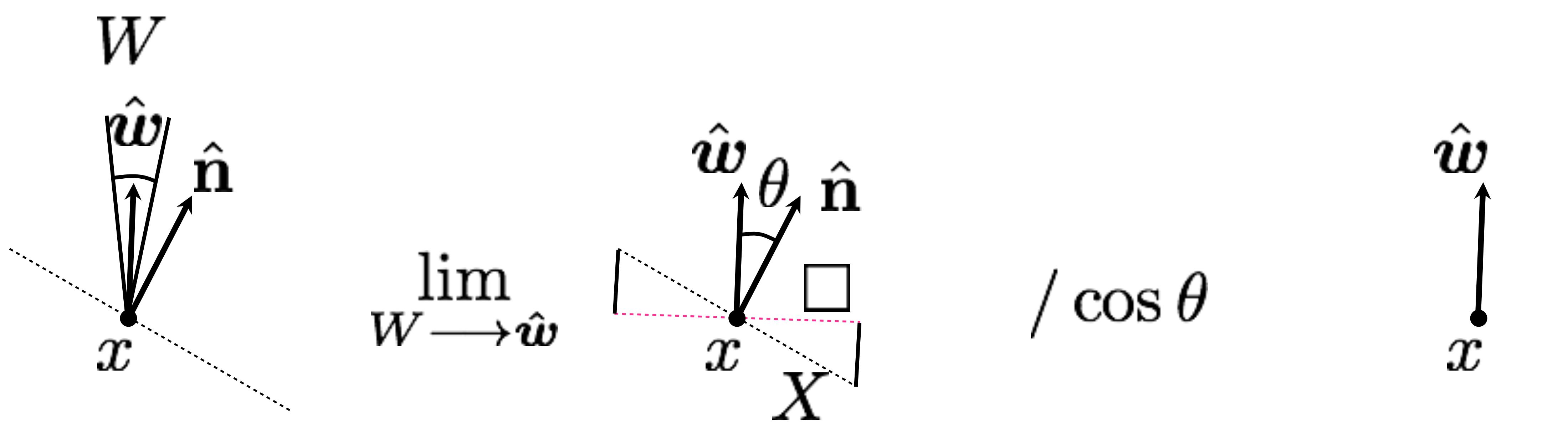

$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \qquad L_{\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}, x) \qquad L(\hat{\boldsymbol{\omega}}, x)$$

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- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
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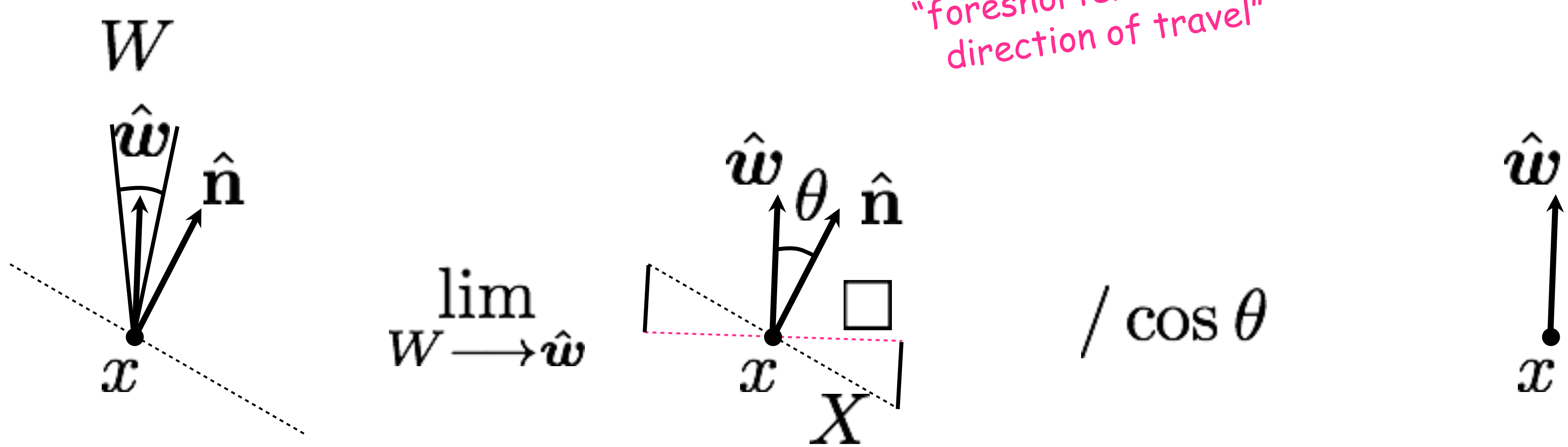
- Units: watts per steradian per square meter [W/(m^2·sr)]

"foreshortened in the direction of travel"



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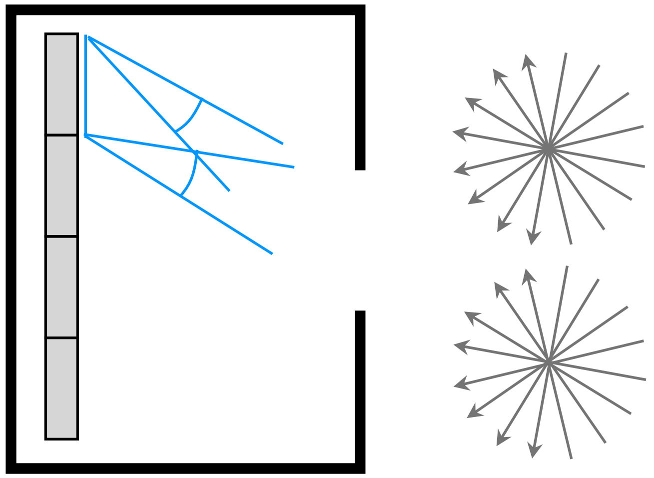
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor



Quantifying light: flux, irradiance, and radiance

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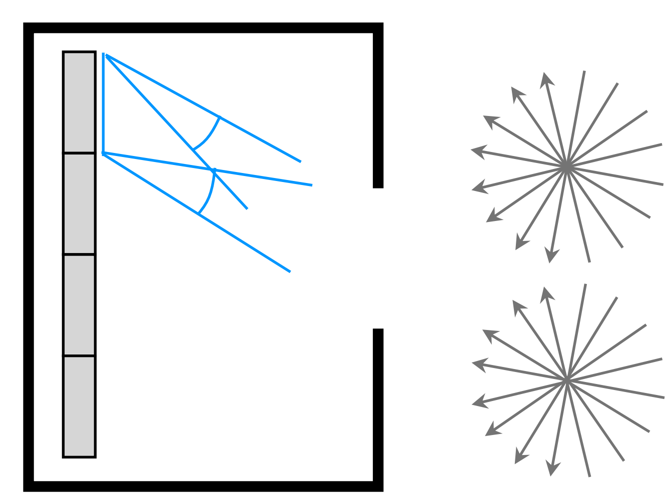
$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos\theta \, d\omega \, dA$$



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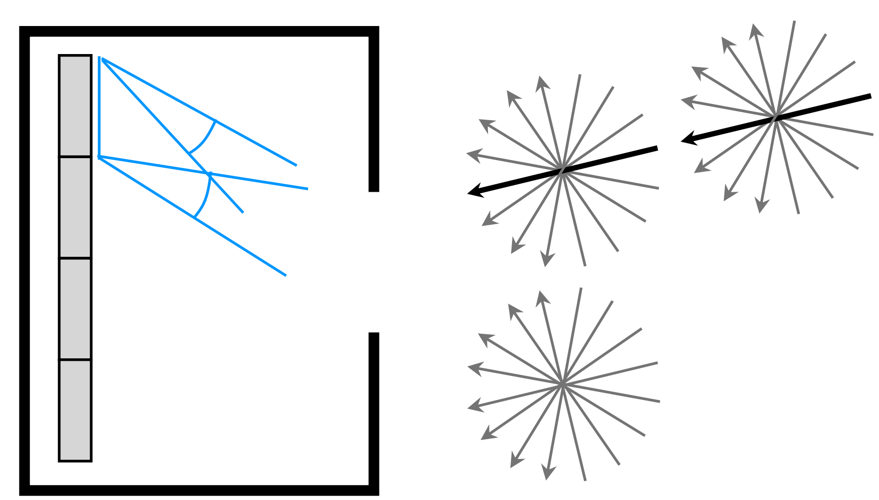
  - Constant along a ray in free space

$$L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})$$



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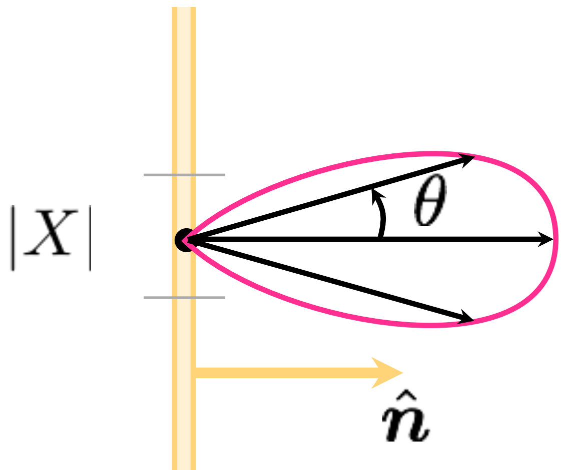
  - Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$

  - A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.
    - "Processed" images (like PNG and JPEG) are not linear radiance measurements!!

Question

- Most light sources, like a heated metal sheet, follow Lambert's Law



"Lambertian area source"

$$J(\hat{\boldsymbol{\omega}}) = J_o \langle \hat{\boldsymbol{\omega}}, \hat{\boldsymbol{n}} \rangle = J_o \cos\theta$$

radiant intensity [W/sr]

- What is the radiance $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x})$ of an infinitesimal patch [W/sr·m^2]?

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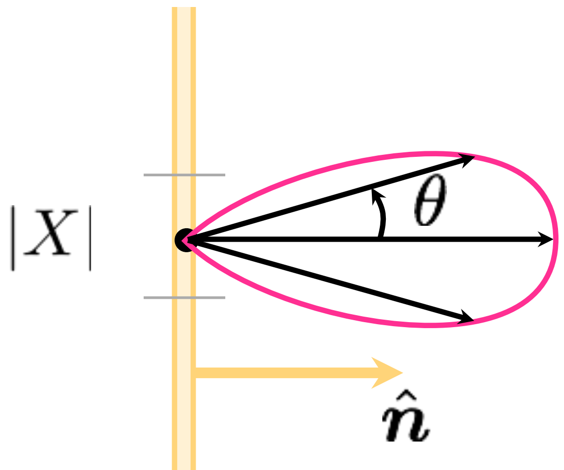

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Answer: $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x}) = J_o / |X|$(independent of direction)

Question

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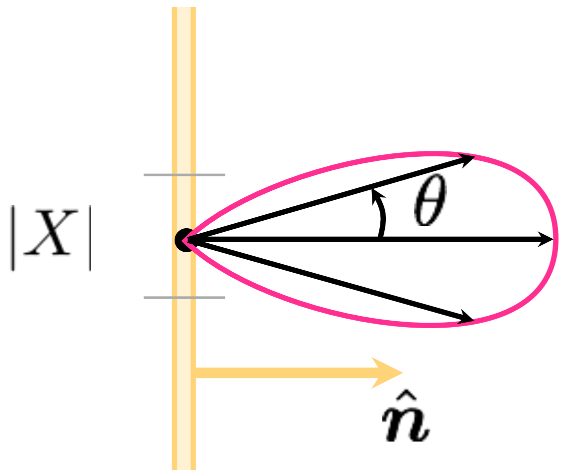

