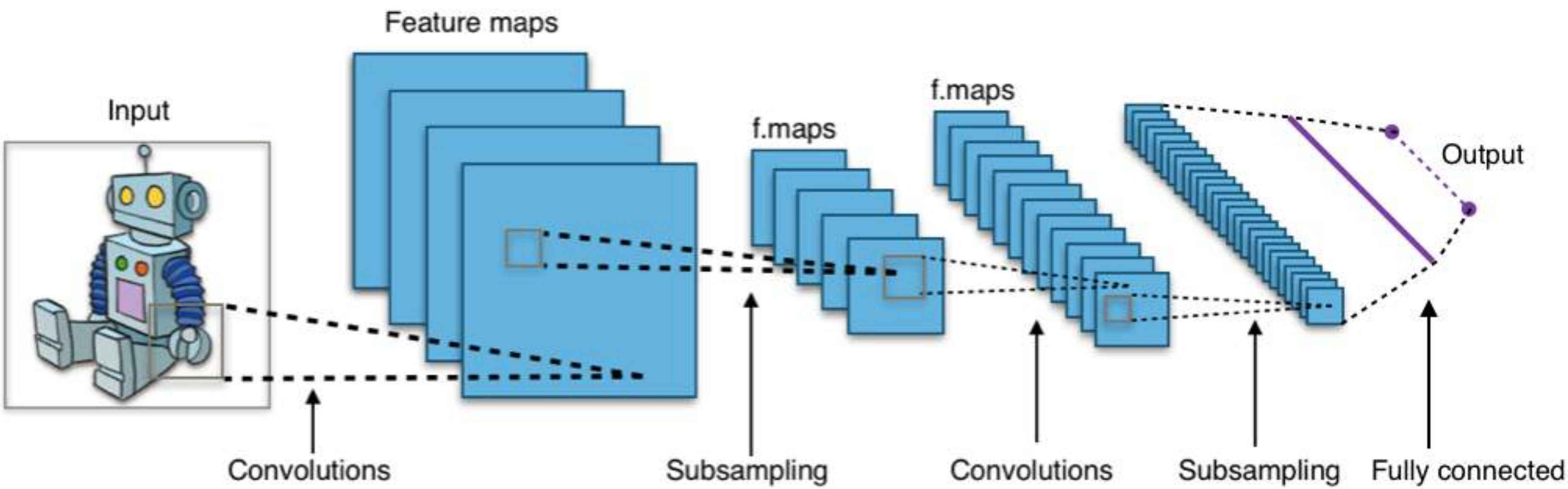
Convolutional neural networks



http://16385.courses.cs.cmu.edu

16-385 Computer Vision
Spring 2021, Lecture 22 & 23

Overview of today's lecture

- Some notes on optimization.
- Convolutional neural networks.
- Training ConvNets.

Slide credits

Most of these slides were adapted from:

- Noah Snavely (Cornell University).
- Fei-Fei Li (Stanford University).
- Andrej Karpathy (Stanford University).

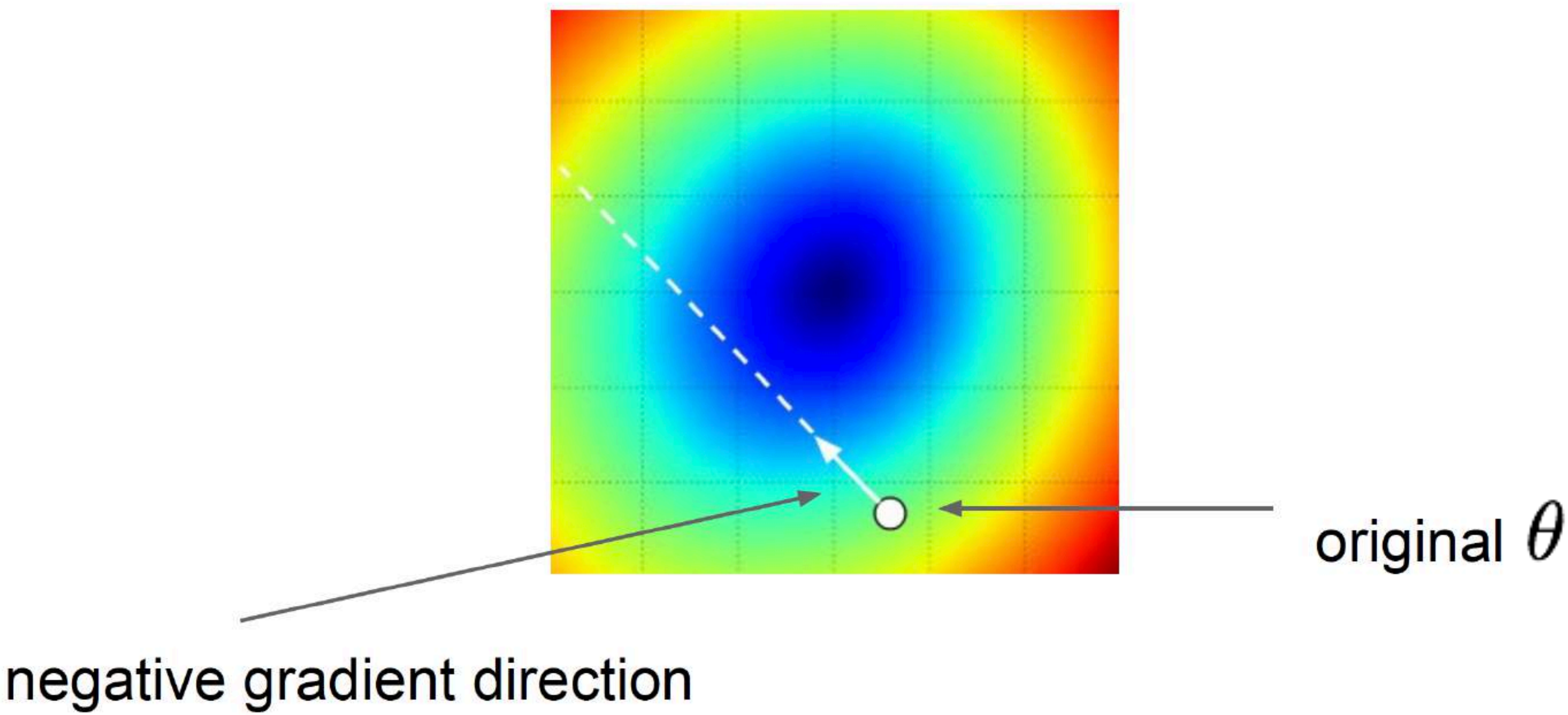
Some notes on optimization

Summary

- Always use mini-batch gradient descent
- Incorrectly refer to it as "doing SGD" as everyone else

  (or call it batch gradient descent)
- The mini-batch size is a hyperparameter, but it is not very common to cross-validate over it (usually based on practical concerns, e.g. space/time efficiency)

Learning rates



$$\theta \leftarrow \theta - \eta \frac{\partial \mathcal{L}}{\partial \theta}$$

Step size: learning rate
Too big: will miss the minimum
Too small: slow convergence

Learning rate scheduling

- Use different learning rate at each iteration.
- Most common choice:

$$\eta_t = \frac{\eta_0}{\sqrt{t}}$$

Need to select initial learning rate η_0, important!

- More modern choice: Adaptive learning rates.

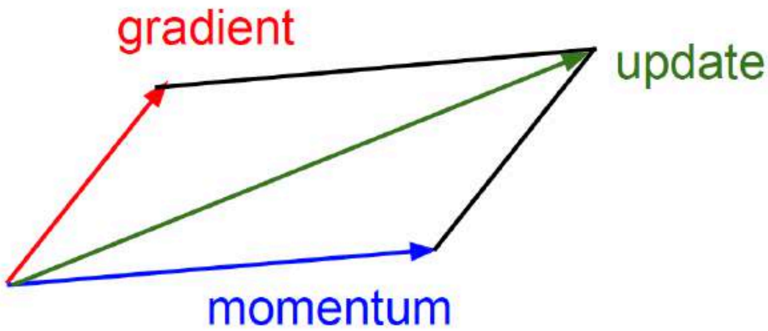
$$\eta_t = G\left(\left\{\frac{\partial L}{\partial \theta}\right\}_{i=0}^{t}\right)$$

Many choices for G (Adam, Adagrad, Adadelta).

Momentum Update

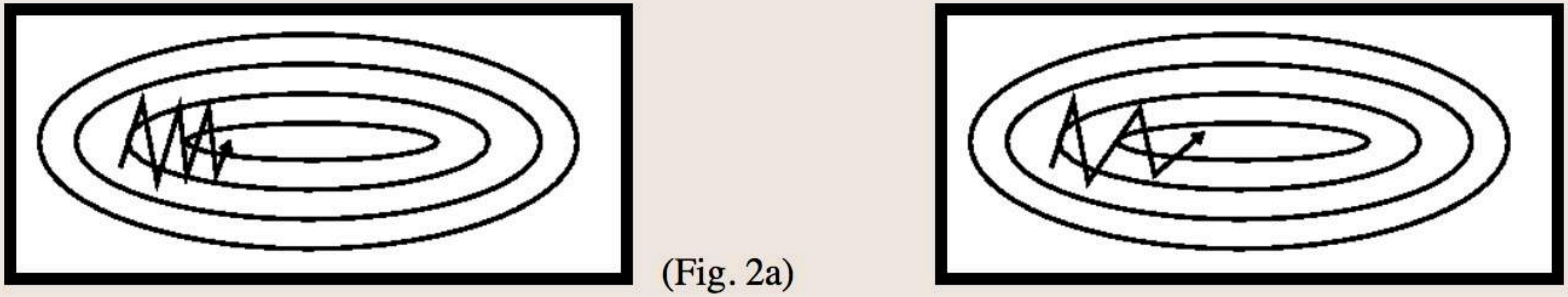
$$\theta \leftarrow \theta - \eta \frac{\partial \mathcal{L}}{\partial \theta}$$

$$\Delta\theta \leftarrow w \frac{\partial L}{\partial \theta} + (1 - w)\Delta\theta$$



Take direction history into account!

```
weights_grad = evaluate_gradient(loss_fun, data, weights)
vel = vel * 0.9 - step_size * weights_grad
weights += vel
```



(Fig. 2a) (Fig. 2b)

Many other ways to perform optimization…

- Second order methods that use the Hessian (or its approximation): BFGS, **LBFGS**, etc.

- Currently, the lesson from the trenches is that well-tuned SGD+Momentum is very hard to beat for CNNs.

- No consensus on Adam etc.: Seem to give faster performance to worse local minima.

Derivatives

- Given f(x), where x is vector of inputs
  - Compute gradient of f at x: $\nabla f(x)$

How do we do differentiation?

Numerical differentiation

Numerical differentiation

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h\,\frac{df(x)}{dx}$$

Numerical differentiation

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = f(x) + h\,\frac{df(x)}{dx}$$

Numerical differentiation is:

– Approximate.

– Slow.

– Numerically unstable.

– Easy to write.

Symbolic differentiation

Symbolic differentiation

- What Mathematica does: Automatically derive *analytical* expressions for derivative.

Symbolic differentiation

- What Mathematica does: Automatically derive *analytical* expressions for derivative.
- Often results in very redundant (and expensive to evaluate) expressions.

```
D[Log[1 + Exp[w * x + b]], w]
```

Out[11]= $$\frac{e^{b+w\,x}\,w}{1+e^{b+w\,x}}$$

```
In[19]:= D[Log[1 + Exp[w2 * Log[1 + Exp[w1 * x + b1]] + b2]], w1]
```

Out[19]= $$\frac{e^{b1+b2+w1\,x+w2\,\mathrm{Log}\left[1+e^{b1+w1\,x}\right]}\,w2\,x}{\left(1+e^{b1+w1\,x}\right)\left(1+e^{b2+w2\,\mathrm{Log}\left[1+e^{b1+w1\,x}\right]}\right)}$$

- Often intractable.

Automatic differentiation (autodiff)

Automatic differentiation (autodiff)

- An autodiff system will convert the program into a sequence of primitive operations which have specified routines for computing derivatives.
- In this representation, backprop can be done in a completely mechanical way.

Original program:

$$z = wx + b$$

$$y = \frac{1}{1 + \exp(-z)}$$

$$\mathcal{L} = \frac{1}{2}(y - t)^2$$

Sequence of primitive operations:

$$\begin{aligned}
t_1 &= wx \\
z &= t_1 + b \\
t_3 &= -z \\
t_4 &= \exp(t_3) \\
t_5 &= 1 + t_4 \\
y &= 1/t_5 \\
t_6 &= y - t \\
t_7 &= t_6^2 \\
\mathcal{L} &= t_7/2
\end{aligned}$$

In summary

- Numerical gradient: easy to implement, bad to use.
- Symbolic gradient: sometimes useful, often intractable.
- Automatic gradient: exact, fast, error-prone.

In practice: Use symbolic gradient for small/trivial programs. Almost always use analytic gradient, but check correctness of implementation with numerical gradient.

- This is called a gradient check.

Convolutional Neural Networks

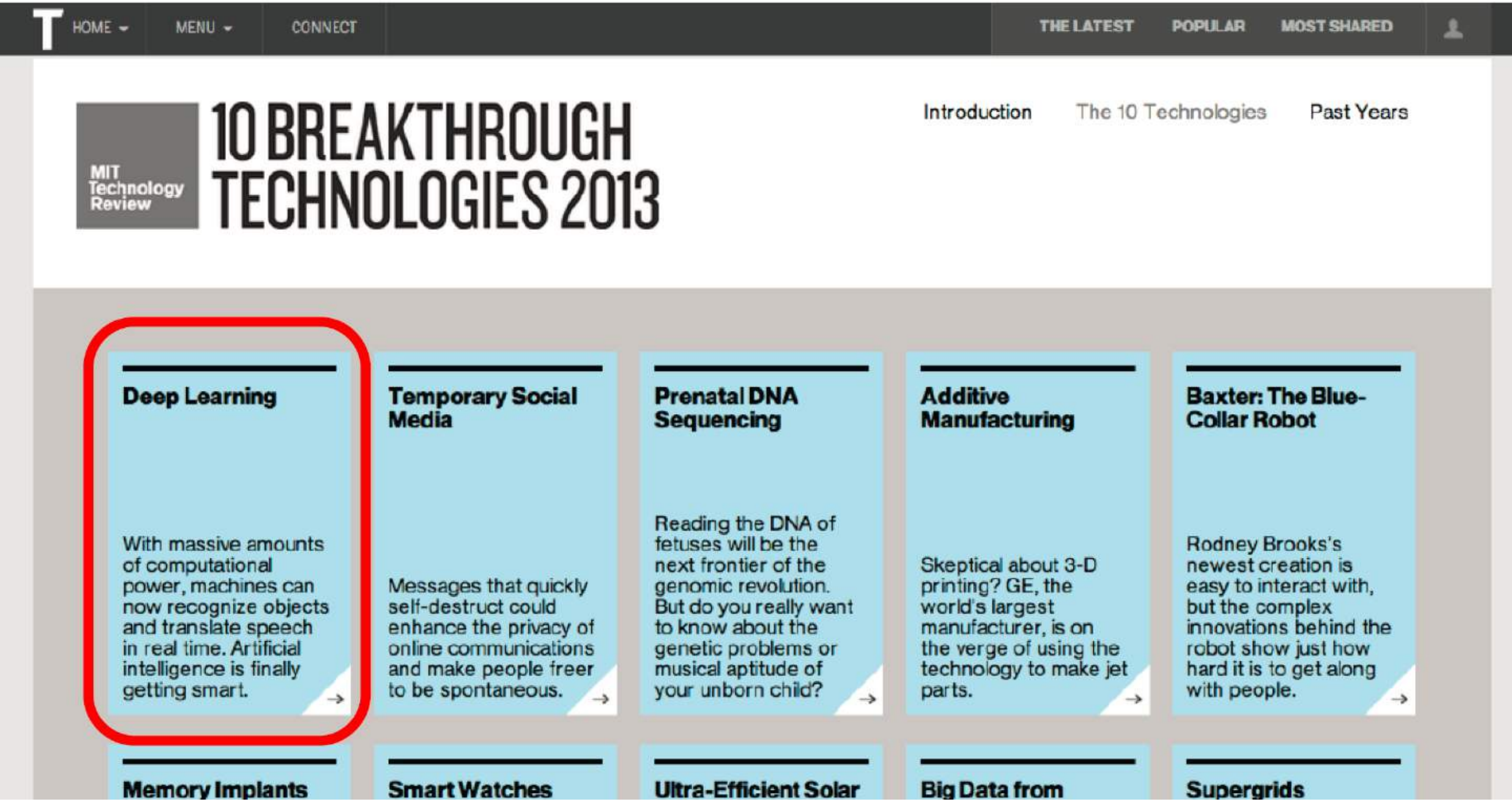
Aside: “CNN” vs “ConvNet”

Note:

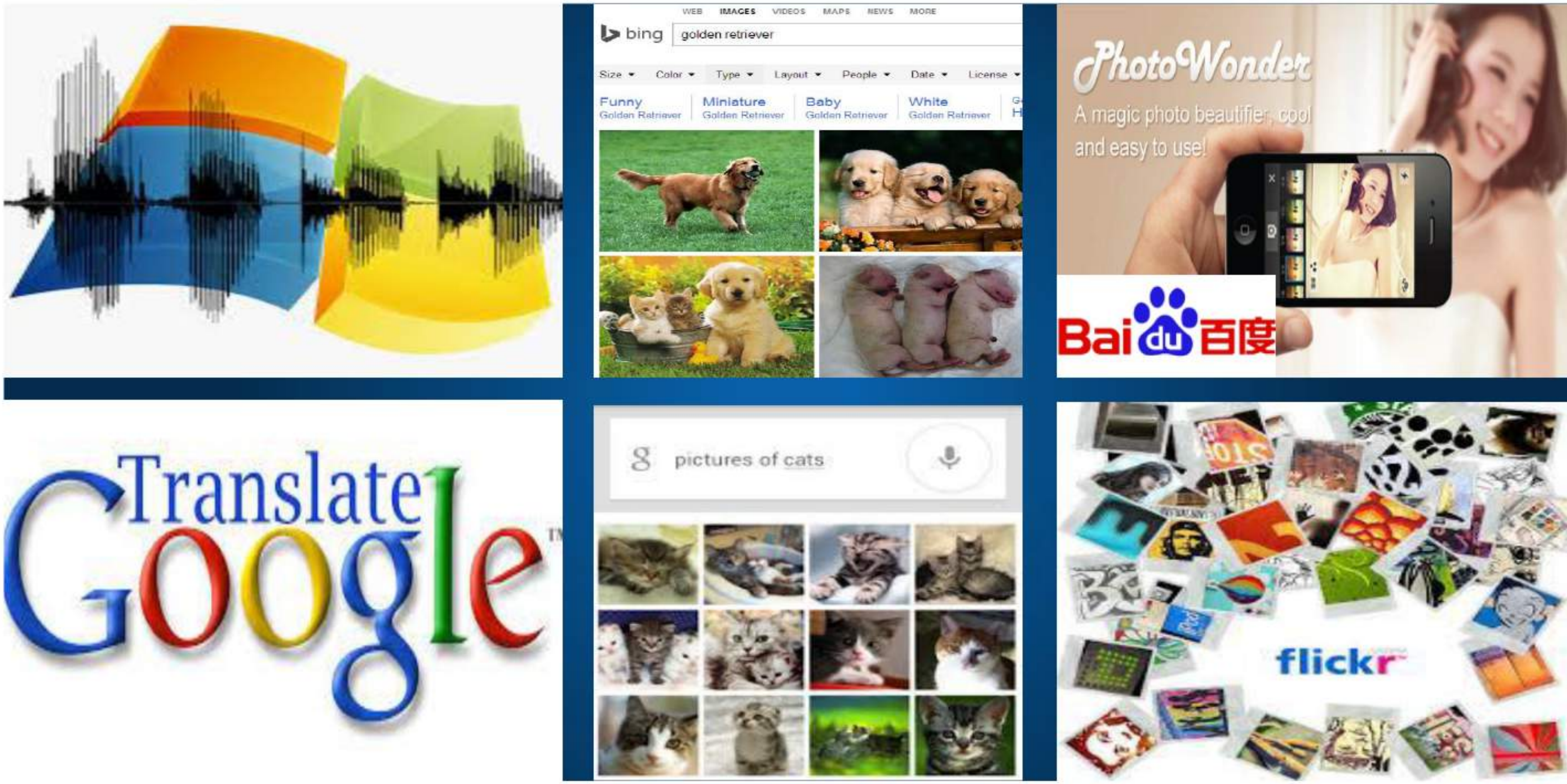
- There are many papers that use either phrase, but
- “ConvNet” is the preferred term, since “CNN” clashes with other things called CNN

Yann LeCun

Motivation

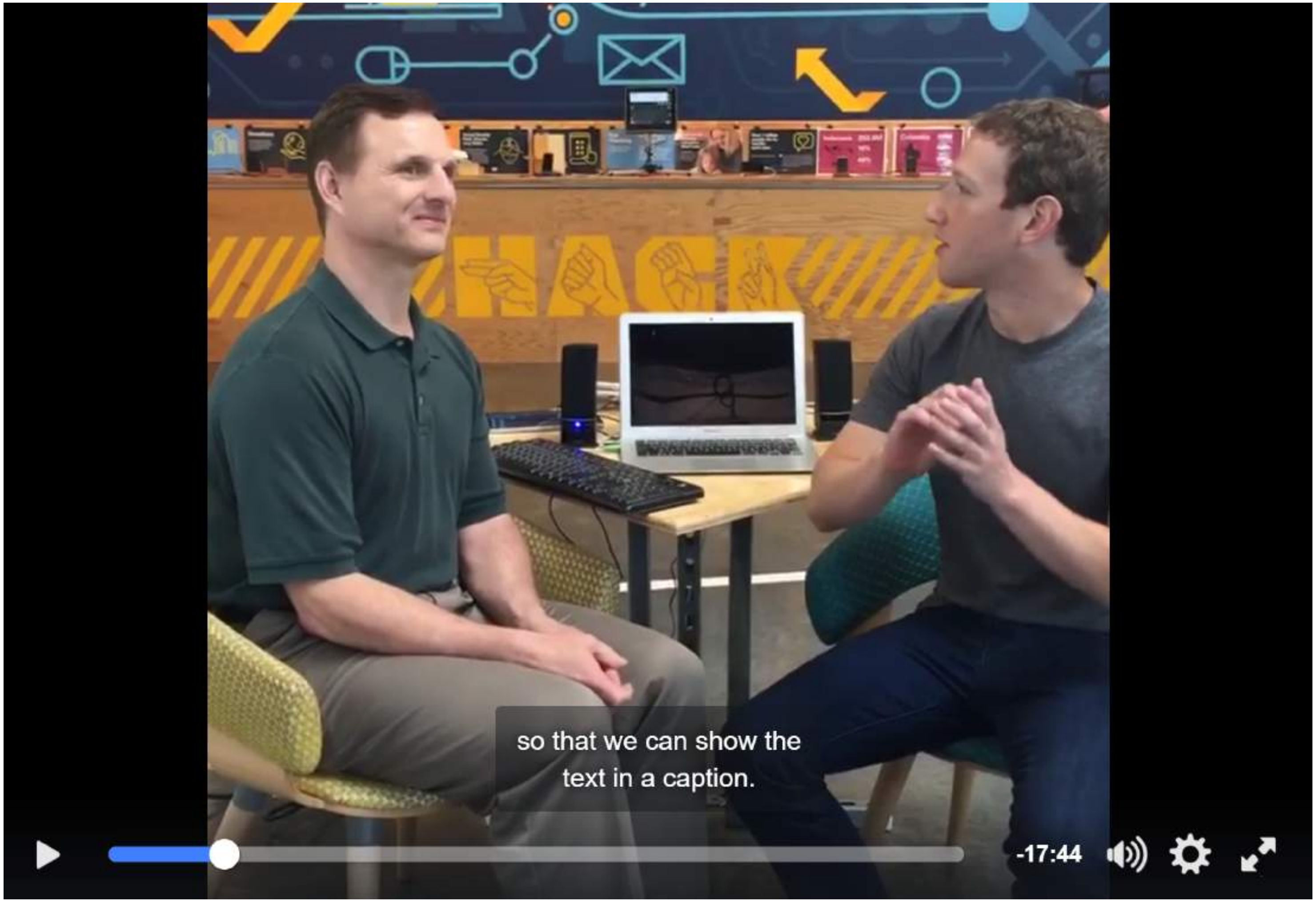

HOME
MENU
CONNECT
THE LATEST
POPULAR
MOST SHARED
MIT Technology Review
10 BREAKTHROUGH TECHNOLOGIES 2013
Introduction
The 10 Technologies
Past Years
Deep Learning
With massive amounts of computational power, machines can now recognize objects and translate speech in real time. Artificial intelligence is finally getting smart.
Temporary Social Media
Messages that quickly self-destruct could enhance the privacy of online communications and make people freer to be spontaneous.
Prenatal DNA Sequencing
Reading the DNA of fetuses will be the next frontier of the genomic revolution. But do you really want to know about the genetic problems or musical aptitude of your unborn child?
Additive Manufacturing
Skeptical about 3-D printing? GE, the world's largest manufacturer, is on the verge of using the technology to make jet parts.
Baxter: The Blue-Collar Robot
Rodney Brooks's newest creation is easy to interact with, but the complex innovations behind the robot show just how hard it is to get along with people.
Memory Implants
Smart Watches
Ultra-Efficient Solar
Big Data from
Supergrids

Products



WEB IMAGES VIDEOS MAPS NEWS MORE
bing
golden retriever
Size Color Type Layout People Date License
Funny Golden Retriever
Miniature Golden Retriever
Baby Golden Retriever
White Golden Retriever
PhotoWonder
A magic photo beautifier, cool and easy to use!
Baidu 百度
Google Translate
pictures of cats
flickr

Helping the Blind

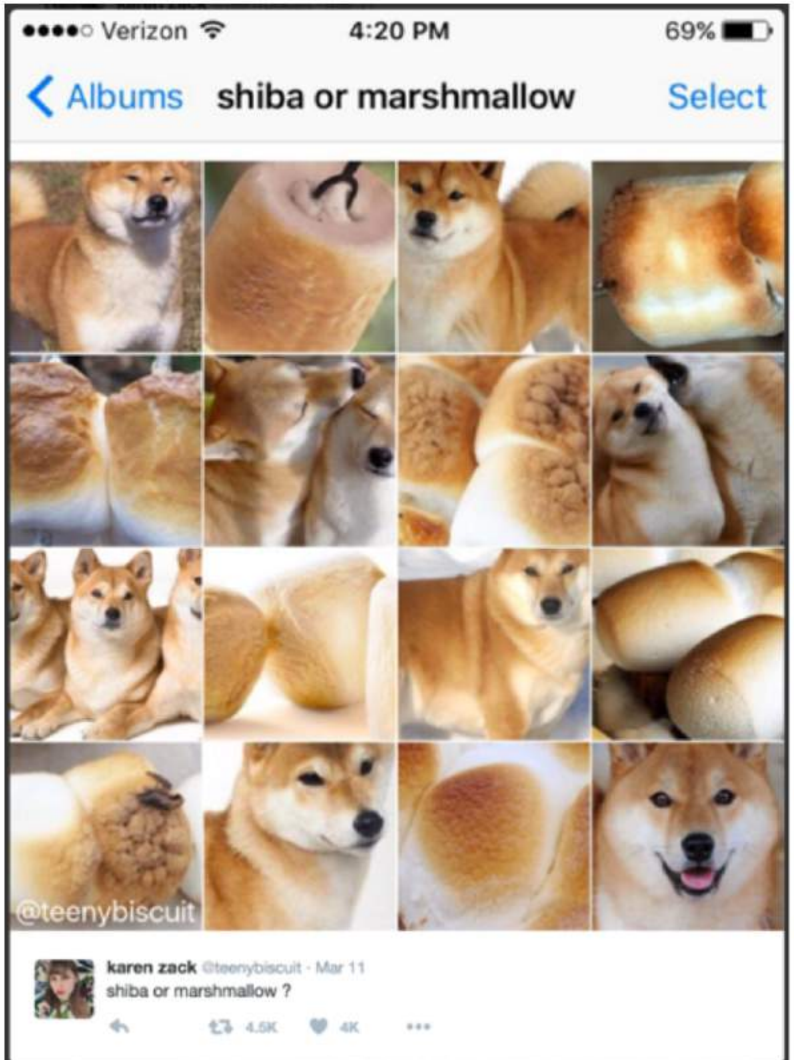


https://www.facebook.com/zuck/videos/10102801434799001/

(Unrelated) Dog vs Food

[Karen Zack, @teenybiscuit]

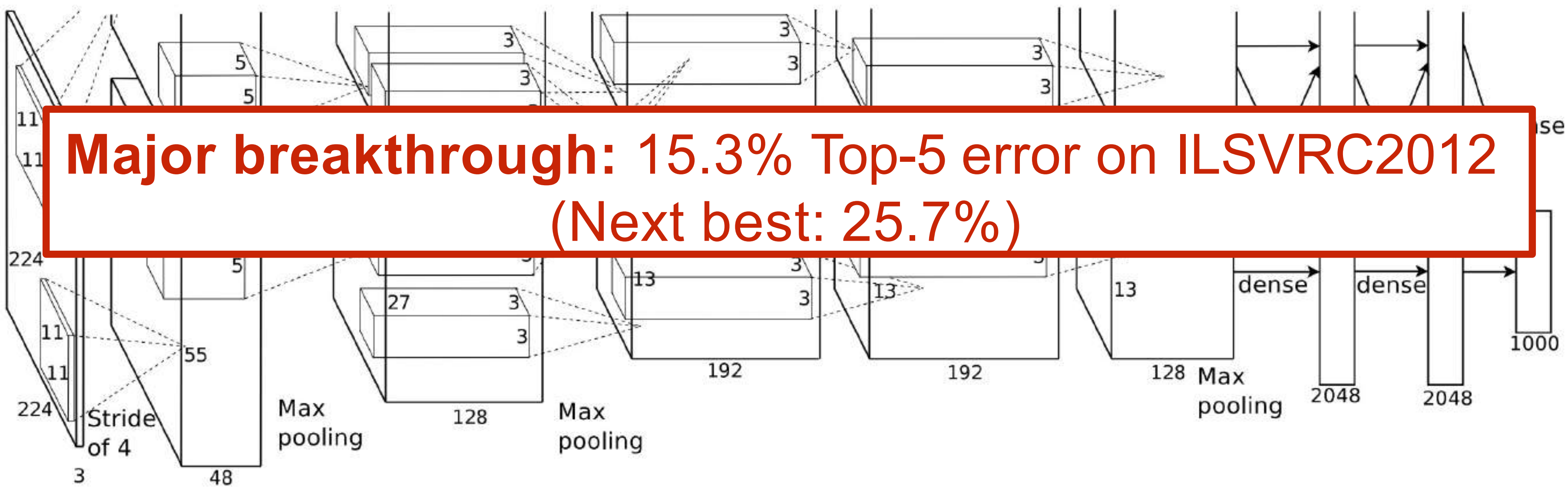
(Unrelated) Dog vs Food



[Karen Zack, @teenybiscuit]

CNNs in 2012: “SuperVision” (aka “AlexNet”)

“AlexNet” — Won the ILSVRC2012 Challenge



Major breakthrough: 15.3% Top-5 error on ILSVRC2012 (Next best: 25.7%)

Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

[Krizhevsky, Sutskever, Hinton. NIPS 2012]

Recap: Before Deep Learning

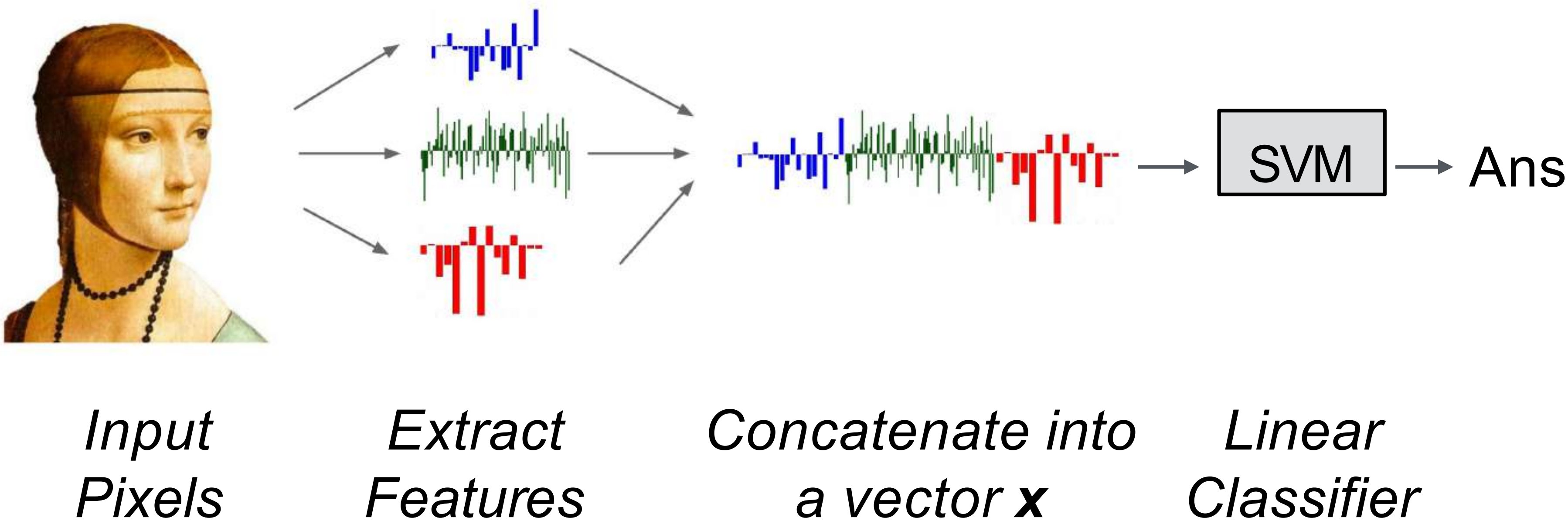
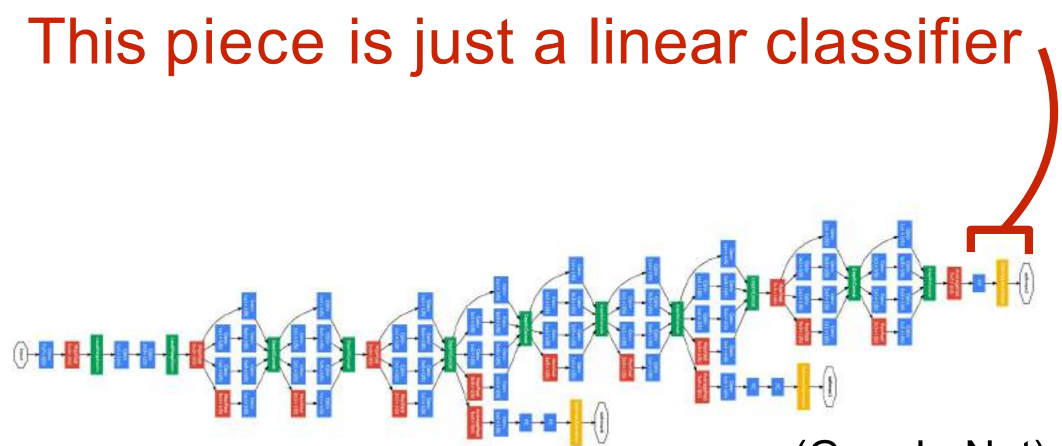


Figure: Karpathy 2016

The last layer of (most) CNNs are linear classifiers



Ans

Input Pixels

Perform everything with a big neural network, trained end-to-end

Key: perform enough processing so that by the time you get to the end of the network, the classes are linearly separable

ConvNets

They're just neural networks with
3D activations and weight sharing

What shape should the activations have?

$$x \rightarrow \boxed{\text{Layer}} \rightarrow h^{(1)} \rightarrow \boxed{\text{Layer}} \rightarrow h^{(2)} \rightarrow \cdots \rightarrow f$$

- The input is an image, which is 3D (RGB channel, height, width)

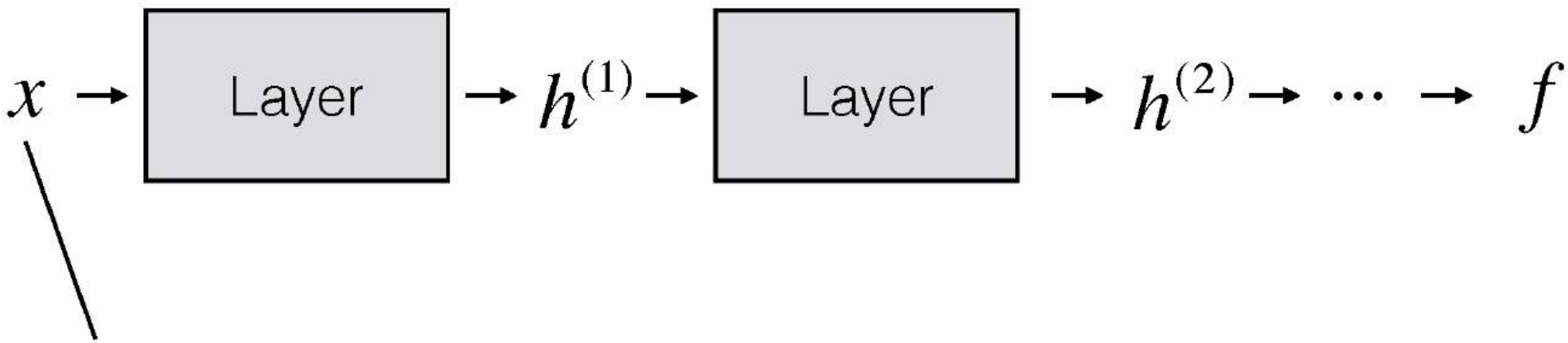
What shape should the activations have?

$$x \to \boxed{\text{Layer}} \to h^{(1)} \to \boxed{\text{Layer}} \to h^{(2)} \to \cdots \to f$$

- The input is an image, which is 3D (RGB channel, height, width)

- We could flatten it to a 1D vector, but then we lose structure

What shape should the activations have?



