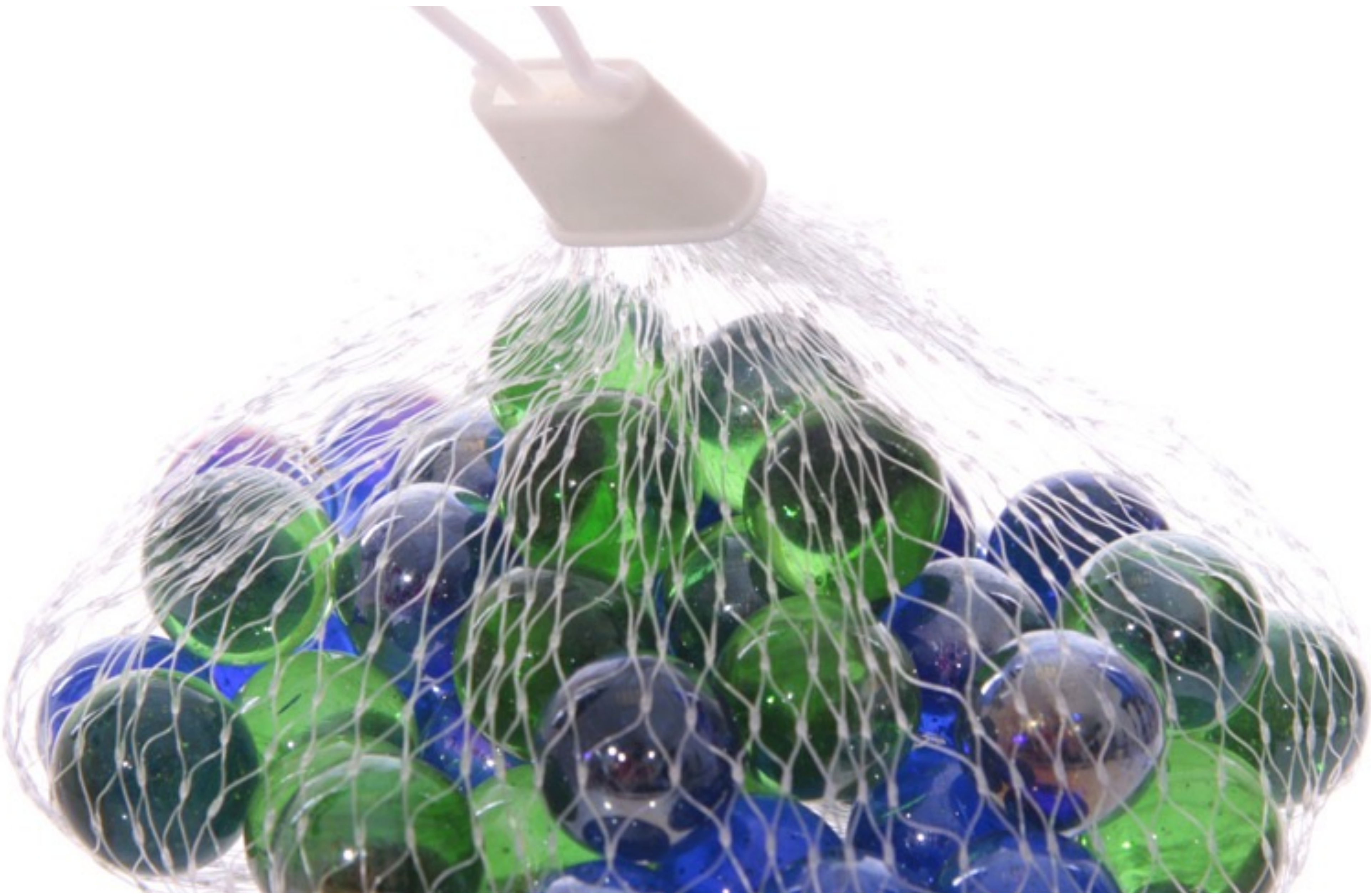
Image classification



16-385 Computer Vision
Spring 2022, Lecture 13 & 14

http://16385.courses.cs.cmu.edu/

Overview of today's lecture

- Introduction to learning-based vision.
- Image classification.
- Bag-of-words.
- K-means clustering.
- Classification.
- K nearest neighbors.
- Naïve Bayes.
- Support vector machine.

Slide credits

Most of these slides were adapted from:

- Kris Kitani (16-385, Spring 2017).
- Noah Snavely (Cornell University).
- Fei-Fei Li (Stanford University).