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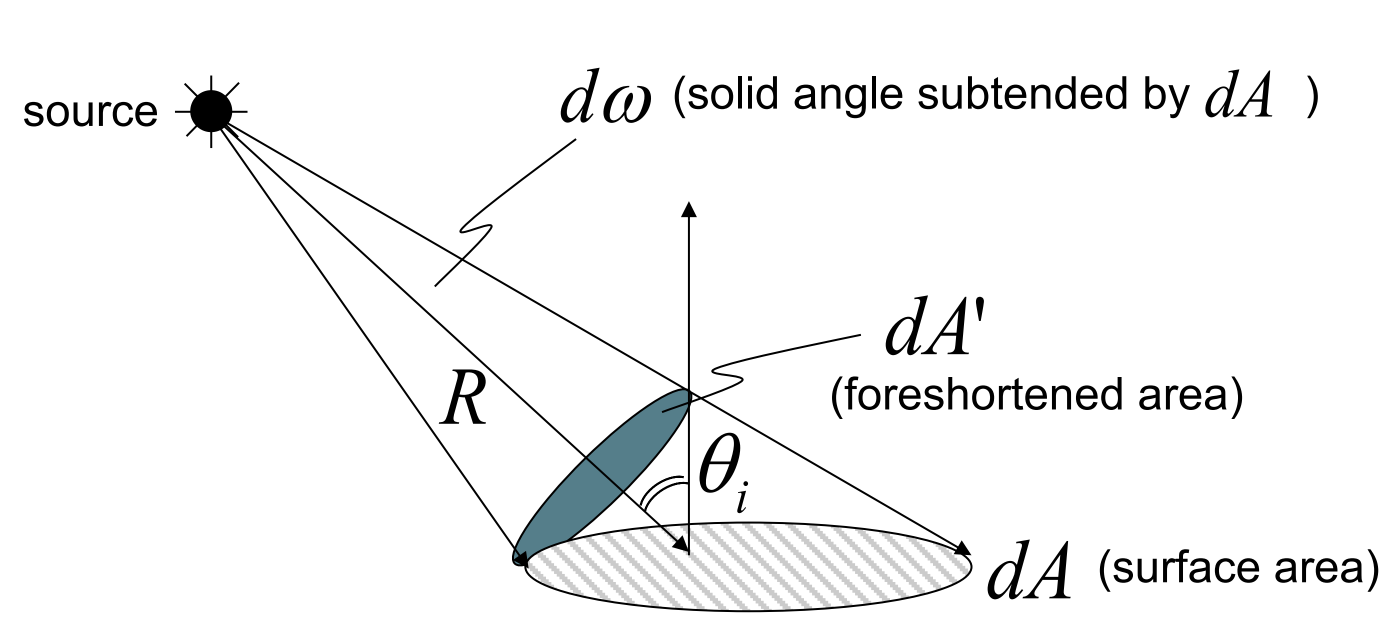
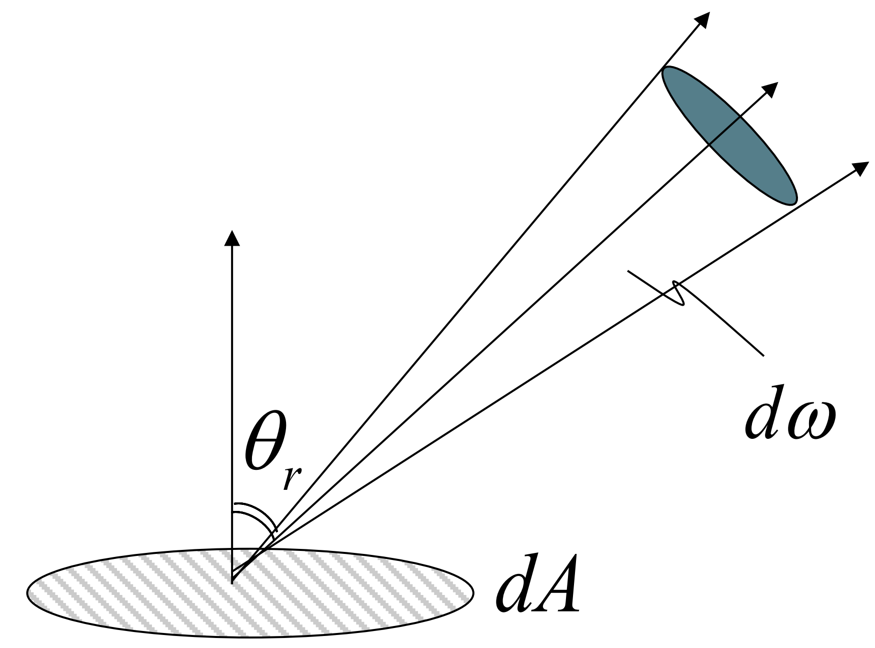
"Looks equally bright when viewed from any direction"

Radiometric concepts – boring…but, important!





(1) Solid Angle : $d\omega = \dfrac{dA'}{R^2} = \dfrac{dA \cos\theta_i}{R^2}$ (steradian)

What is the solid angle subtended by a hemisphere?

(2) Radiant Intensity of Source : $J = \dfrac{d\Phi}{d\omega}$ (watts / steradian)

Light Flux (power) emitted per unit solid angle

(3) Surface Irradiance : $E = \dfrac{d\Phi}{dA}$ (watts / m^2)

Light Flux (power) incident per unit surface area.

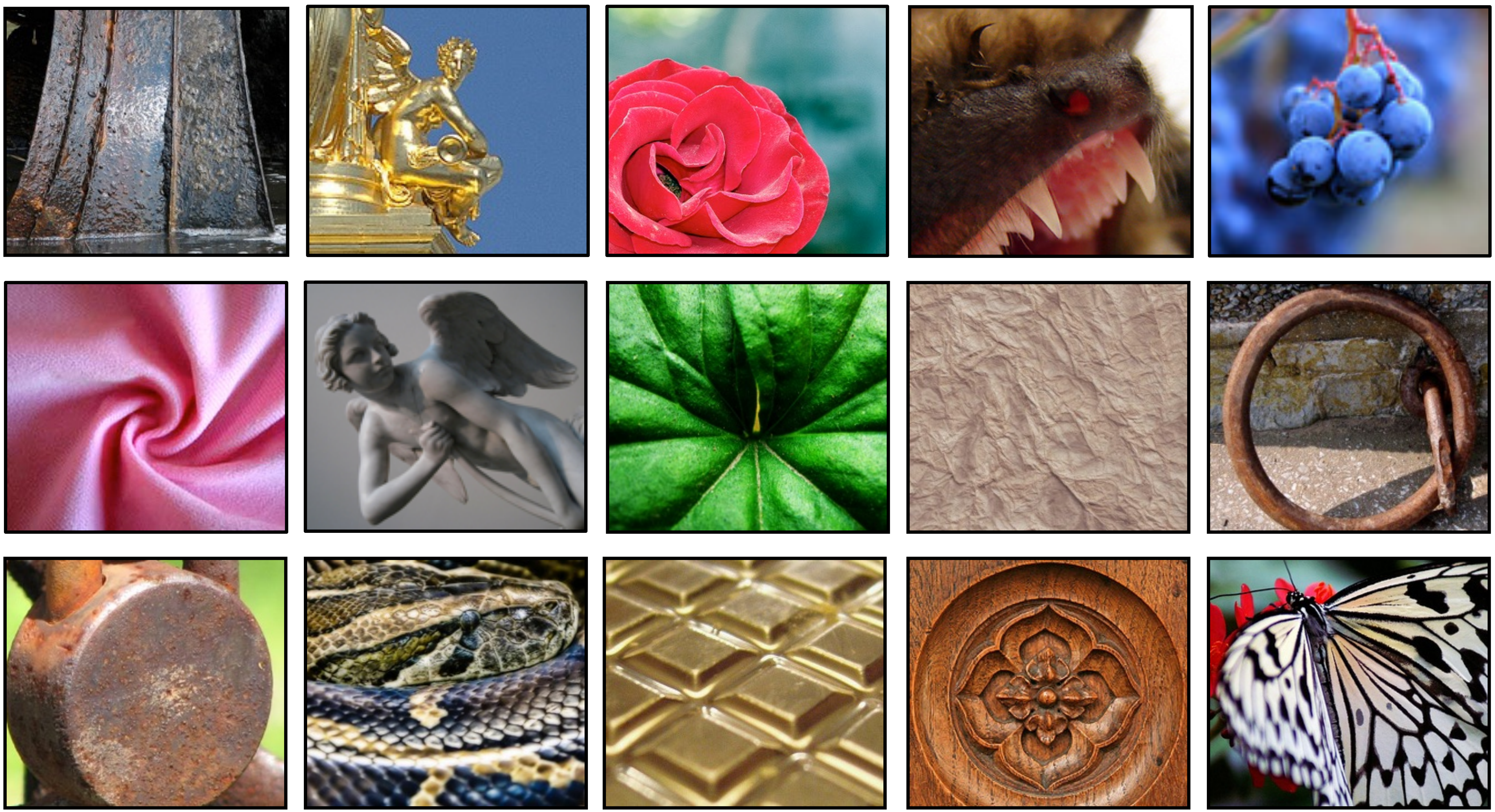
Does not depend on where the light is coming from!

(4) Surface Radiance (tricky) :

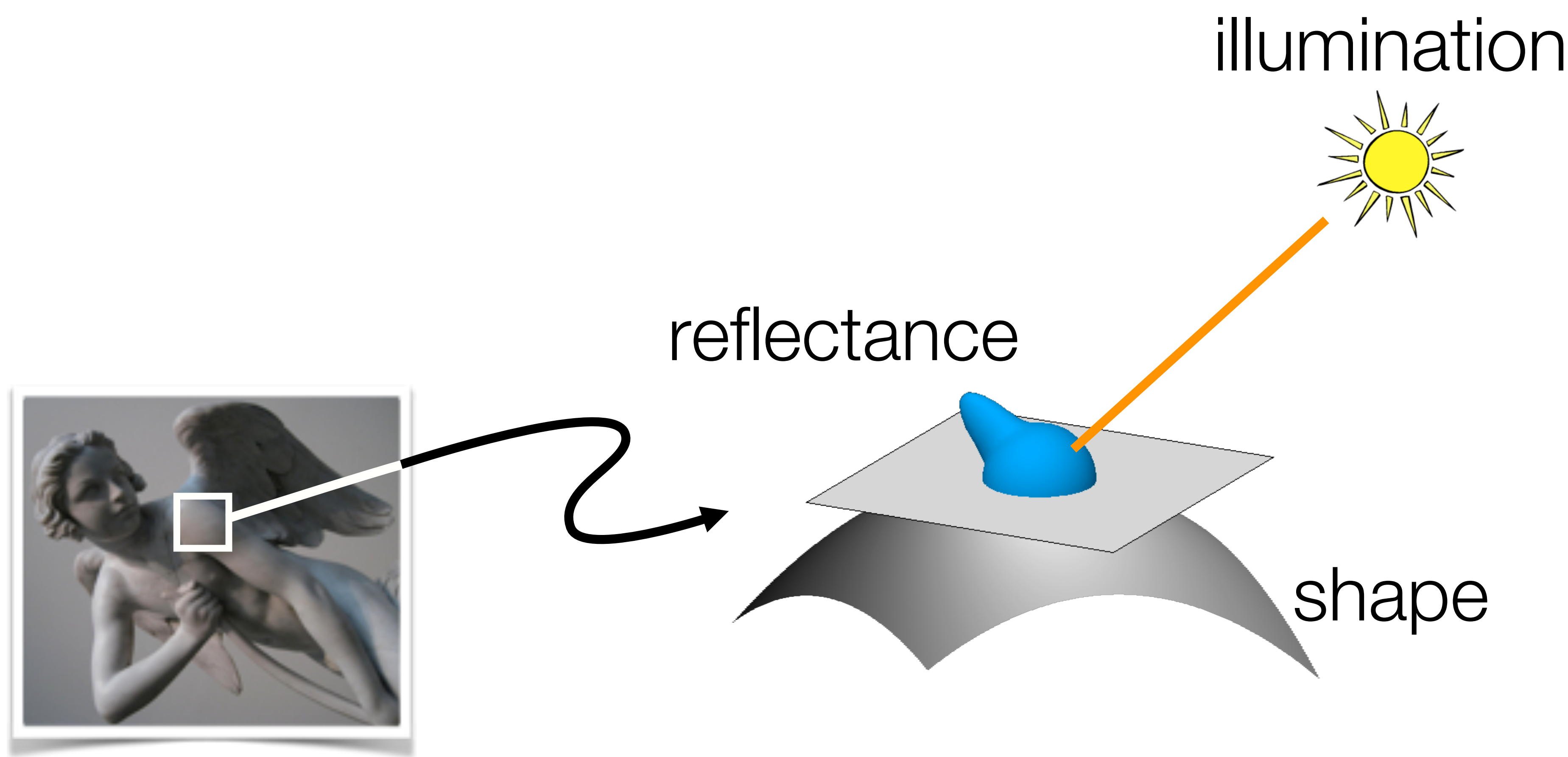
$$L = \frac{d^2\Phi}{(dA \cos\theta_r)\ d\omega}$$ (watts / m^2 steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- L depends on direction θ_r
- Surface can radiate into whole hemisphere.
- L depends on reflectance properties of surface.