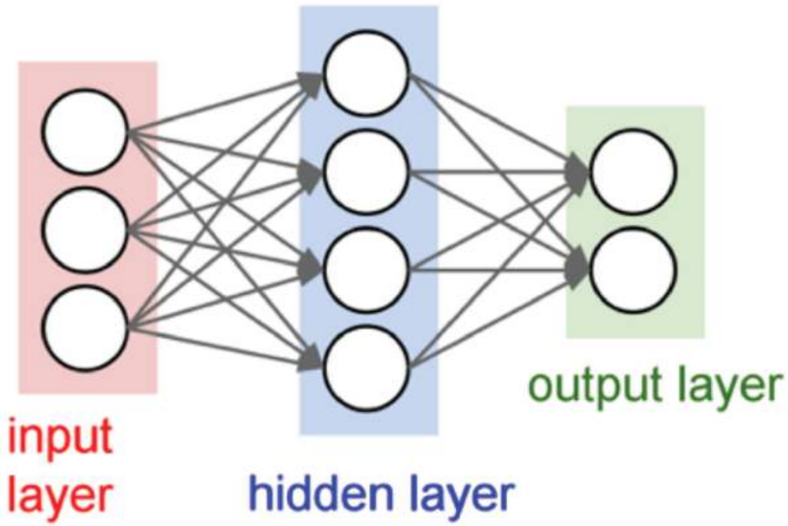
- The input is an image, which is 3D (RGB channel, height, width)

- We could flatten it to a 1D vector, but then we lose structure

- What about keeping everything in 3D?

3D Activations

before:

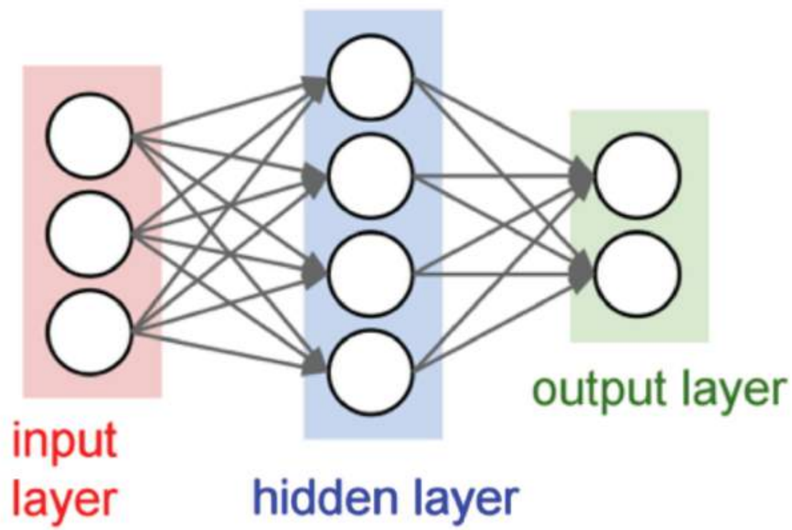


(1D vectors)

Figure: Andrej Karpathy

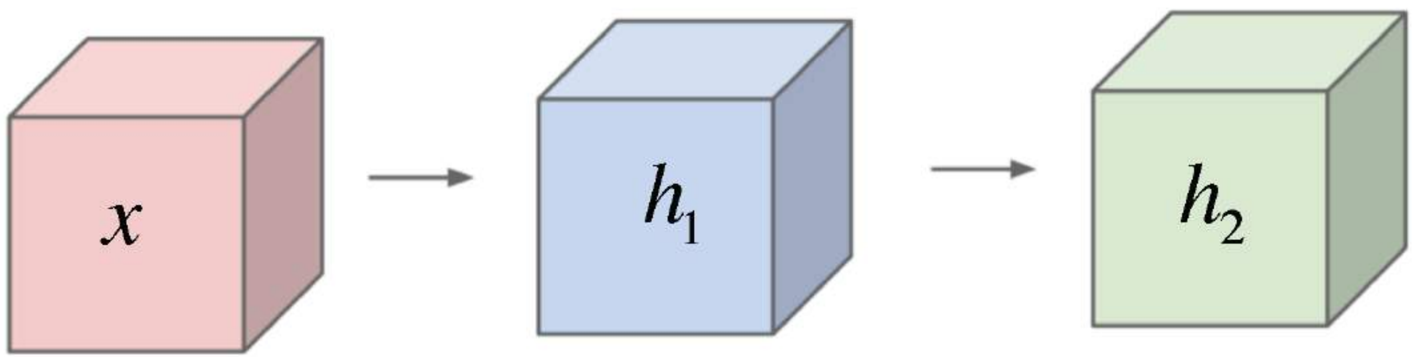
3D Activations

before:



(1D vectors)

now:



(3D arrays)

Figure: Andrej Karpathy

3D Activations

All Neural Net activations arranged in **3 dimensions:**

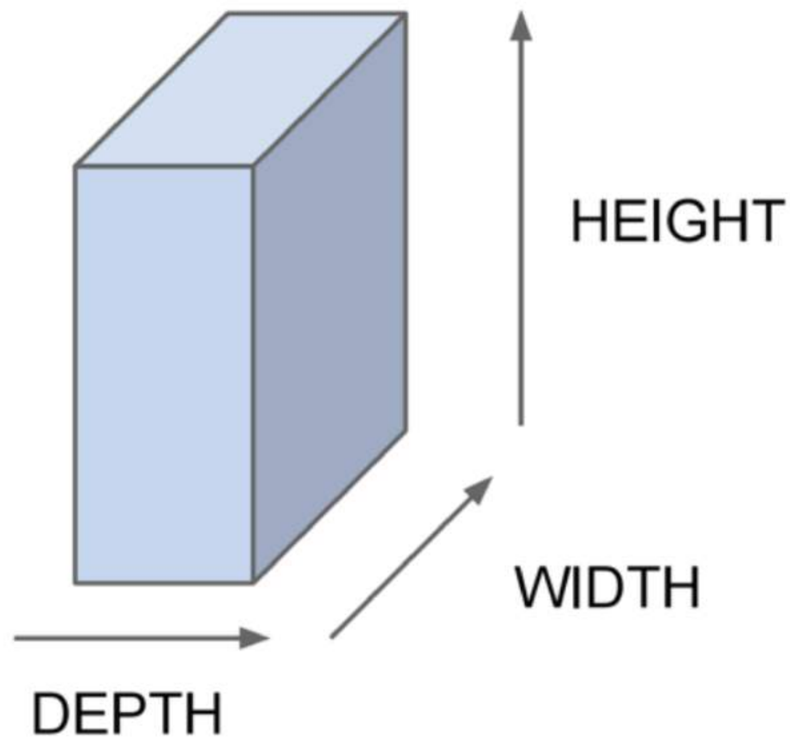
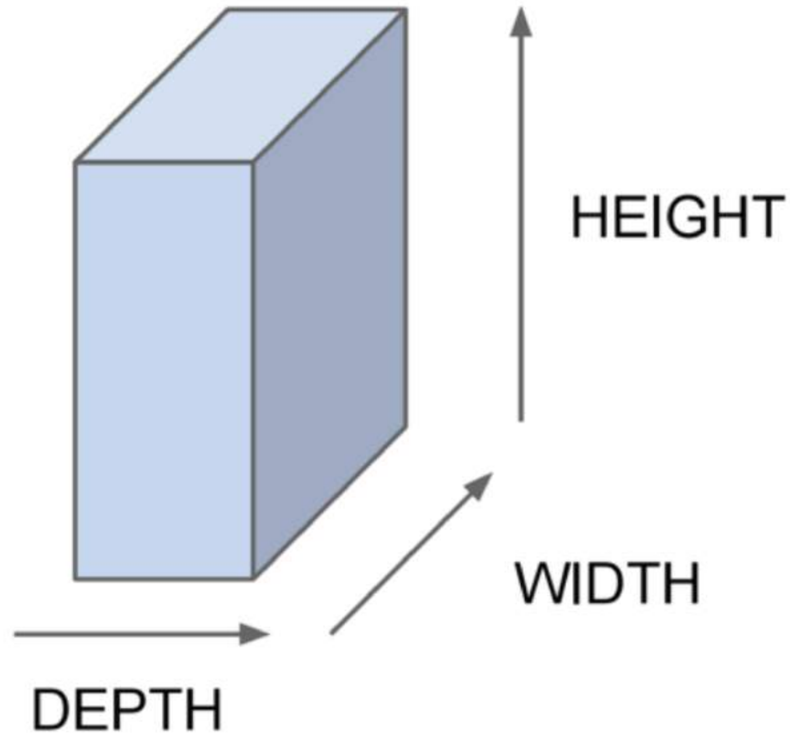


Figure: Andrej Karpathy

3D Activations

All Neural Net activations arranged in **3 dimensions:**



For example, a CIFAR-10 image is a 3x32x32 volume (3 depth — RGB channels, 32 height, 32 width)

Figure: Andrej Karpathy

3D Activations

1D Activations:

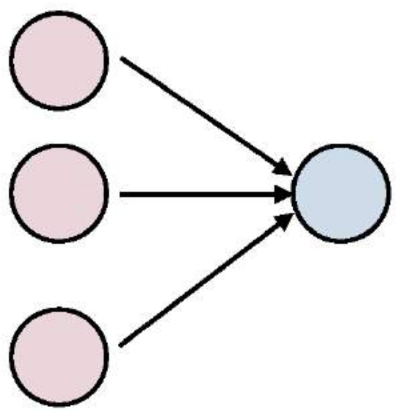
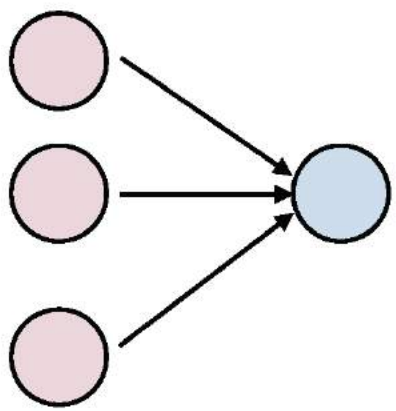


Figure: Andrej Karpathy

3D Activations

1D Activations:



3D Activations:

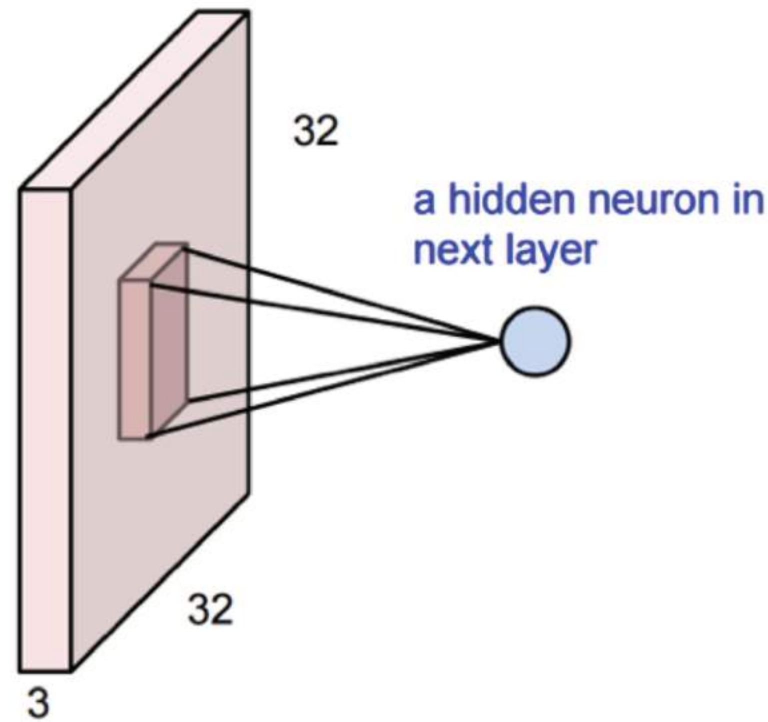
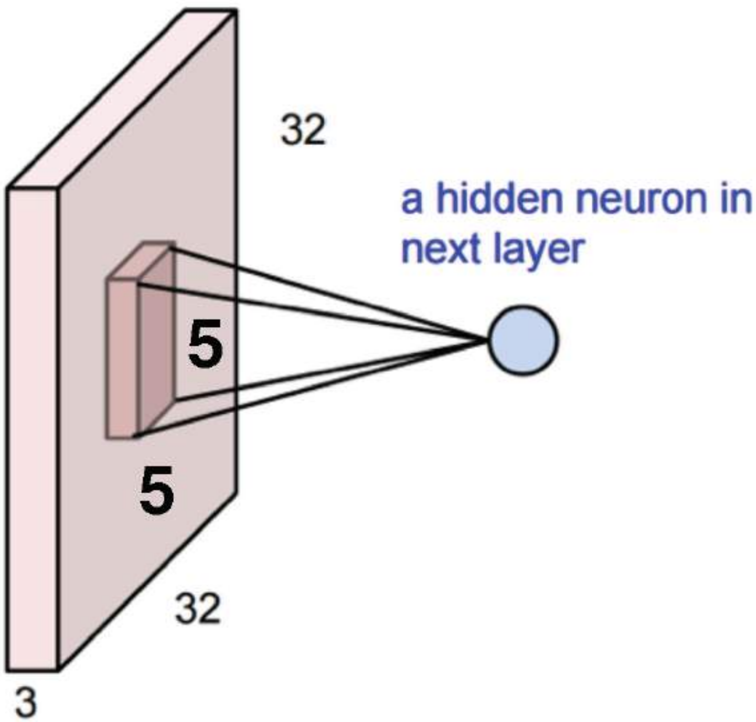


Figure: Andrej Karpathy

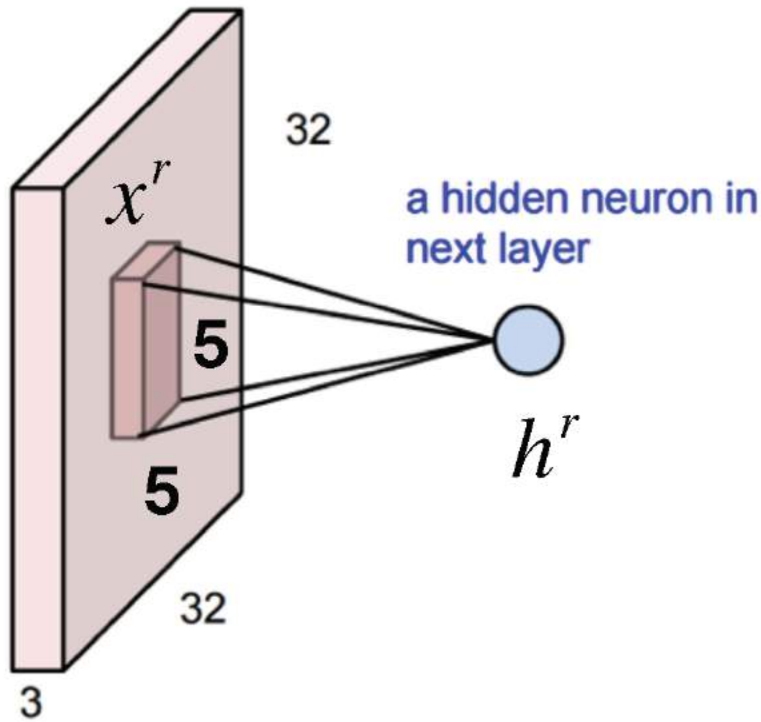
3D Activations



- The input is 3x32x32
- This neuron depends on a 3x5x5 chunk of the input
- The neuron also has a 3x5x5 set of weights and a bias (scalar)

Figure: Andrej Karpathy

3D Activations

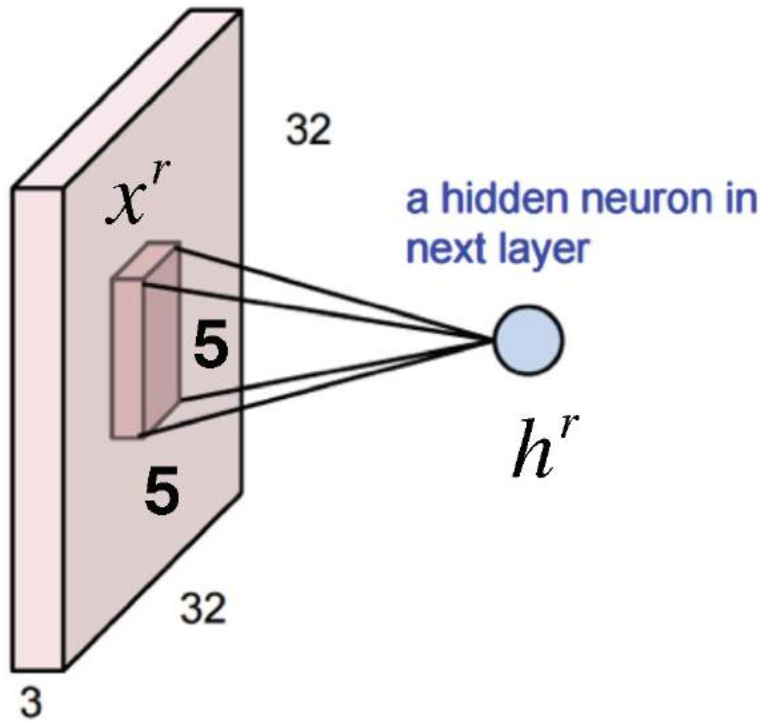


Example: consider the region of the input "x^r"

With output neuron h^r

Figure: Andrej Karpathy

3D Activations



Example: consider the region of the input "x^r"

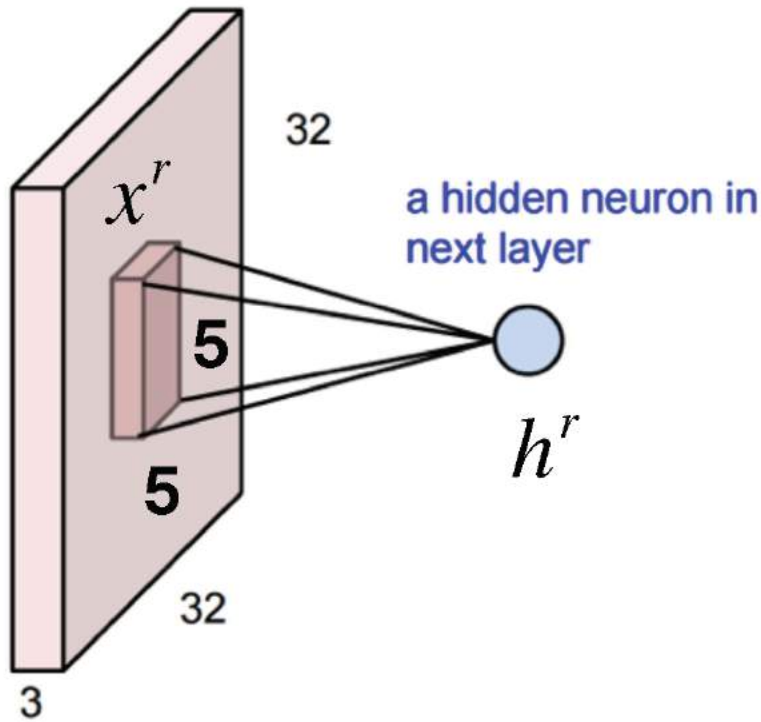
With output neuron h^r

Then the output is:

$$h^r = \sum_{ijk} x^r{}_{ijk} W_{ijk} + b$$

Figure: Andrej Karpathy

3D Activations



Example: consider the region of the input "x^r"

With output neuron h^r

Then the output is:

$$h^r = \sum_{ijk} x^r{}_{ijk} W_{ijk} + b$$

Sum over 3 axes

Figure: Andrej Karpathy

3D Activations

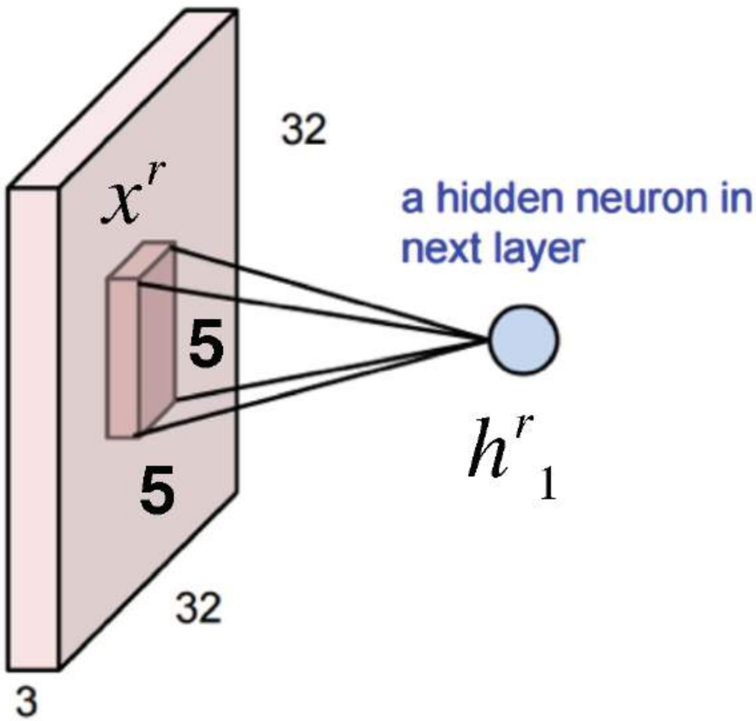


Figure: Andrej Karpathy

3D Activations

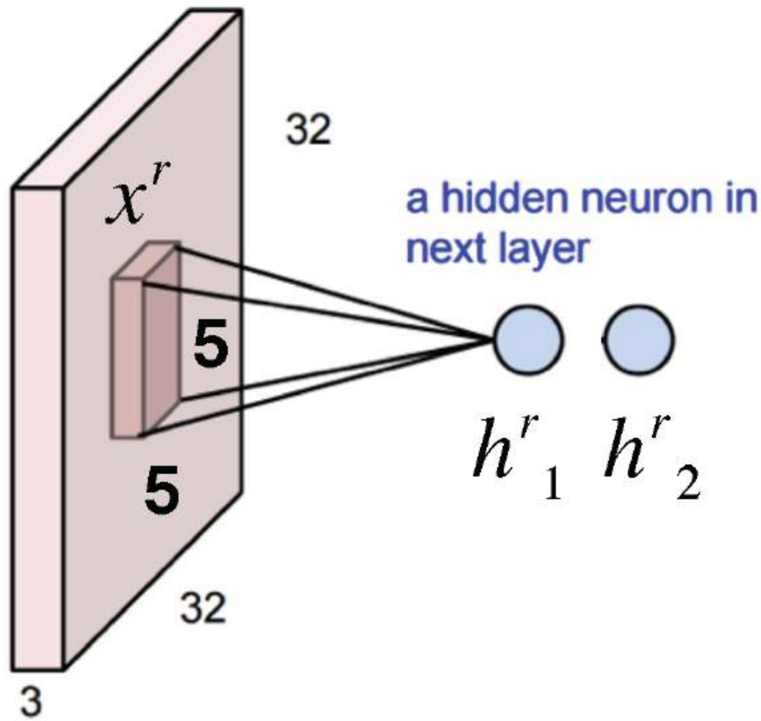
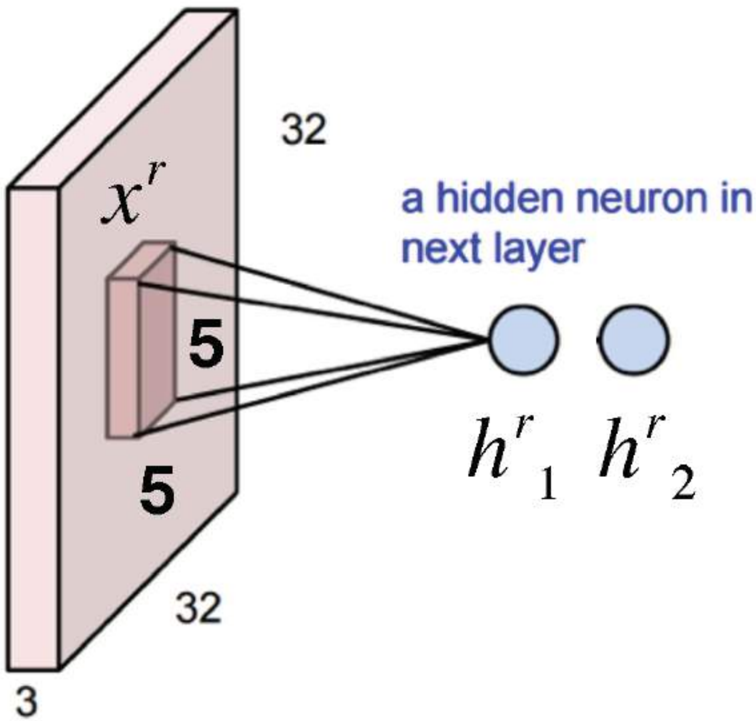


Figure: Andrej Karpathy

3D Activations



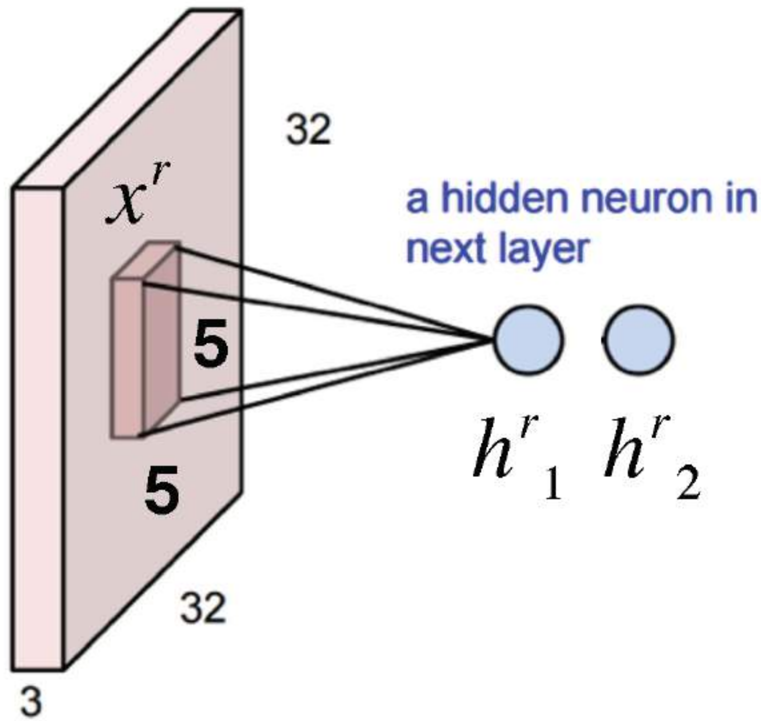
With **2** output neurons

$$h^r{}_1 = \sum_{ijk} x^r{}_{ijk} W_{1ijk} + b_1$$

$$h^r{}_2 = \sum_{ijk} x^r{}_{ijk} W_{2ijk} + b_2$$

Figure: Andrej Karpathy

3D Activations



With **2** output neurons

$$h^r{}_1 = \sum_{ijk} x^r{}_{ijk} W_{\boxed{1}ijk} + b_{\boxed{1}}$$

$$h^r{}_2 = \sum_{ijk} x^r{}_{ijk} W_{\boxed{2}ijk} + b_{\boxed{2}}$$

Figure: Andrej Karpathy

3D Activations

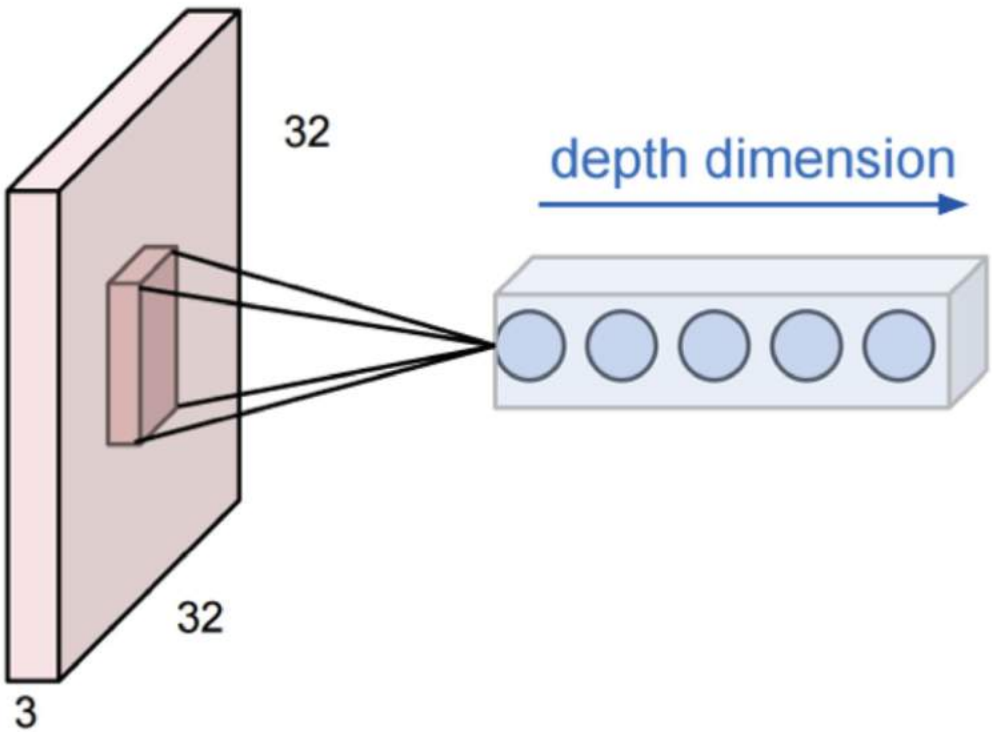
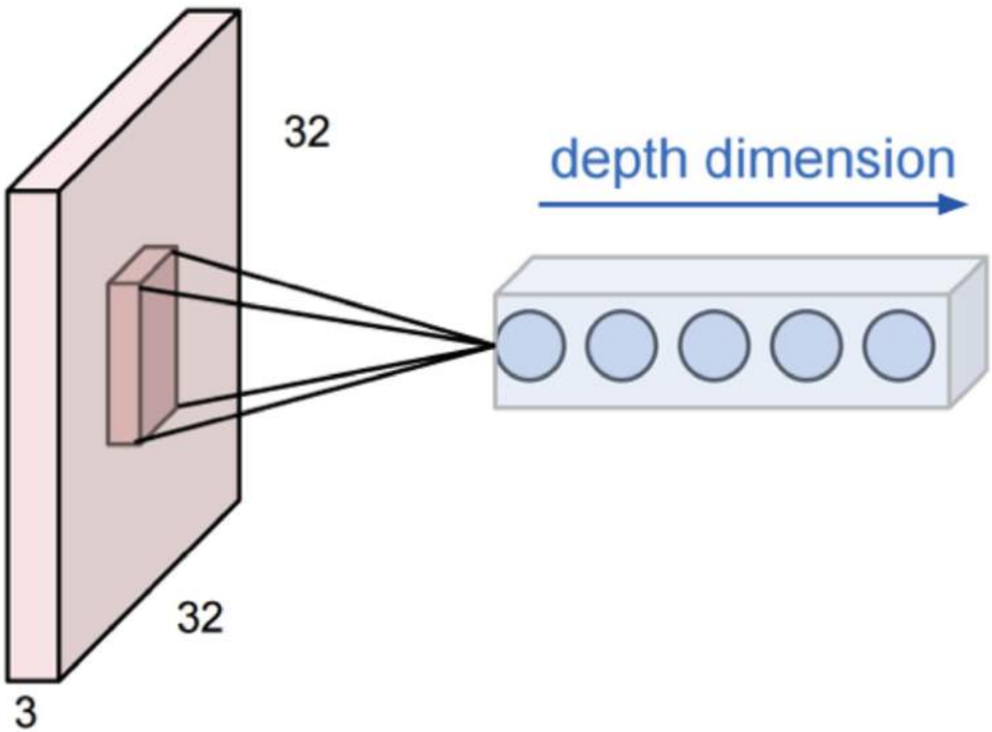