Course overview

1. Image processing. ⟵ Lectures 1 – 6
   See also 18-793: Image and Video Processing

2. Geometry-based vision. ⟵ Lectures 7 – 12
   See also 16-822: Geometry-based Methods in Vision

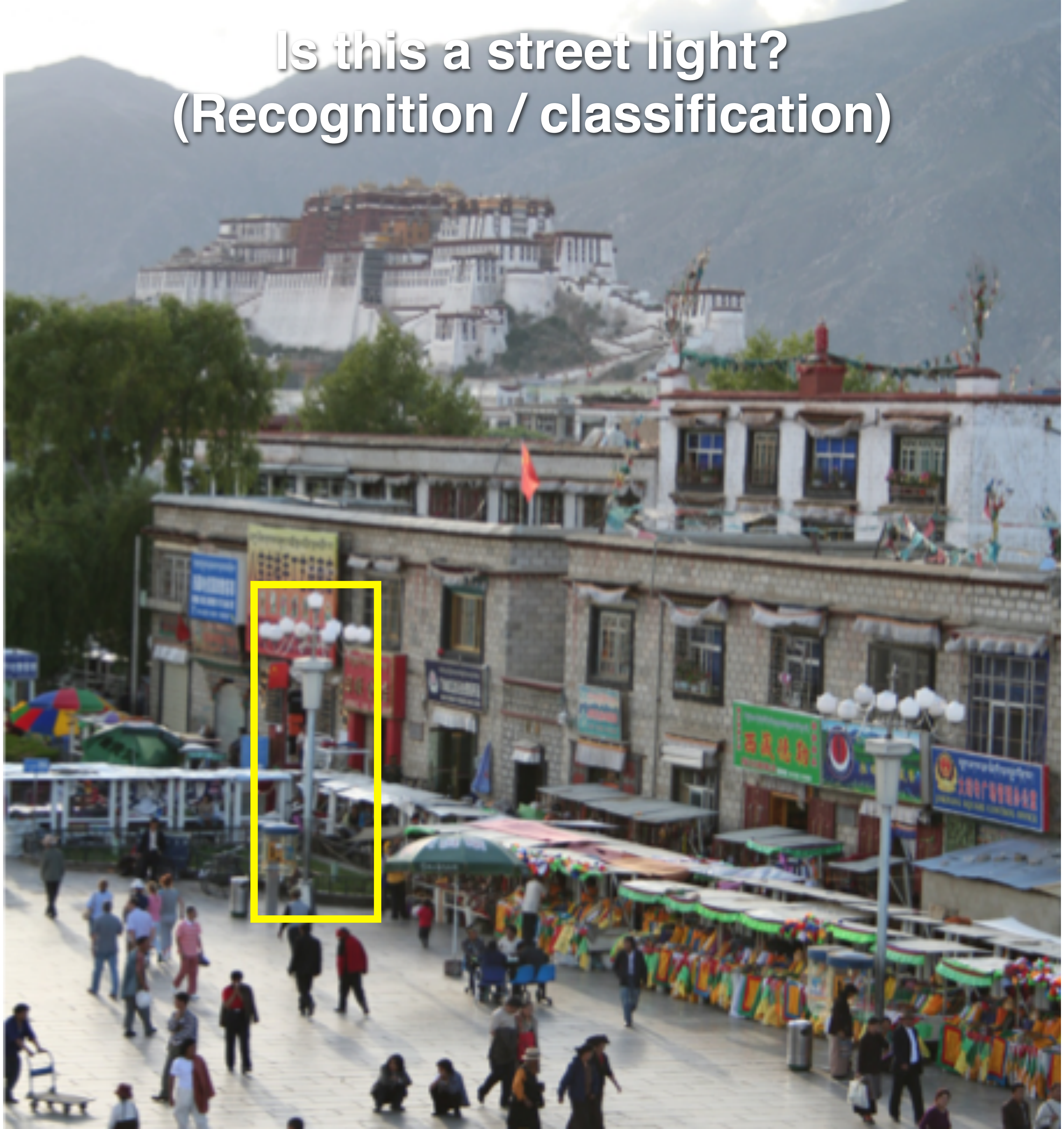
3. Learning-based vision. ⟵ We are starting this part now

4. Dealing with motion.

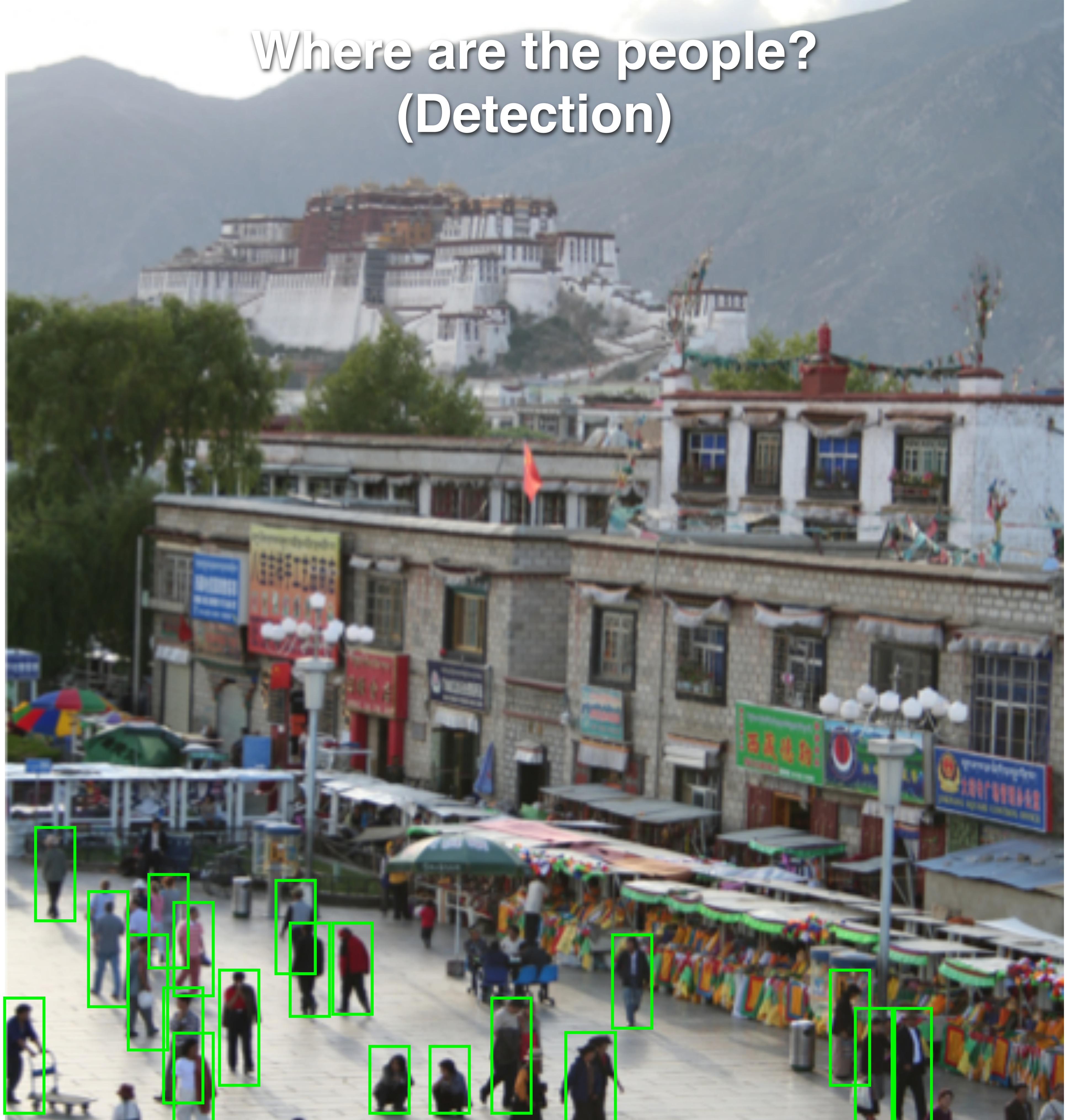
5. Physics-based vision.

What do we mean by learning-based vision or ‘semantic vision’?