Appearance



“Physics-based” computer vision (a.k.a “inverse optics”)



$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Reflectance and BRDF

Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)

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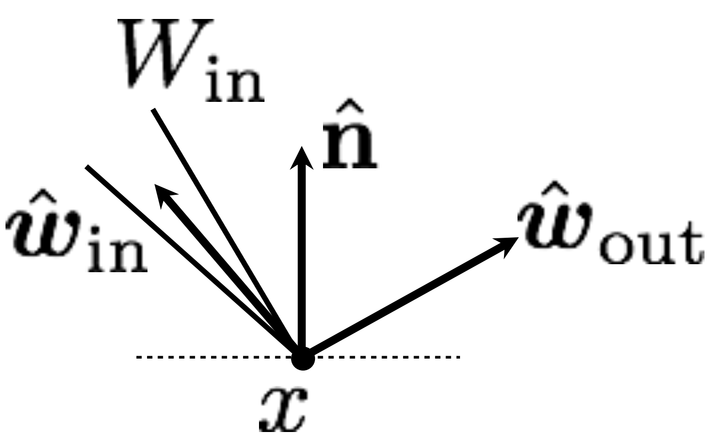
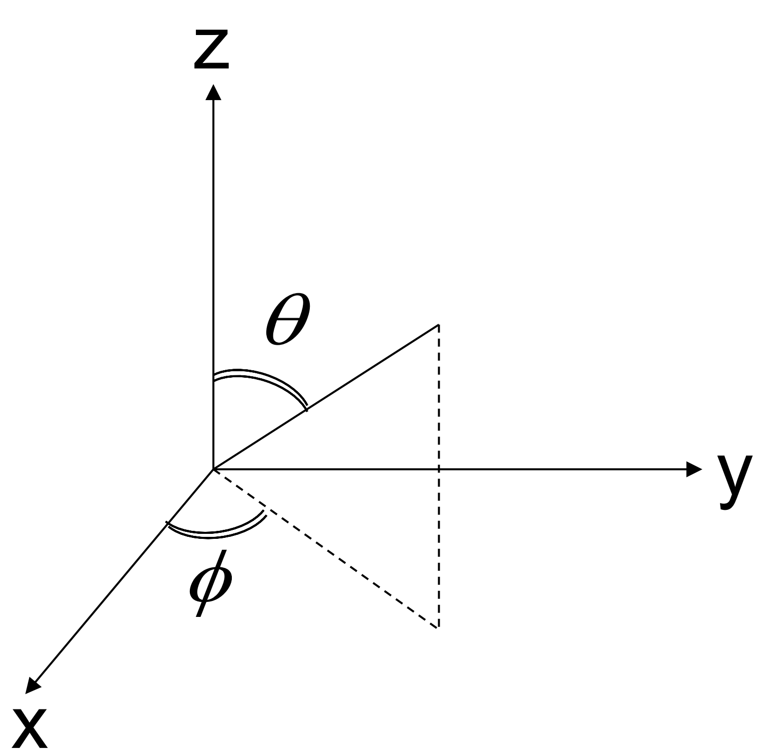


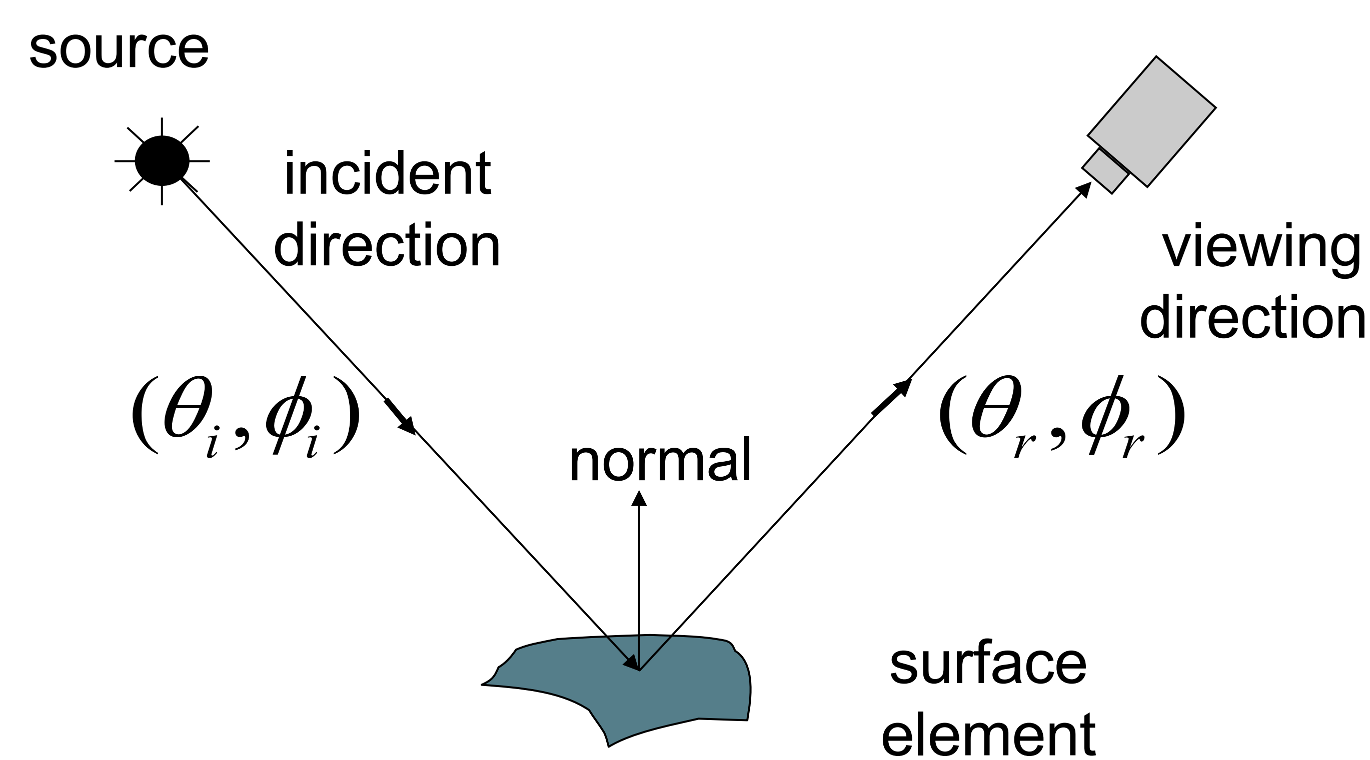
$$\lim_{W_{\text{in}} \longrightarrow \hat{\boldsymbol{w}}_{\text{in}}} \qquad f_{x,\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}})$$

$$f_{x,\hat{\mathbf{n}}}(W_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) = \frac{L^{\text{out}}(x, \hat{\boldsymbol{\omega}}_{\text{out}})}{E^{\text{in}}_{\hat{\mathbf{n}}}(W_{\text{in}}, x)}$$

- Notations x and n often implied by context and omitted; directions \omega are expressed in local coordinate system defined by normal n (and some chosen tangent vector)
- Units: sr^{-1}
- Called Bidirectional Reflectance Distribution Function (BRDF)

BRDF: Bidirectional Reflectance Distribution Function





$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$\boxed{\text{BRDF}: f(\theta_i, \phi_i; \theta_r, \phi_r) \quad = \quad \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}}$$