Figure: Andrej Karpathy

3D Activations

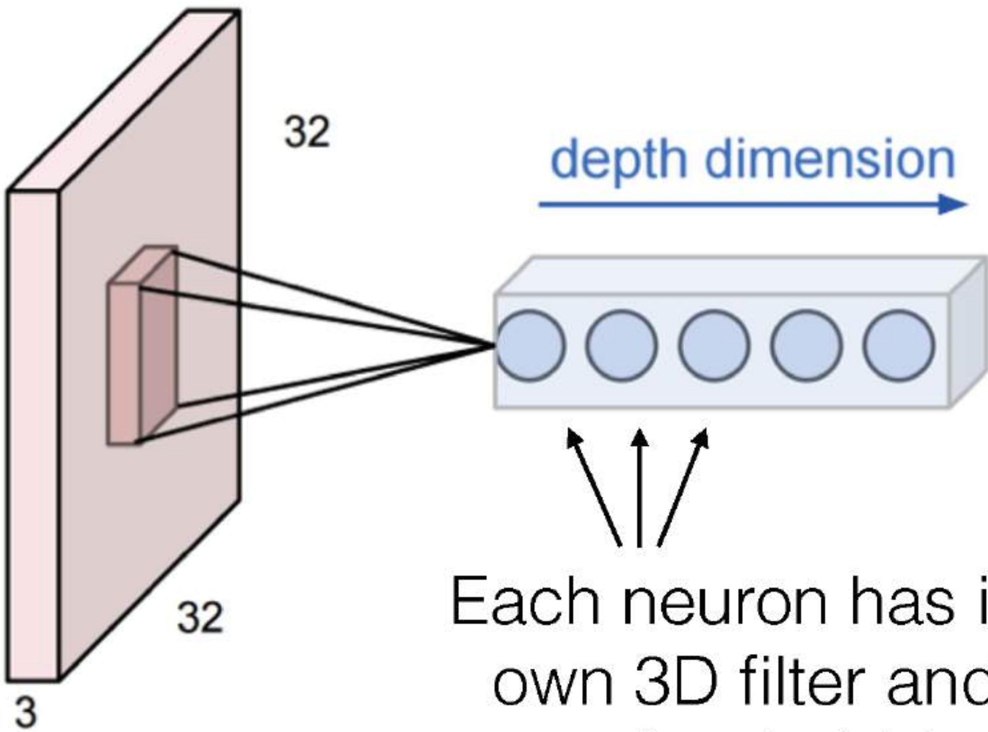


We can keep adding more outputs

These form a column in the output volume: [depth x 1 x 1]

Figure: Andrej Karpathy

3D Activations



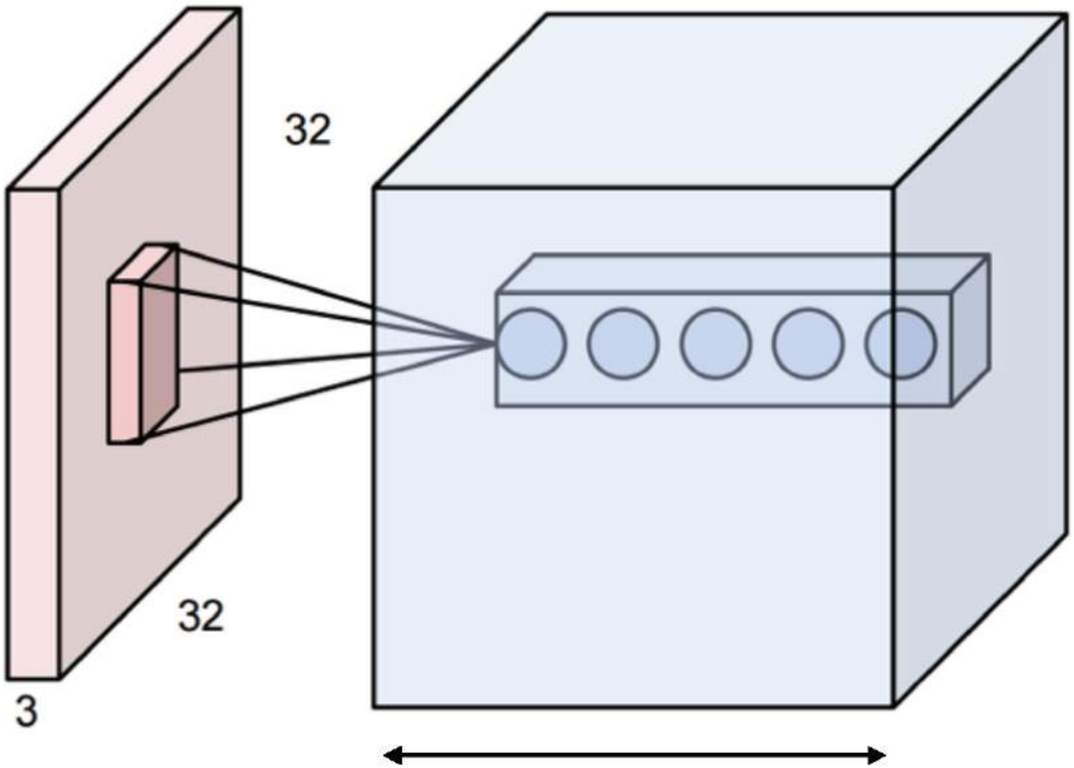
Each neuron has its own 3D filter and own (scalar) bias

We can keep adding more outputs

These form a column in the output volume: [depth x 1 x 1]

Figure: Andrej Karpathy

3D Activations

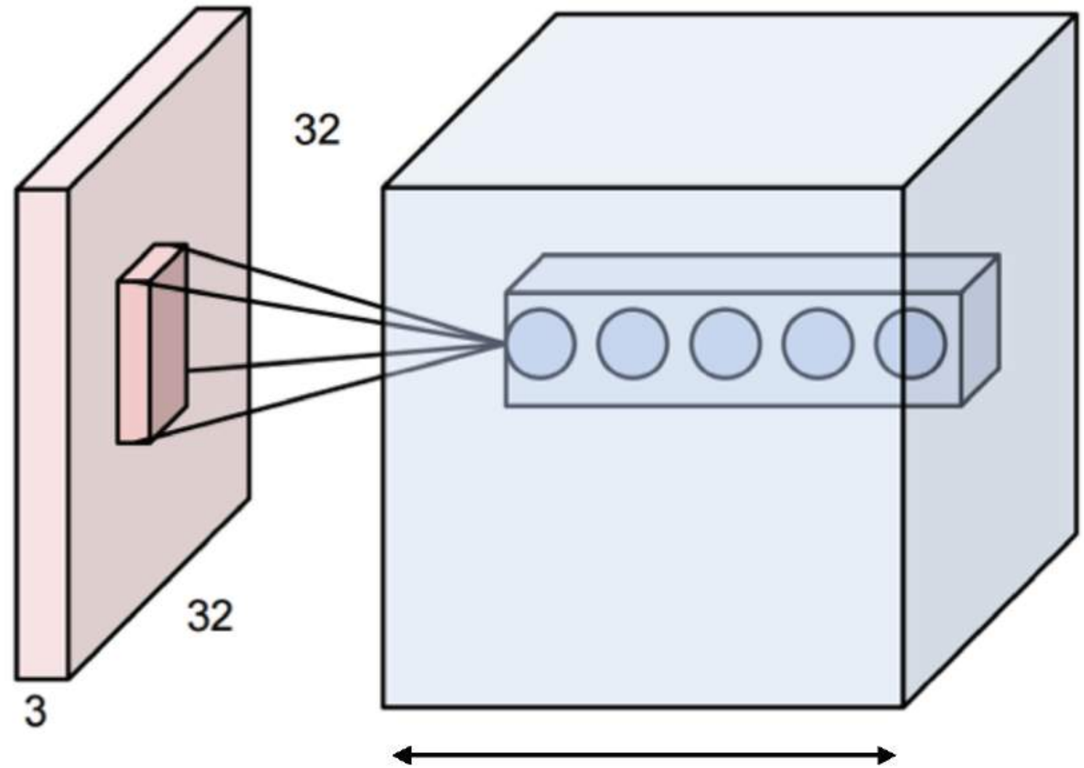


D sets of weights
(also called filters)

Now repeat this across the input

Figure: Andrej Karpathy

3D Activations



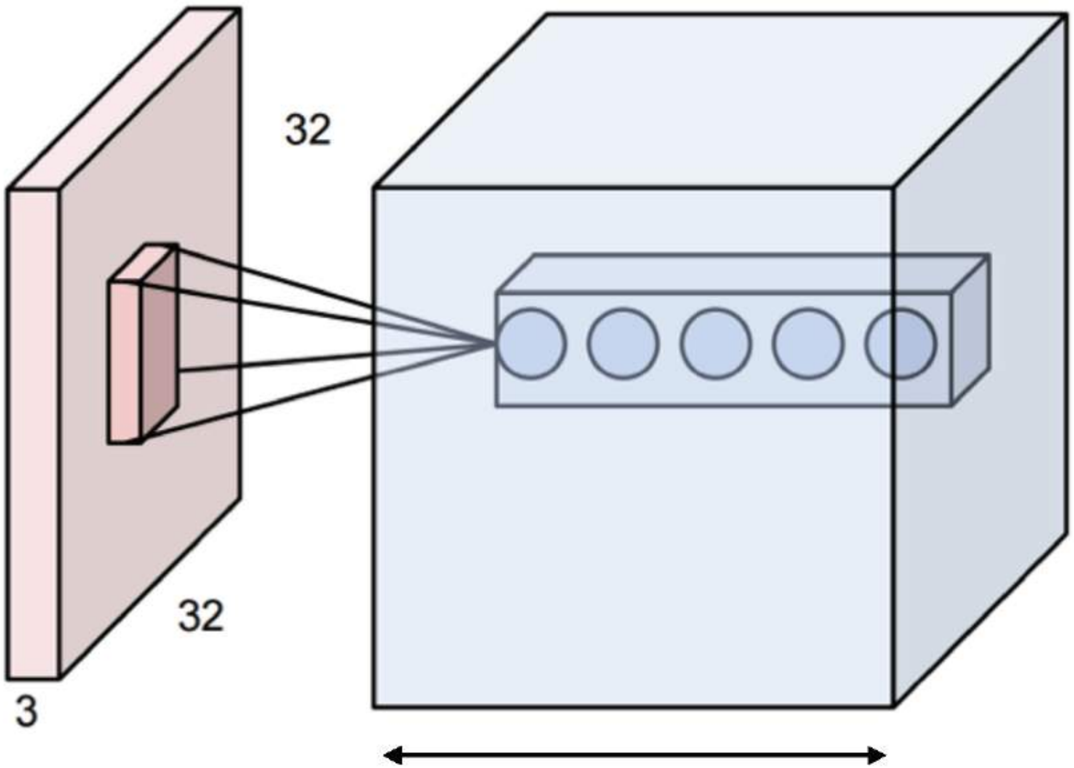
D sets of weights
(also called filters)

Now repeat this across the input

Weight sharing:
Each filter shares the same weights (but each depth index has its own set of weights)

Figure: Andrej Karpathy

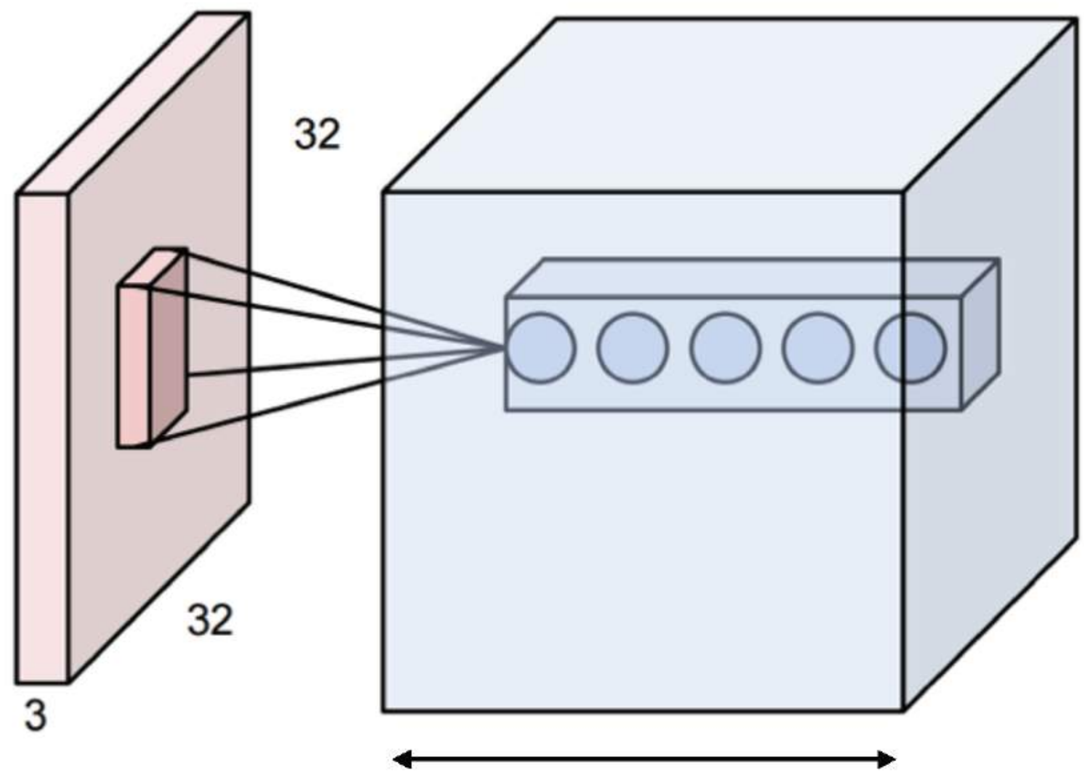
3D Activations



D sets of weights
(also called filters)

Figure: Andrej Karpathy

3D Activations

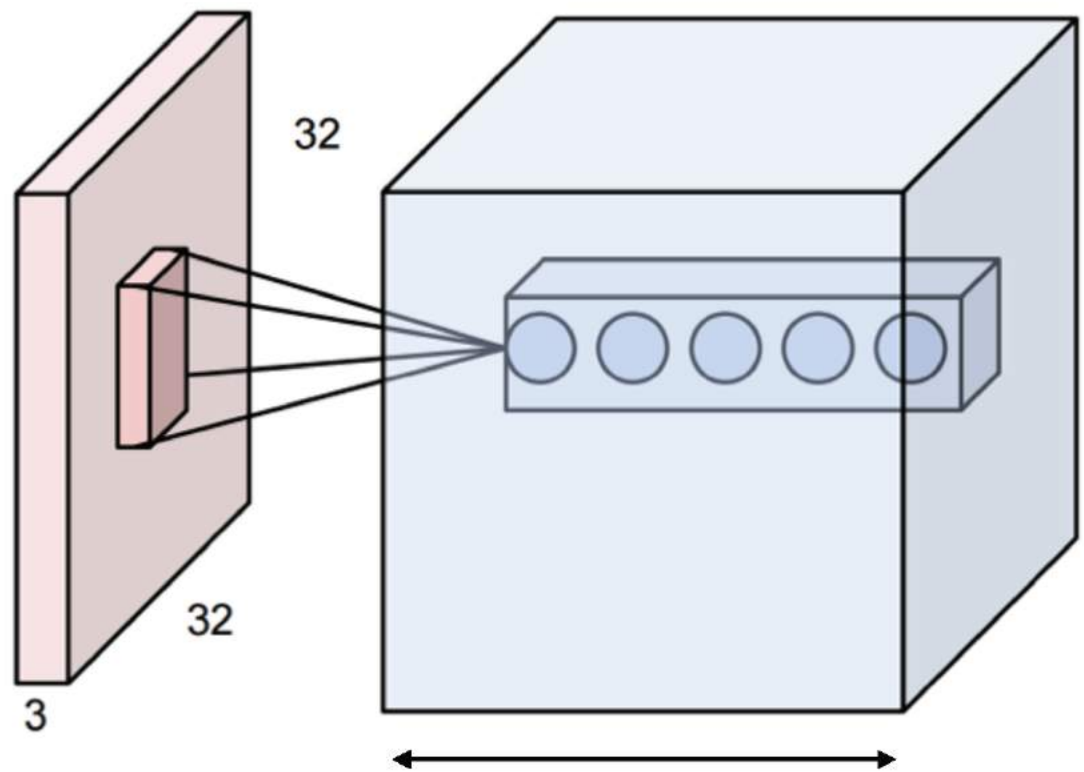


With weight sharing, this is called **convolution**

D sets of weights (also called filters)

Figure: Andrej Karpathy

3D Activations



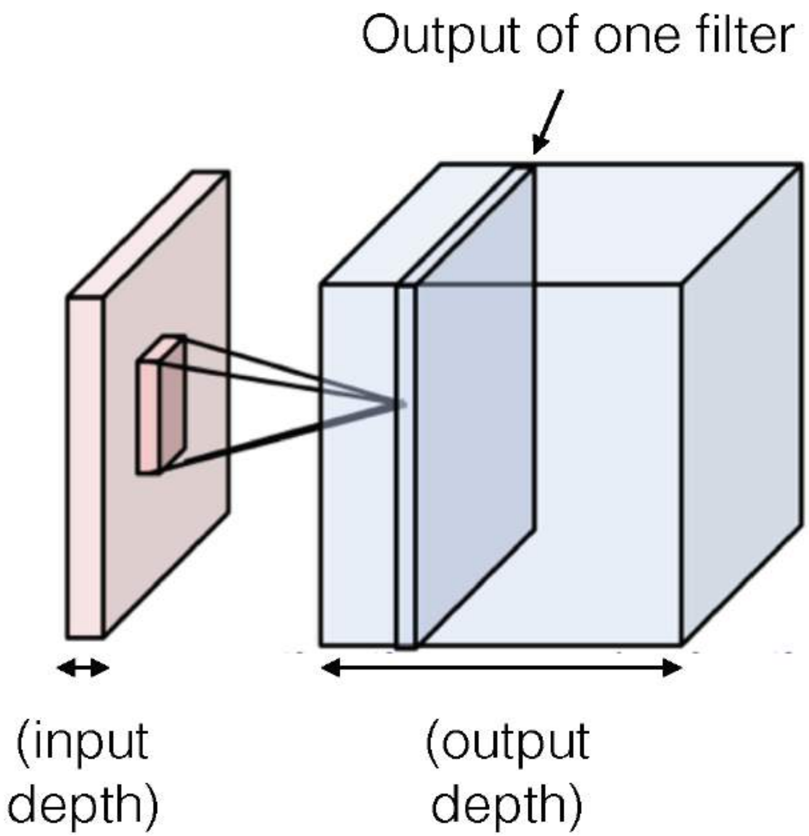
With weight sharing, this is called **convolution**

D sets of weights (also called filters)

Without weight sharing, this is called a **locally connected layer**

Figure: Andrej Karpathy

3D Activations

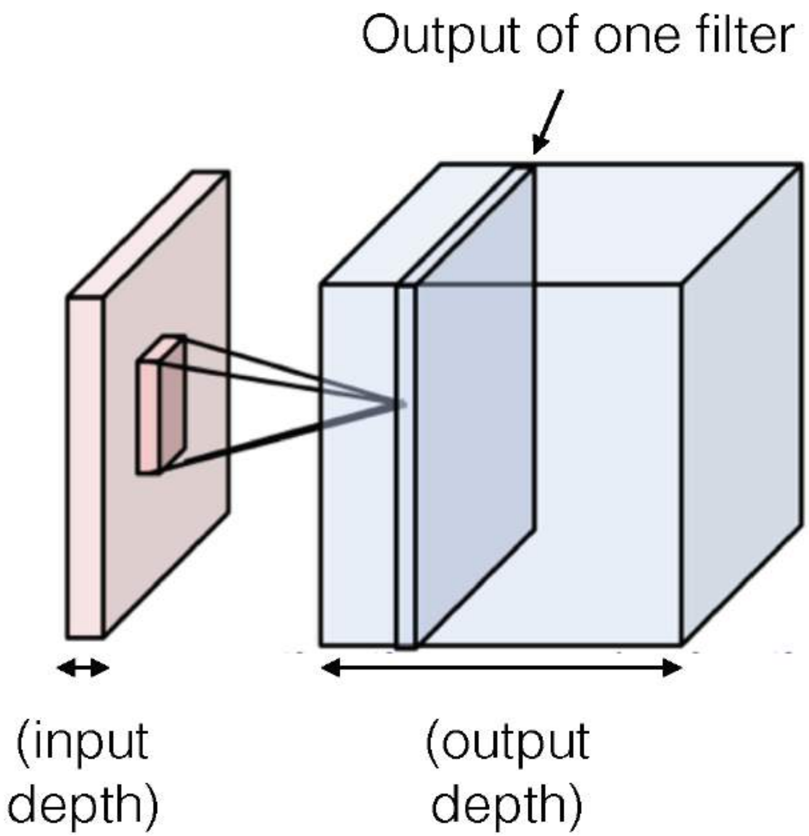


One set of weights gives one slice in the output

To get a 3D output of depth *D*, use *D* different filters

In practice, ConvNets use many filters (~64 to 1024)

3D Activations



One set of weights gives one slice in the output

To get a 3D output of depth *D*, use *D* different filters

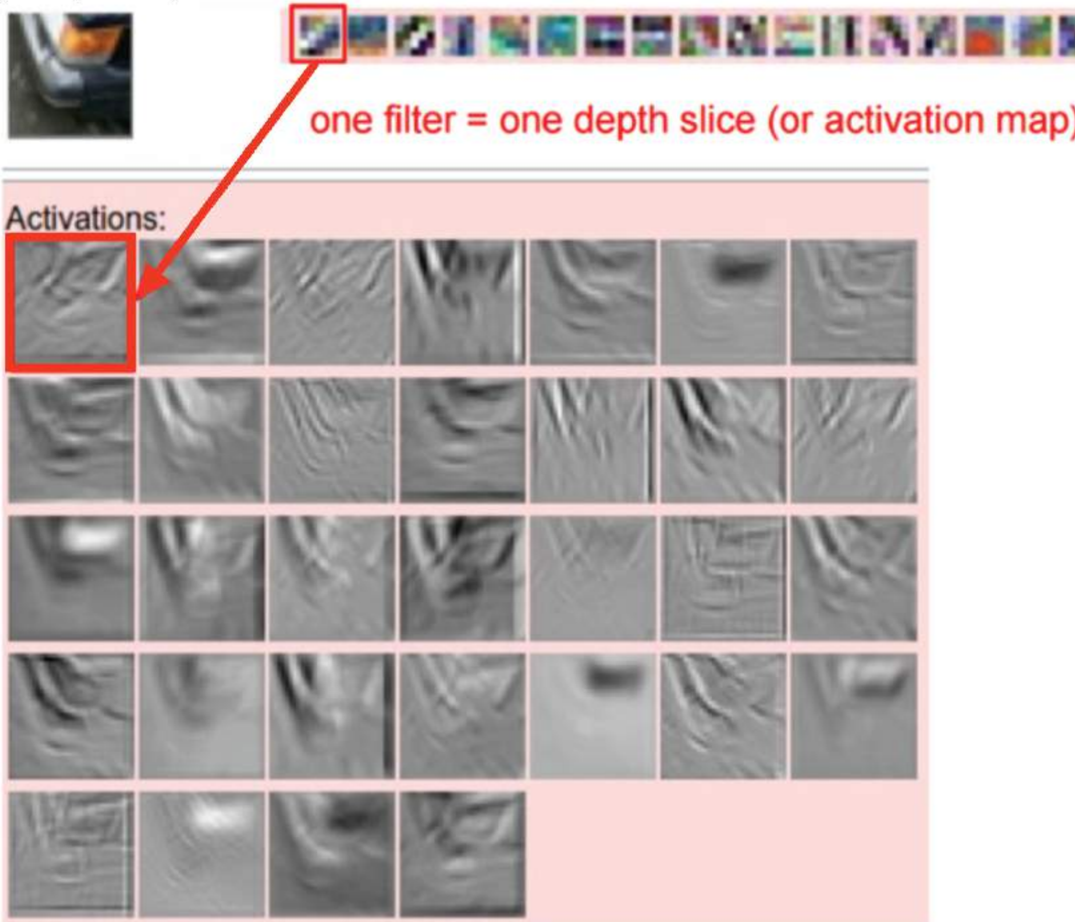
In practice, ConvNets use many filters (~64 to 1024)

All together, the weights are **4** dimensional:
(output depth, input depth, kernel height, kernel width)

3D Activations

We can unravel the 3D cube and show each layer separately:

(Input)



(32 filters, each 3x5x5)

Figure: Andrej Karpathy

3D Activations

We can unravel the 3D cube and show each layer separately:

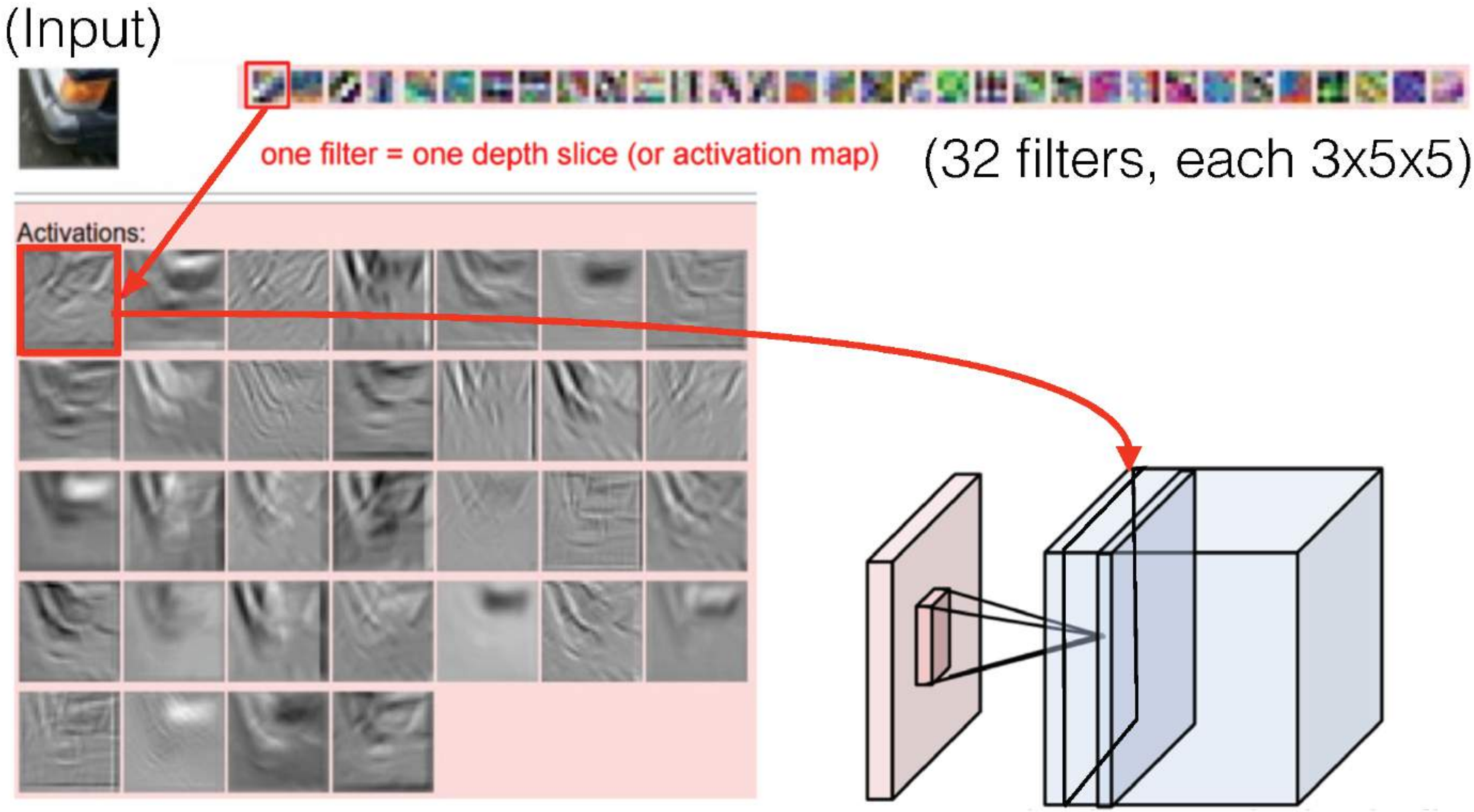


Figure: Andrej Karpathy

3D Activations

We can unravel the 3D cube and show each layer separately:

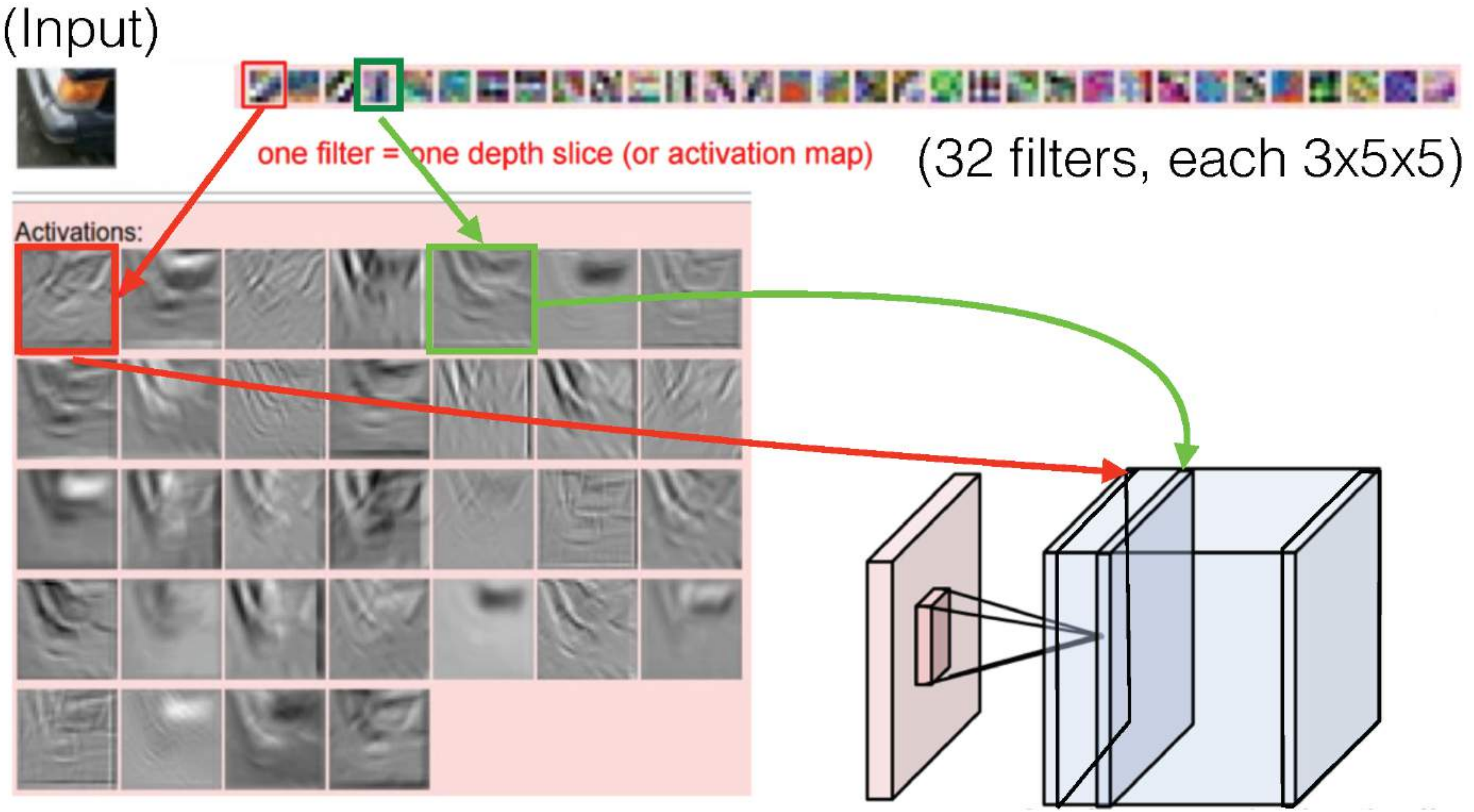


Figure: Andrej Karpathy

3D Activations

We can unravel the 3D cube and show each layer separately:

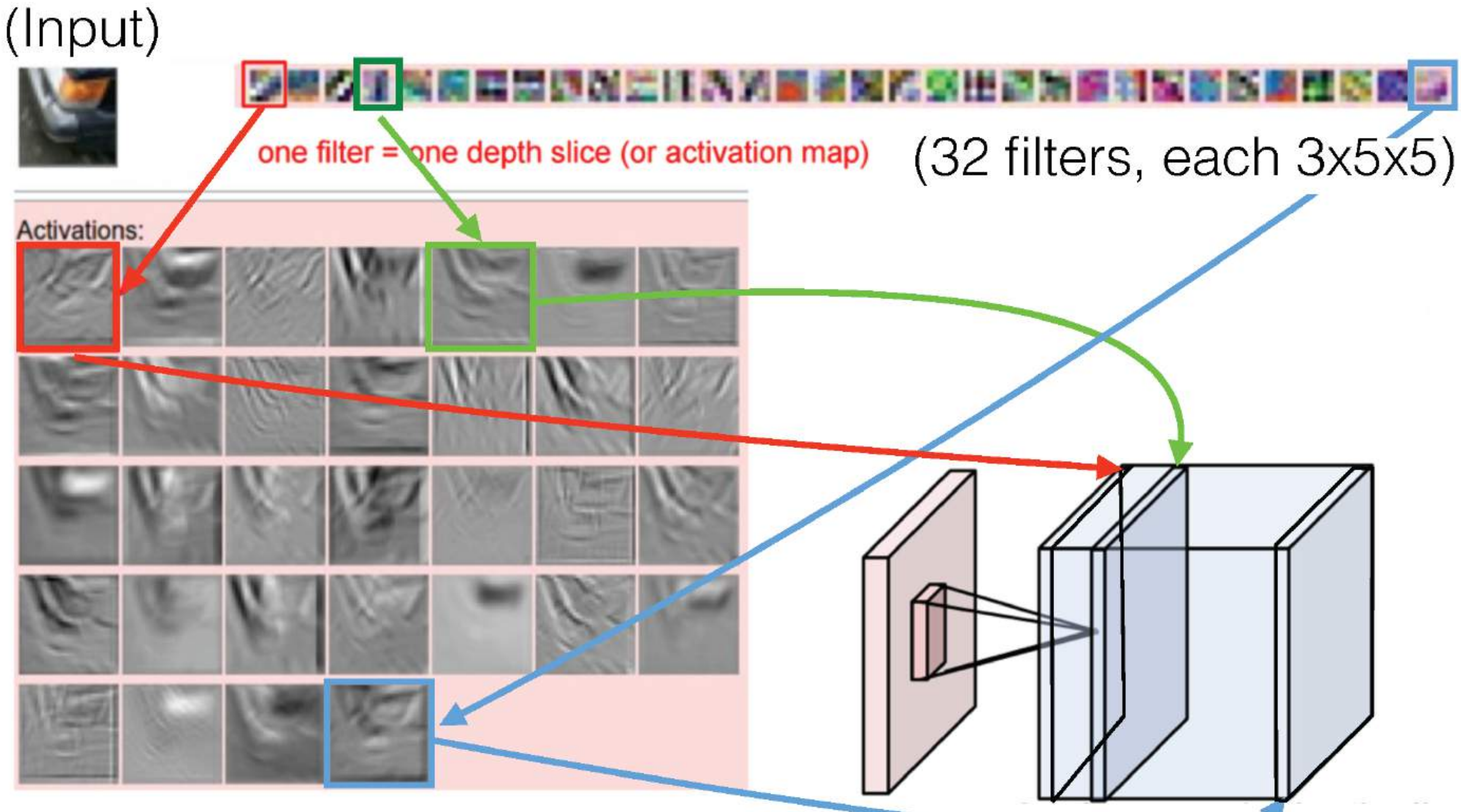
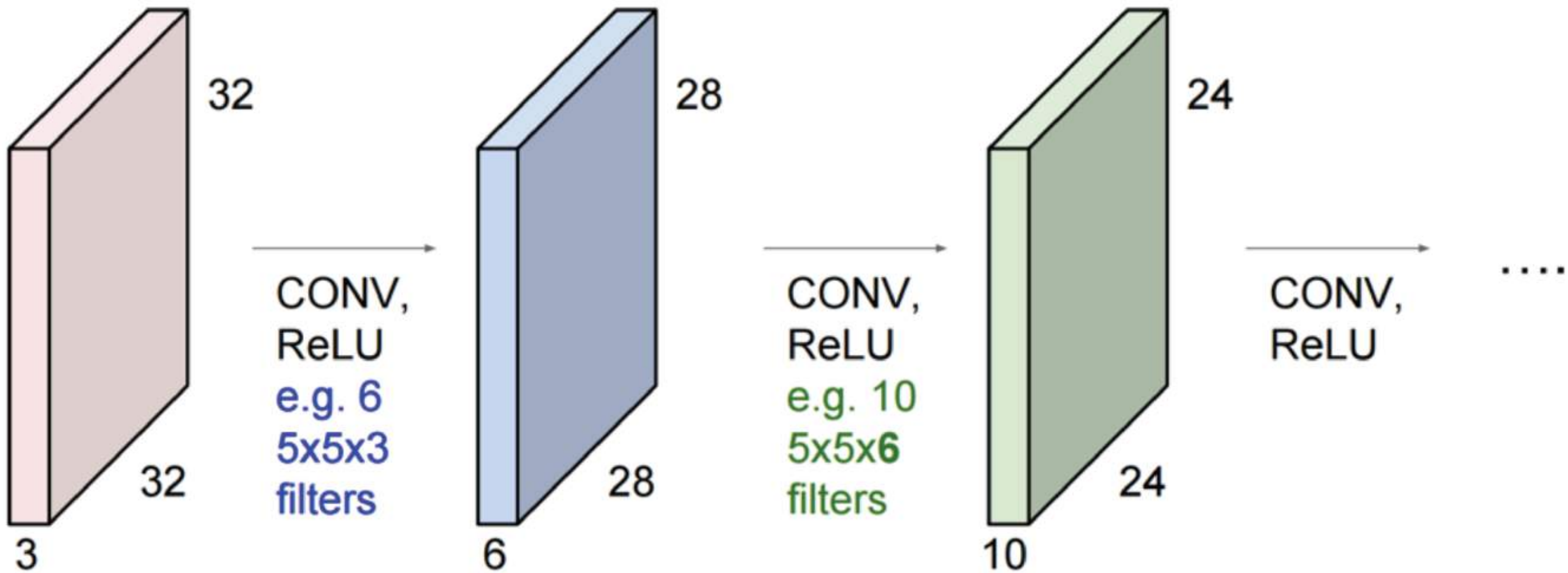