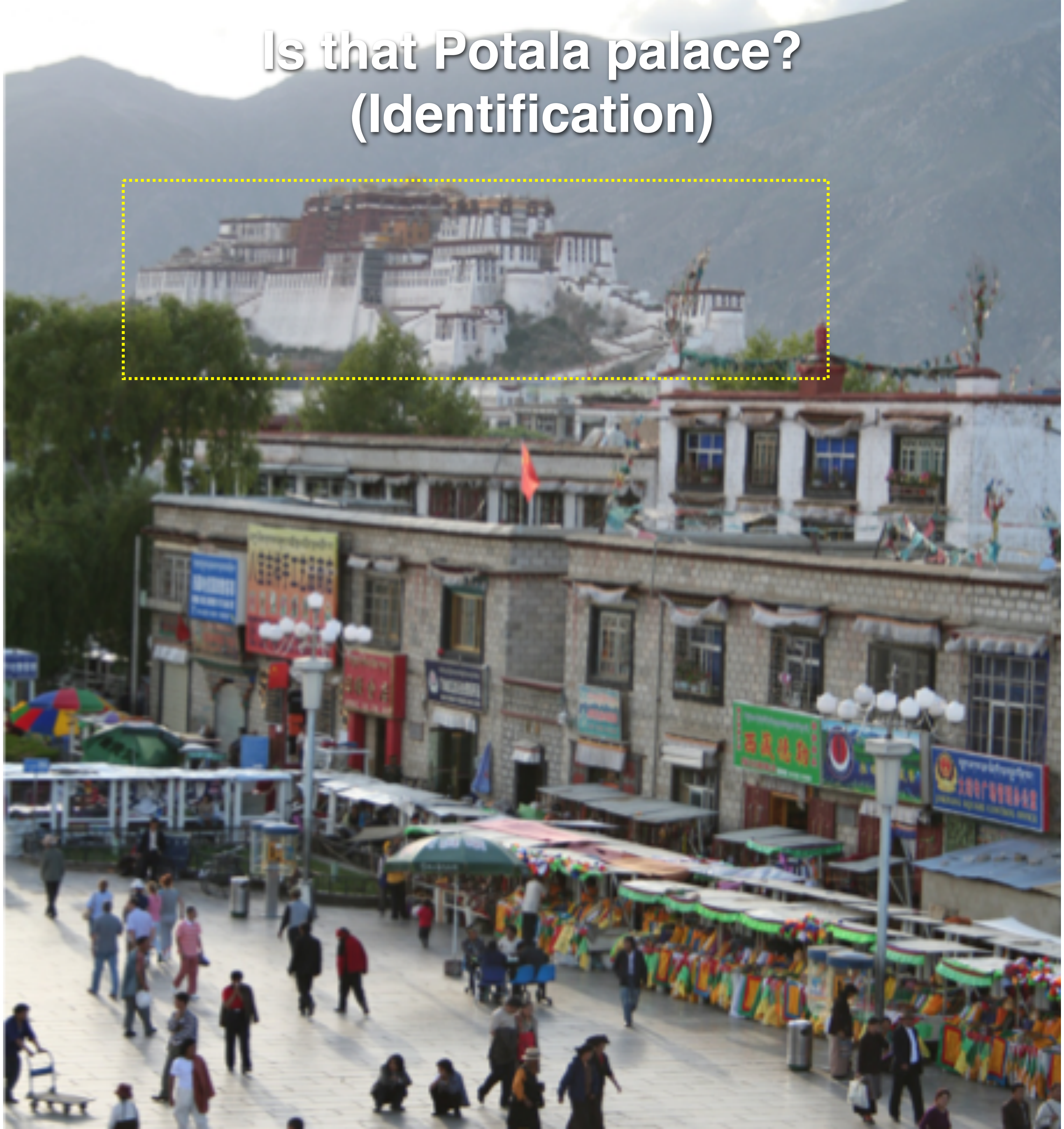
Is this a street light?
(Recognition / classification)



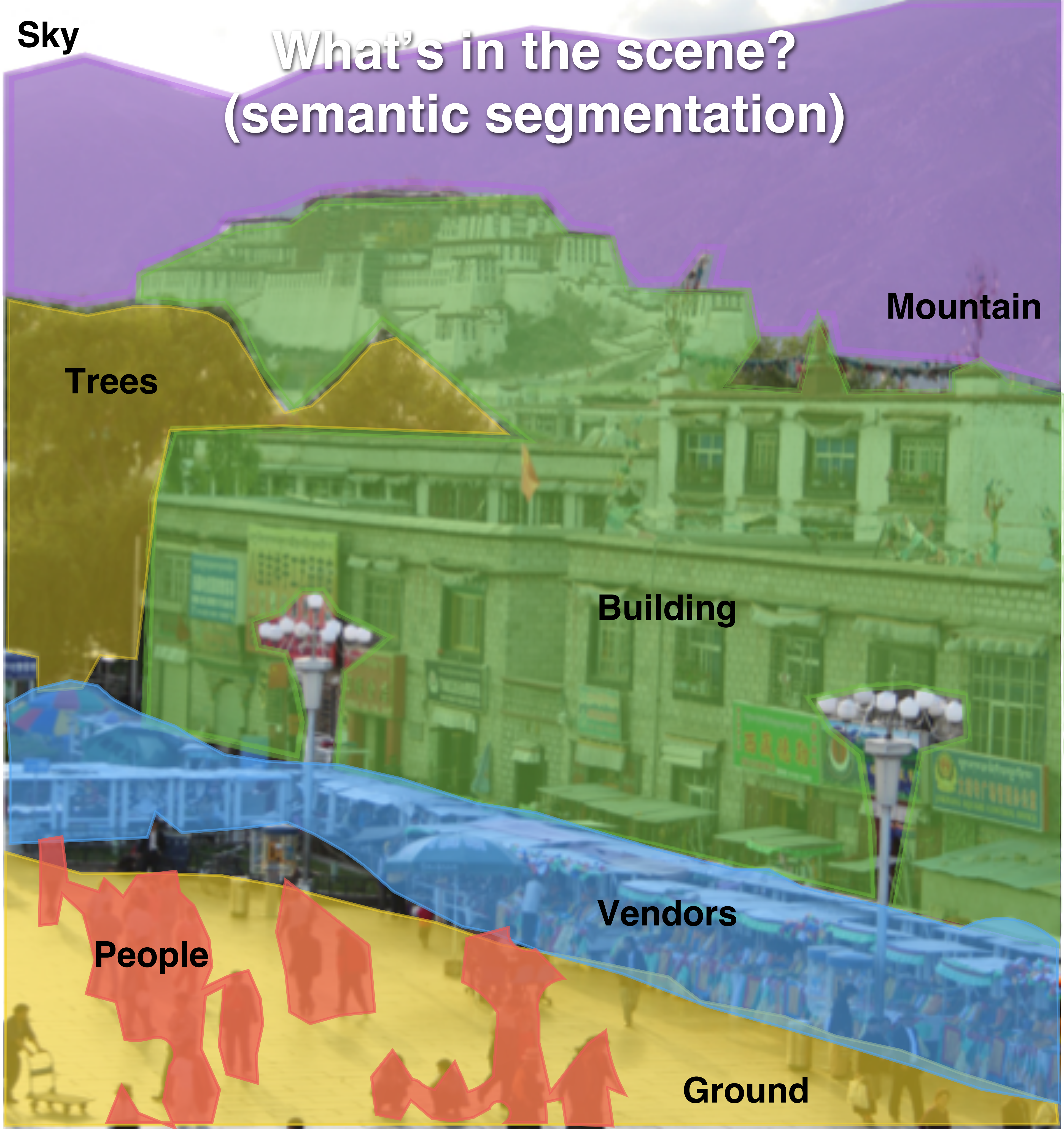
Where are the people?
(Detection)