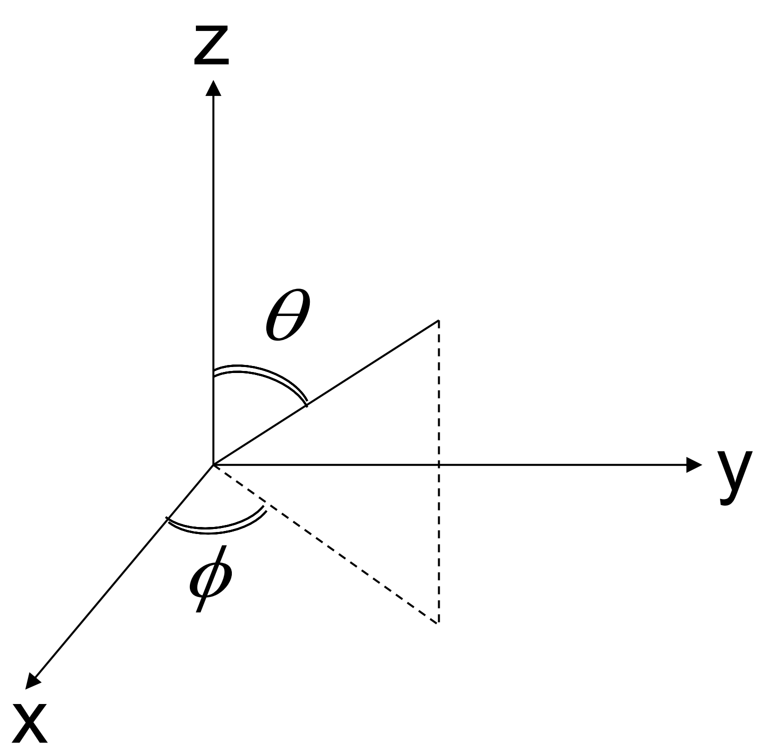
Reflectance: BRDF

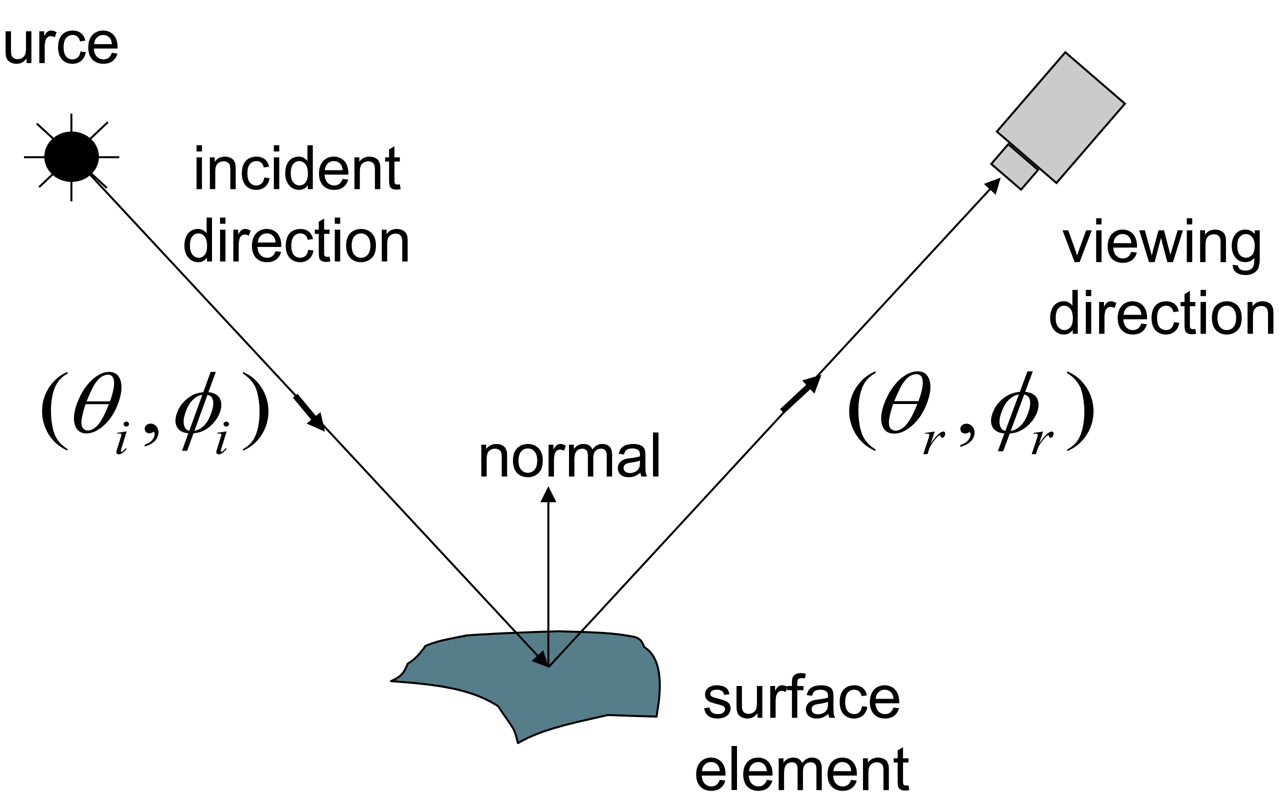
- Units: sr^{-1}

- Real-valued function defined on the double-hemisphere

- Has many useful properties

Important Properties of BRDFs



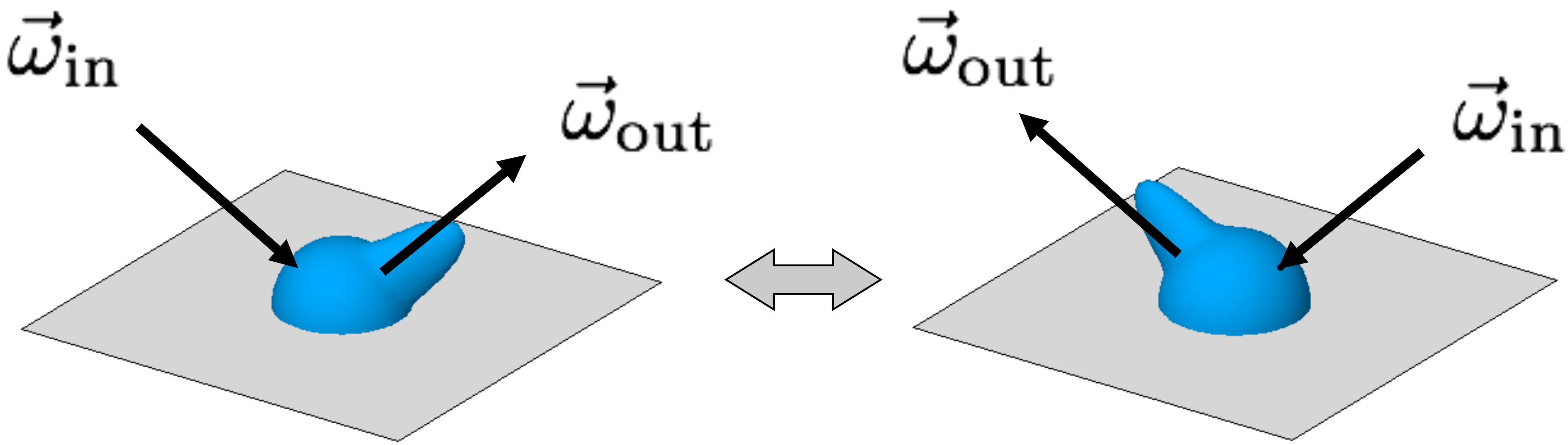


- Conservation of Energy:

$$\forall \hat{\boldsymbol{\omega}}_{\text{in}}, \quad \int_{\Omega_{\text{out}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) \cos\theta_{\text{out}} d\hat{\boldsymbol{\omega}}_{\text{out}} \leq 1$$

Why smaller
than or equal?

Property: "Helmholtz reciprocity"

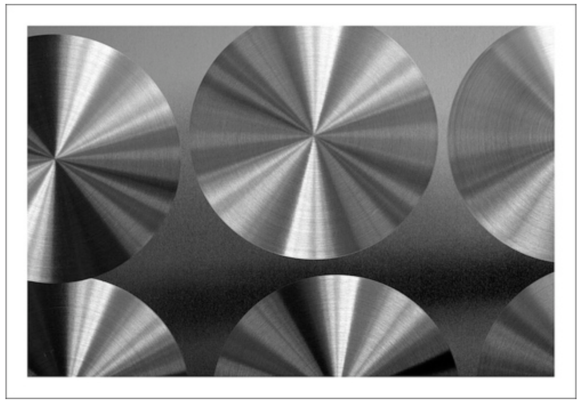


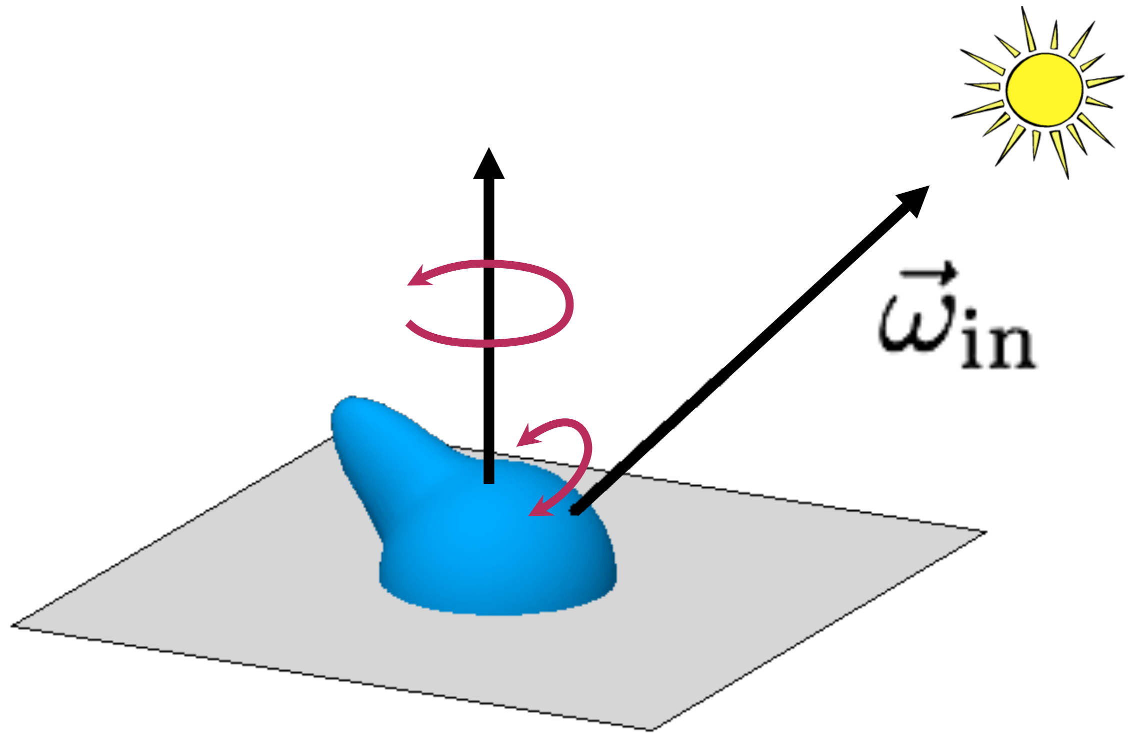
- Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}})$$

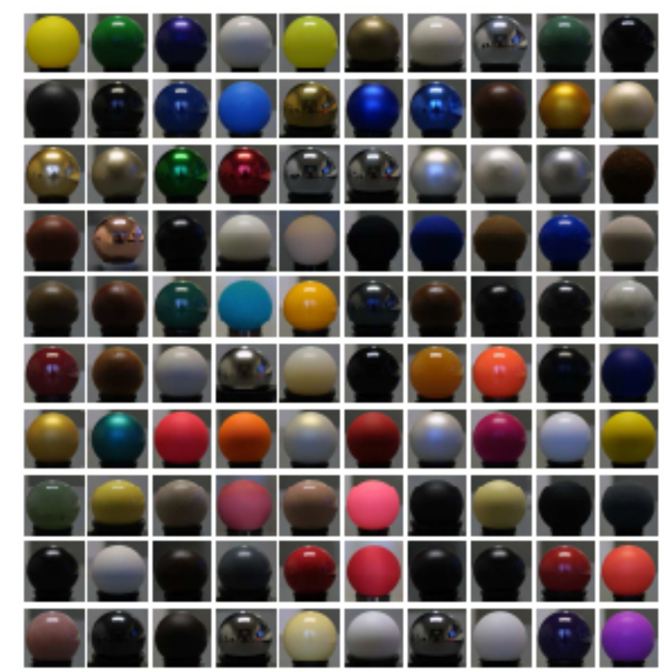
Common <u>assumption</u>: Isotropy





$$f_r(\vec{\omega}_{\text{in}}, \cdot)$$

BRDF does not change when surface is rotated about the normal.



[Matusik et al., 2003]

4D → 3D

$$\boxed{f_r(\cancel{\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

Reflectance: BRDF

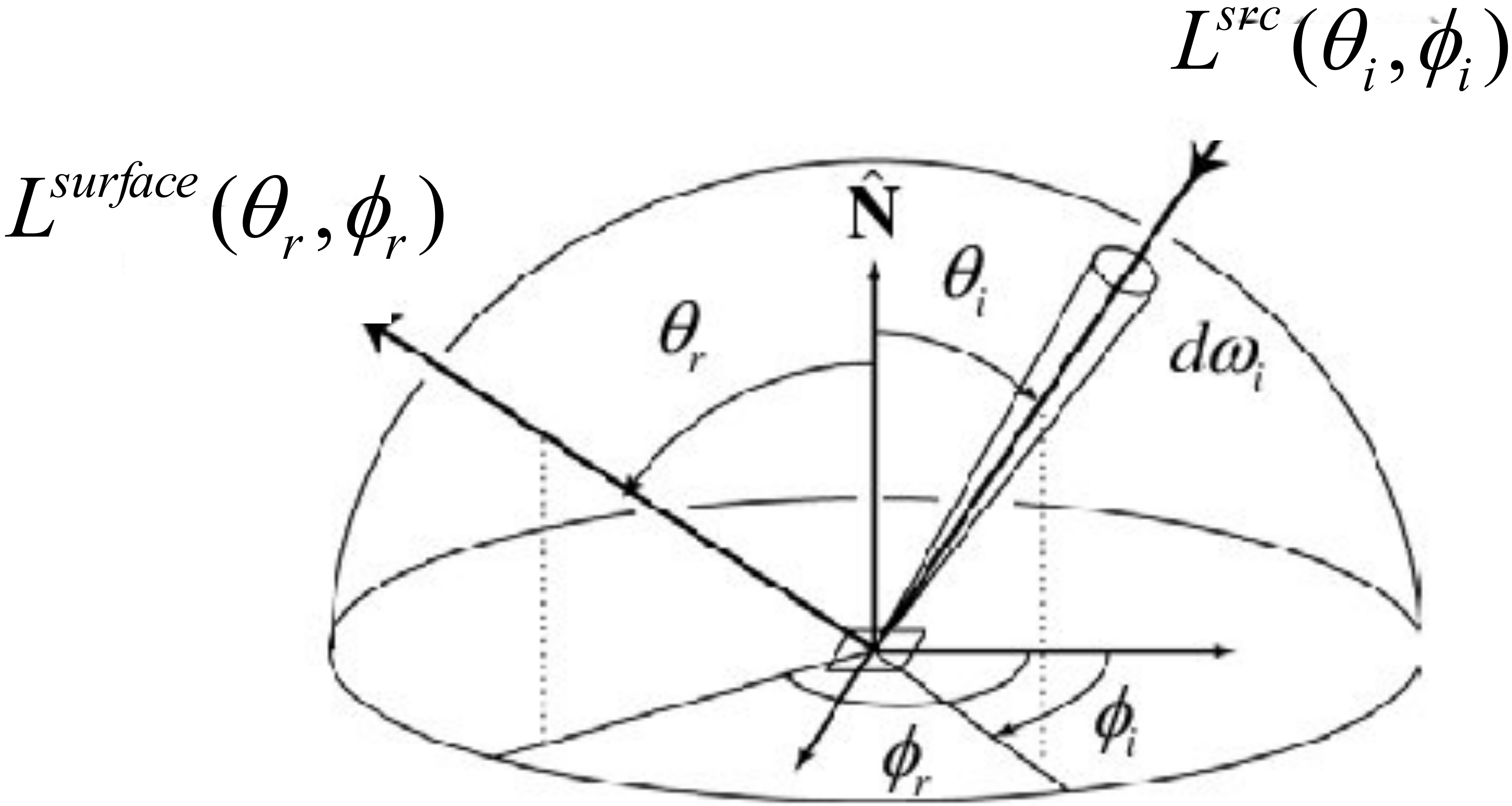
- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for *any* configuration of lights and viewpoint

$$L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\omega}_{\text{in}}$$

reflectance equation

Why is there a cosine in the reflectance equation?