Figure: Andrej Karpathy

(Recap)

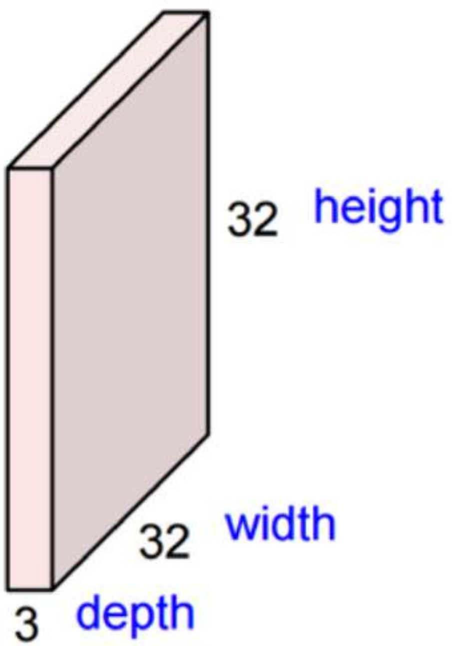
A **ConvNet** is a sequence of convolutional layers, interspersed with activation functions (and possibly other layer types)



(Recap)

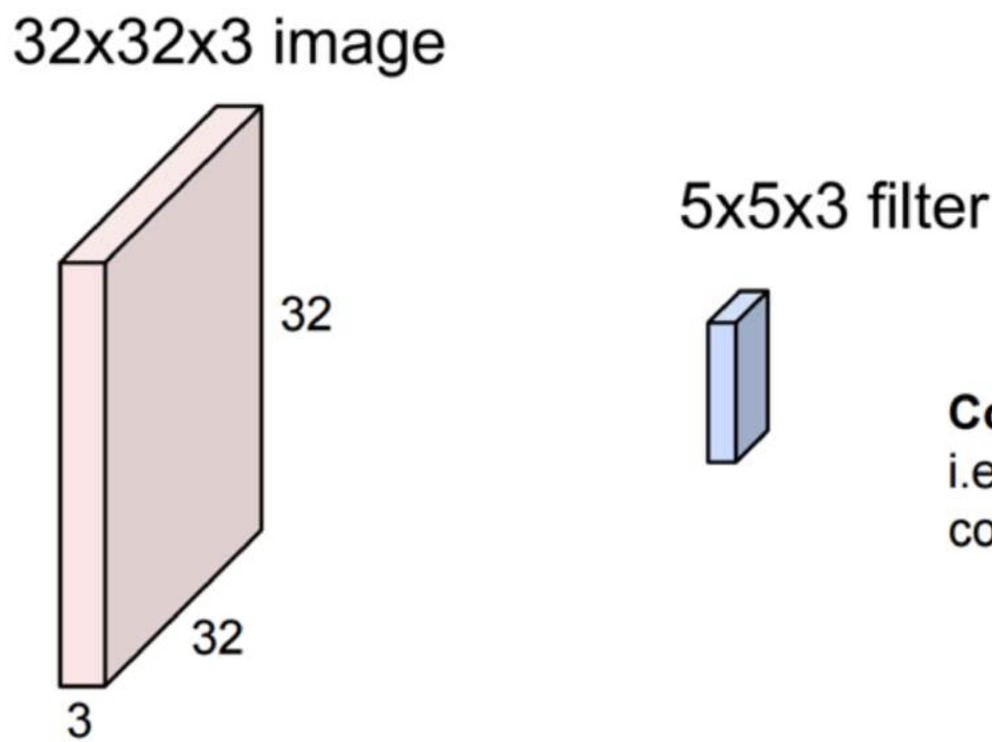
Convolution Layer

32x32x3 image



(Recap)

Convolution Layer



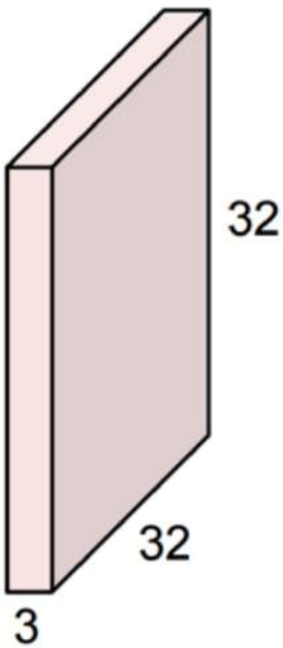
Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

(Recap)

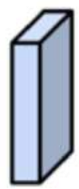
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image



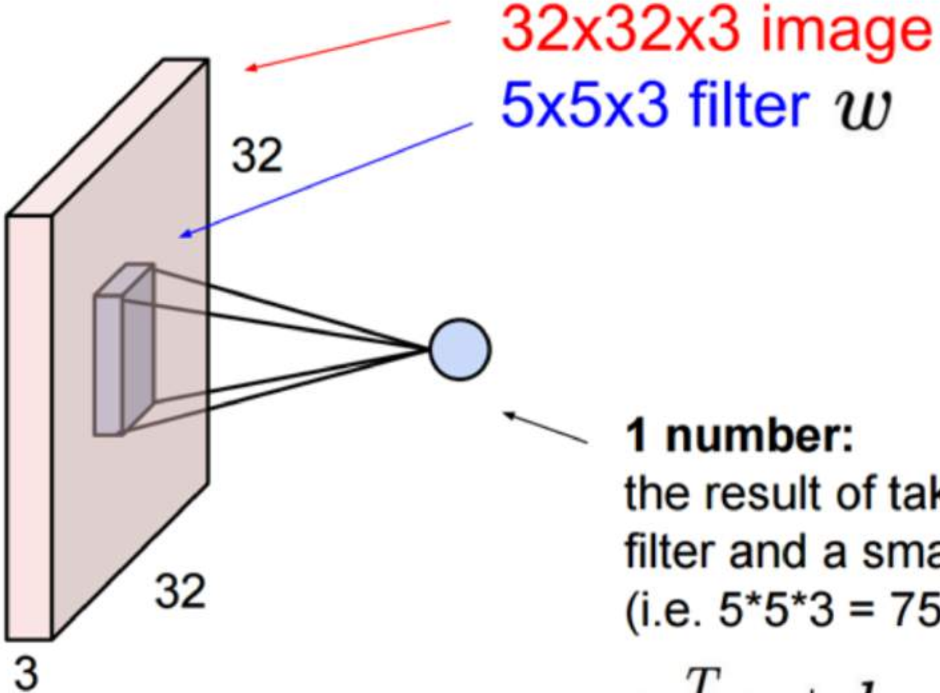
5x5x3 filter



Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

(Recap)

Convolution Layer

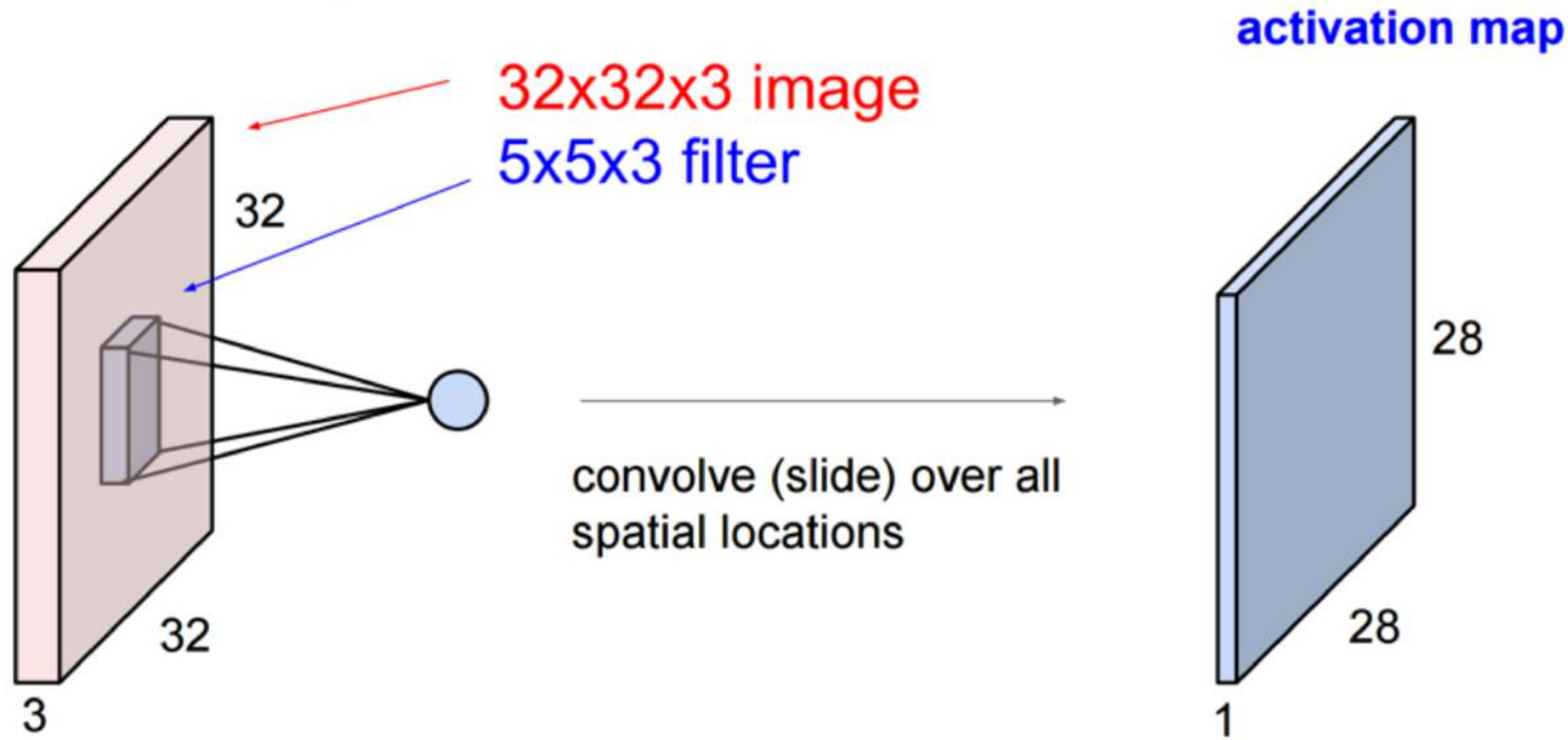


1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

(Recap)

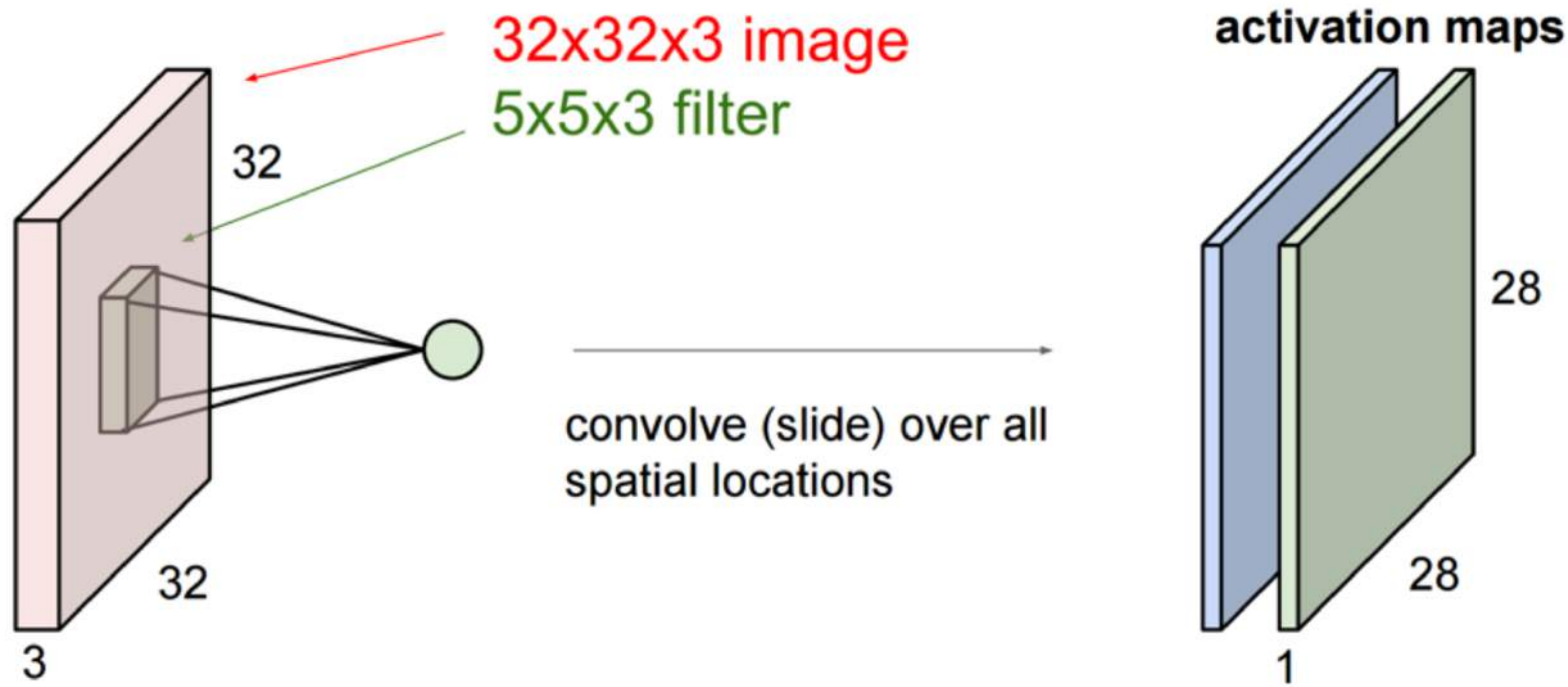
Convolution Layer



(Recap)

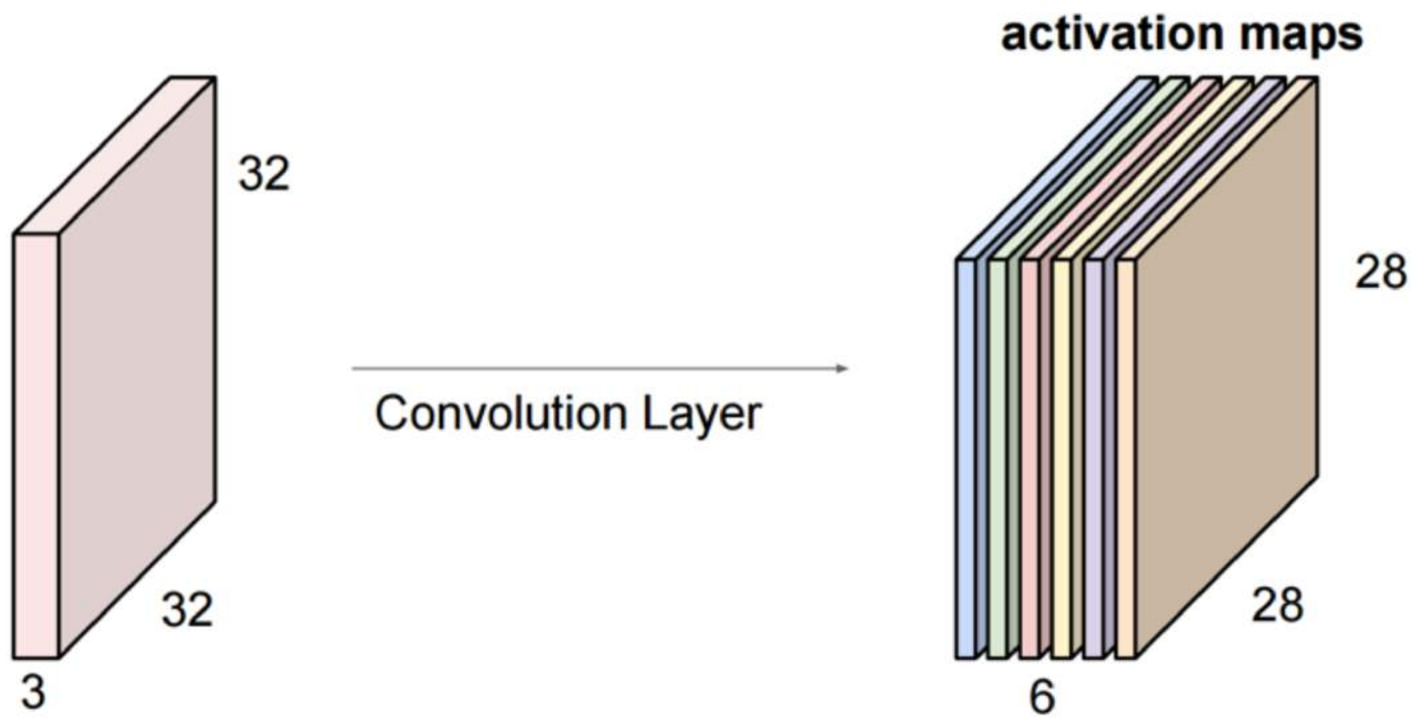
Convolution Layer

consider a second, green filter



(Recap)

For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



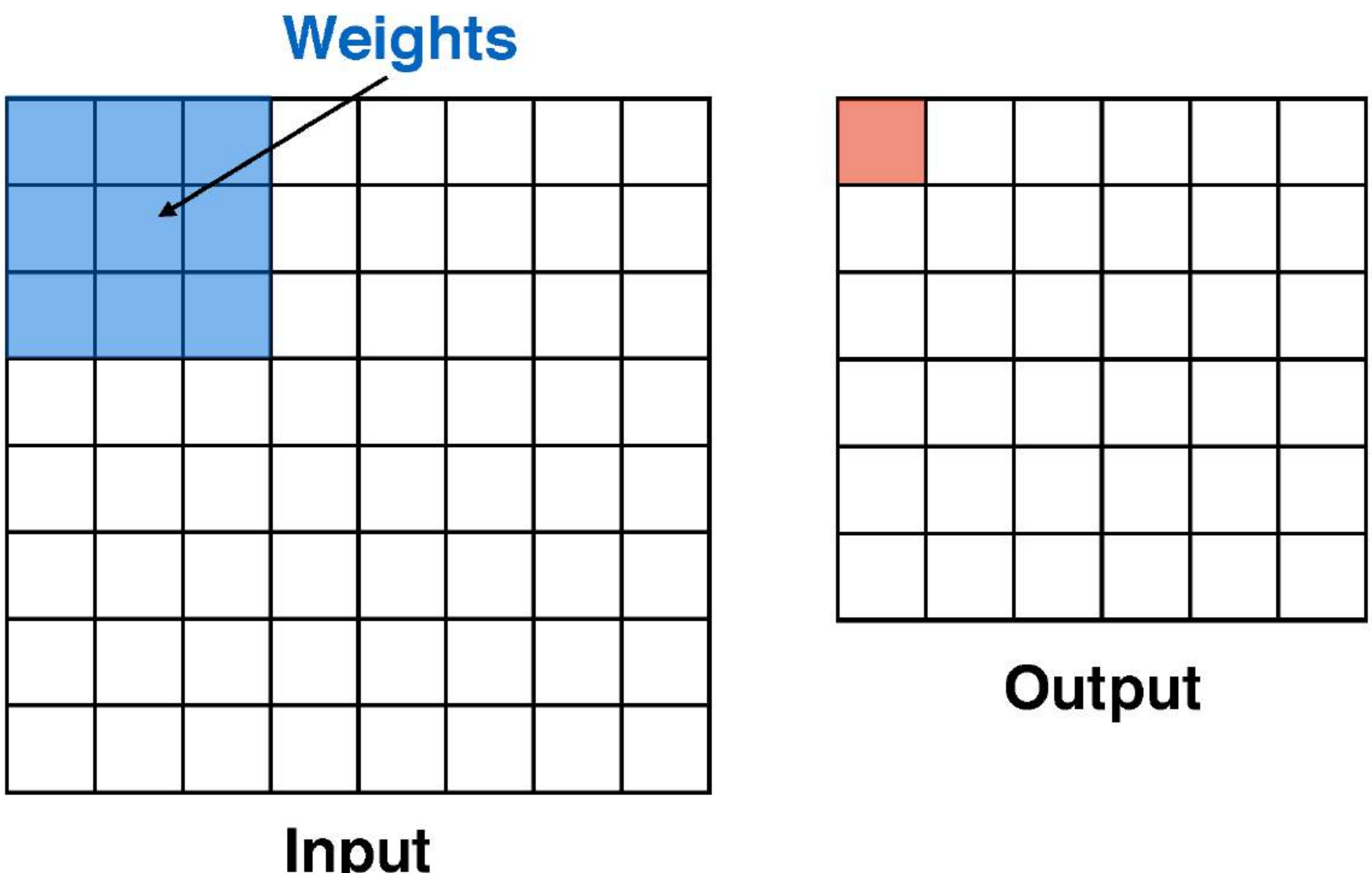
We stack these up to get a "new image" of size 28x28x6!

Demos

- http://cs231n.stanford.edu/
- http://cs.stanford.edu/people/karpathy/convnetjs/demo/mnist.html