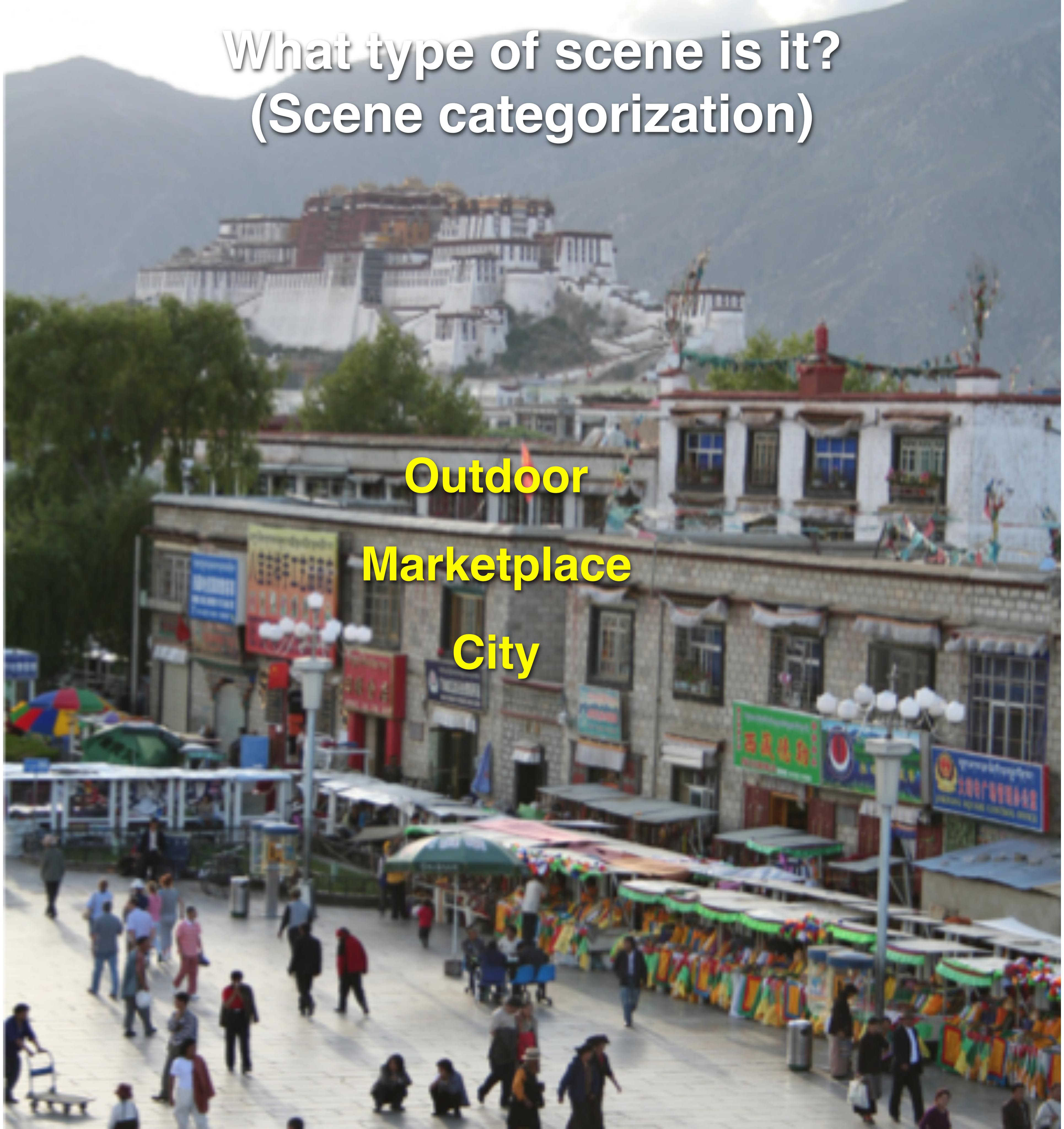
Is that Potala palace?
(Identification)