Derivation of the Reflectance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) = E^{surface}(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \underline{E^{surface}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \underline{L^{src}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)\, \underline{\cos\theta_i d\omega}_i$$

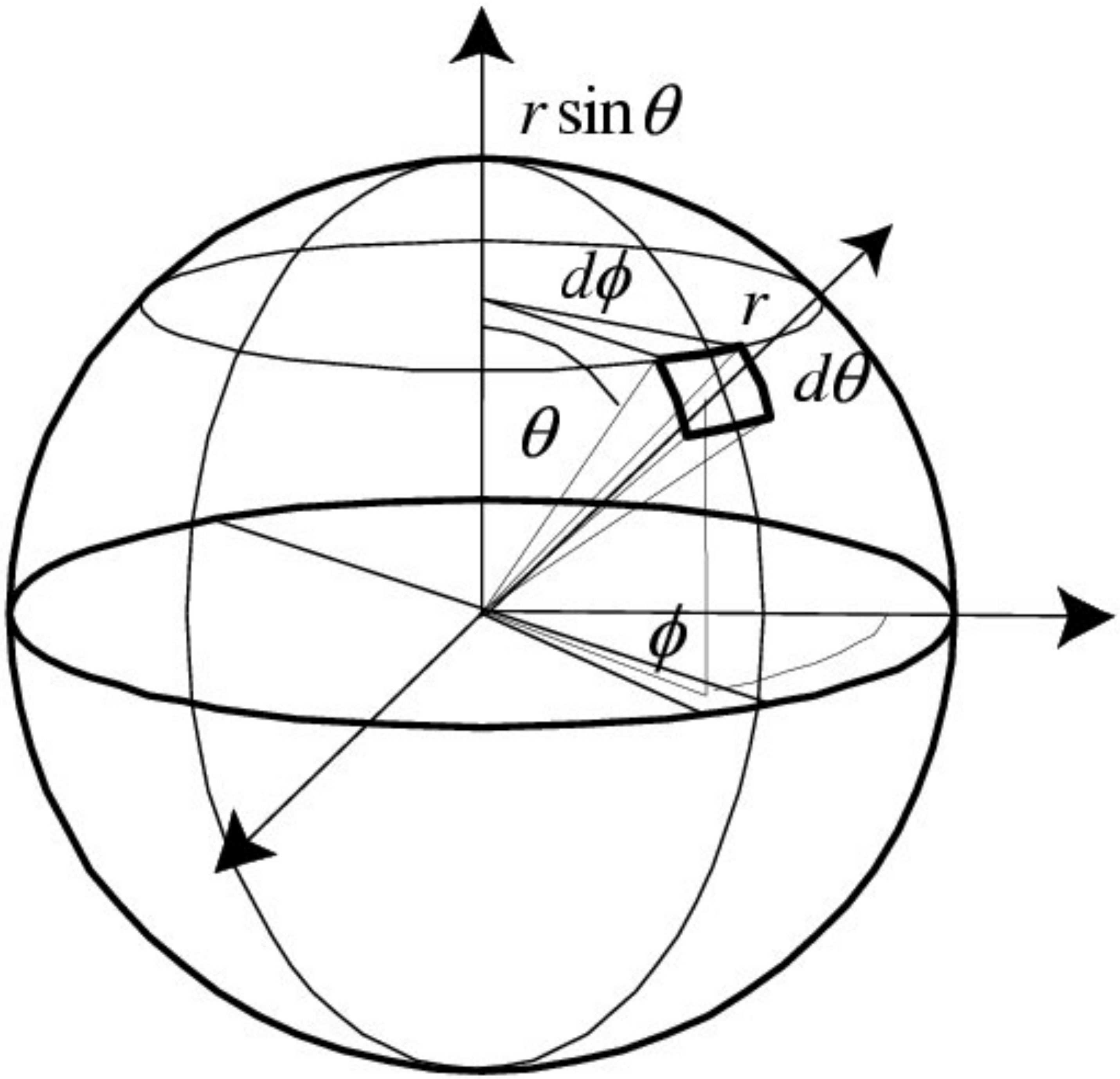
Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{2\pi} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\ \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r,\phi_r) \quad = \quad \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\ \underline{\sin\theta_i d\theta_i d\phi_i}$$

Differential Solid Angles

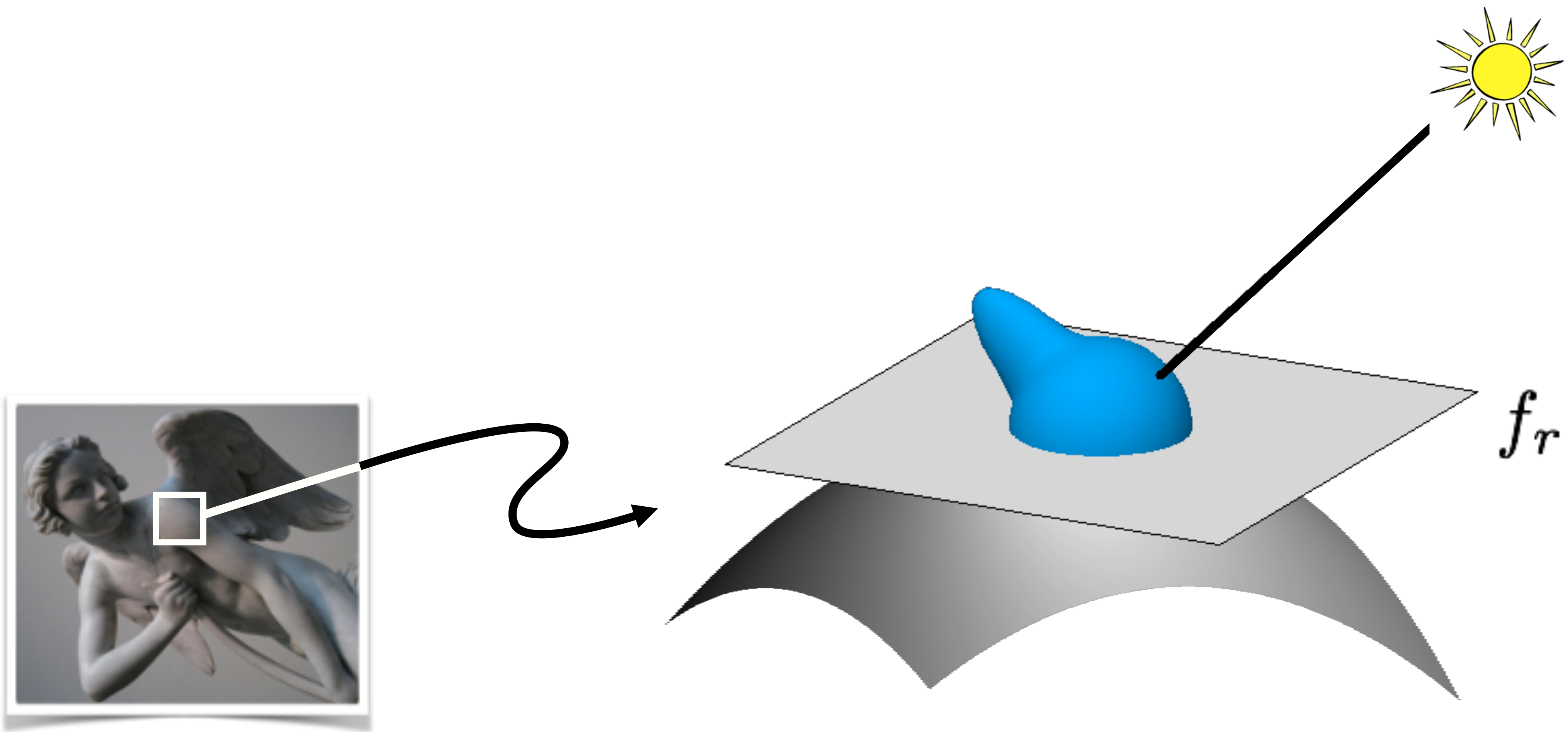


$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

BRDF



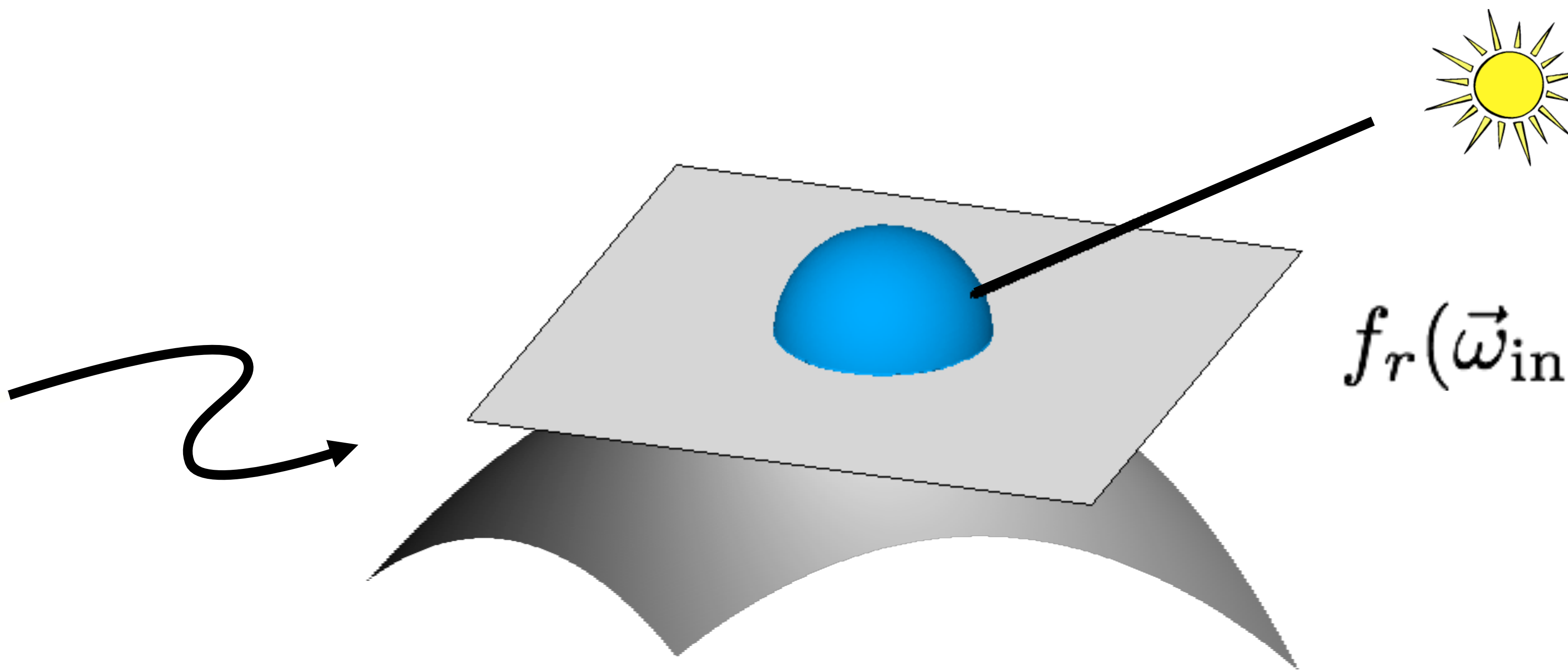
$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions

What does the BRDF equal in this case?