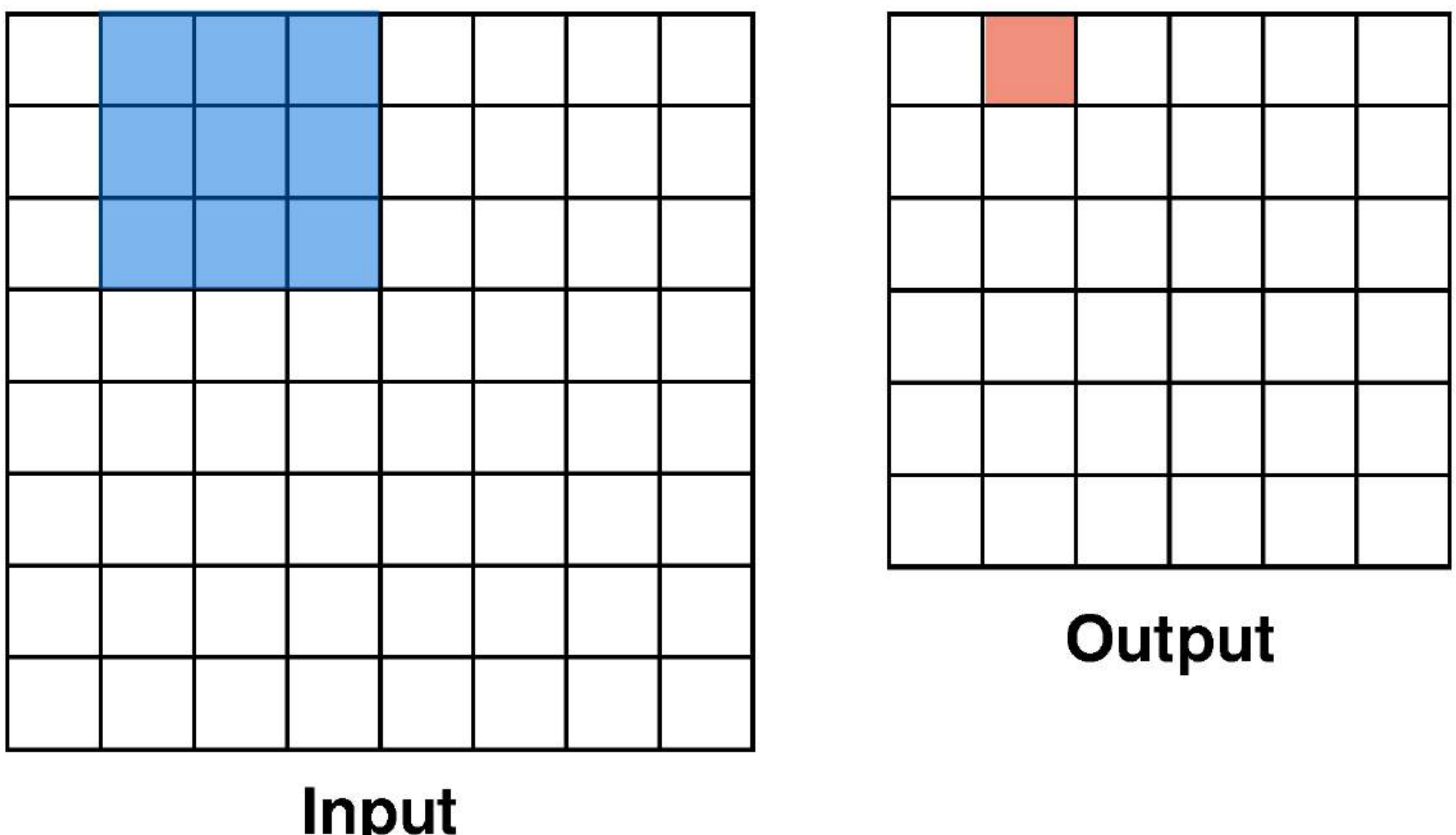
Convolution: Stride

During convolution, the weights "slide" along the input to generate each output



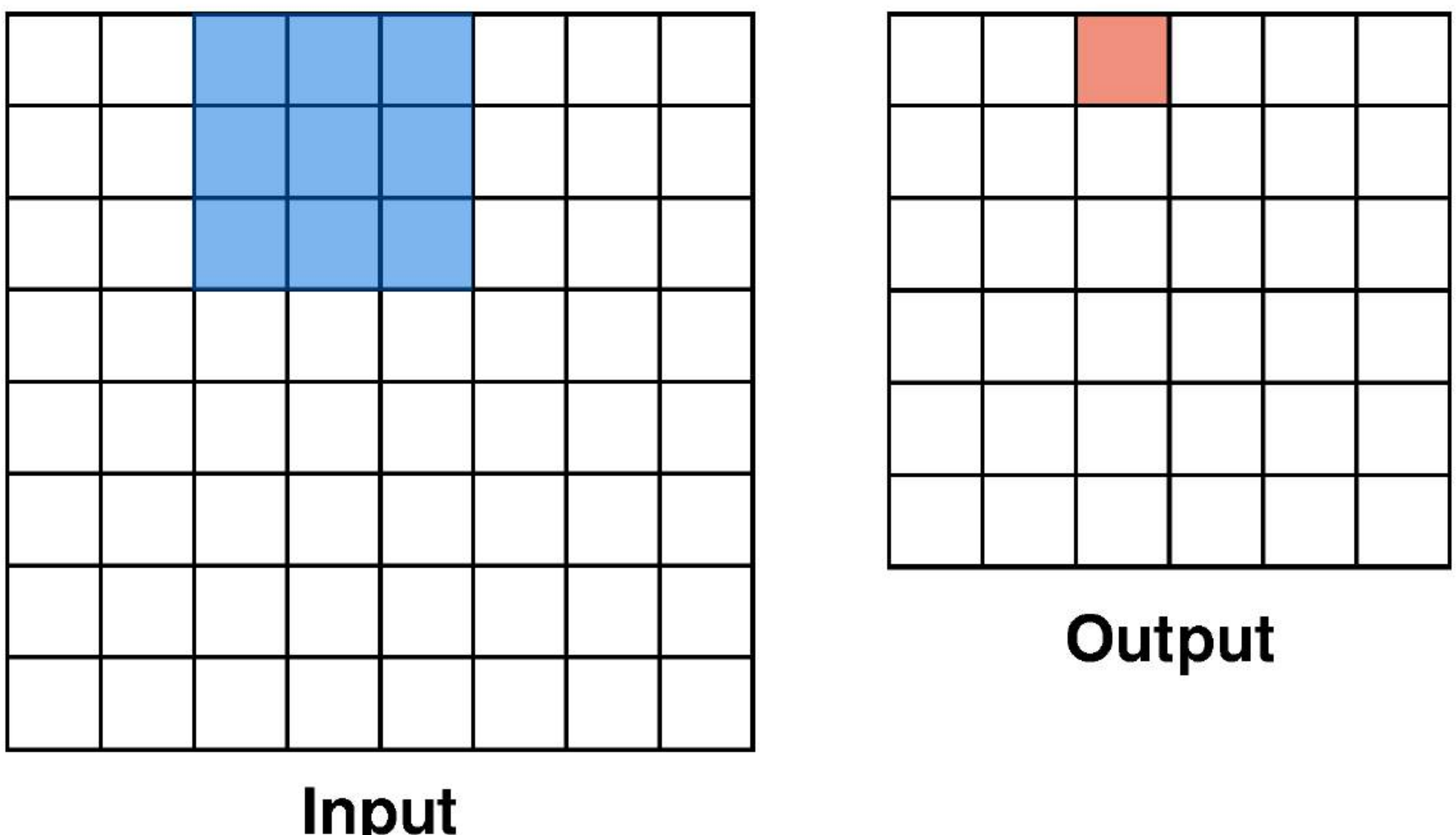
Convolution: Stride

During convolution, the weights “slide” along the input to generate each output



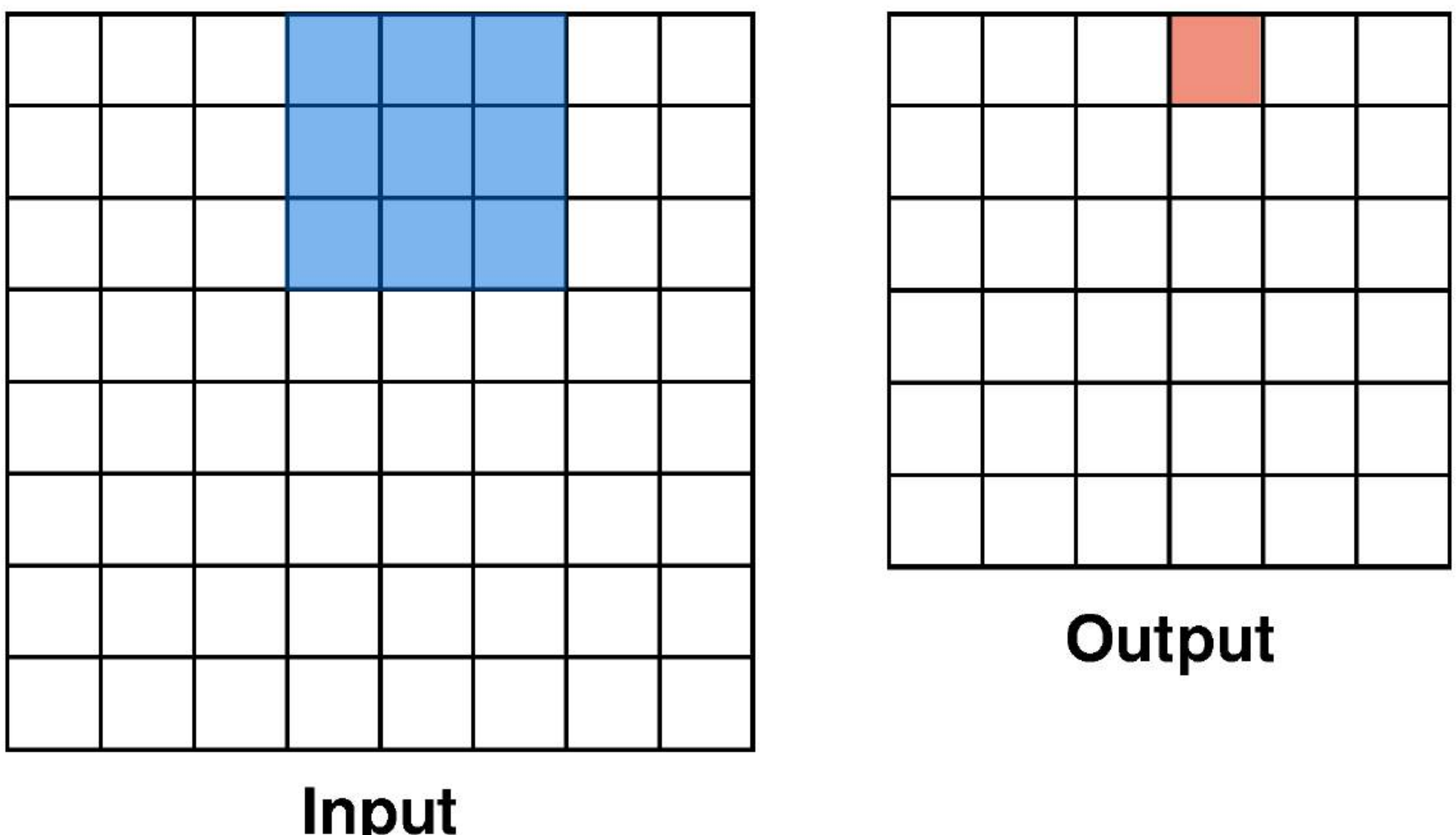
Convolution: Stride

During convolution, the weights "slide" along the input to generate each output



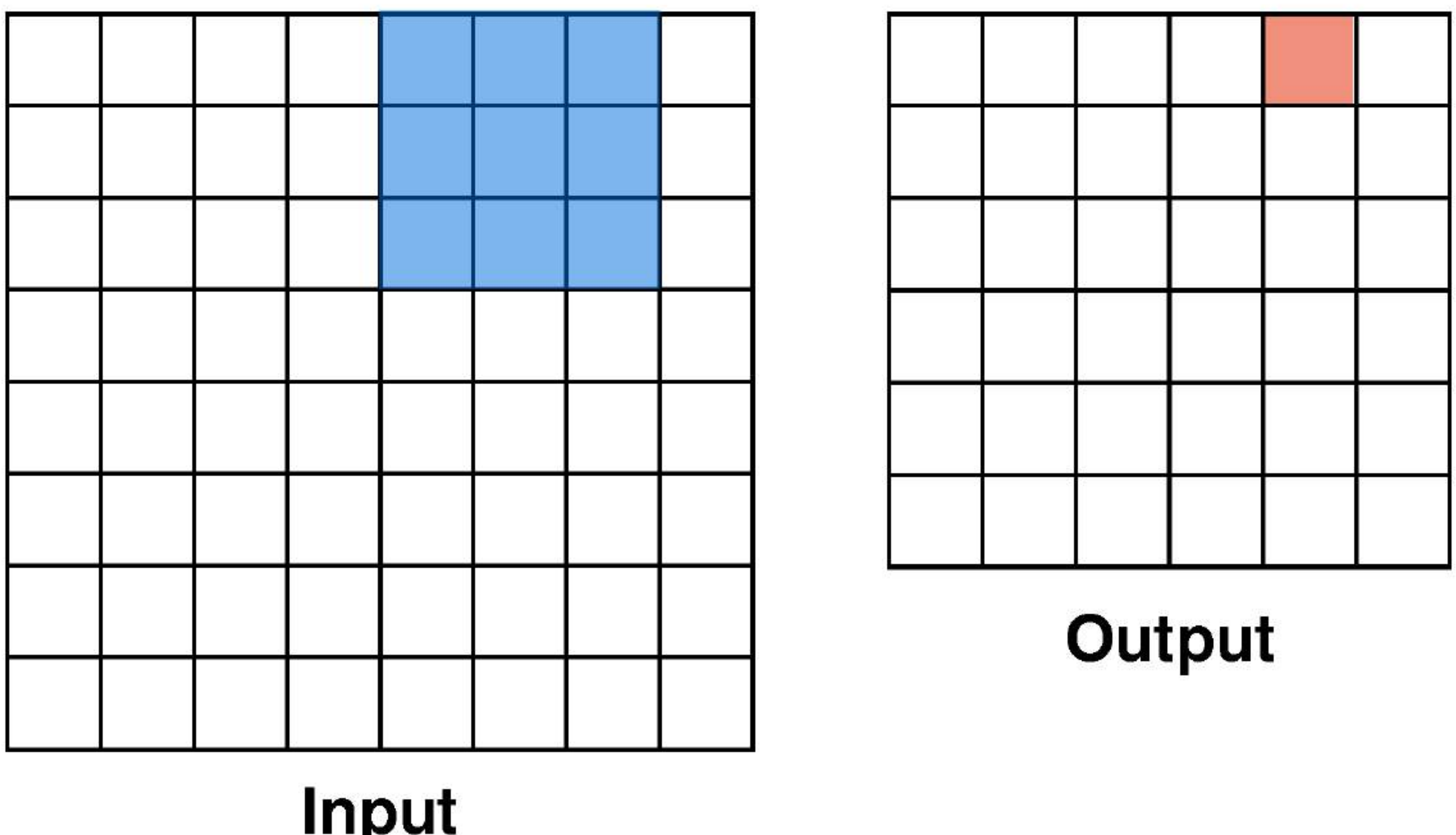
Convolution: Stride

During convolution, the weights "slide" along the input to generate each output



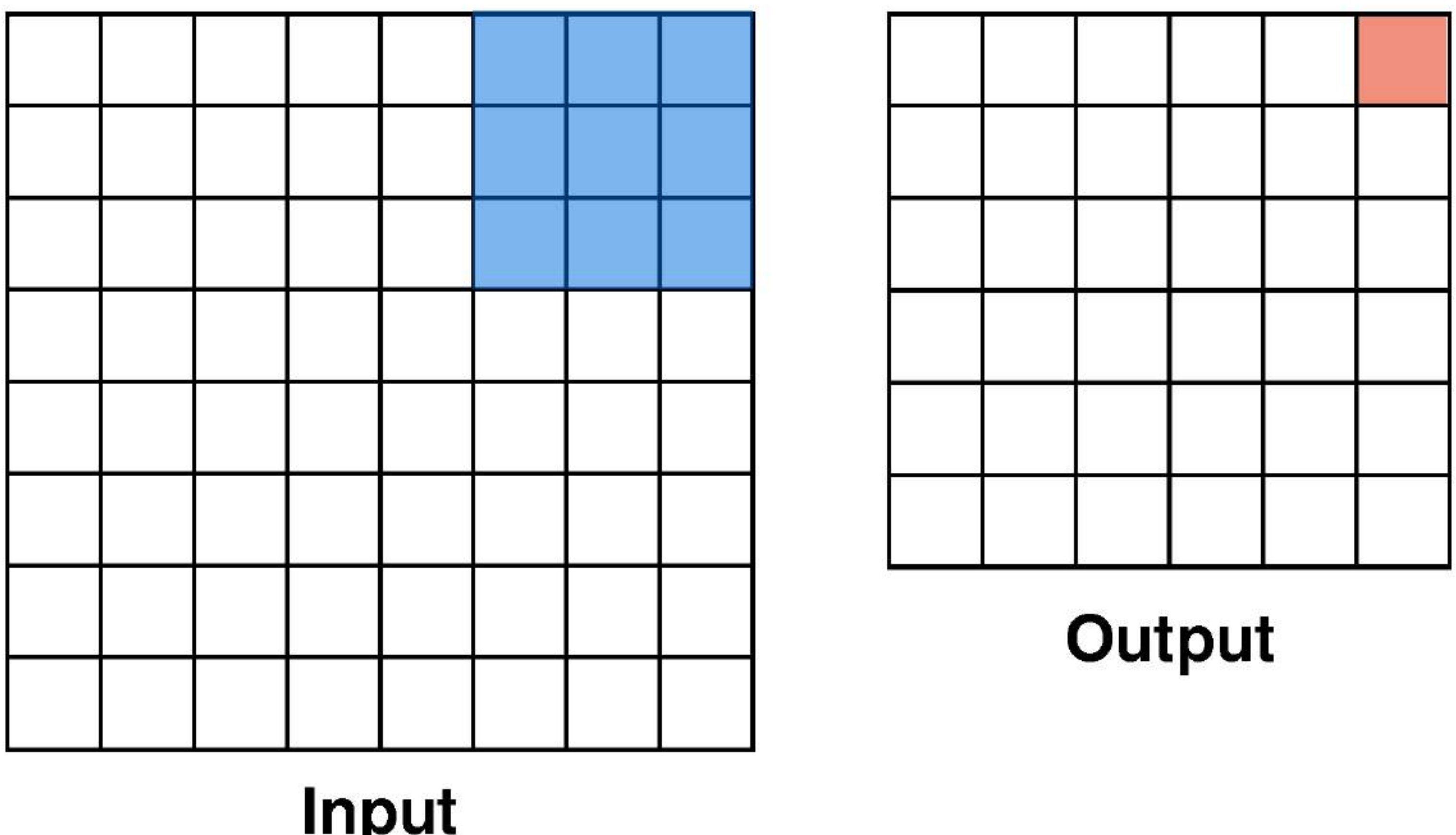
Convolution: Stride

During convolution, the weights "slide" along the input to generate each output



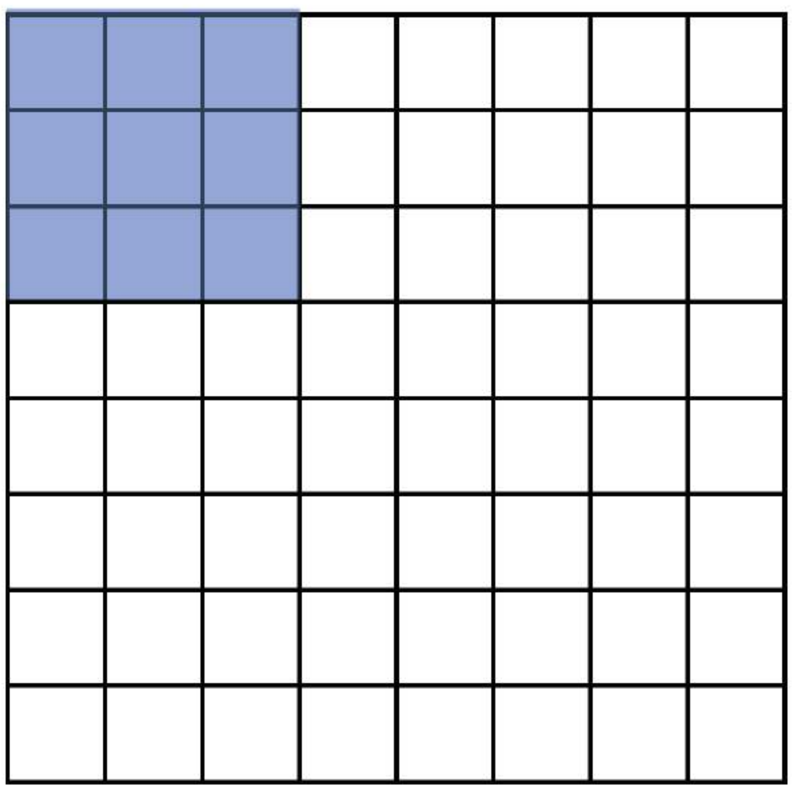
Convolution: Stride

During convolution, the weights "slide" along the input to generate each output



Convolution: Stride

During convolution, the weights "slide" along the input to generate each output



Input

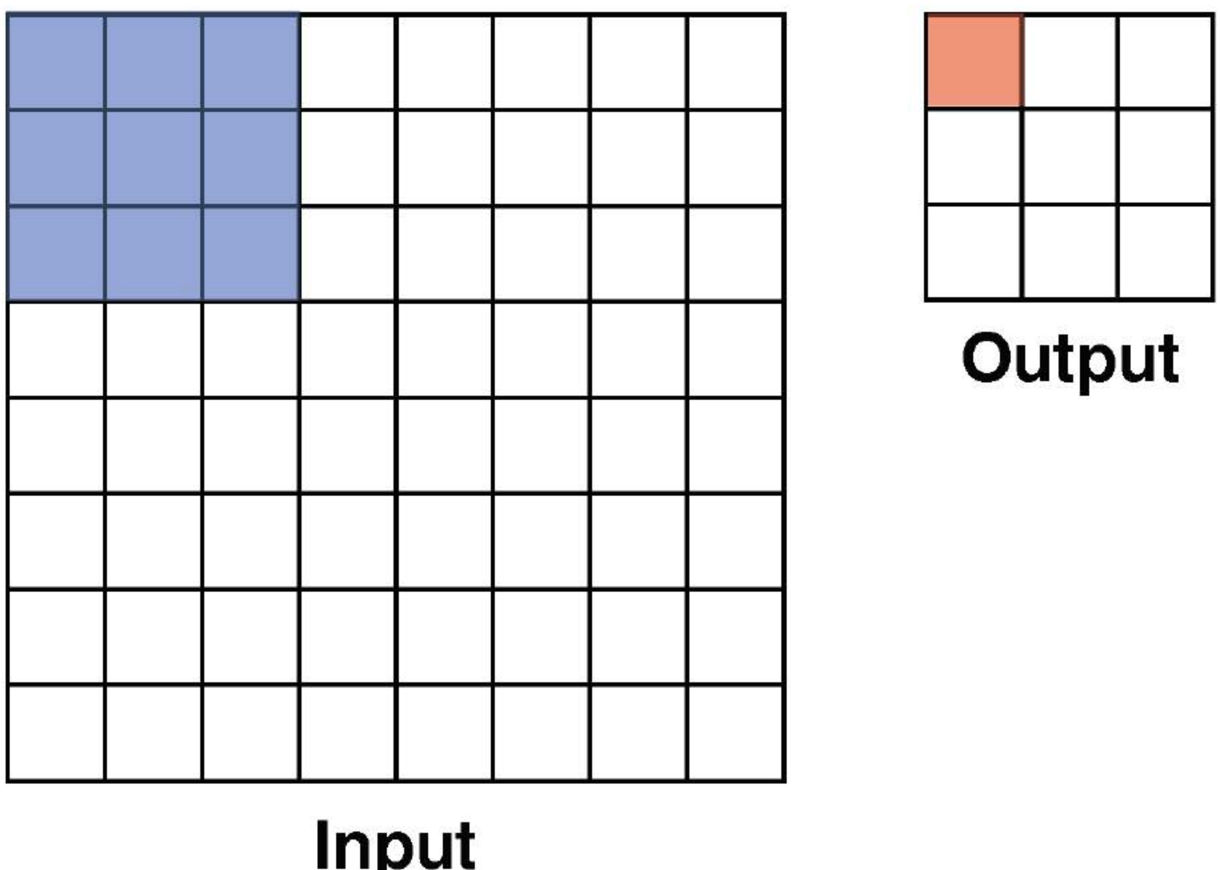
Recall that at each position, we are doing a **3D** sum:

$$h^r = \sum_{ijk} x^r{}_{ijk} W_{ijk} + b$$

(channel, row, column)

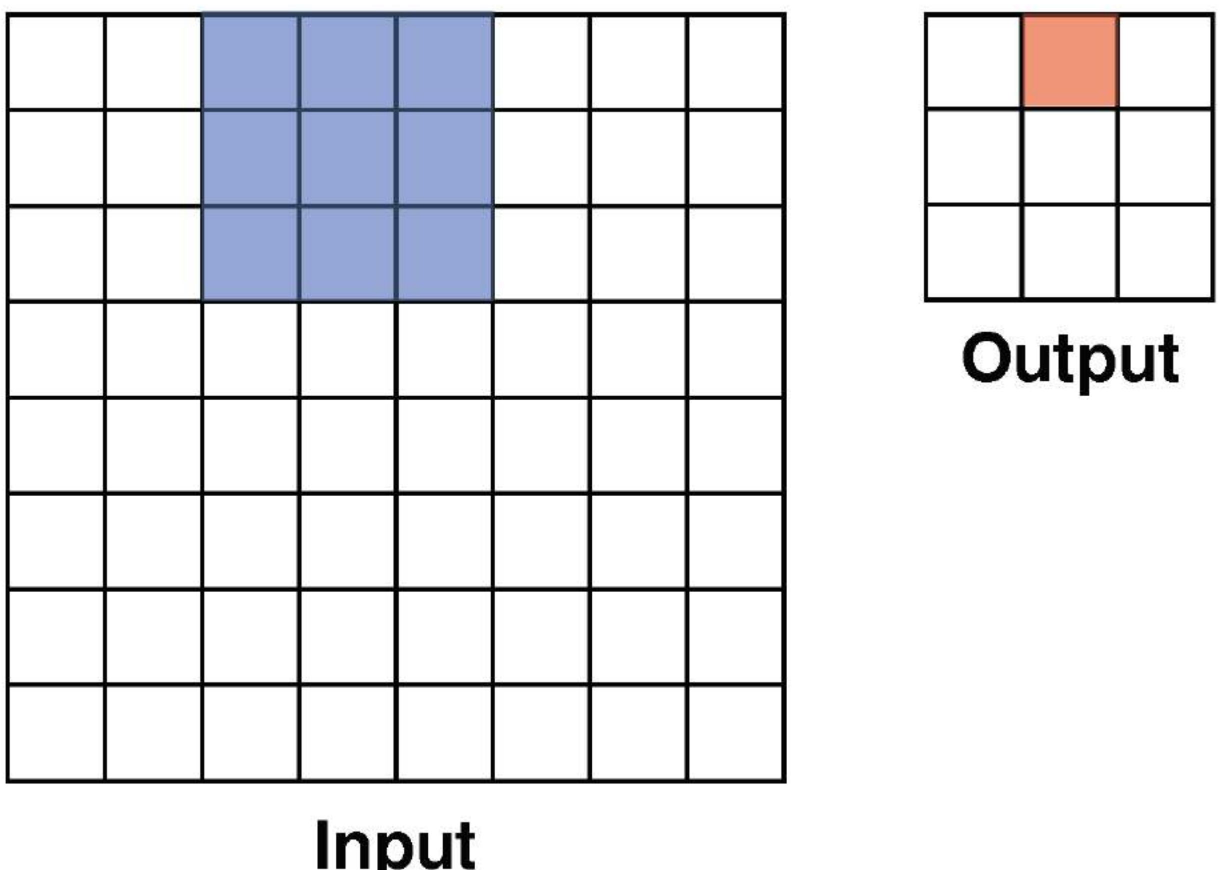
Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2



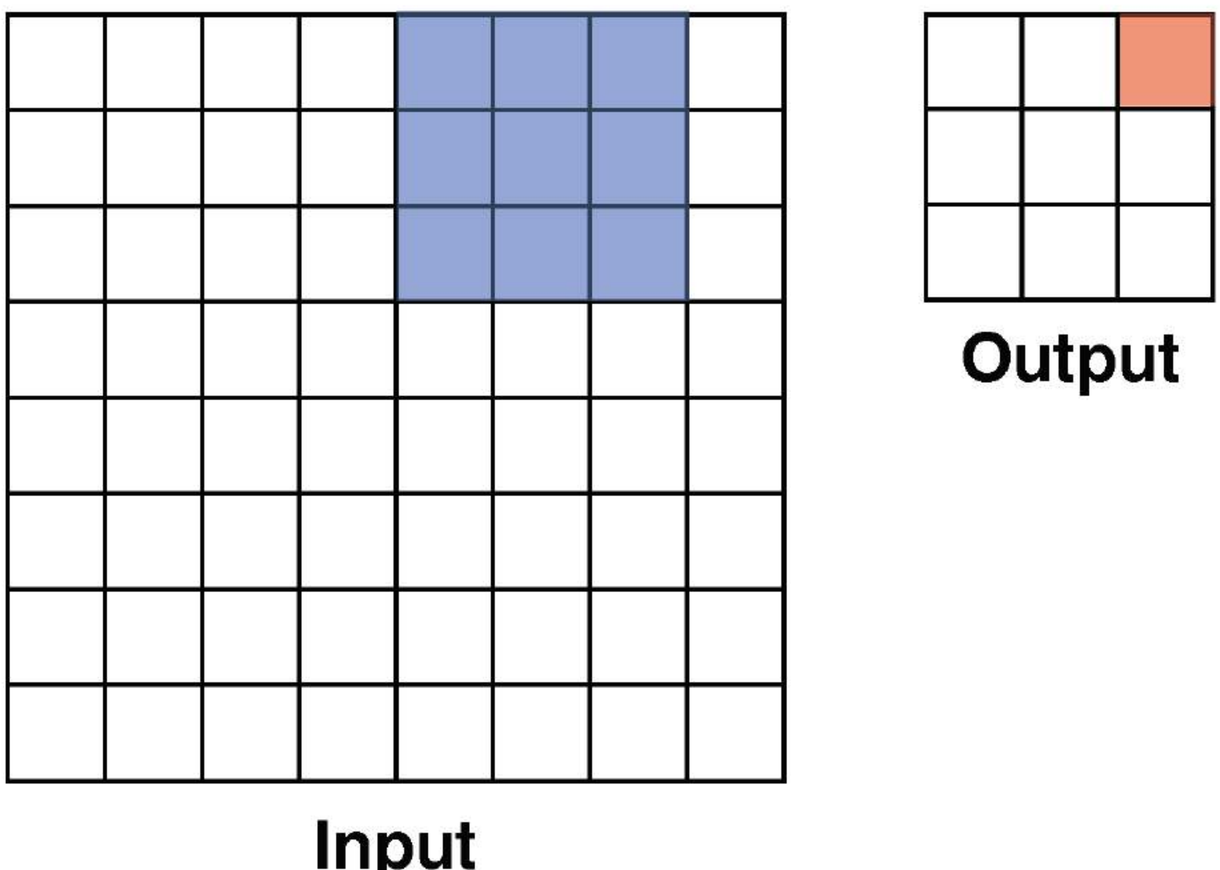
Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2



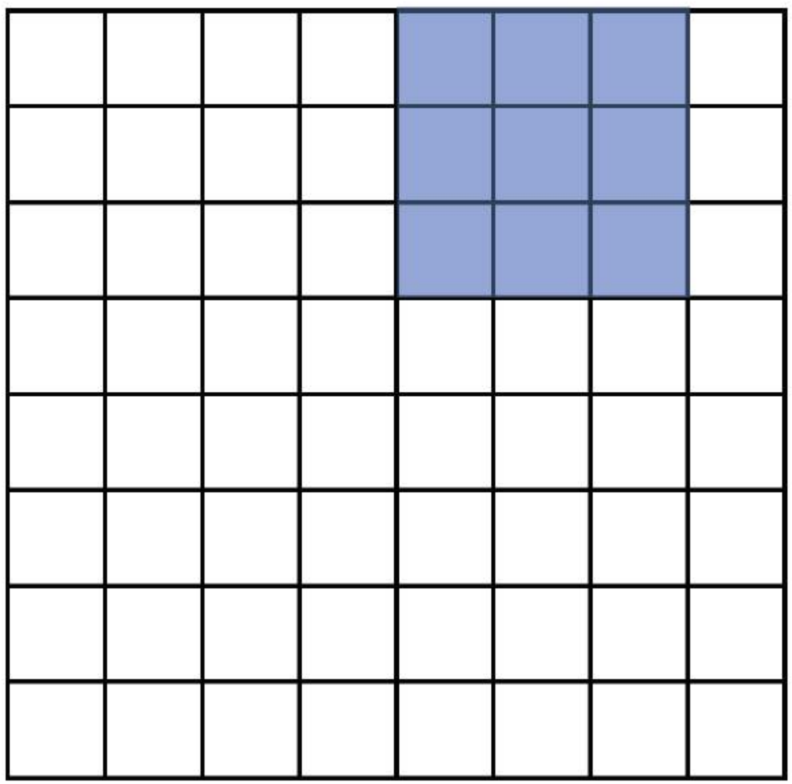
Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2

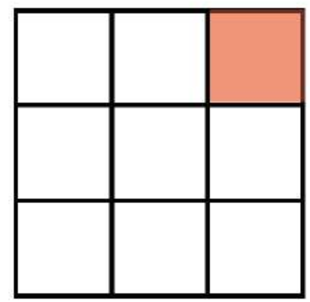


Convolution: Stride

But we can also convolve with a **stride**, e.g. stride = 2



Input



Output

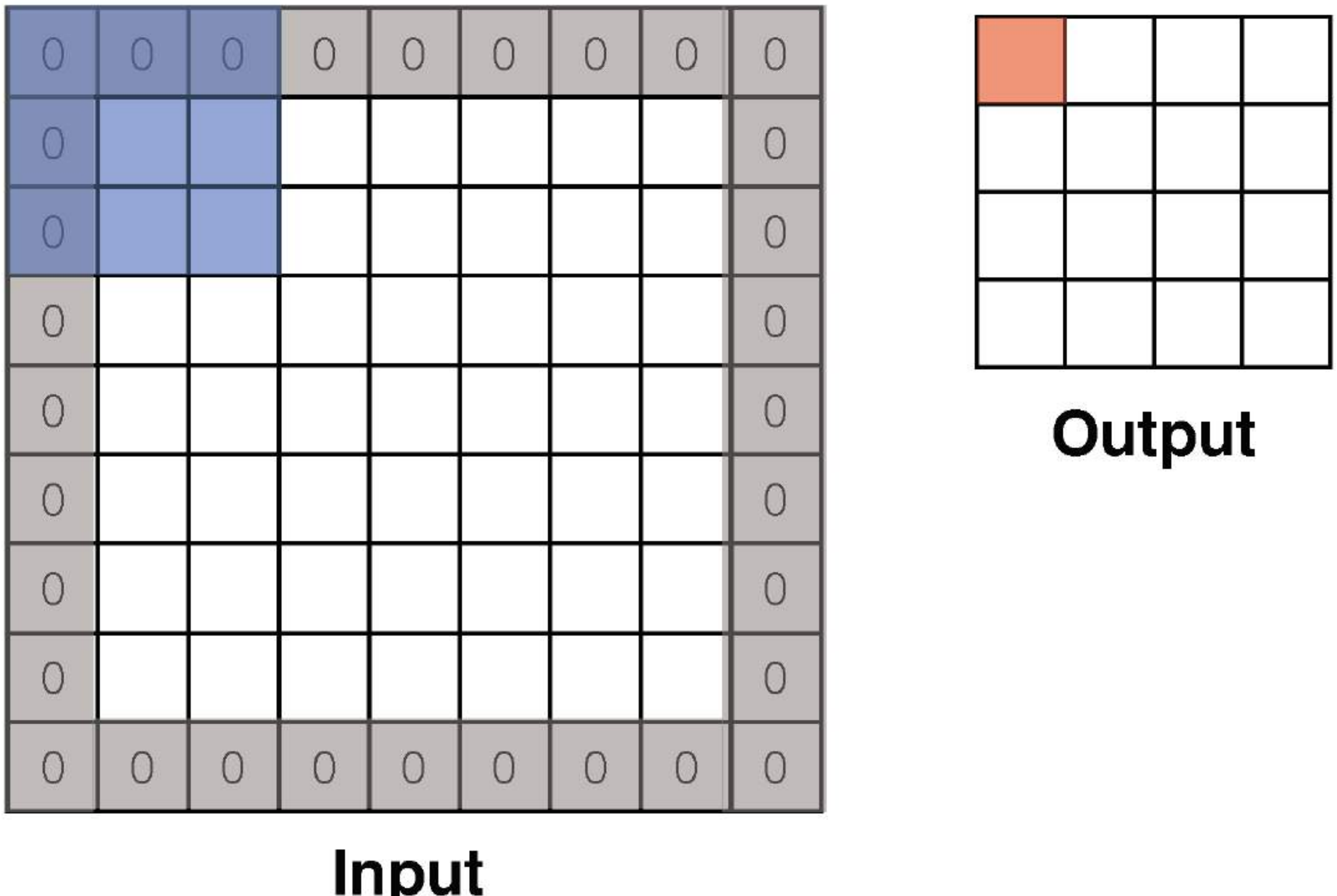
- Notice that with certain strides, we may not be able to cover all of the input

- The output is also half the size of the input

Convolution: Padding

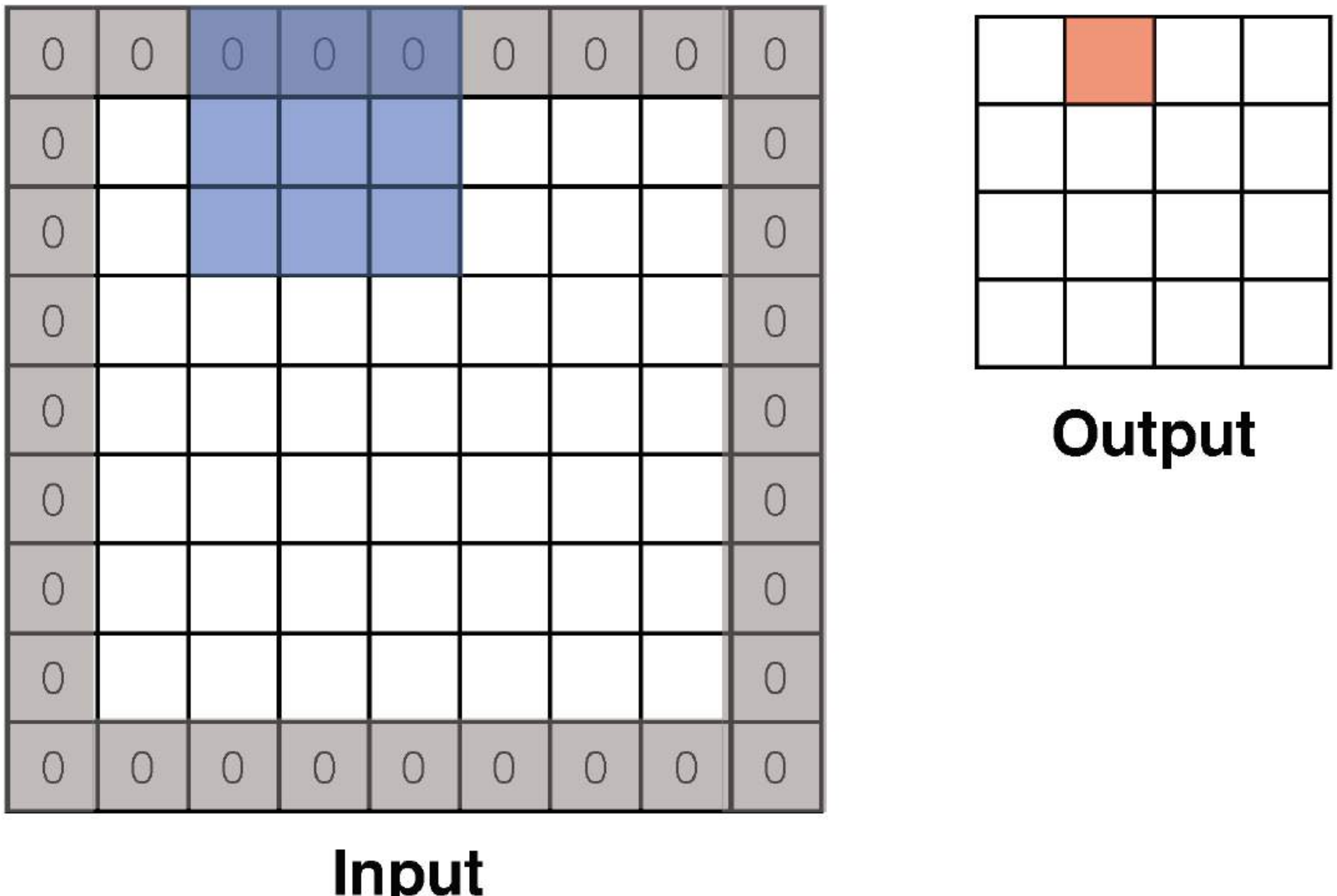
We can also pad the input with zeros.

Here, **pad = 1, stride = 2**



Convolution: Padding

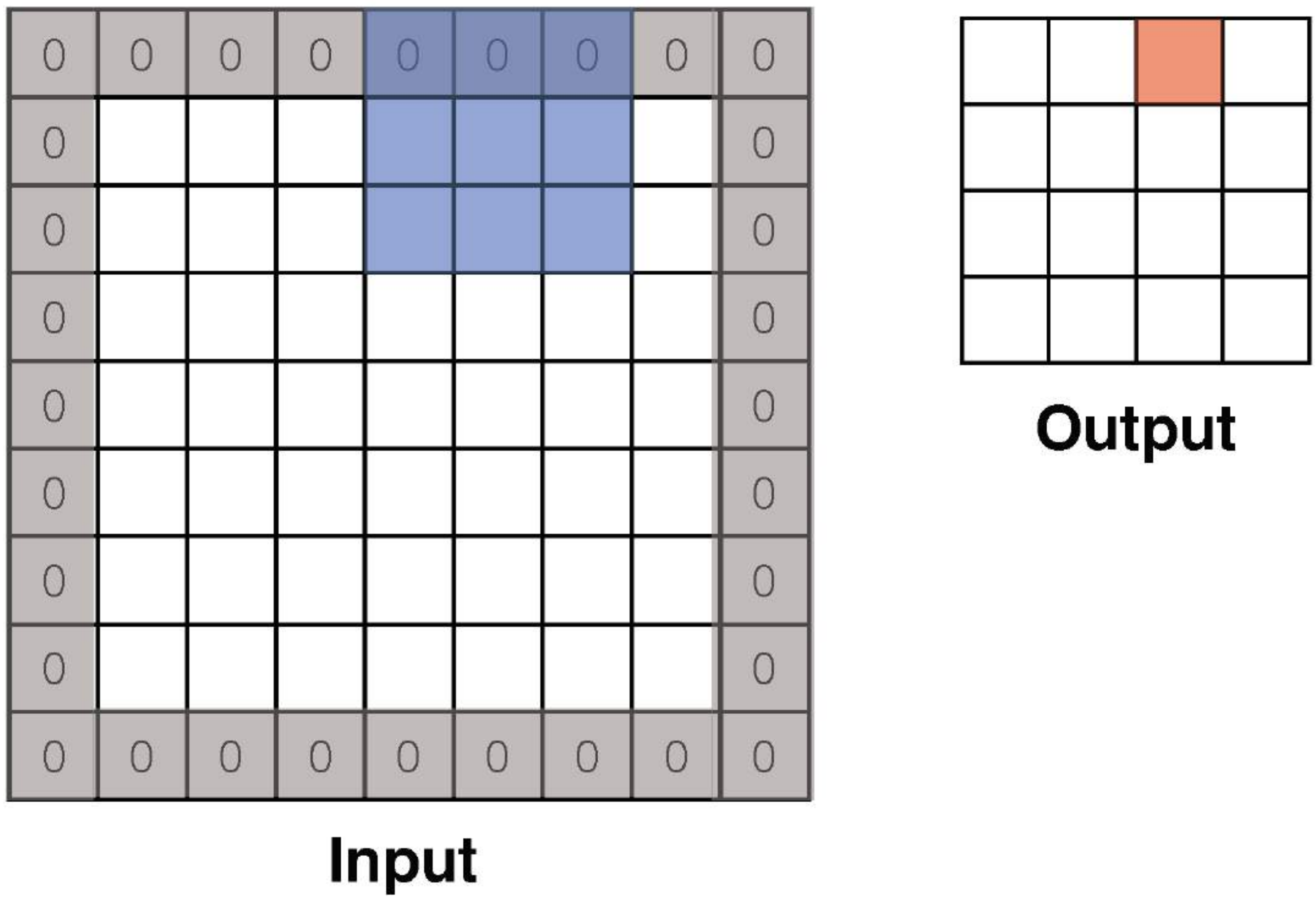
We can also pad the input with zeros.
Here, **pad = 1, stride = 2**



Convolution: Padding

We can also pad the input with zeros.