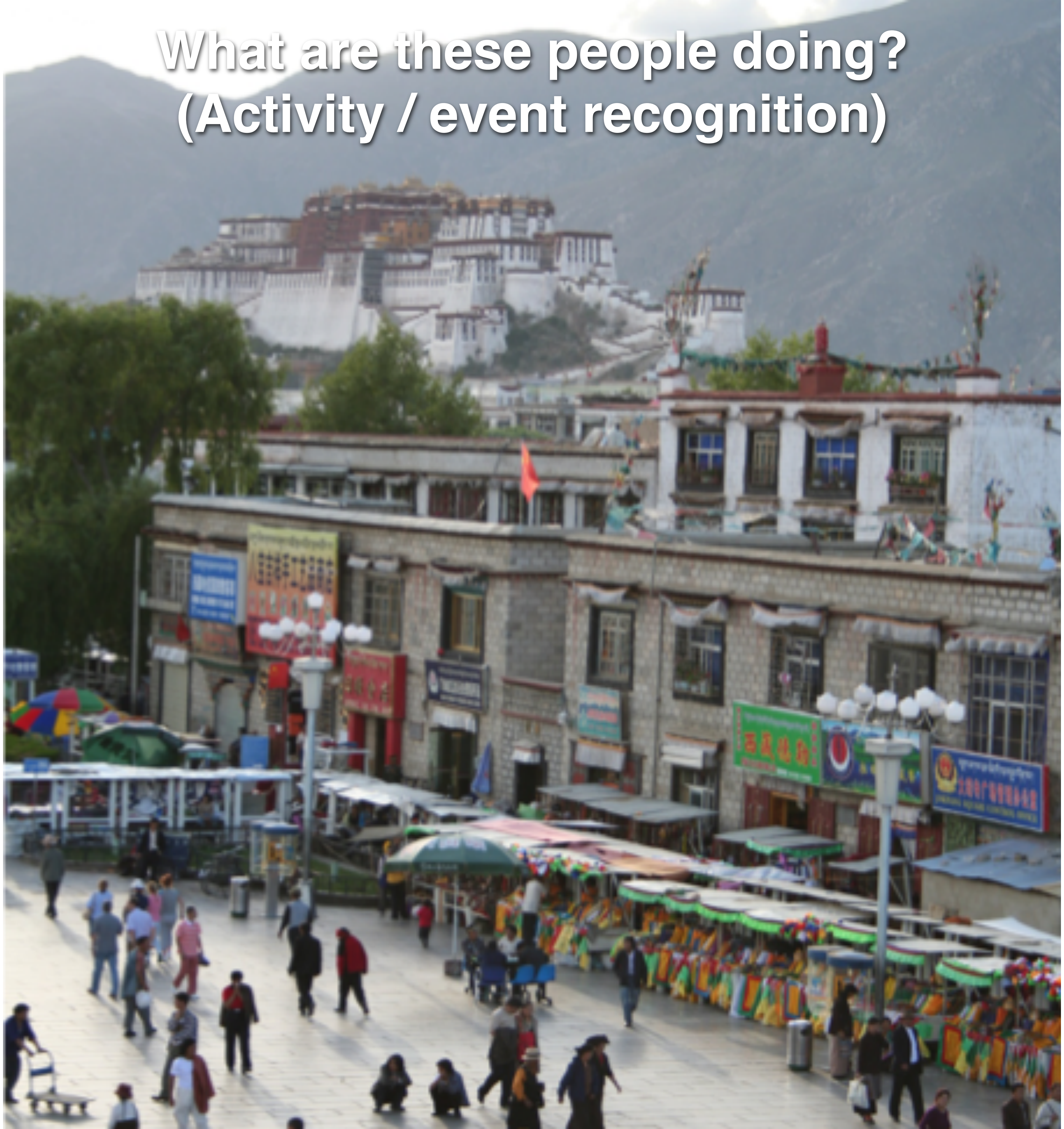
What's in the scene? (semantic segmentation)



What type of scene is it?
(Scene categorization)

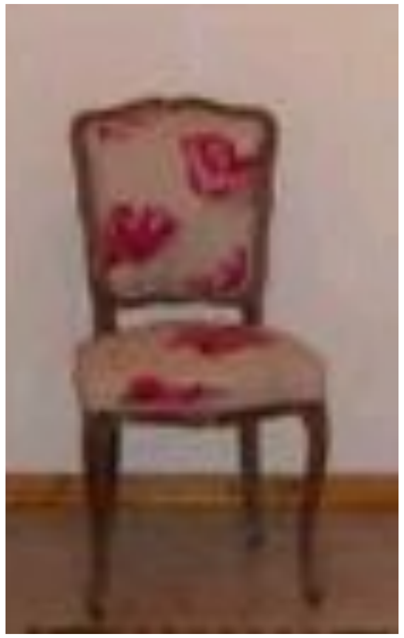


What are these people doing?
(Activity / event recognition)



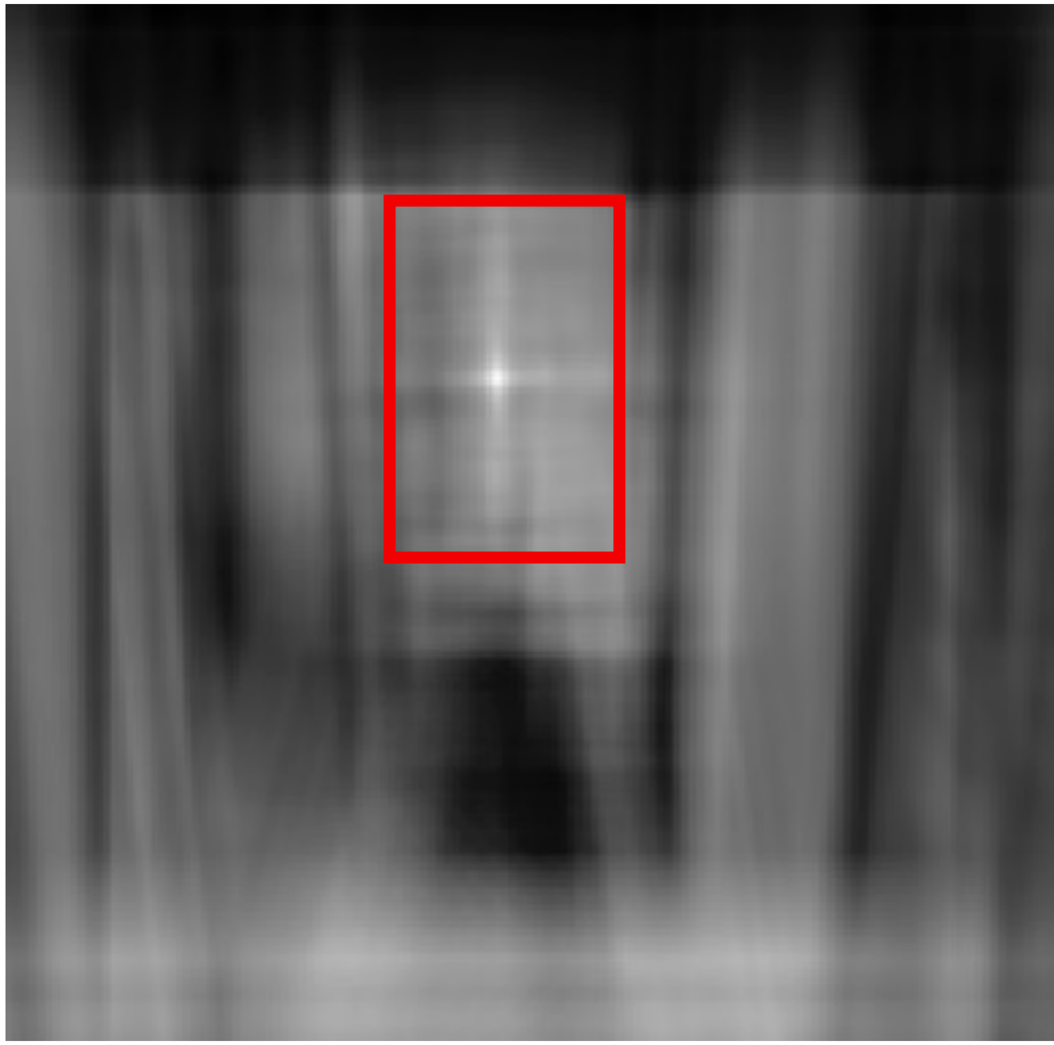
Object recognition
Is it really so hard?