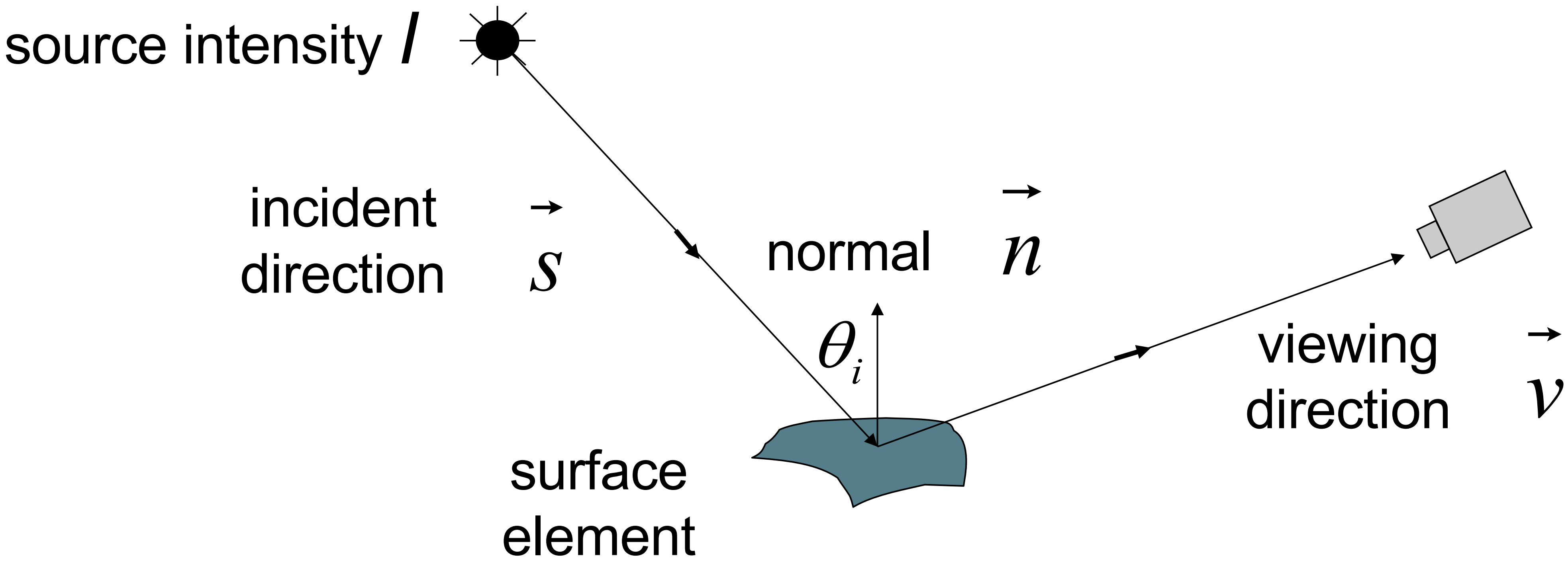


$f_r(\vec{\omega}_{\text{in}}, \cdot)$

$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Diffuse Reflection and Lambertian BRDF



- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)

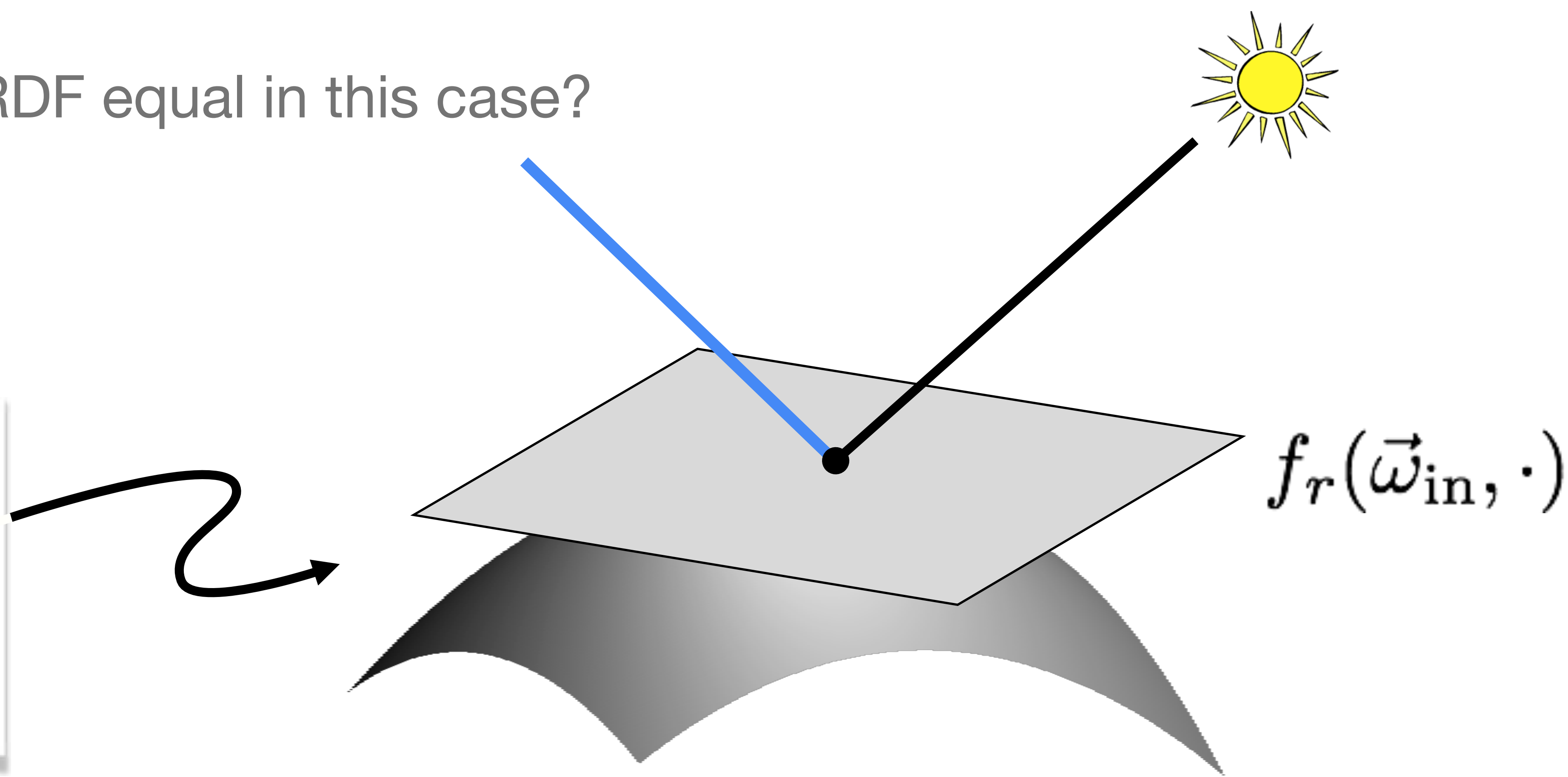
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \dfrac{\rho_d}{\pi}$ albedo

- Most commonly used BRDF in Vision and Graphics!

BRDF

Specular BRDF: all energy concentrated in mirror direction

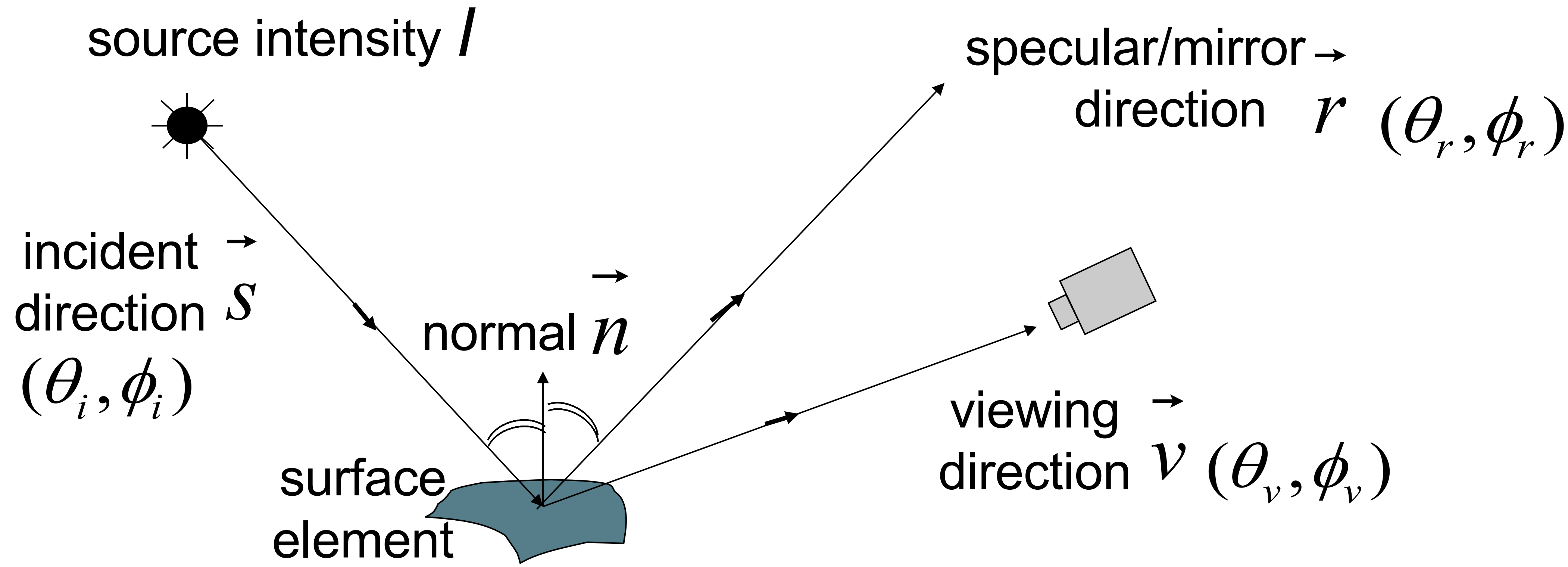
What does the BRDF equal in this case?



$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.

- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).

- Mirror BRDF is simply a double-delta function :

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \ \delta(\theta_i - \theta_v) \ \delta(\phi_i + \pi - \phi_v)$$

Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc

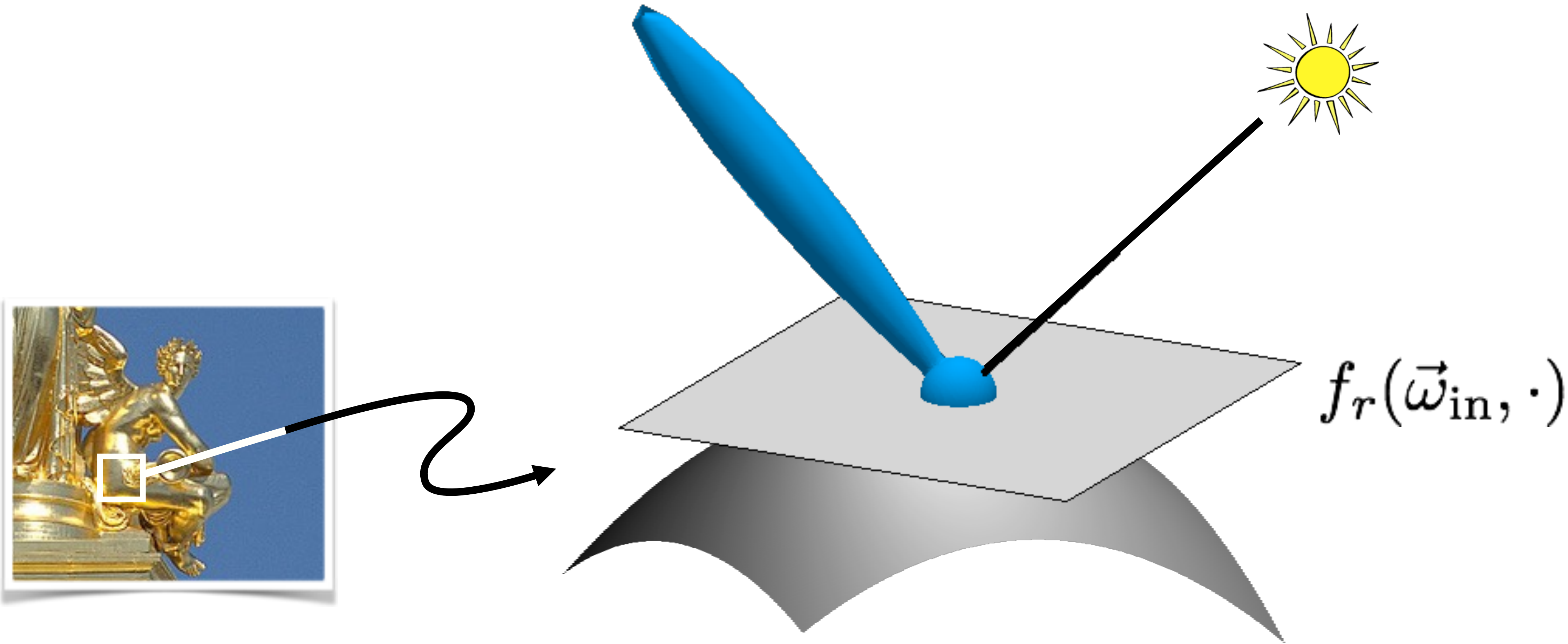
Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:

BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere



$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Trick for dielectrics (non-metals)

- BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components
- The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

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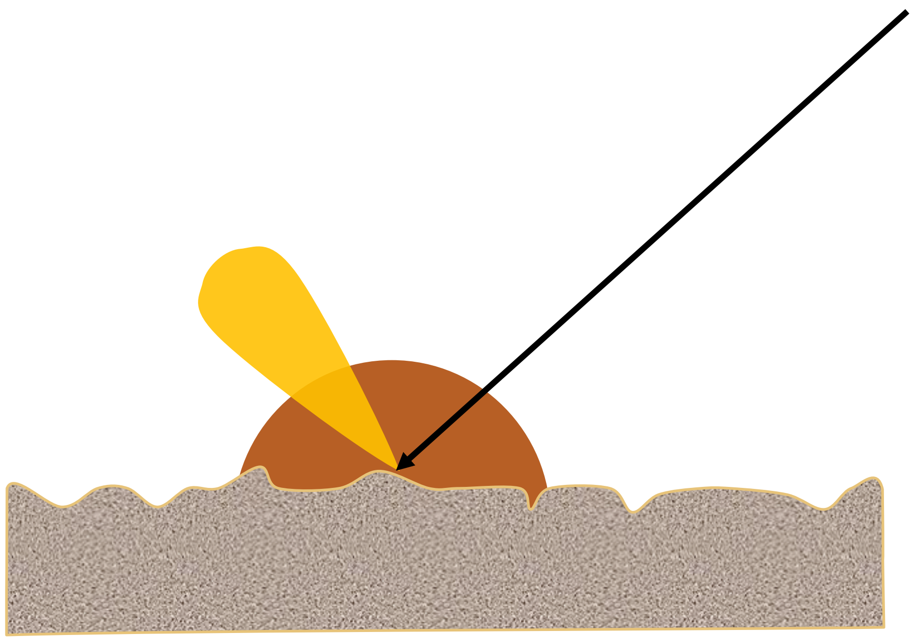
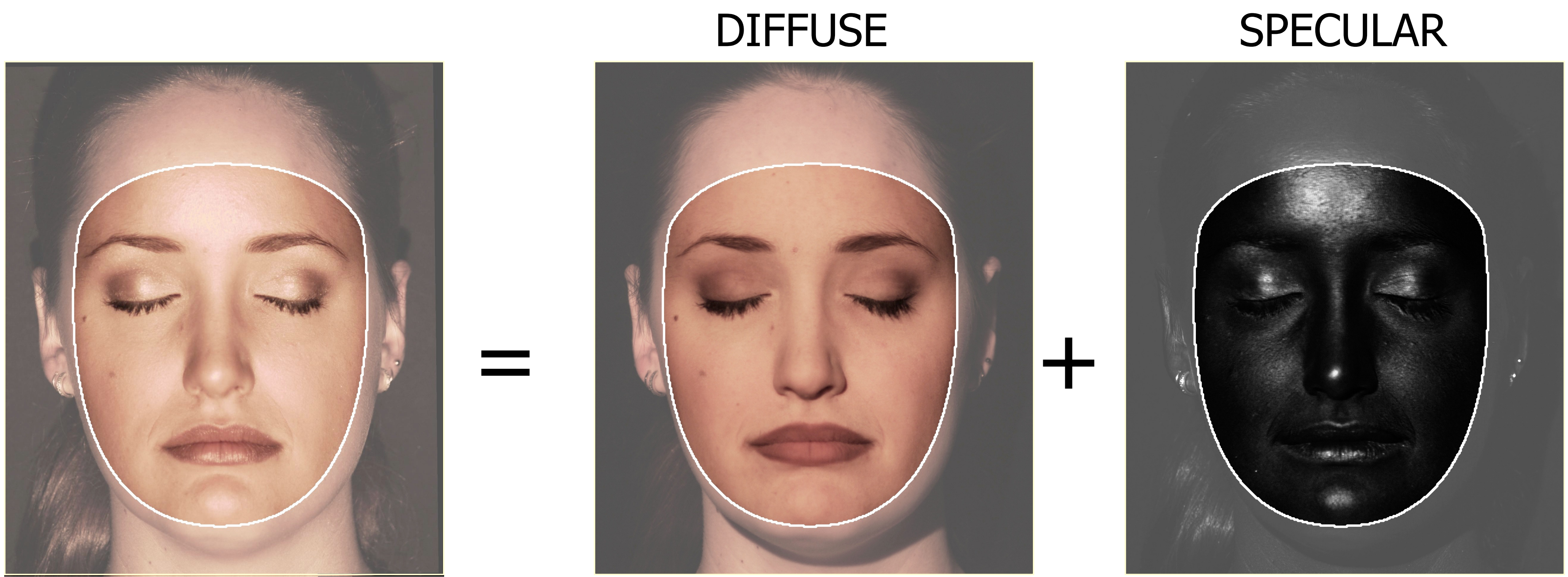
$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

Often called the *dichromatic BRDF*:

- Diffuse term varies with wavelength, constant with polarization
- Specular term constant with wavelength, varies with polarization

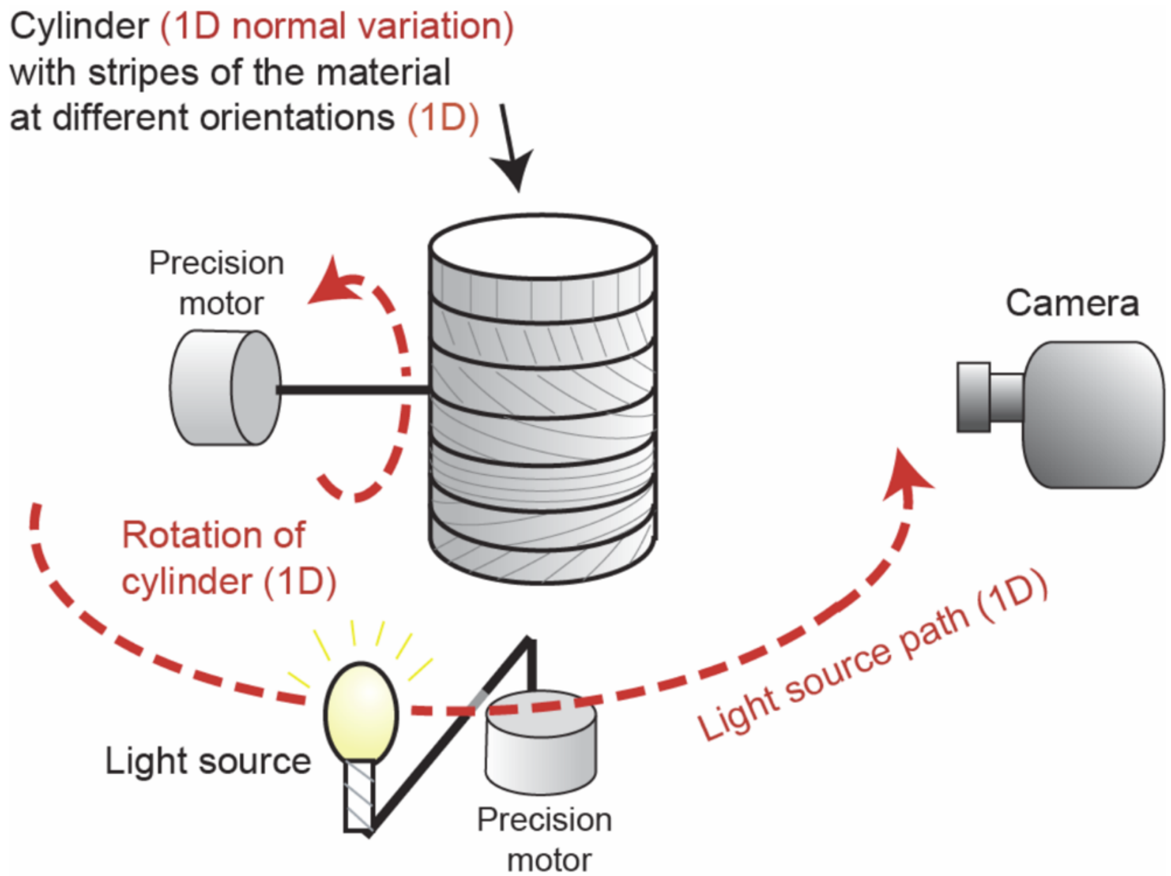


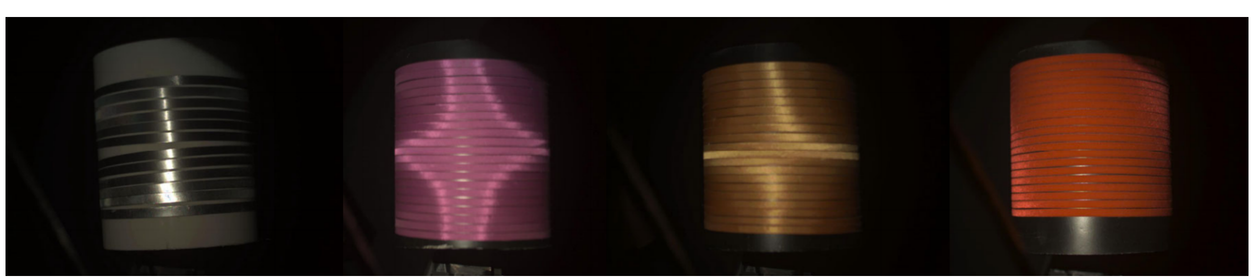
Trick for dielectrics (non-metals)



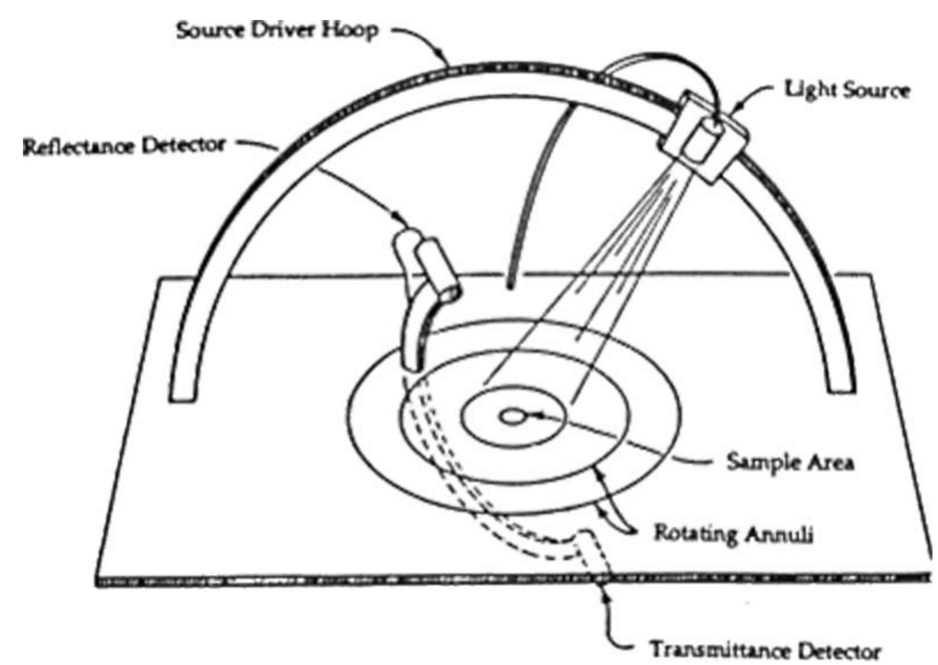
- In this example, the two components were separated using linear polarizing filters on the camera and light source.

Tabulated 4D BRDFs (hard to measure)





[Ngan et al., 2005]



Gonioreflectometer

Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D,3D, or 4D) function
- Except for Lambertian, the BRDF is non-linear in these parameters
- Examples:

Where do these constants come from?