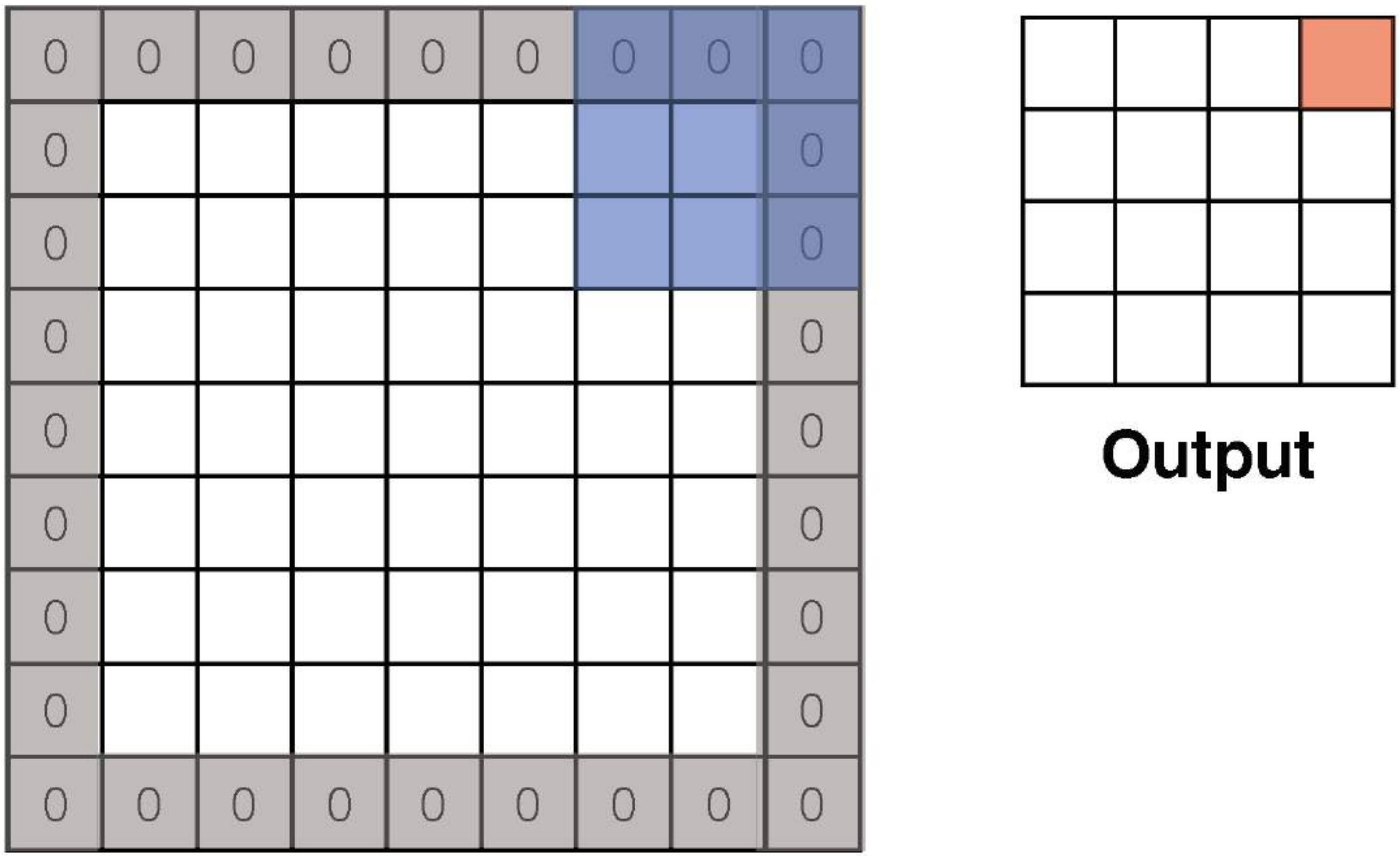
Here, **pad = 1, stride = 2**



Convolution: Padding

We can also pad the input with zeros.

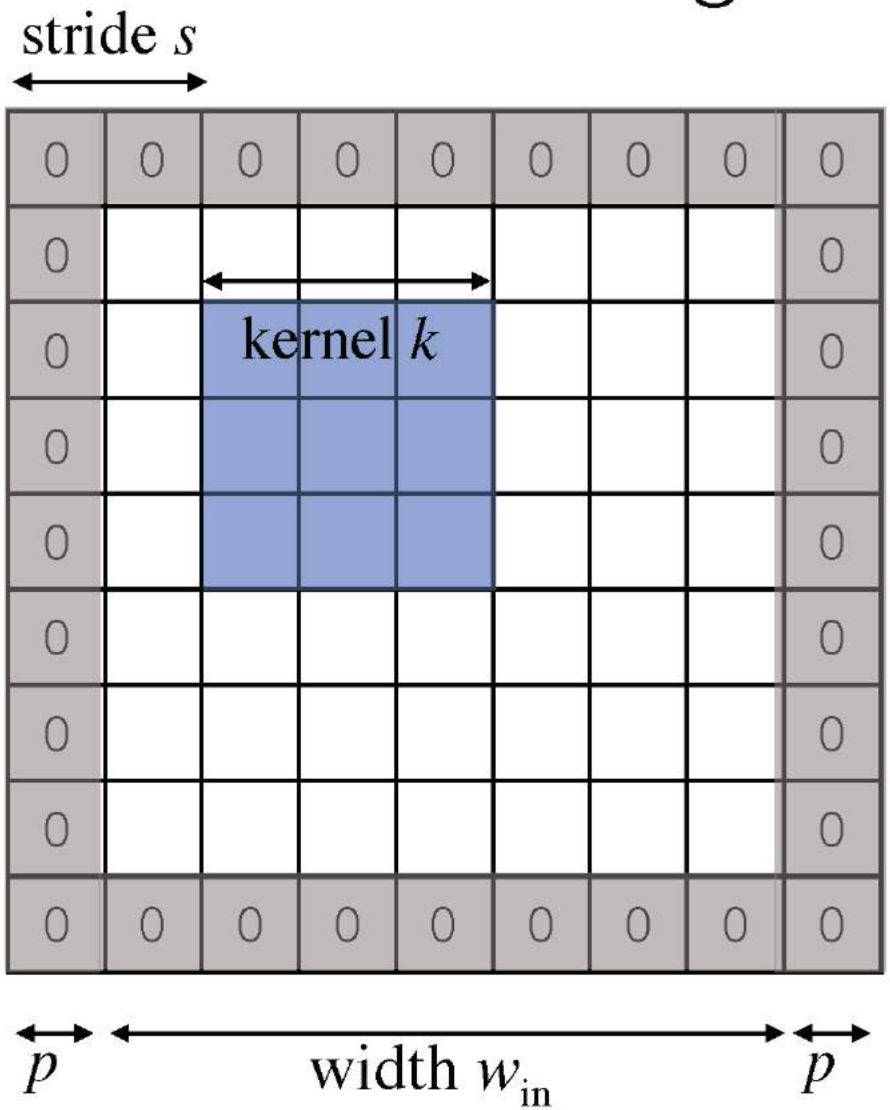
Here, **pad = 1, stride = 2**



Input

Convolution:

How big is the output?

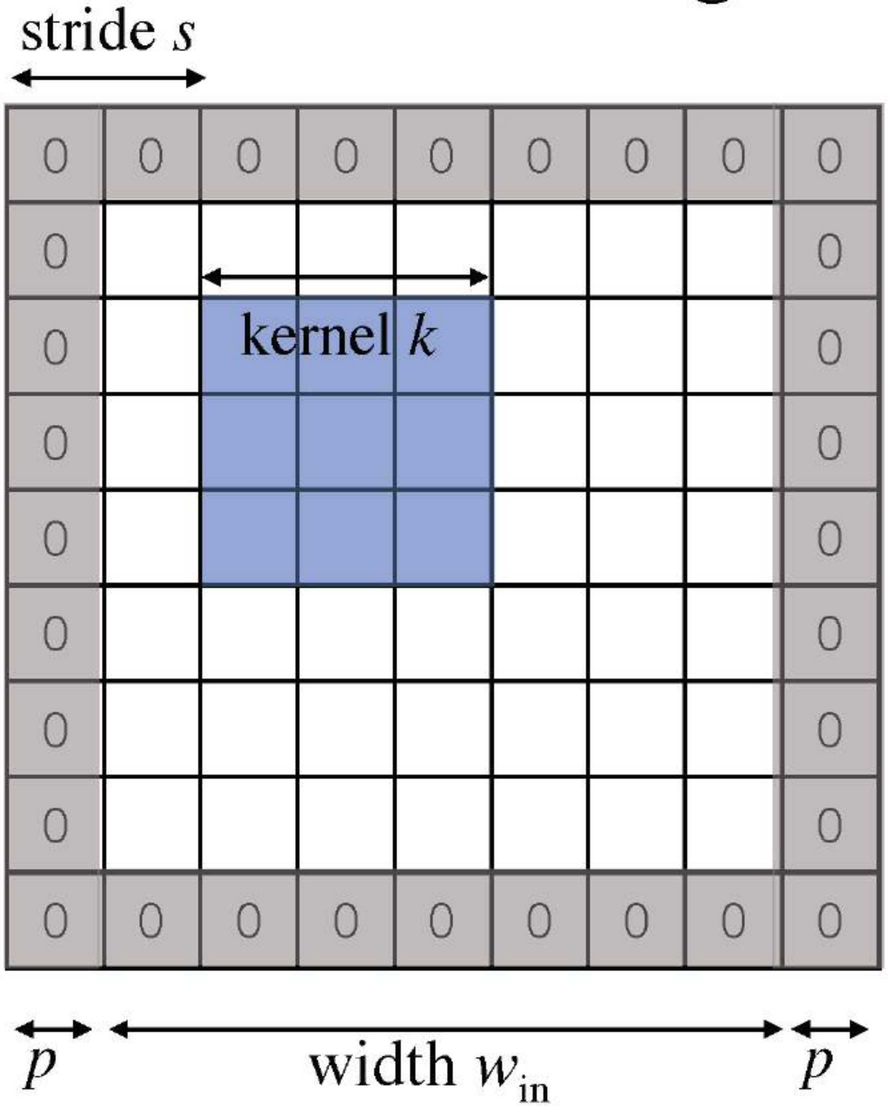


In general, the output has size:

$$w_{\text{out}} = \left\lfloor \frac{w_{\text{in}} + 2p - k}{s} \right\rfloor + 1$$

Convolution:

How big is the output?



Example: k=3, s=1, p=1

$$w_{\text{out}} = \left\lfloor \frac{w_{\text{in}} + 2p - k}{s} \right\rfloor + 1$$

$$= \left\lfloor \frac{w_{\text{in}} + 2 - 3}{1} \right\rfloor + 1$$

$$= w_{\text{in}}$$

VGGNet [Simonyan 2014] uses filters of this shape

Pooling

For most ConvNets, **convolution** is often followed by **pooling**:

- Creates a smaller representation while retaining the most important information
- The "max" operation is the most common
- Why might "avg" be a poor choice?

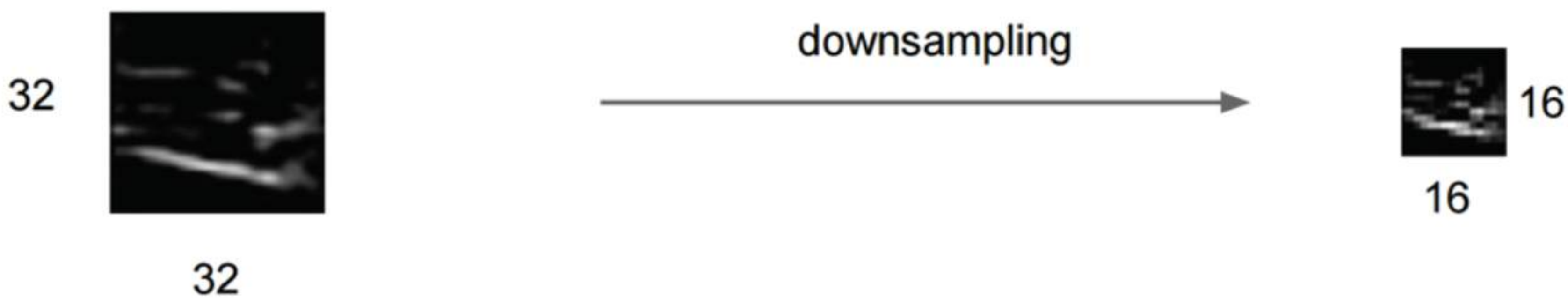
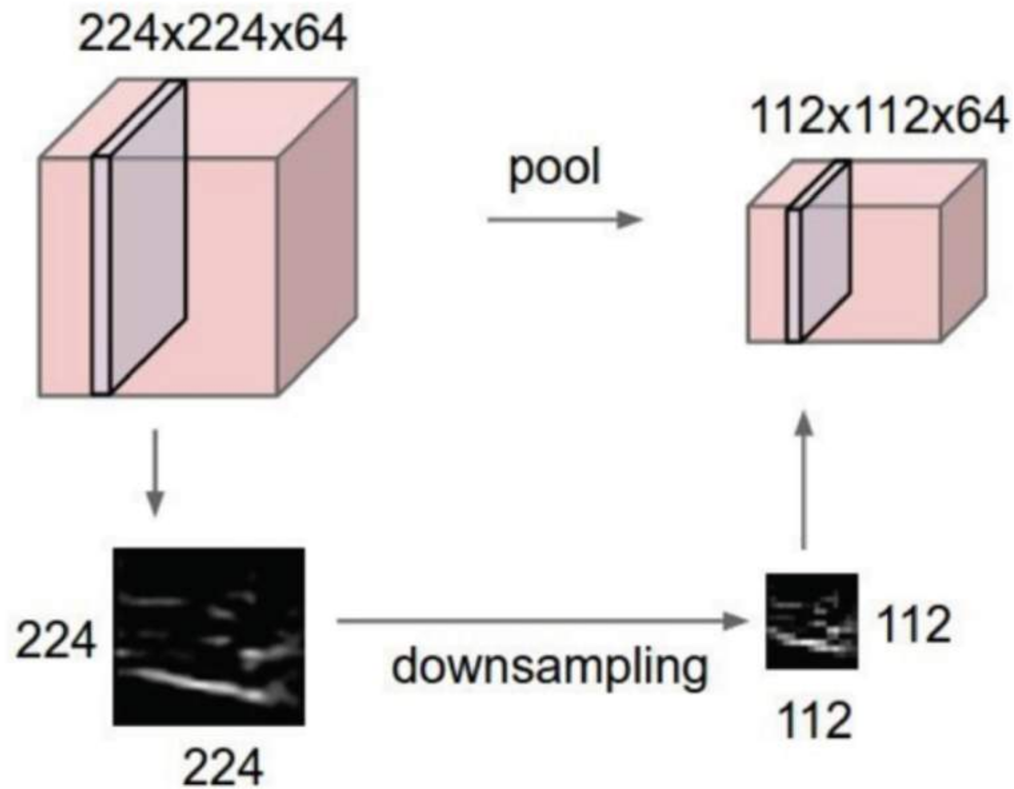


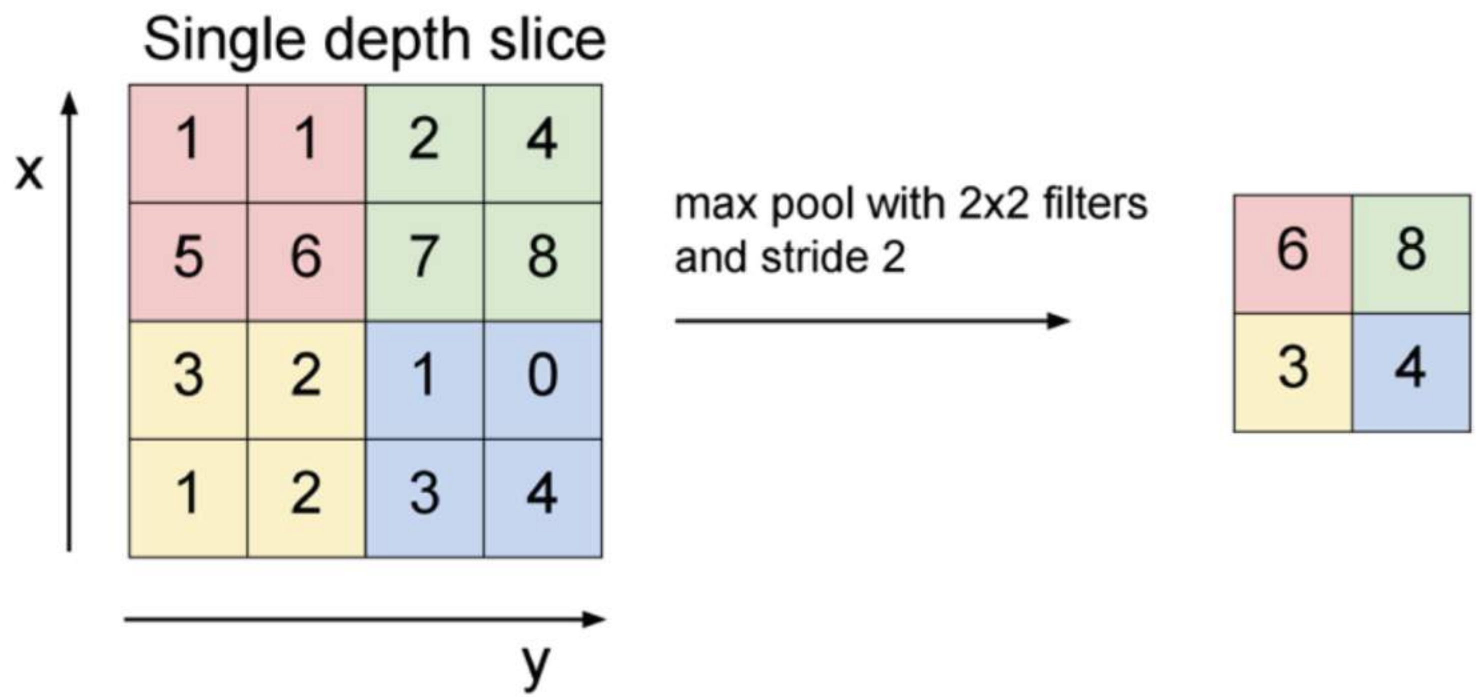
Figure: Andrej Karpathy

Pooling

- makes the representations smaller and more manageable
- operates over each activation map independently:



Max Pooling



What's the backprop rule for max pooling?

- In the forward pass, store the index that took the max
- The backprop gradient is the input gradient at that index

Figure: Andrej Karpathy

Example ConvNet

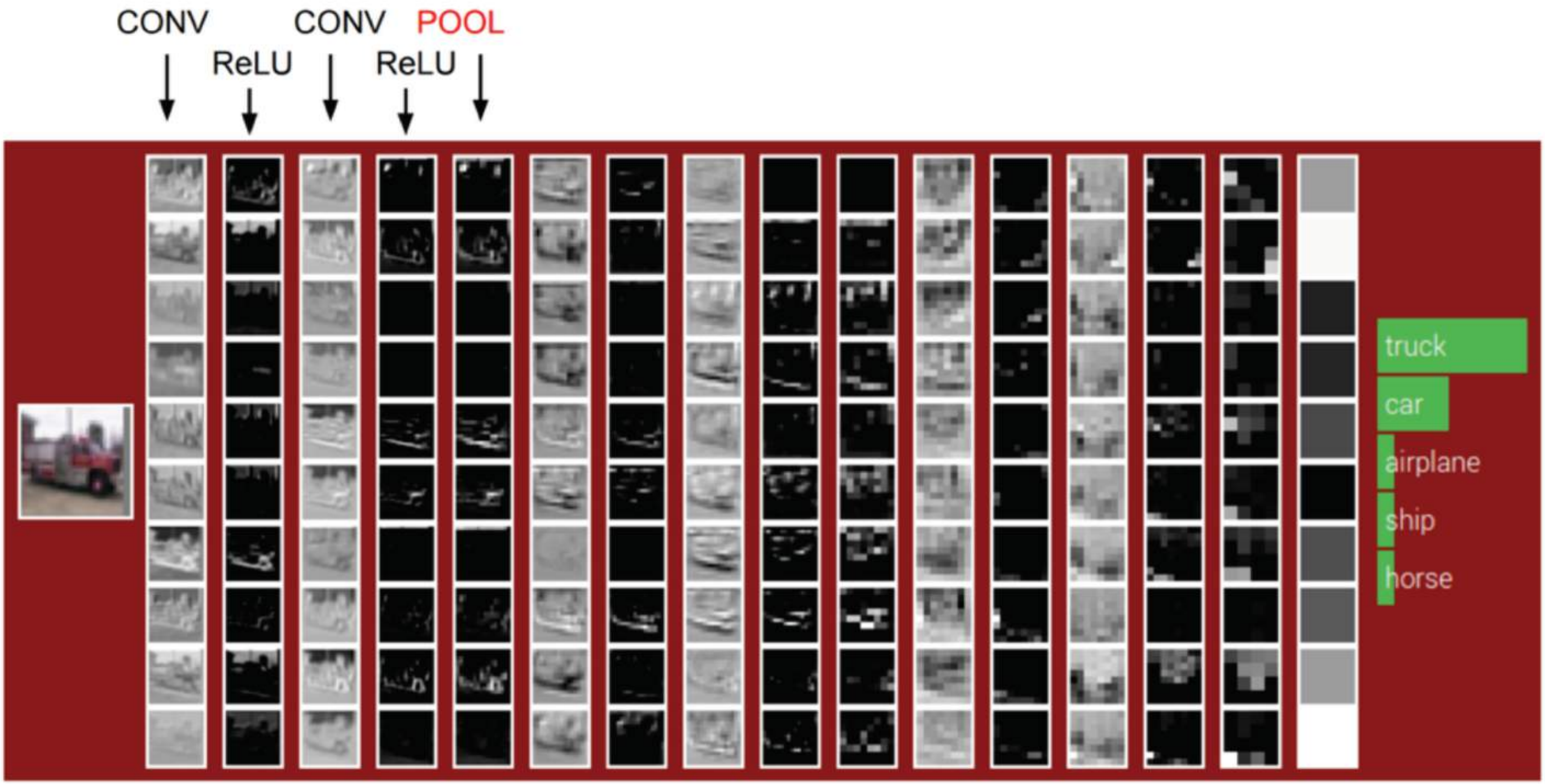


Figure: Andrej Karpathy

Example ConvNet

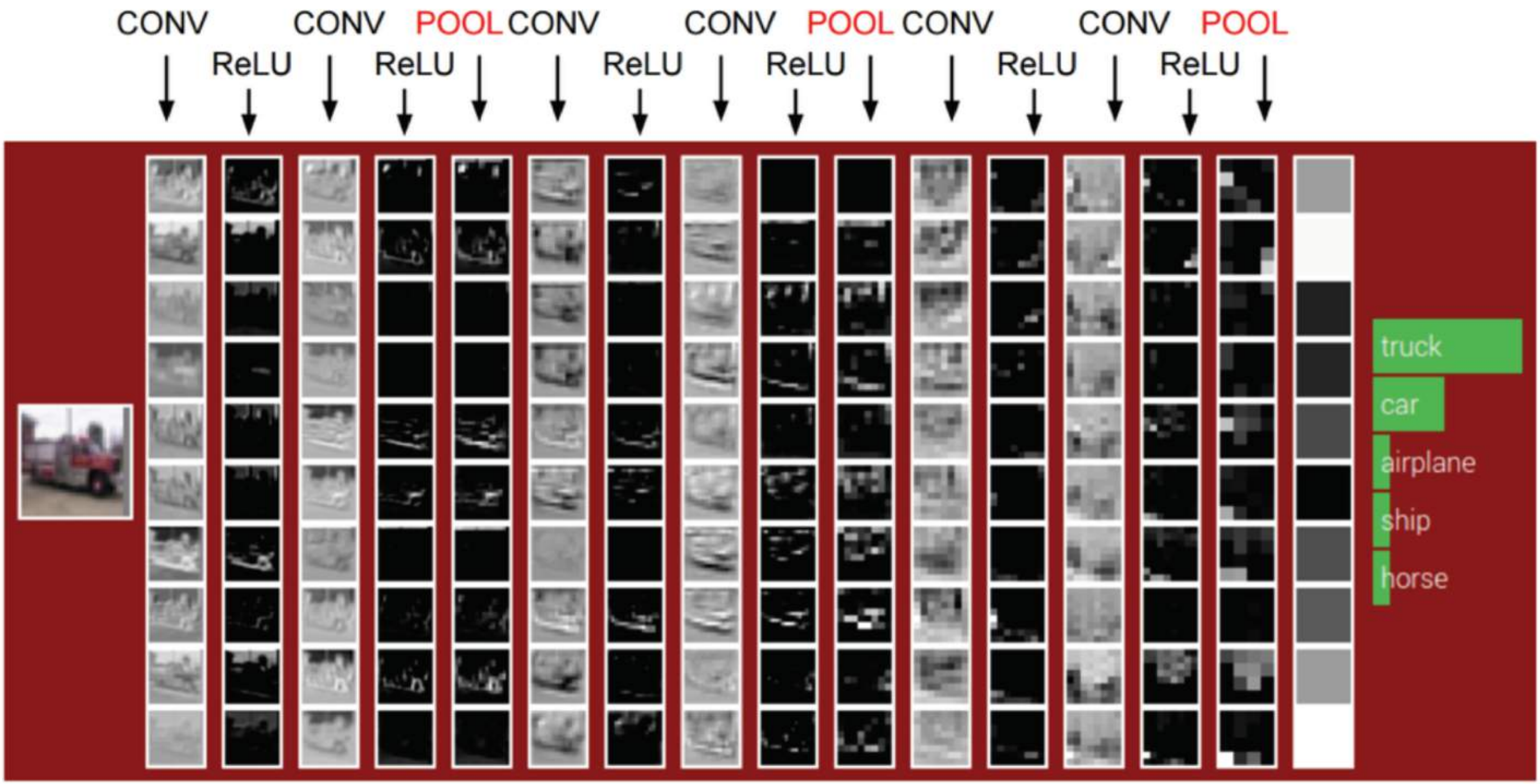


Figure: Andrej Karpathy

Example ConvNet

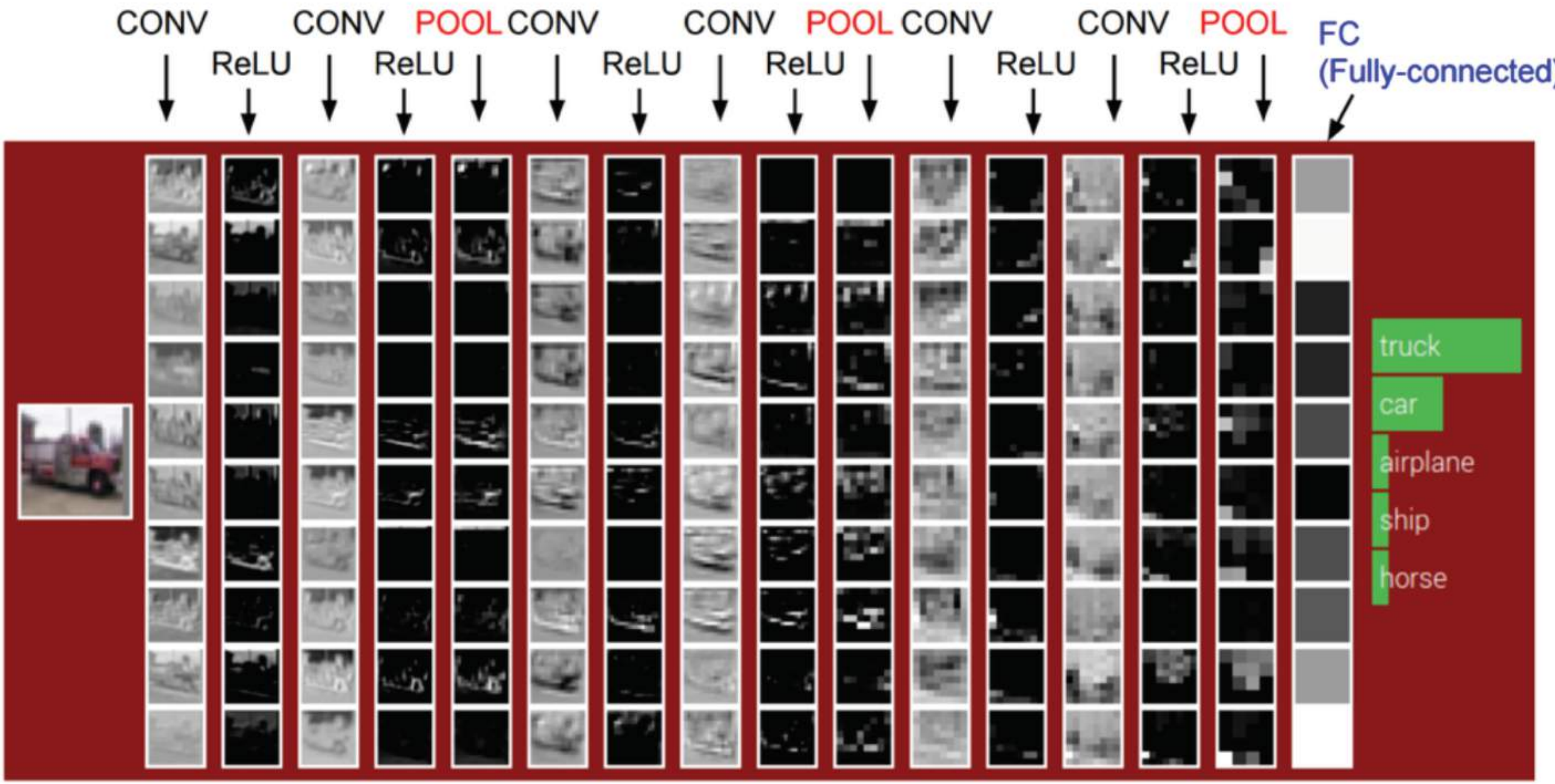
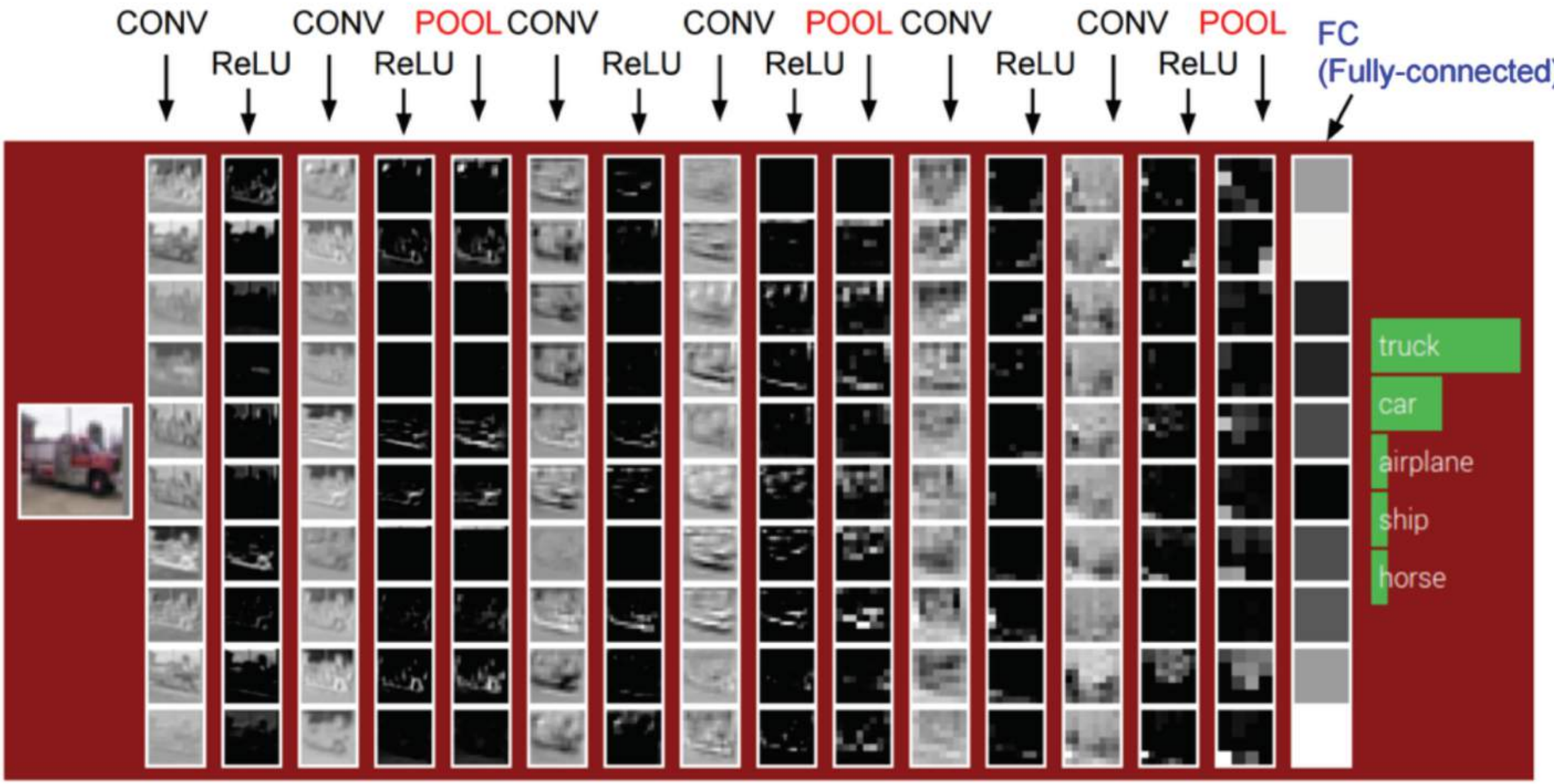