This is a chair



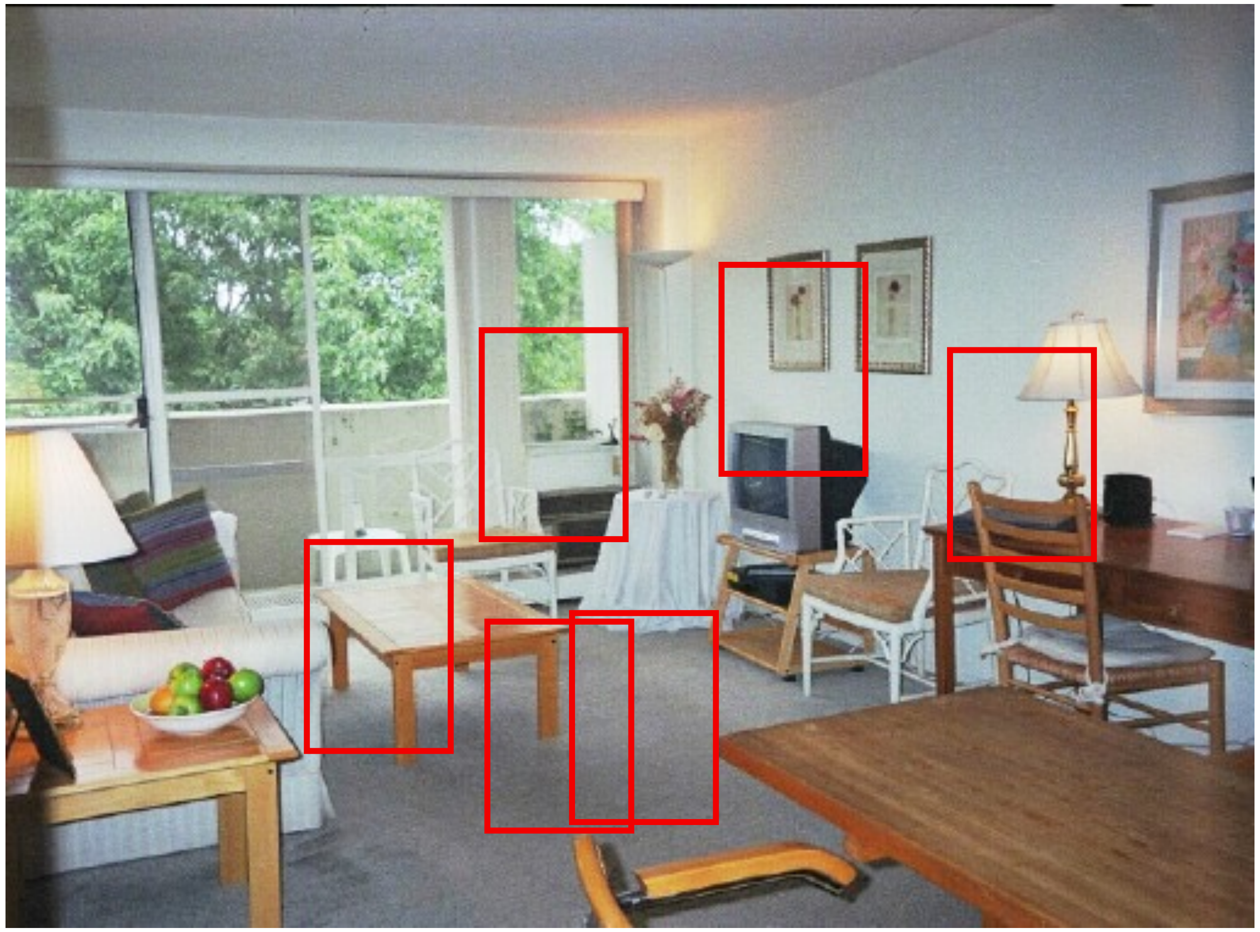
Find the chair in this image

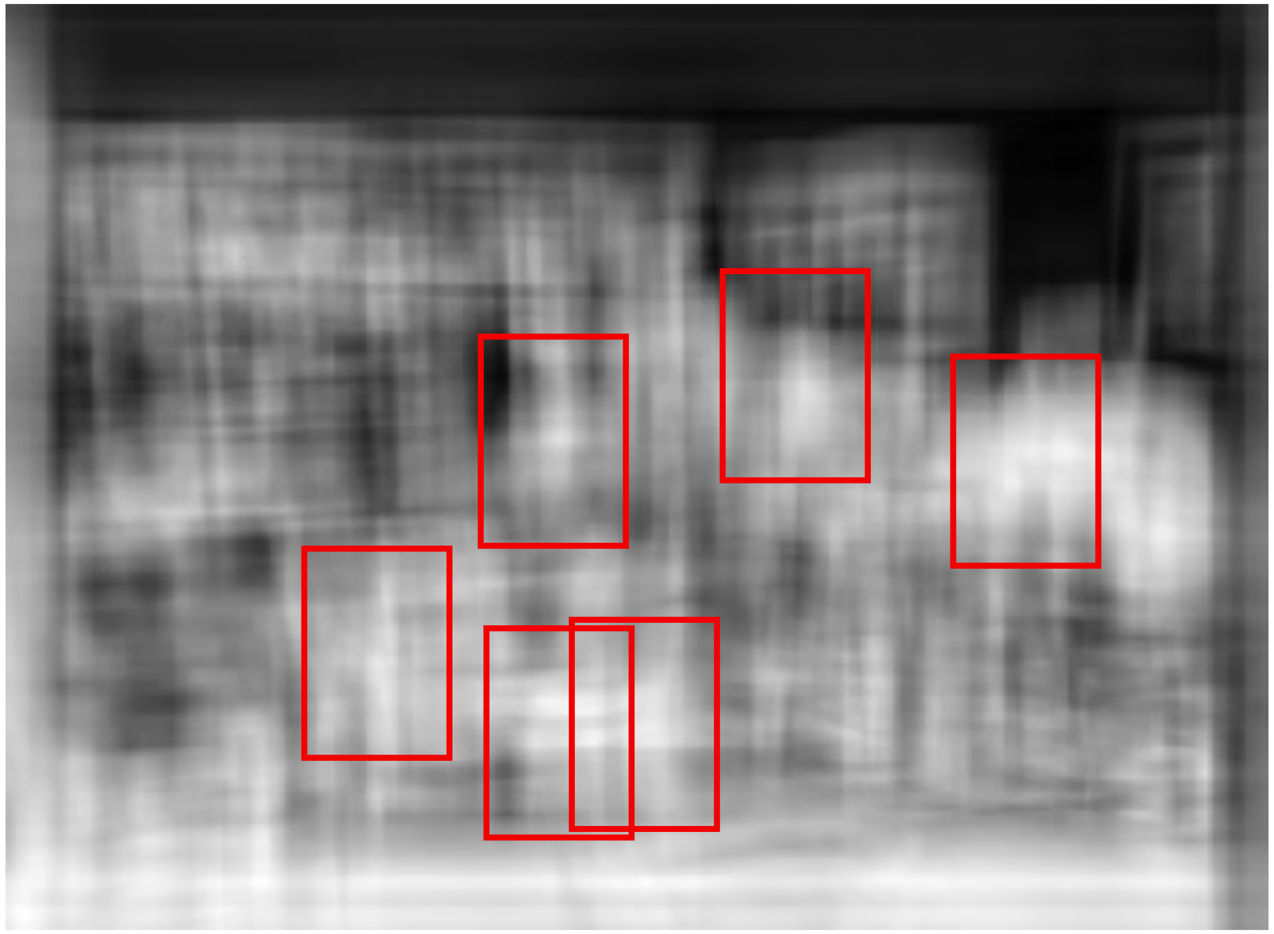
Output of normalized correlation



Object recognition
Is it really so hard?

Find the chair in this image





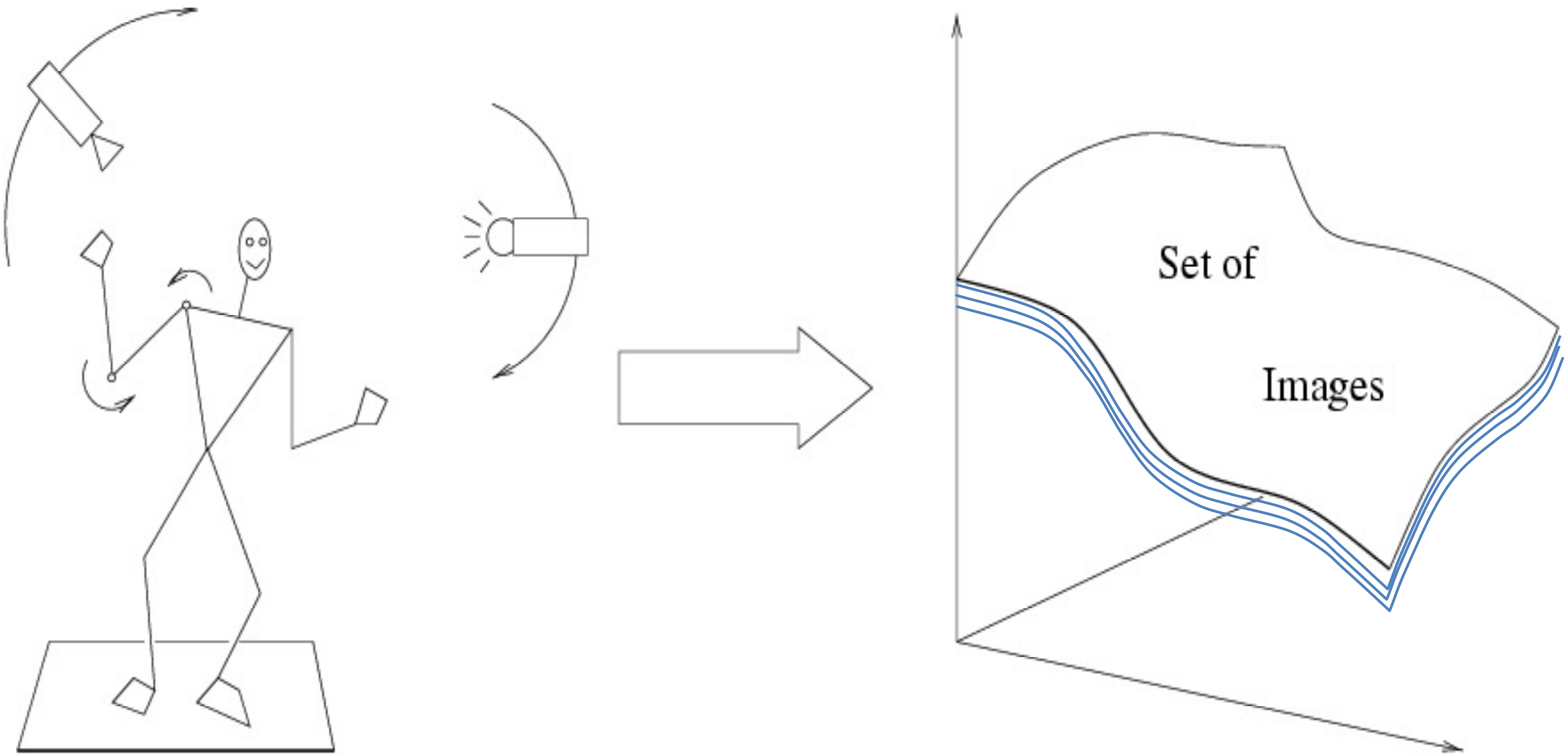
Pretty much garbage
Simple template matching is not going to make it

A "popular method is that of template matching, by point to point correlation of a model pattern with the image pattern. These techniques are inadequate for three-dimensional scene analysis for many reasons, such as occlusion, changes in viewing angle, and articulation of parts." Nivatia & Binford, 1977.

And it can get a lot harder

Brady, M. J., & Kersten, D. (2003). Bootstrapped learning of novel objects. J Vis, 3(6), 413-422

Why is this hard?



Variability: Camera position
Illumination
Shape parameters

How many object categories are there?