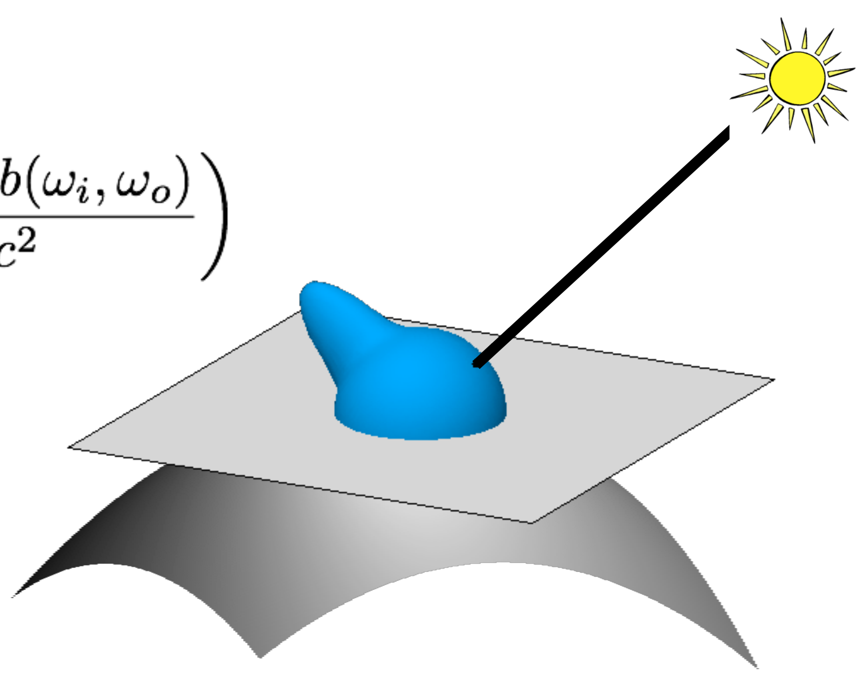
Lambertian: $f(\omega_i, \omega_o) = \frac{a}{\pi}$

Phong: $f(\omega_i, \omega_o) = \frac{a}{\pi} + b\cos^c\left(2\langle\omega_i, n\rangle\langle\omega_o, n\rangle - \langle\omega_i, \omega_o\rangle\right)$

Blinn: $f(\omega_i, \omega_o) = \frac{a}{\pi} + b\cos^c b(\omega_i, \omega_o)$

Lafortune: $f(\omega_i, \omega_o) = \frac{a}{\pi} + b(-\omega_i^\top A\omega_o)^k$

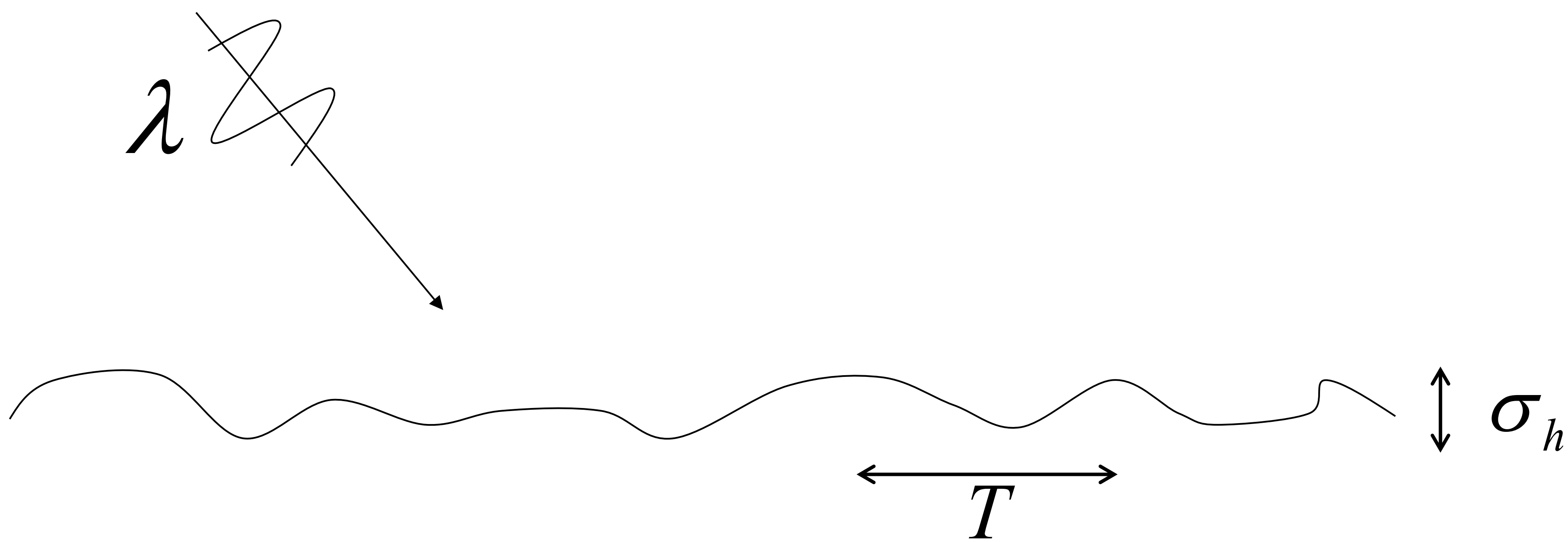
Ward: $f(\omega_i, \omega_o) = \frac{a}{\pi} + \frac{b}{4\pi c^2\sqrt{\langle n, \omega_i\rangle\langle n, \omega_o\rangle}} \exp\left(\frac{-\tan^2 b(\omega_i, \omega_o)}{c^2}\right)$



α is called the *albedo*

Reflectance Models

Reflection: An Electromagnetic Phenomenon



Two approaches to derive Reflectance Models:

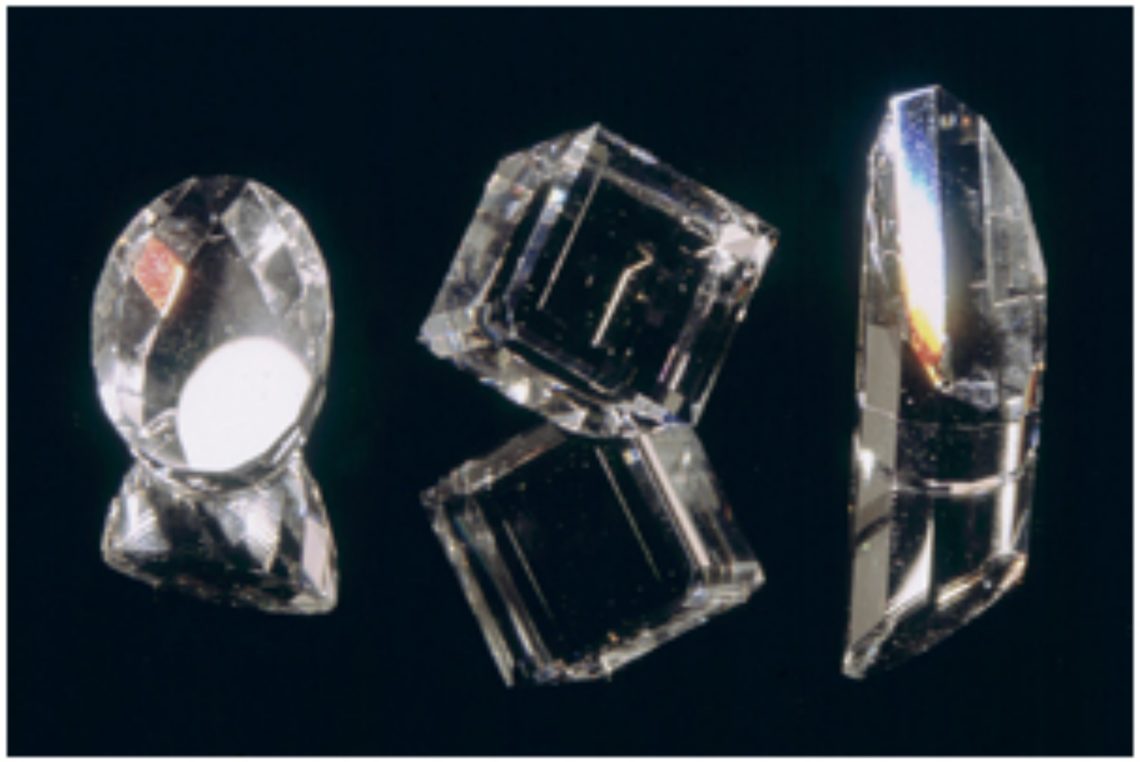
– Physical Optics (Wave Optics)

– Geometrical Optics (Ray Optics)

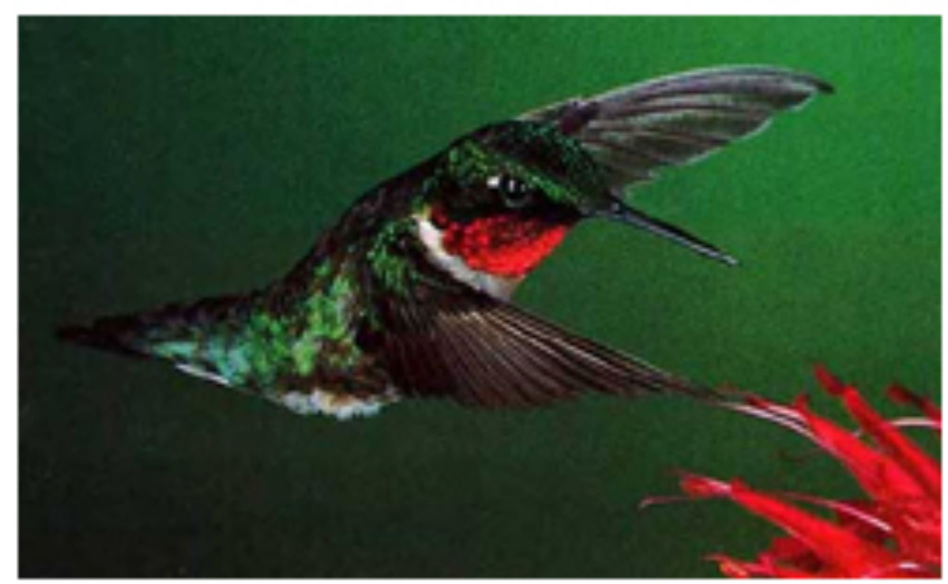
Geometrical models are approximations to physical models

But they are easier to use!

Reflectance that Require Wave Optics





References

Basic reading:

- Szeliski, Section 2.2.
- Gortler, Chapter 21.
  This book by Steven Gortler has a great *introduction* to radiometry, reflectance, and their use for image formation.

Additional reading:

- Arvo, "Analytic Methods for Simulated Light Transport," Yale 1995.
- Veach, "Robust Monte Carlo Methods for Light Transport Simulation," Stanford 1997.
  These two thesis are foundational for modern computer graphics. Among other things, they include a thorough derivation (starting from wave optics and measure theory) of all radiometric quantities and associated integro-differential equations. You can also look at them if you are interested in physics-based rendering.
- Dutre et al., "Advanced Global Illumination," 2006.
  A book discussing modeling and simulation of other appearance effects beyond single-bounce reflectance.
- Weyrich et al., "Principles of Appearance Acquisition and Representation," FTCGV 2009.
  A very thorough review of everything that has to do with modeling and measuring BRDFs.
- Walter et al., "Microfacet models for refraction through rough surfaces," EGSR 2007.
  This paper has a great review of physics-based models for reflectance and refraction.
- Matusik, "A data-driven reflectance model," MIT 2003.
  This thesis introduced the largest measured dataset of 4D reflectances. It also provides detailed discussion of many topics relating to modelling reflectance.
- Rusinkiewicz, "A New Change of Variables for Efficient BRDF Representation," 1998.
- Romeiro and Zickler, "Inferring reflectance under real-world illumination," Harvard TR 2010.
  These two papers discuss the isotropy and other properties of common BRDFs, and how one can take advantage of them using alternative parameterizations.
- Shafer, "Using color to separate reflection components," 1984.
  The paper introducing the dichromatic reflectance model.
- Stam, "Diffraction Shaders," SIGGRAPH 1999.
- Levin et al., "Fabricating BRDFs at high spatial resolution using wave optics," SIGGRAPH 2013.
- Cuypers et al., "Reflectance model for diffraction," TOG 2013.
  These three papers describe reflectance effects that can only be modeled using wave optics (and in particular diffraction).