Figure: Andrej Karpathy

Example ConvNet

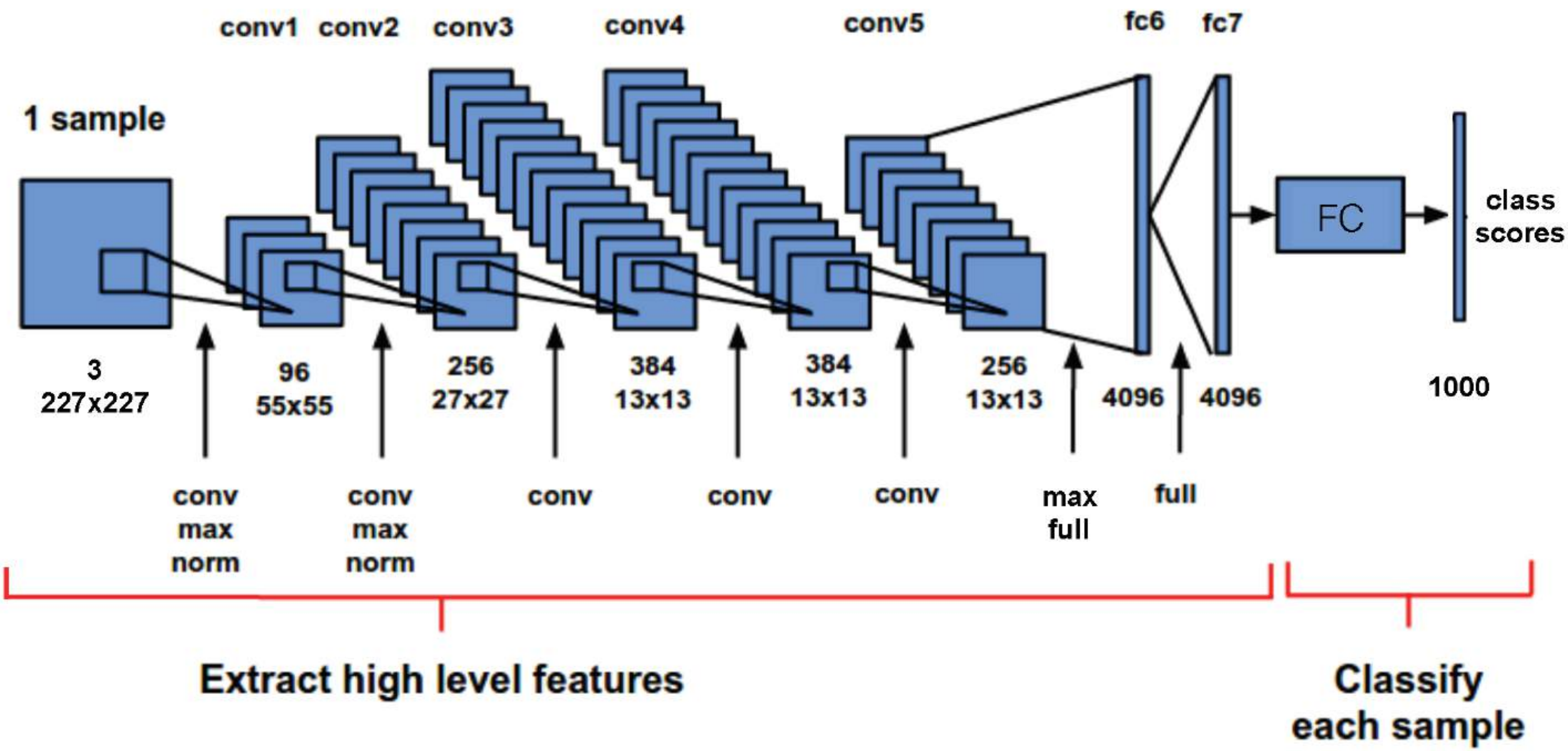


10x3x3 conv filters, stride 1, pad 1

2x2 pool filters, stride 2

Figure: Andrej Karpathy

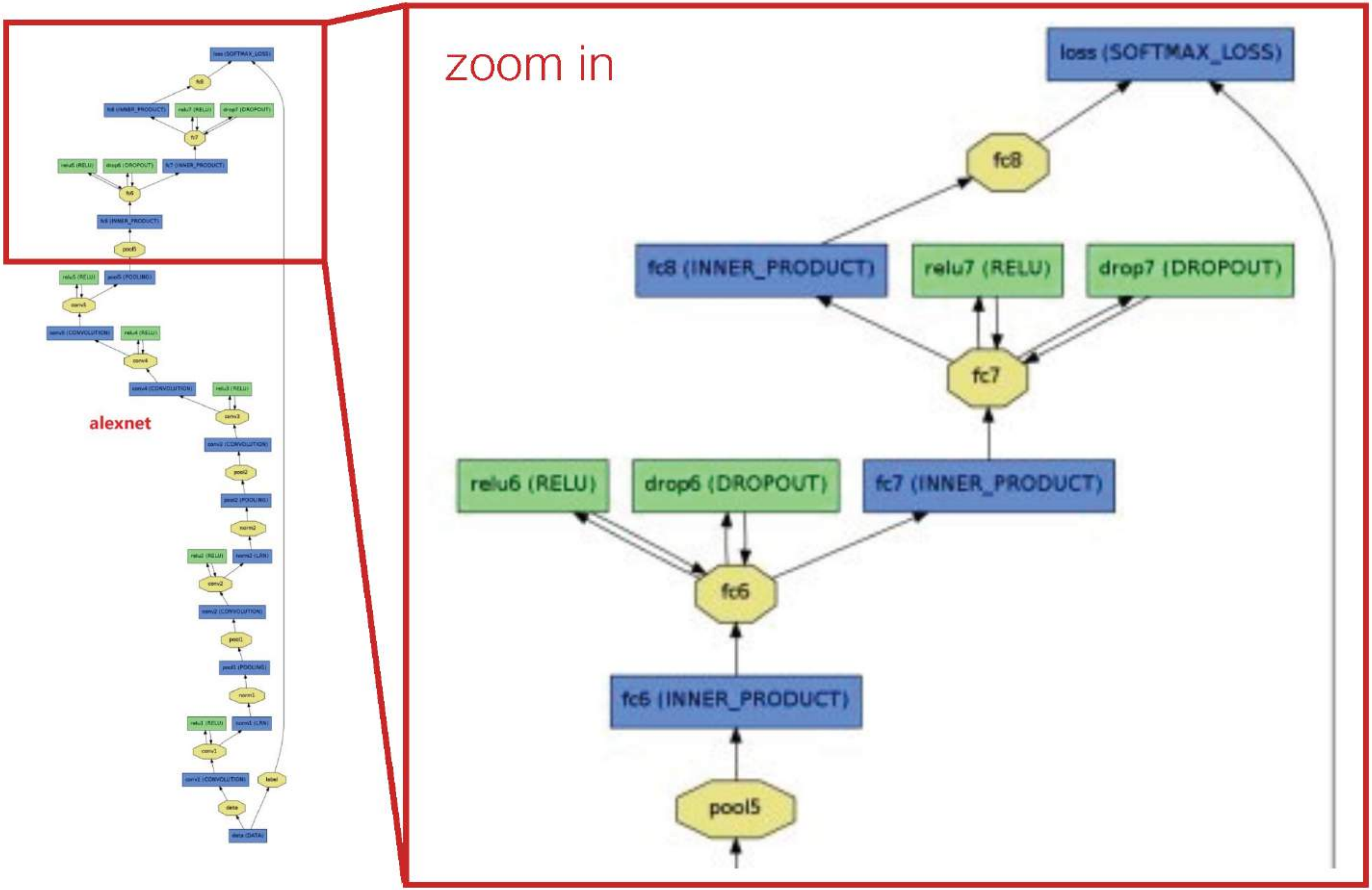
Example: AlexNet [Krizhevsky 2012]



"max": max pooling
"norm": local response normalization
"full": fully connected

Figure: [Karnowski 2015] *(with corrections)*

Example: AlexNet [Krizhevsky 2012]

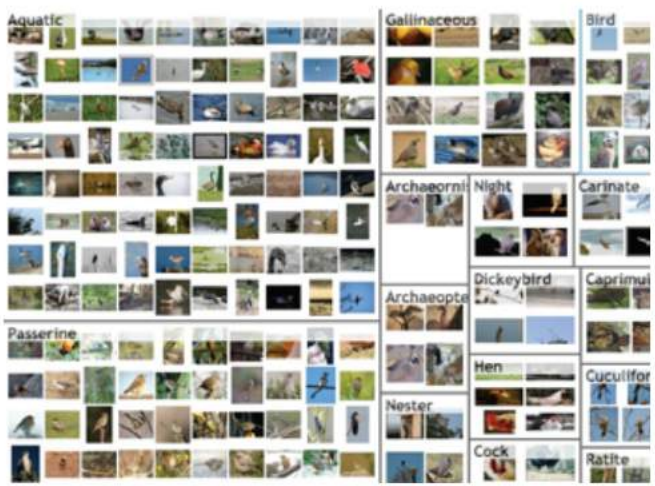


Training ConvNets

How do you actually train these things?

Roughly speaking:

Gather labeled data



Find a ConvNet architecture

Minimize the loss

Training a convolutional neural network

- Split and preprocess your data
- Choose your network architecture
- Initialize the weights
- Find a learning rate and regularization strength
- Minimize the loss and monitor progress
- Fiddle with knobs

Mini-batch Gradient Descent

Loop:

1. Sample a batch of training data (~100 images)

2. Forwards pass: compute loss (avg. over batch)

3. Backwards pass: compute gradient

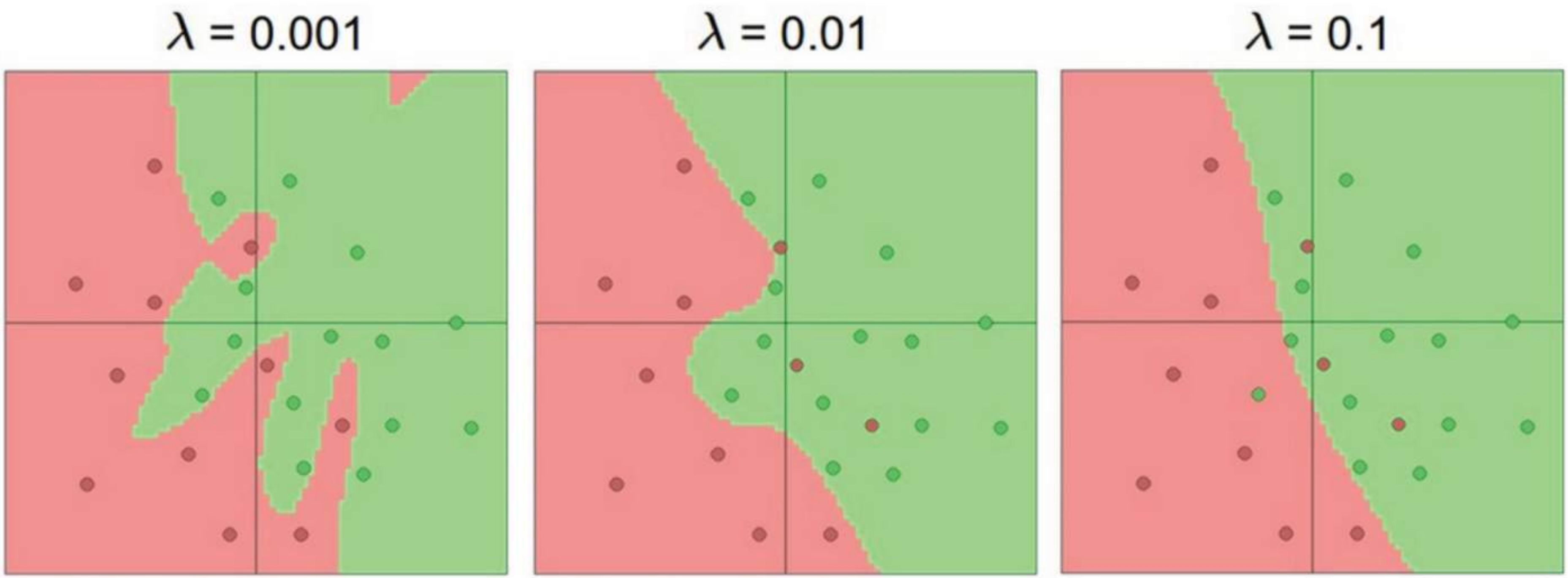
4. Update all parameters

Note: usually called “stochastic gradient descent” even though SGD has a batch size of 1

Regularization

Regularization reduces overfitting:

$$L = L_{\text{data}} + L_{\text{reg}} \qquad L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2$$



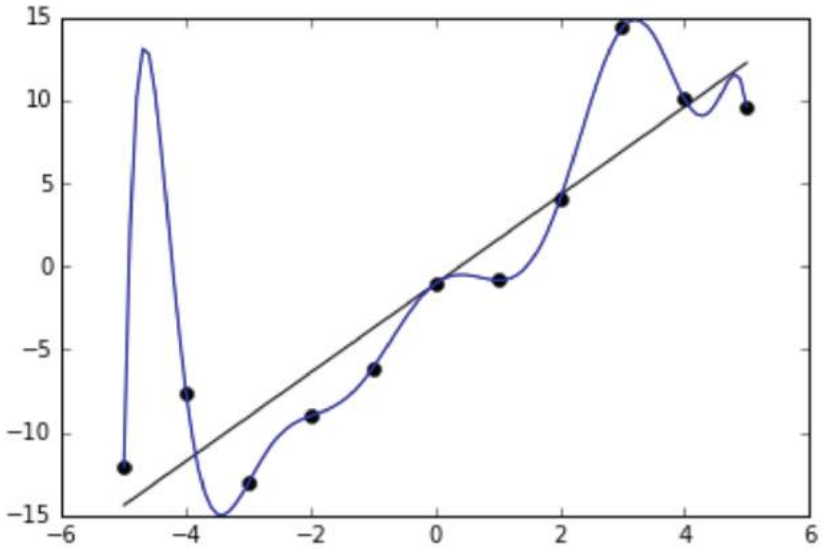
[Andrej Karpathy http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html]

Overfitting

Overfitting: modeling noise in the training set instead of the “true” underlying relationship

Underfitting: insufficiently modeling the relationship in the training set

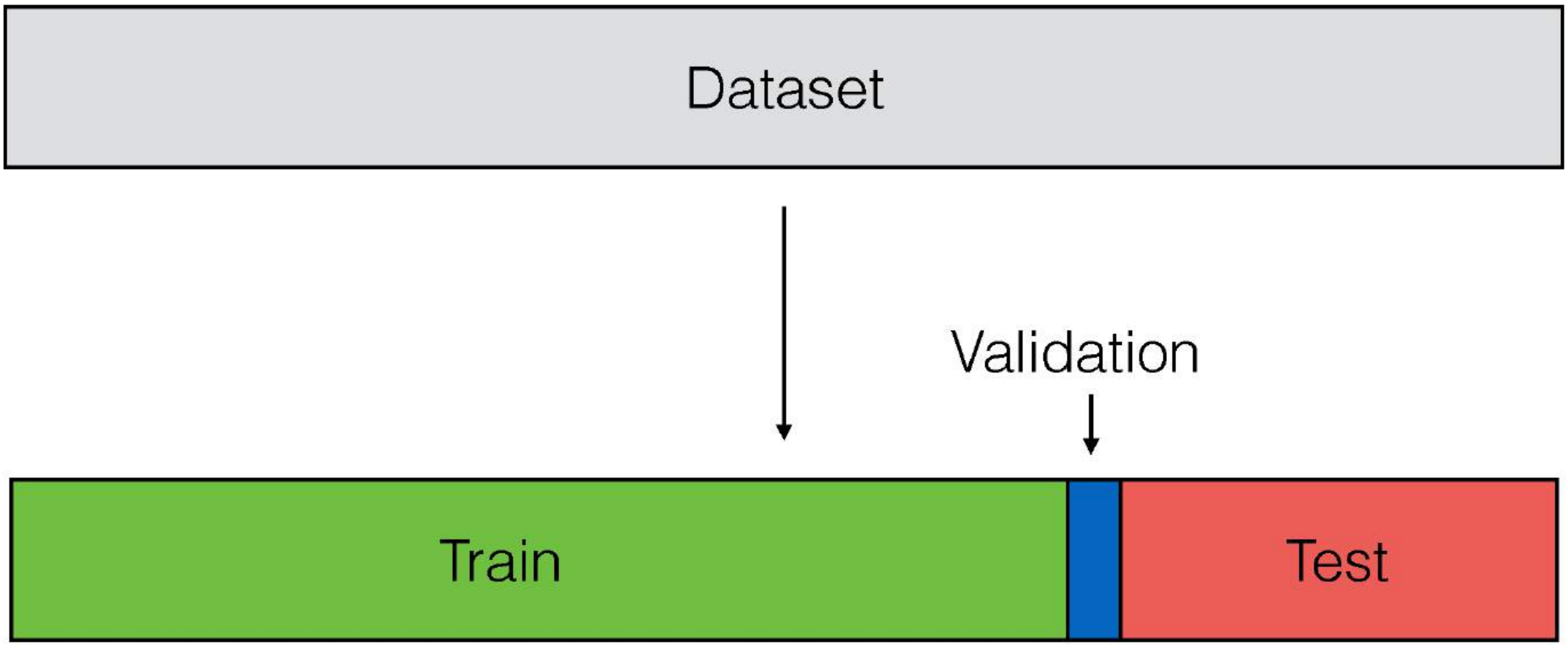
General rule: models that are “bigger” or have more capacity are more likely to overfit



[Image: https://en.wikipedia.org/wiki/File:Overfitted_Data.png]

(0) Dataset split

Split your data into “train”, “validation”, and “test”:



(0) Dataset split

Train: gradient descent and fine-tuning of parameters

Validation: determining hyper-parameters (learning rate, regularization strength, etc) and picking an architecture

Test: estimate real-world performance
(e.g. accuracy = fraction correctly classified)

(0) Dataset split

Be careful with false discovery:

To avoid false discovery, once we have used a test set once, we should *not use it again* (but nobody follows this rule, since it's expensive to collect datasets)

Instead, try and avoid looking at the test score until the end

(1) Data preprocessing

Preprocess the data so that learning is better conditioned:

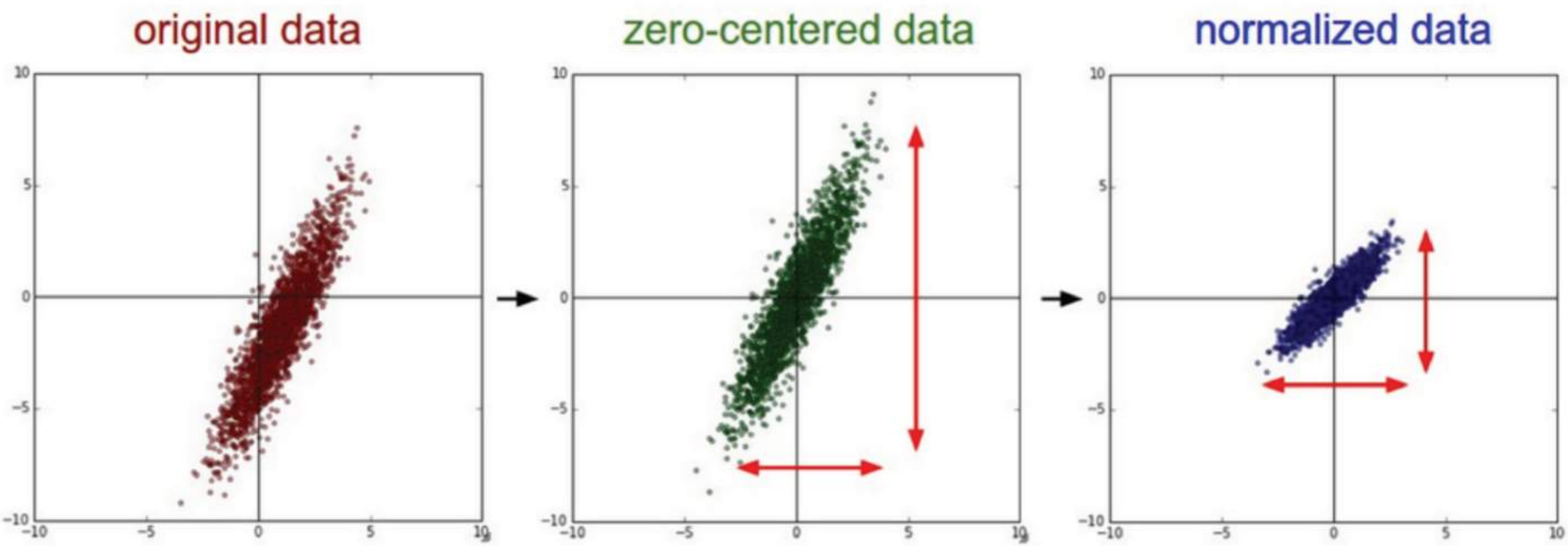
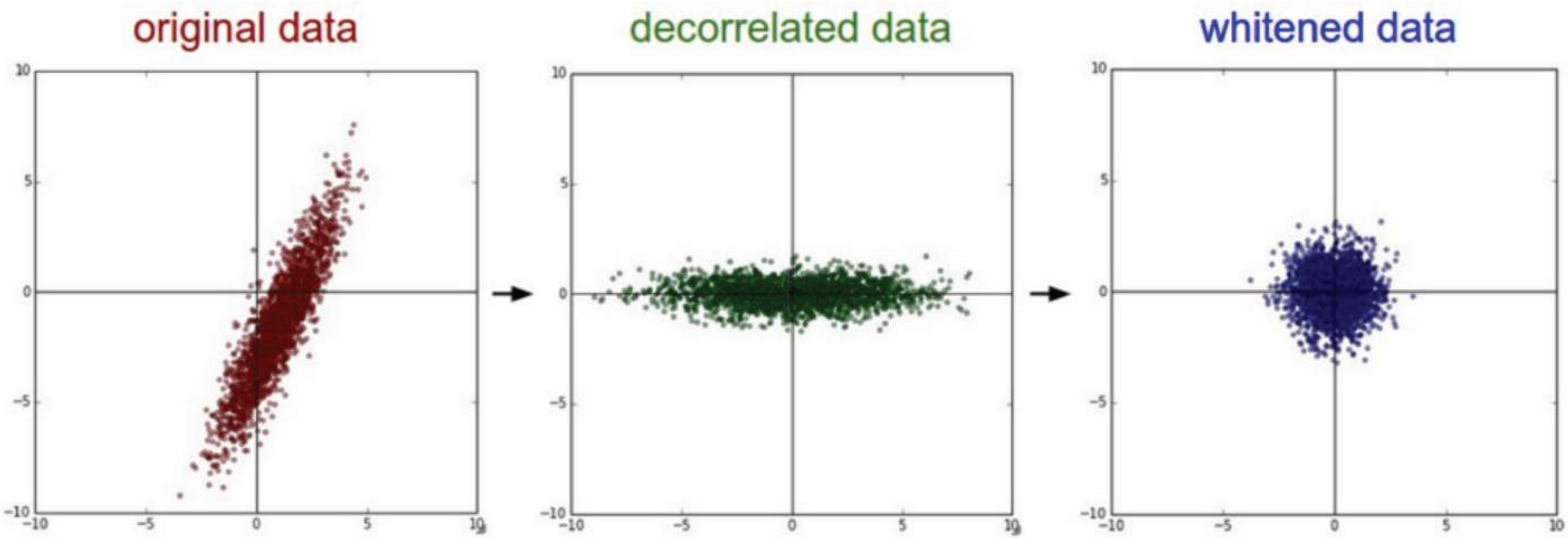


Figure: Andrej Karpathy

(1) Data preprocessing

In practice, you may also see **PCA** and **Whitening** of the data:



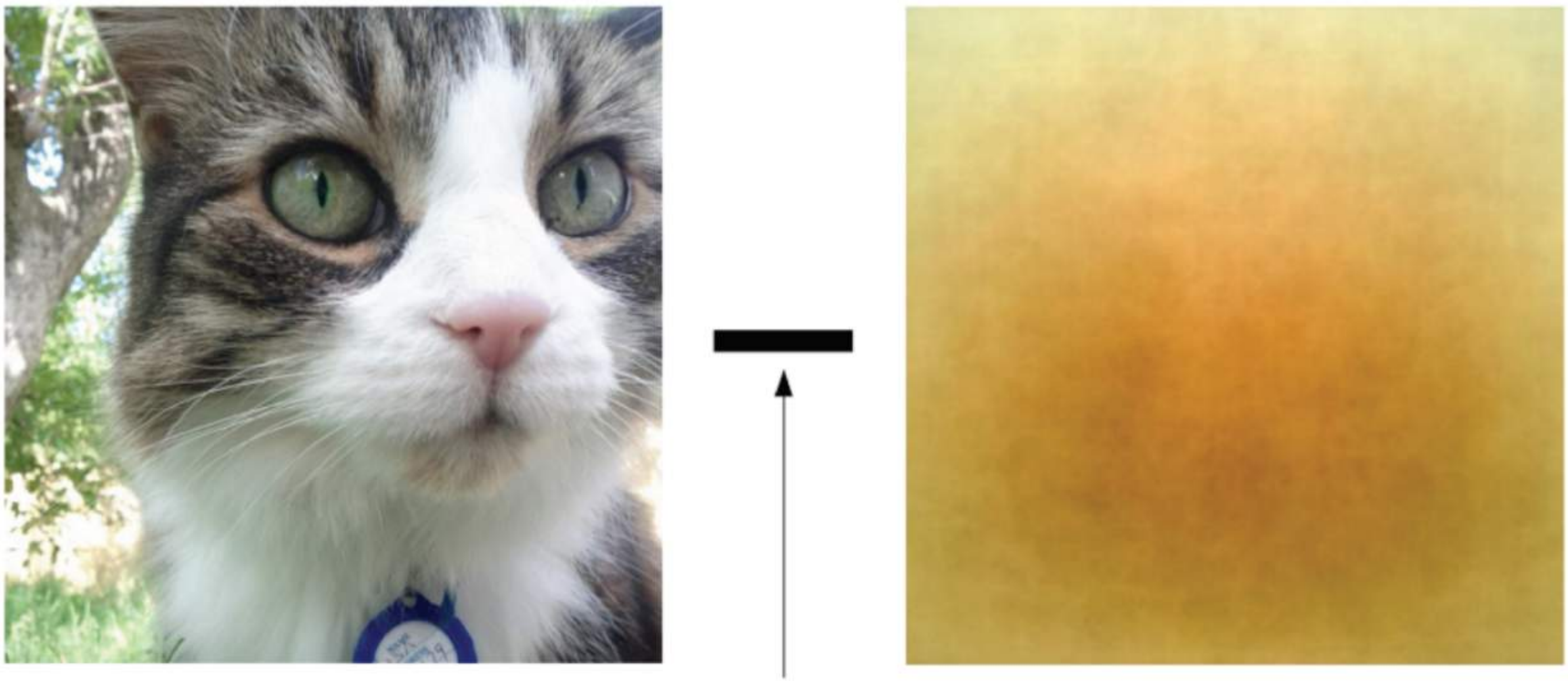
(data has diagonal covariance matrix)

(covariance matrix is the identity matrix)

Slide: Andrej Karpathy

(1) Data preprocessing

For ConvNets, typically only the mean is subtracted.



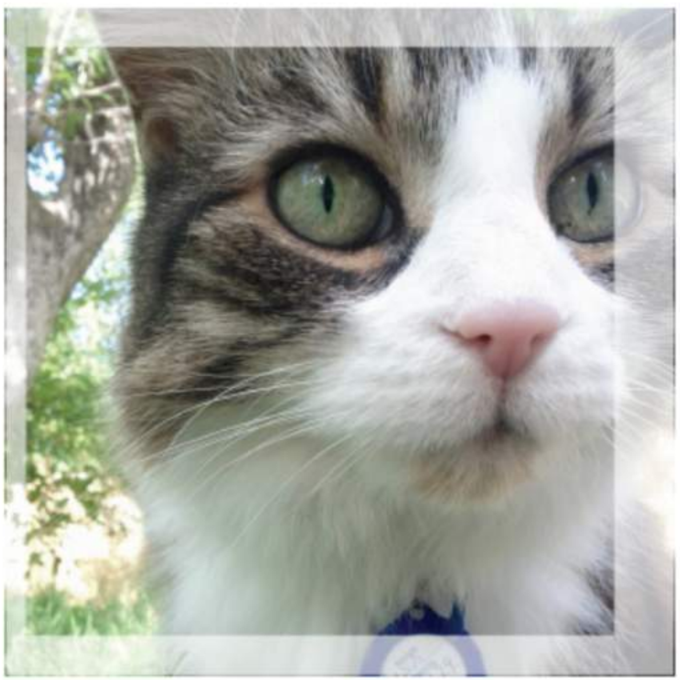
An input image (256x256) — Minus sign — The mean input image

A per-channel mean also works (one value per R,G,B).

Figure: Alex Krizhevsky

(1) Data preprocessing

Augment the data — extract random crops from the input, with slightly jittered offsets. Without this, typical ConvNets (e.g. [Krizhevsky 2012]) overfit the data.



E.g. 224x224 patches extracted from 256x256 images

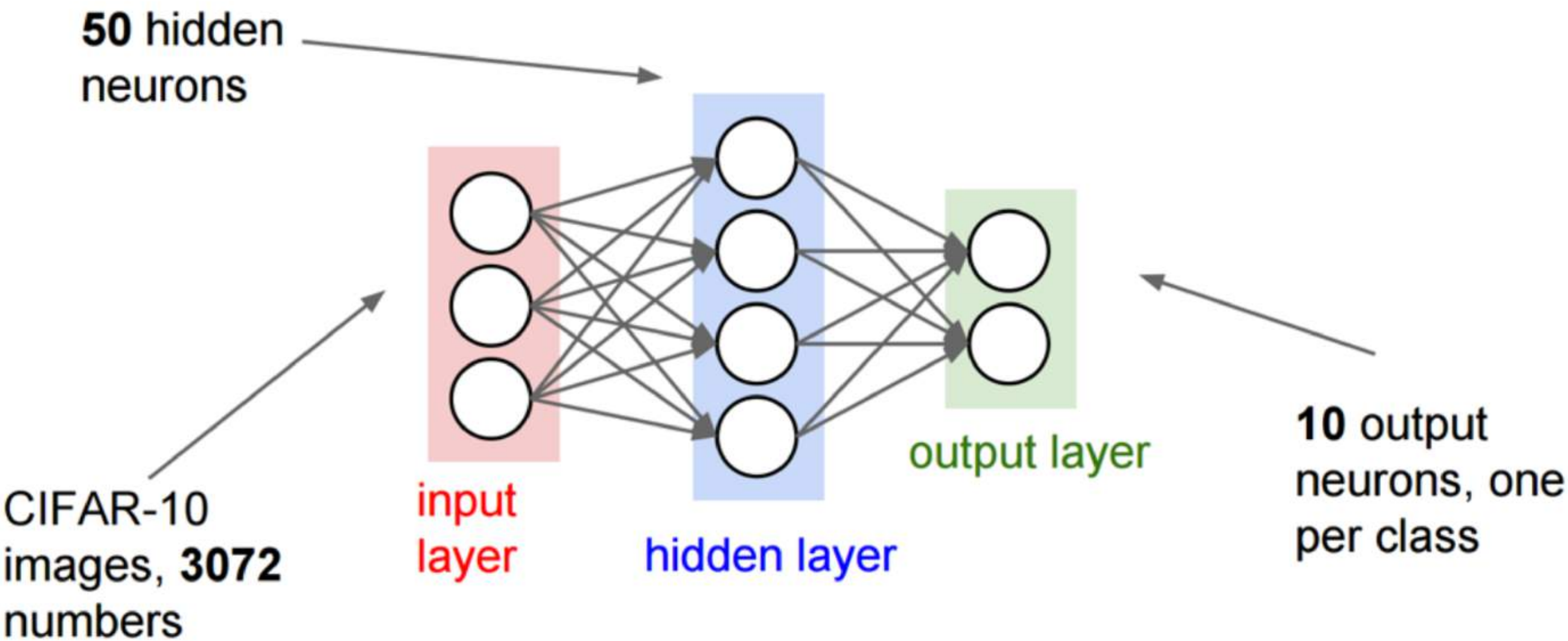
Randomly reflect horizontally

Perform the augmentation live during training

Figure: Alex Krizhevsky

(2) Choose your architecture

Toy example: one hidden layer of size 50



Slide: Andrej Karpathy

(3) Initialize your weights

Set the weights to small random numbers:

```
W = np.random.randn(D, H) * 0.001
```

(matrix of small random numbers drawn from a Gaussian distribution)

(the magnitude is important and this is not optimal — more on this later)

Set the bias to zero (or small nonzero):

```
b = np.zeros(H)
```

Slide: Andrej Karpathy

(3) Check that the loss is reasonable

```
def init_two_layer_model(input_size, hidden_size, output_size):
  # initialize a model
  model = {}
  model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)
  model['b1'] = np.zeros(hidden_size)
  model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)
  model['b2'] = np.zeros(output_size)
  return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
loss, grad = two_layer_net(X_train, model, y_train, 0.0)
print loss
```

disable regularization

returns the loss and the gradient for all parameters

Slide: Andrej Karpathy

(3) Check that the loss is reasonable

```python
def init_two_layer_model(input_size, hidden_size, output_size):
  # initialize a model
  model = {}
  model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)
  model['b1'] = np.zeros(hidden_size)
  model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)
  model['b2'] = np.zeros(output_size)
  return model
```

```python
model = init_two_layer_model(32*32*3, 50, 10) # input_size, hidden size, number of classes
loss, grad = two_layer_net(X_train, model, y_train, 1e3)
print loss
```

crank up regularization

loss went up, good. (sanity check)

Slide: Andrej Karpathy

(4) Overfit a small portion of the data

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
                                  model, two_layer_net,
                                  num_epochs=200, reg=0.0,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = False,
                                  learning_rate=1e-3, verbose=True)
```

Details:

'sgd': vanilla gradient descent (no momentum etc)

learning_rate_decay = 1: constant learning rate

sample_batches = False (full gradient descent, no batches)

epochs = 200: number of passes through the data

Slide: Andrej Karpathy

(4) Overfit a small portion of the data

100% accuracy on the training set (good)

```
Finished epoch 1 / 200: cost 2.302603, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 2 / 200: cost 2.302258, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 3 / 200: cost 2.301849, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 4 / 200: cost 2.301196, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 5 / 200: cost 2.300044, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 6 / 200: cost 2.297864, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 7 / 200: cost 2.293595, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 8 / 200: cost 2.285096, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 9 / 200: cost 2.268094, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 10 / 200: cost 2.234787, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 11 / 200: cost 2.173187, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 12 / 200: cost 2.076862, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 13 / 200: cost 1.974090, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 14 / 200: cost 1.895885, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 15 / 200: cost 1.820876, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 16 / 200: cost 1.737430, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 17 / 200: cost 1.642356, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 18 / 200: cost 1.535239, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 19 / 200: cost 1.421527, train: 0.600000, val 0.600000, lr 1.000000e-03
```

```
Finished epoch 195 / 200: cost 0.002694, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 196 / 200: cost 0.002674, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 197 / 200: cost 0.002655, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 198 / 200: cost 0.002635, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 199 / 200: cost 0.002617, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 200 / 200: cost 0.002597, train: 1.000000, val 1.000000, lr 1.000000e-03
finished optimization. best validation accuracy: 1.000000
```

Slide: Andrej Karpathy

(4) Find a learning rate

Let's start with small regularization and find the learning rate that makes the loss decrease:

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e-6, verbose=True)
```

(4) Find a learning rate

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e-6, verbose=True)
```

```
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best validation accuracy: 0.192000
```

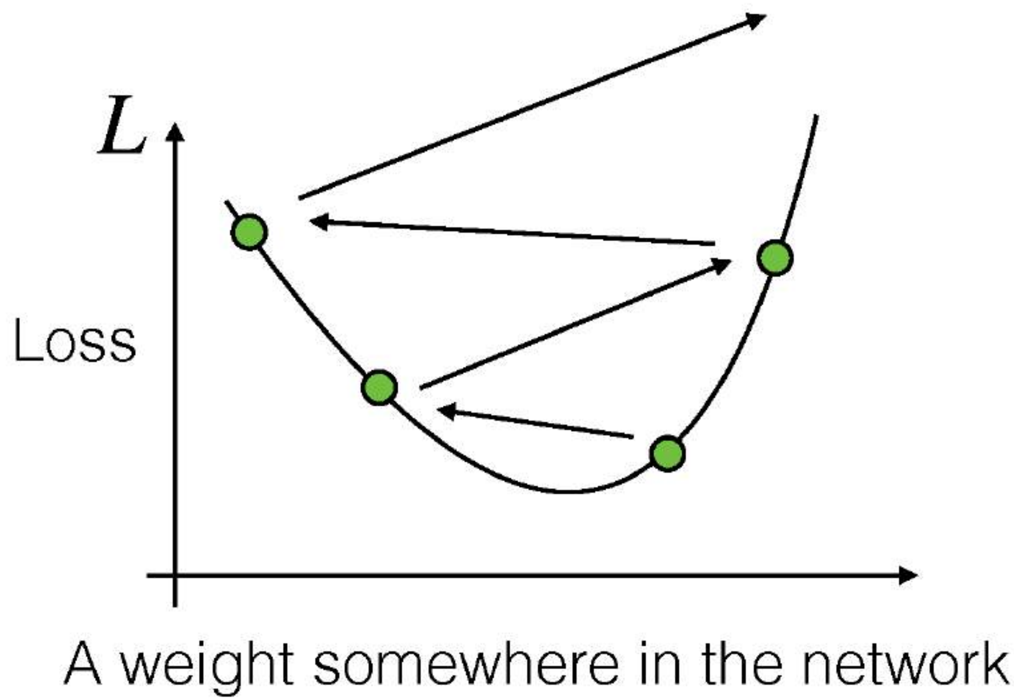
Loss barely changes **Why is the accuracy 20%?**

(learning rate is too low or regularization too high)

Slide: Andrej Karpathy

(4) Find a learning rate

Learning rate: 1e6 — what could go wrong?



(4) Find a learning rate

Coarse to fine search

First stage: only a few epochs (passes through the data) to get a rough idea

Second stage: longer running time, finer search

Tip: if loss > 3 * original loss, quit early
(learning rate too high)

Slide: Andrej Karpathy

(4) Find a learning rate

Normally, you don't have the budget for lots of cross-validation —> visualize as you go

Plot the loss

For very small learning rates, the loss decreases linearly and slowly

(Why linearly?)

Larger learning rates tend to look more exponential

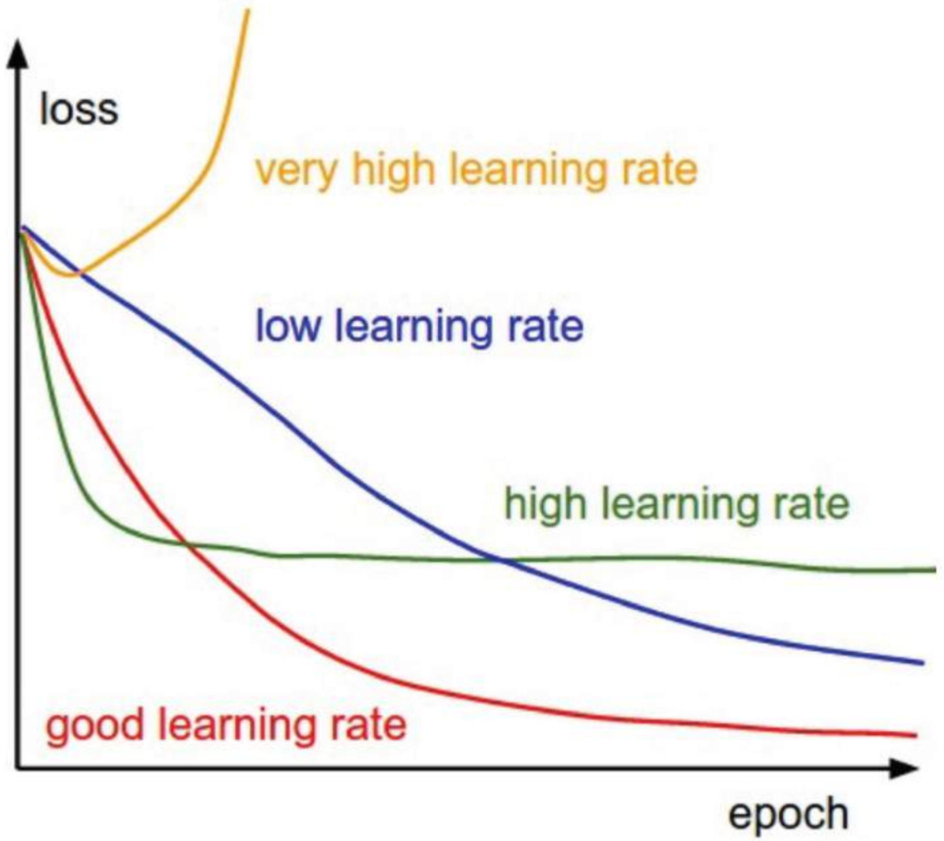


Figure: Andrej Karpathy

(4) Find a learning rate

Normally, you don't have the budget for lots of cross-validation —> visualize as you go

Typical training loss:

Why is it varying so rapidly?

The width of the curve is related to the batchsize — if too noisy, increase the batch size

Possibly too linear (learning rate too small)

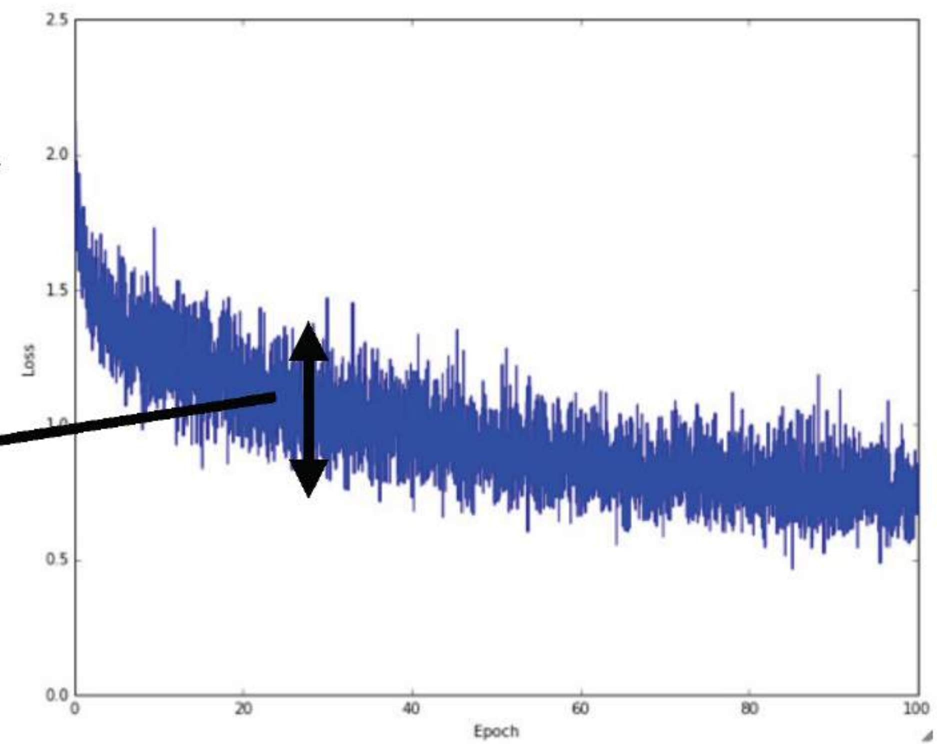
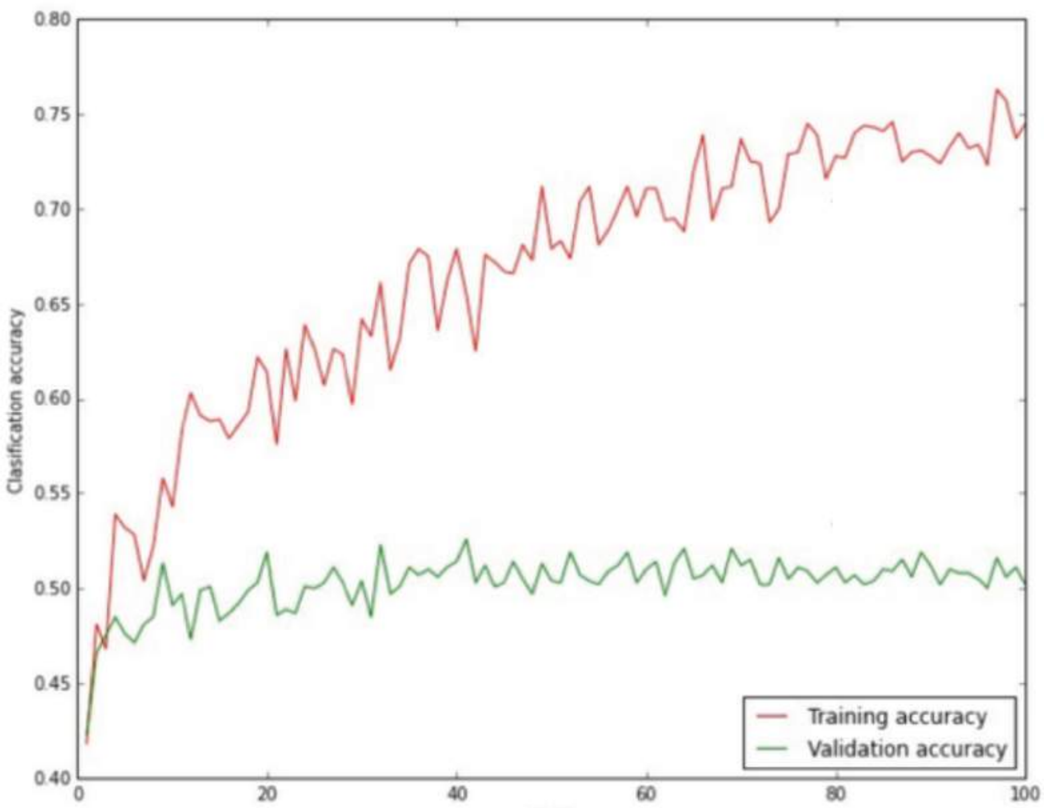


Figure: Andrej Karpathy

(4) Find a learning rate

Visualize the accuracy



Big gap: overfitting
(increase regularization)

No gap: underfitting
(increase model capacity,
make layers bigger
or decrease regularization)

Figure: Andrej Karpathy

(4) Find a learning rate

Visualize the weights

Noisy weights: possibly regularization not strong enough

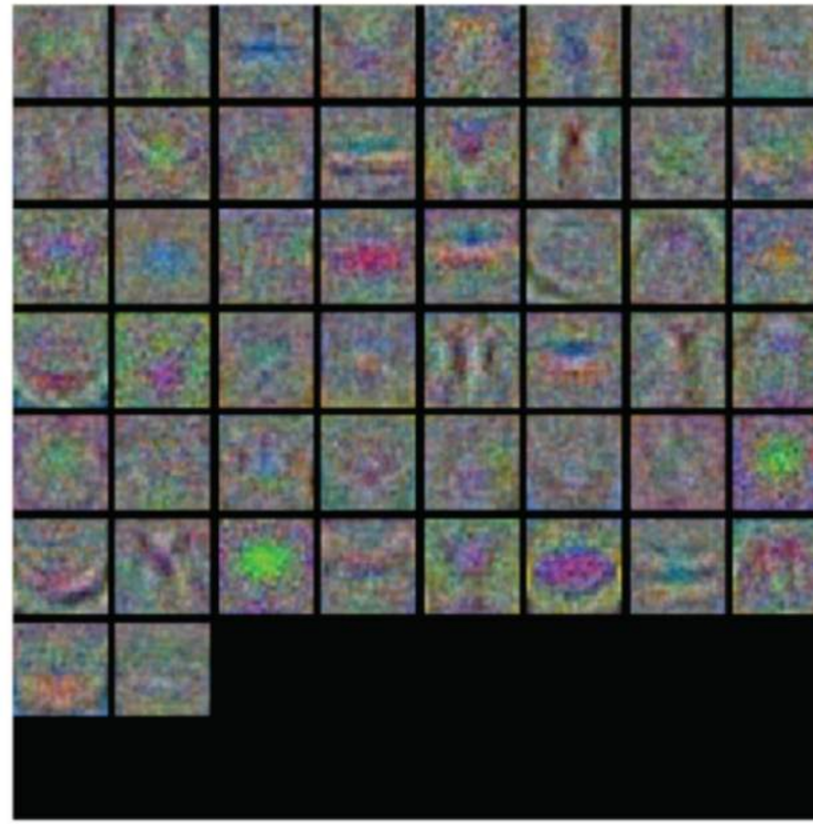
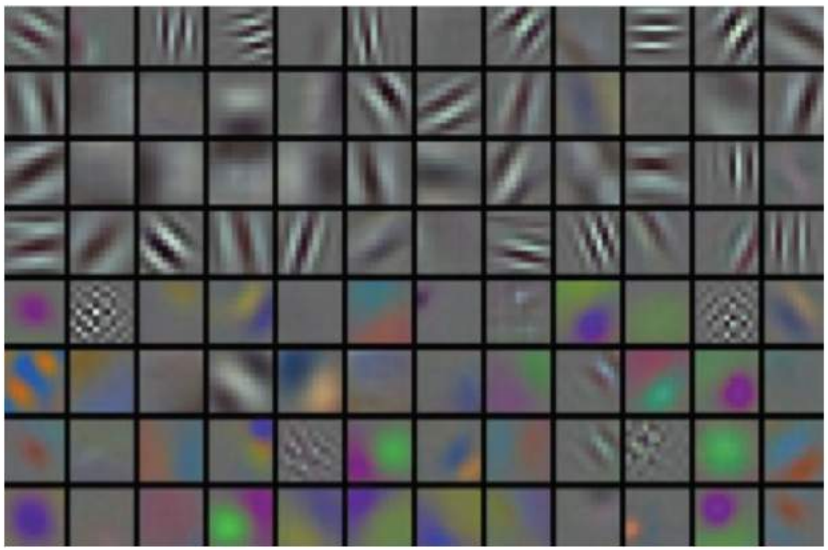


Figure: Andrej Karpathy

(4) Find a learning rate

Visualize the weights



Nice clean weights:
training is proceeding well

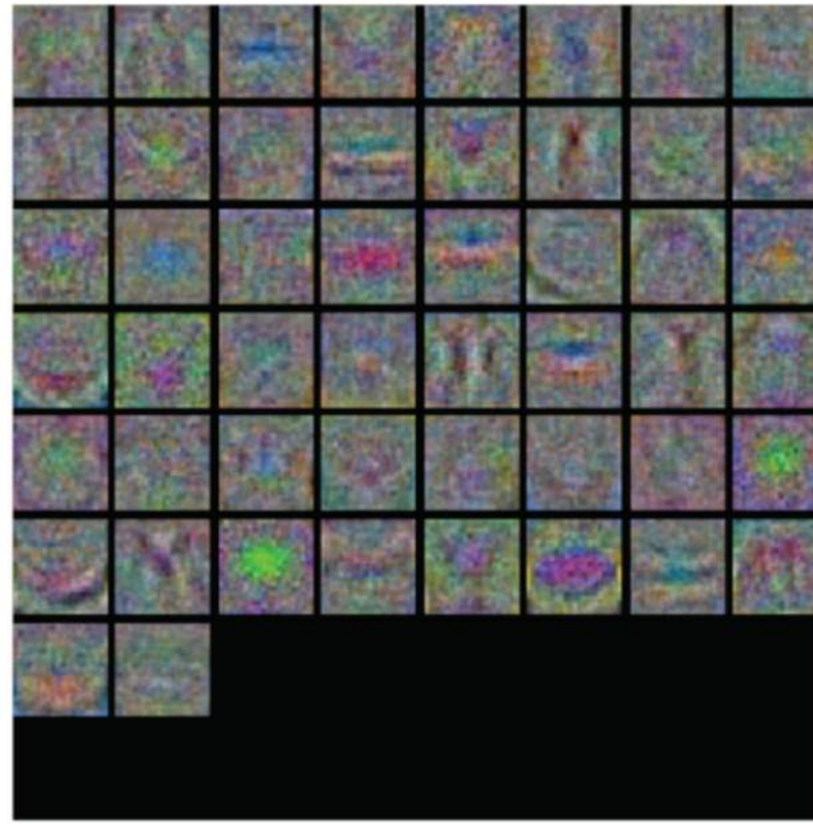


Figure: Alex Krizhevsky , Andrej Karpathy

Learning rate schedule

How do we change the learning rate over time?

Various choices:

- Step down by a factor of 0.1 every 50,000 mini-batches (used by SuperVision [Krizhevsky 2012])
- Decrease by a factor of 0.97 every epoch (used by GoogLeNet [Szegedy 2014])
- Scale by sqrt(1-t/max_t) (used by BVLC to re-implement GoogLeNet)
- Scale by 1/t
- Scale by exp(-t)

Summary of things to fiddle

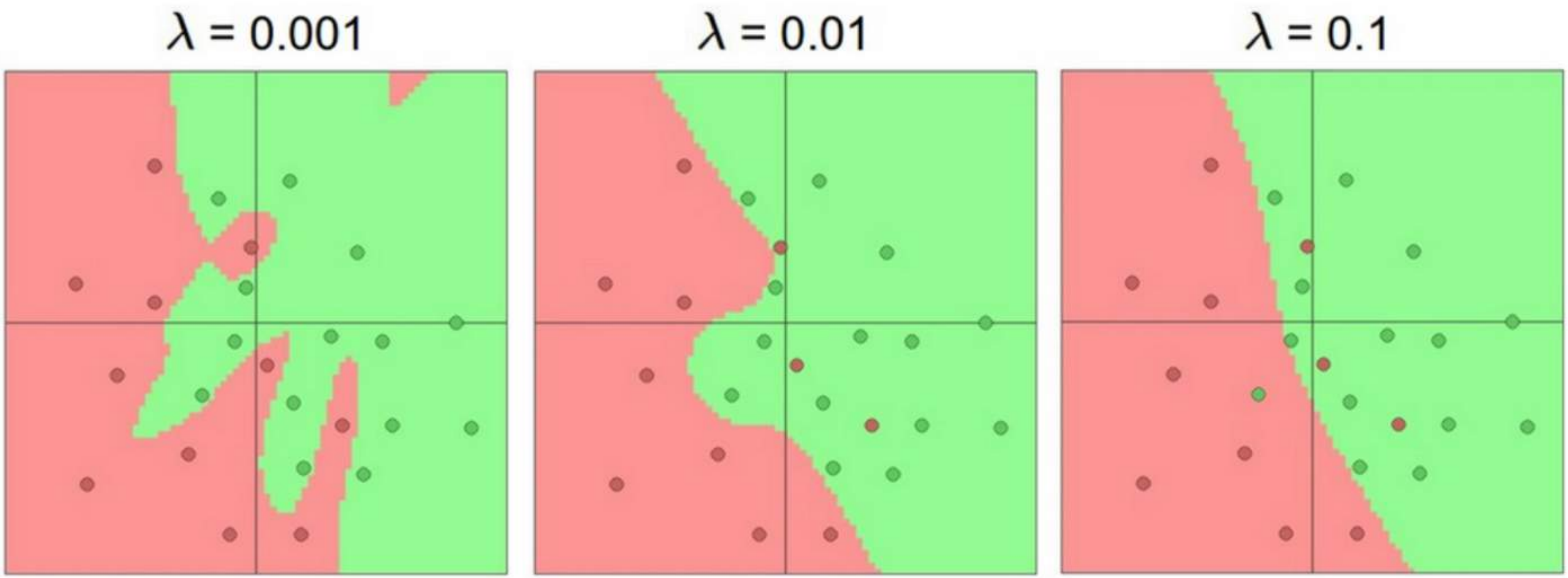
- Network architecture
- Learning rate, decay schedule, update type
- Regularization (L2, L1, maxnorm, dropout, …)
- Loss function (softmax, SVM, …)
- Weight initialization

Neural network parameters

(Recall) Regularization reduces overfitting

$$L = L_{\text{data}} + L_{\text{reg}}$$

$$L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2$$



[Andrej Karpathy http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html]

Example Regularizers

L2 regularization

$$L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2$$

(L2 regularization encourages small weights)

L1 regularization

$$L_{\text{reg}} = \lambda \|W\|_1 = \lambda \sum_{ij} |W_{ij}|$$

(L1 regularization encourages sparse weights: weights are encouraged to reduce to exactly zero)

"Elastic net"

$$L_{\text{reg}} = \lambda_1 \|W\|_1 + \lambda_2 \|W\|_2^2$$

(combine L1 and L2 regularization)

Max norm

Clamp weights to some max norm

$$\|W\|_2^2 \leq c$$

"Weight decay"

Regularization is also called "weight decay" because the weights "decay" each iteration:

$$L_{\text{reg}} = \lambda \frac{1}{2} \|W\|_2^2 \longrightarrow \frac{\partial L}{\partial W} = \lambda W$$

Gradient descent step:

$$W \leftarrow W - \alpha\lambda W - \frac{\partial L_{\text{data}}}{\partial W}$$

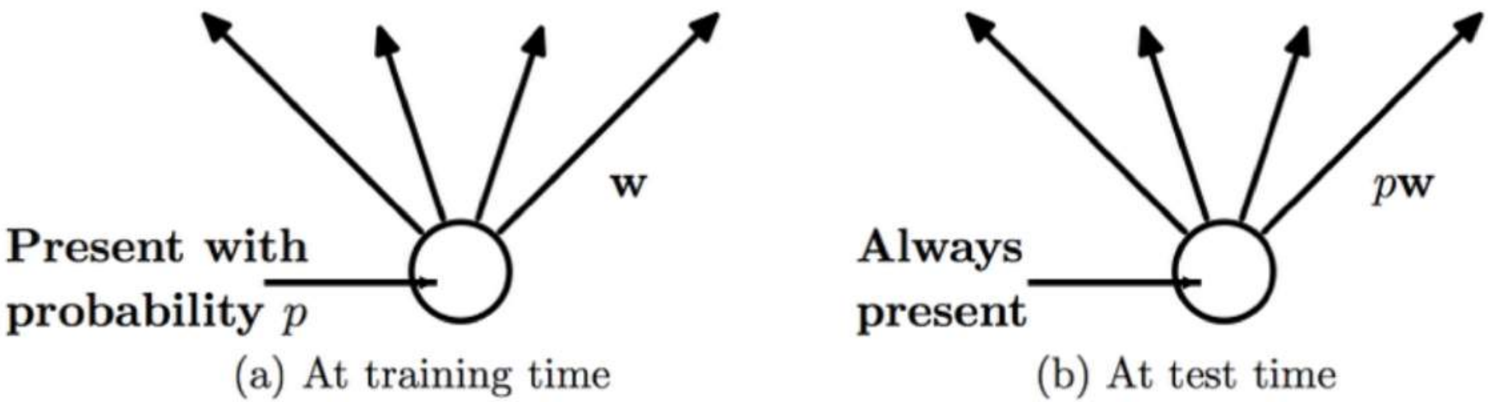
Weight decay: $\alpha\lambda$ (weights always decay by this amount)

Note: biases are sometimes excluded from regularization

[Andrej Karpathy http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html]

Dropout

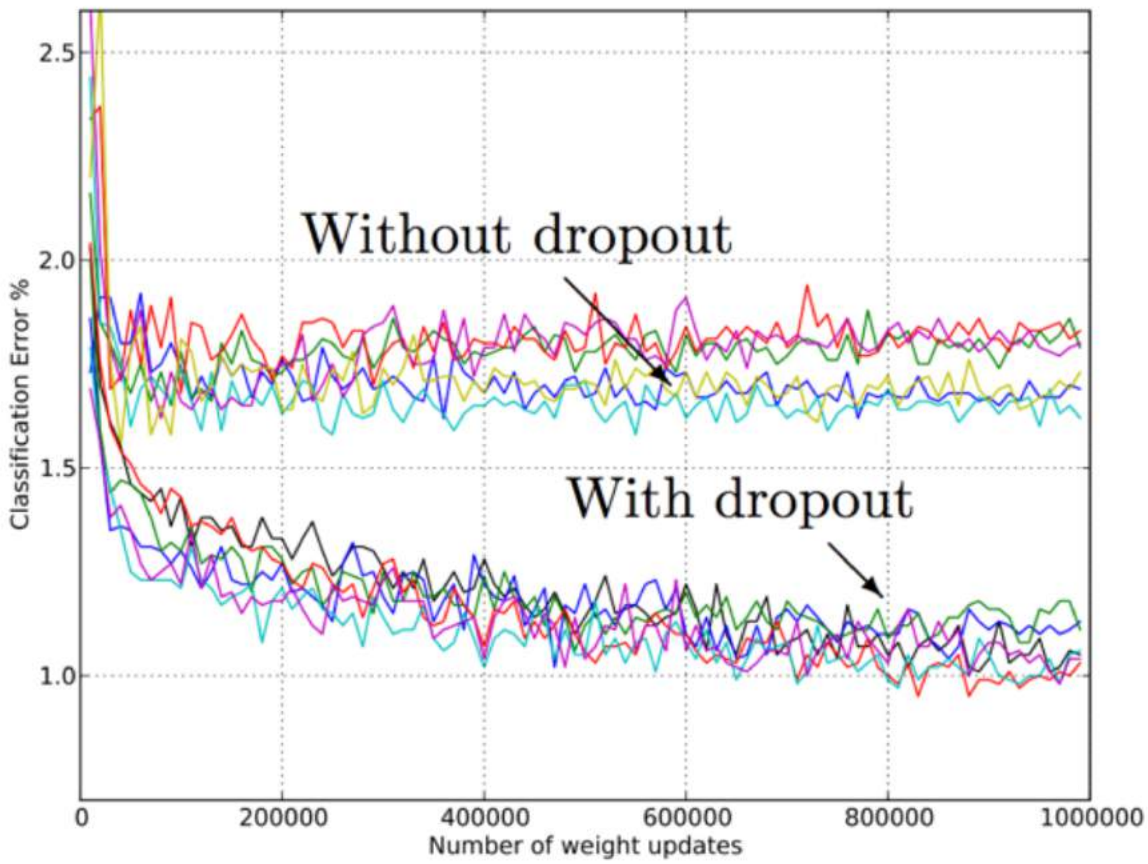
Simple but powerful technique to reduce overfitting:



[Srivasta et al, "Dropout: A Simple Way to Prevent Neural Networks from Overfitting", JMLR 2014]

Dropout

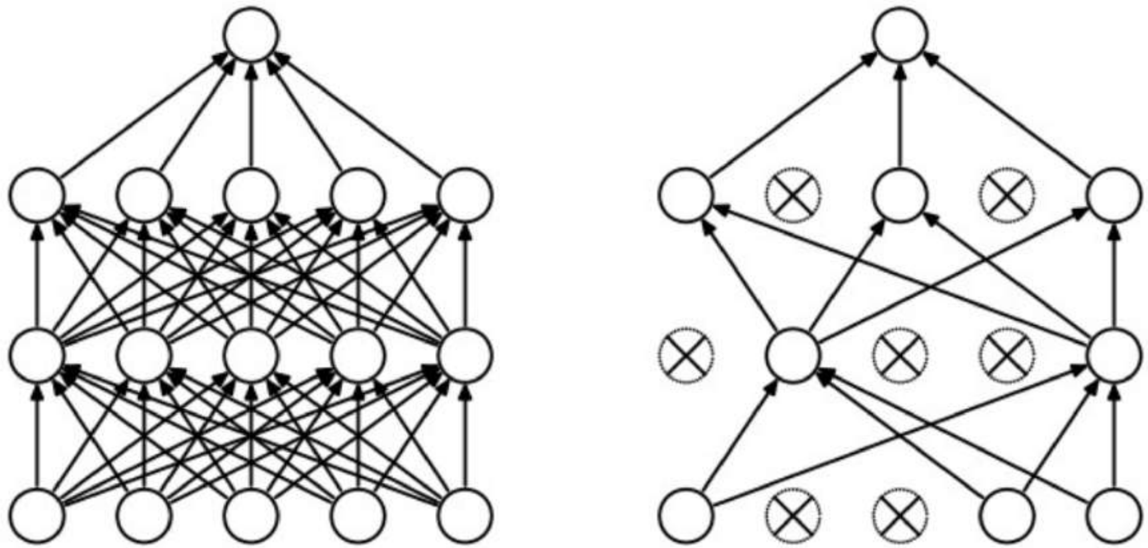
Simple but powerful technique to reduce overfitting:



[Srivasta et al, "Dropout: A Simple Way to Prevent Neural Networks from Overfitting", JMLR 2014]

Dropout

Simple but powerful technique to reduce overfitting:



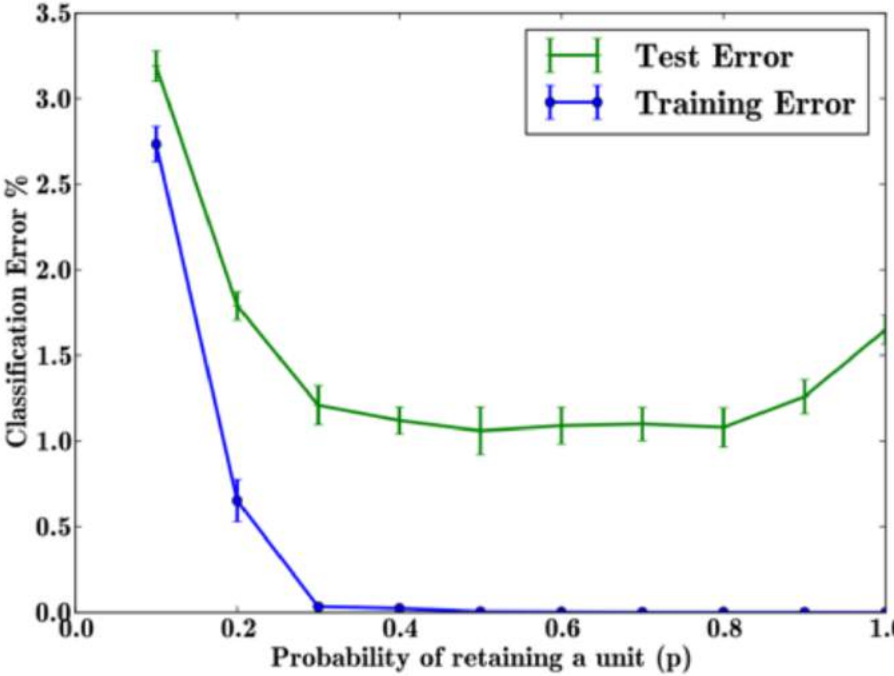
(a) Standard Neural Net (b) After applying dropout.

Note: Dropout can be interpreted as an approximation to taking the geometric mean of an ensemble of exponentially many models

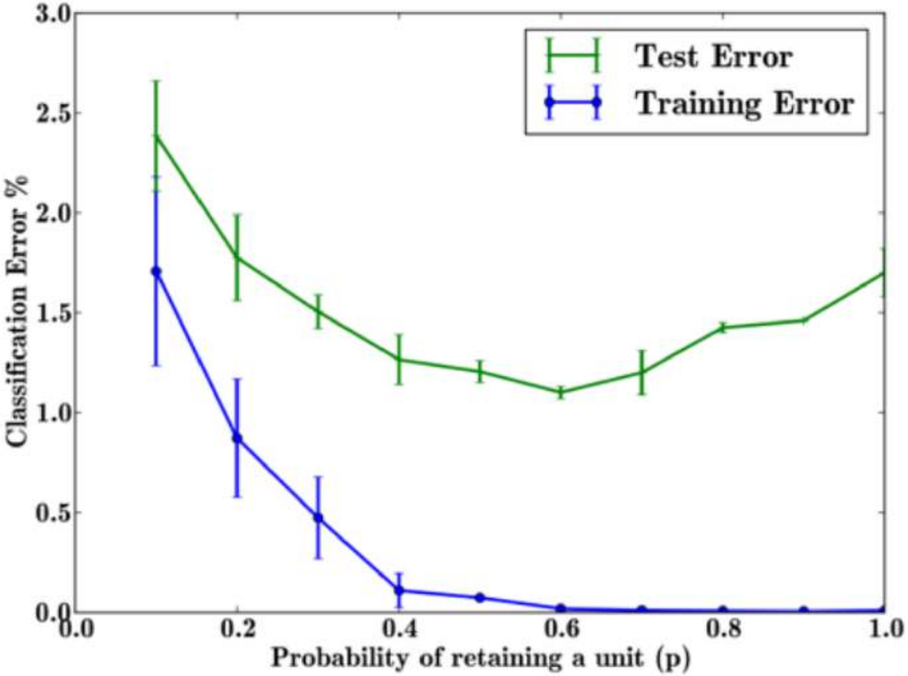
[Srivasta et al, “Dropout: A Simple Way to Prevent Neural Networks from Overfitting”, JMLR 2014]

Dropout

How much dropout? Around p = 0.5



(a) Keeping n fixed.



(b) Keeping pn fixed.

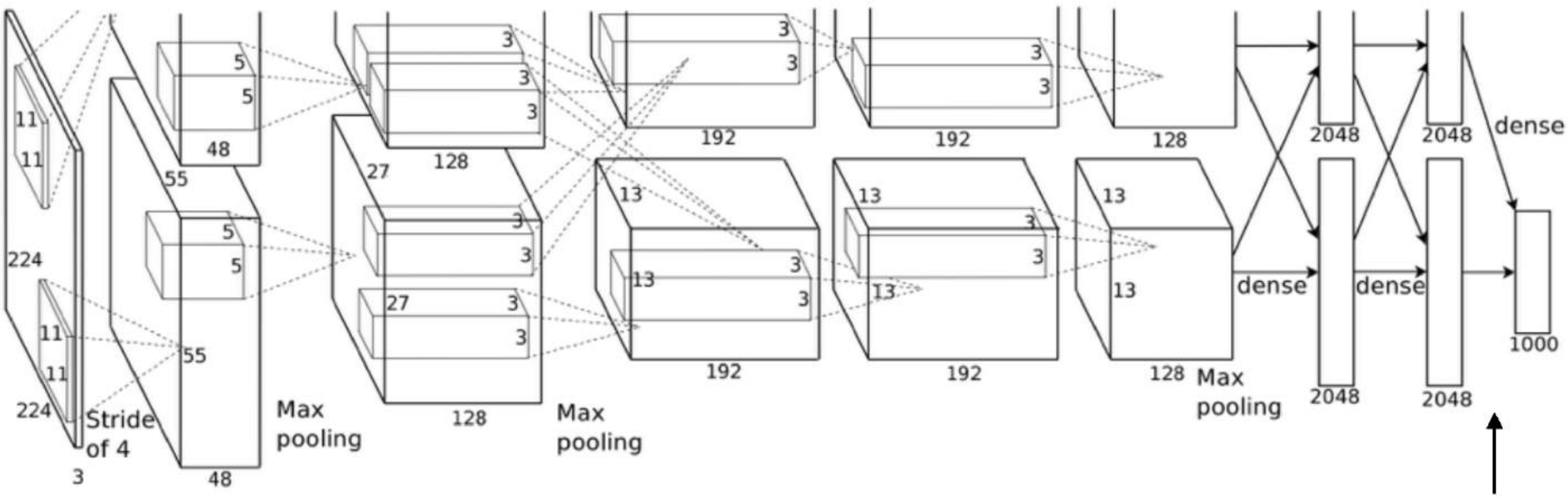
[Srivasta et al, “Dropout: A Simple Way to Prevent Neural Networks from Overfitting”, JMLR 2014]

Dropout

Case study: [Krizhevsky 2012]

“Without dropout, our network exhibits substantial overfitting.”

Dropout here



But not here — why?

[Krizhevsky et al, “ImageNet Classification with Deep Convolutional Neural Networks”, NIPS 2012]

Dropout

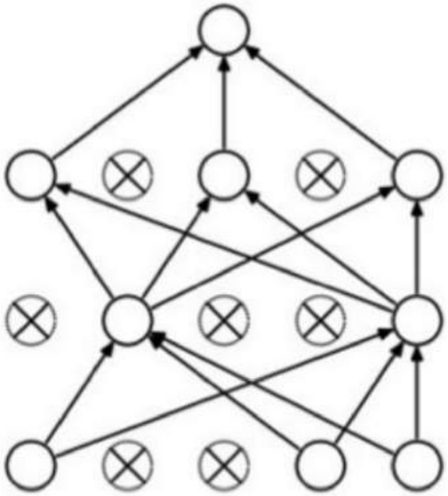
```python
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  """ X contains the data """

  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = np.random.rand(*H1.shape) < p # first dropout mask
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = np.random.rand(*H2.shape) < p # second dropout mask
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)
```

Example forward pass with a 3-layer network using dropout



(note, here X is a single input)

Figure: Andrej Karpathy

Dropout

Test time: scale the activations

Expected value of a neuron *h* with dropout:

$$E[h] = ph + (1-p)0 = ph$$

```python
def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
  H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
  out = np.dot(W3, H2) + b3
```

We want to keep the same expected value

Figure: Andrej Karpathy

Summary

- Preprocess the data (subtract mean, sub-crops)
- Initialize weights carefully
- Use Dropout
- Use SGD + Momentum
- Fine-tune from ImageNet
- Babysit the network as it trains