~10,000 to 30,000

Challenge: variable viewpoint

Michelangelo 1475-1564

Challenge: variable illumination

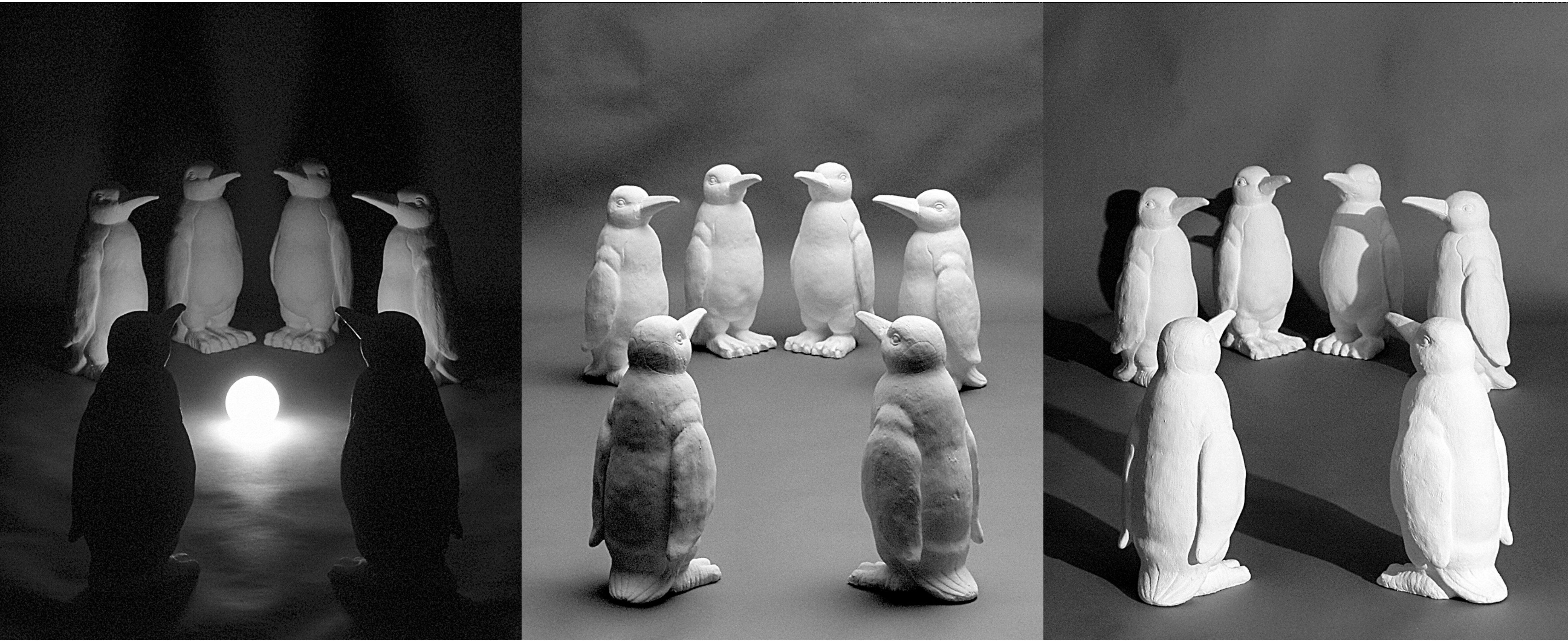


image credit: J. Koenderink

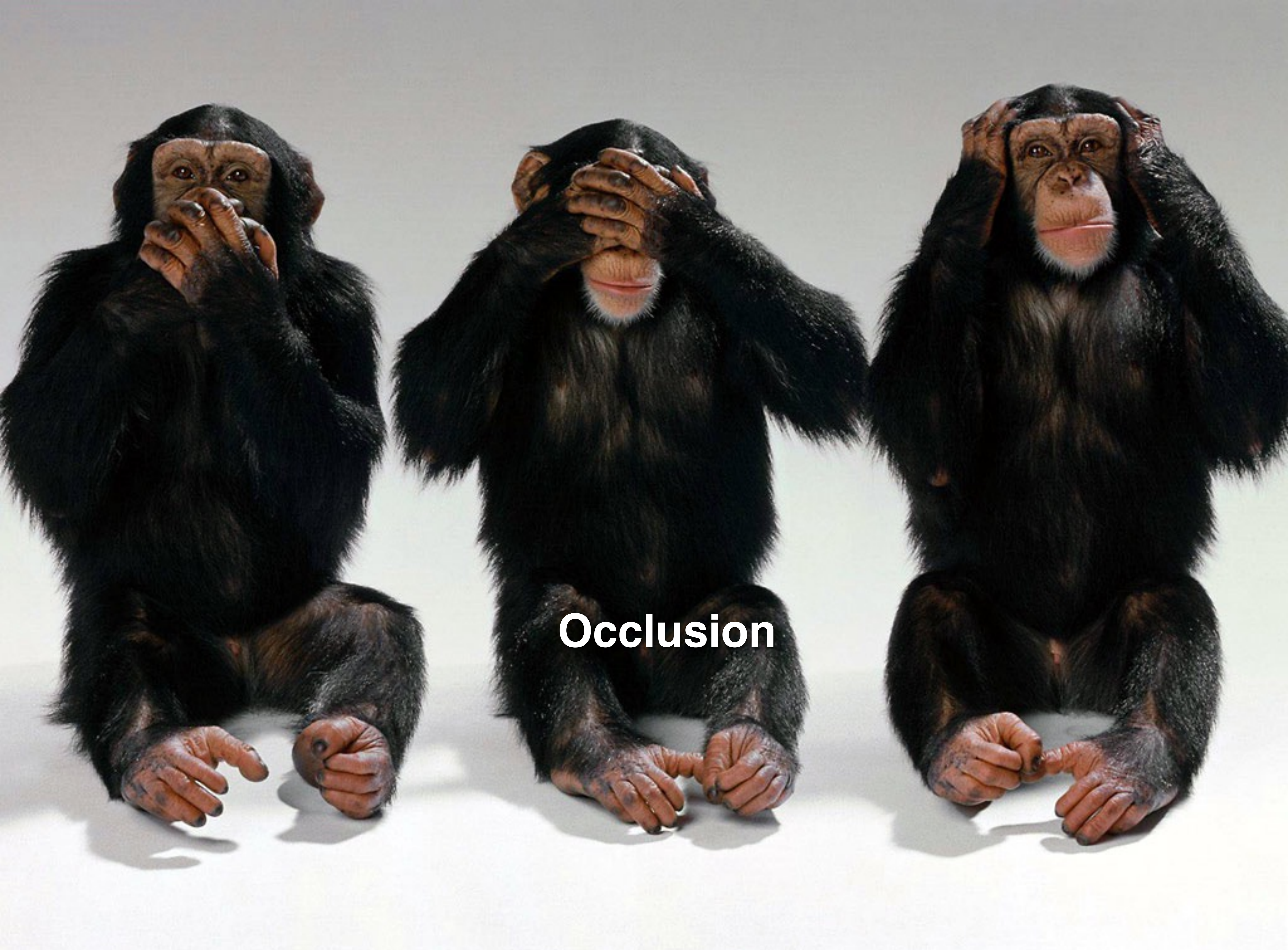
Challenge: scale

Challenge: deformation

Deformation

Challenge: Occlusion

Magritte, 1957


Occlusion

Challenge: background clutter

Kilmeny Niland. 1995

Challenge: Background clutter

Challenge: intra-class variations





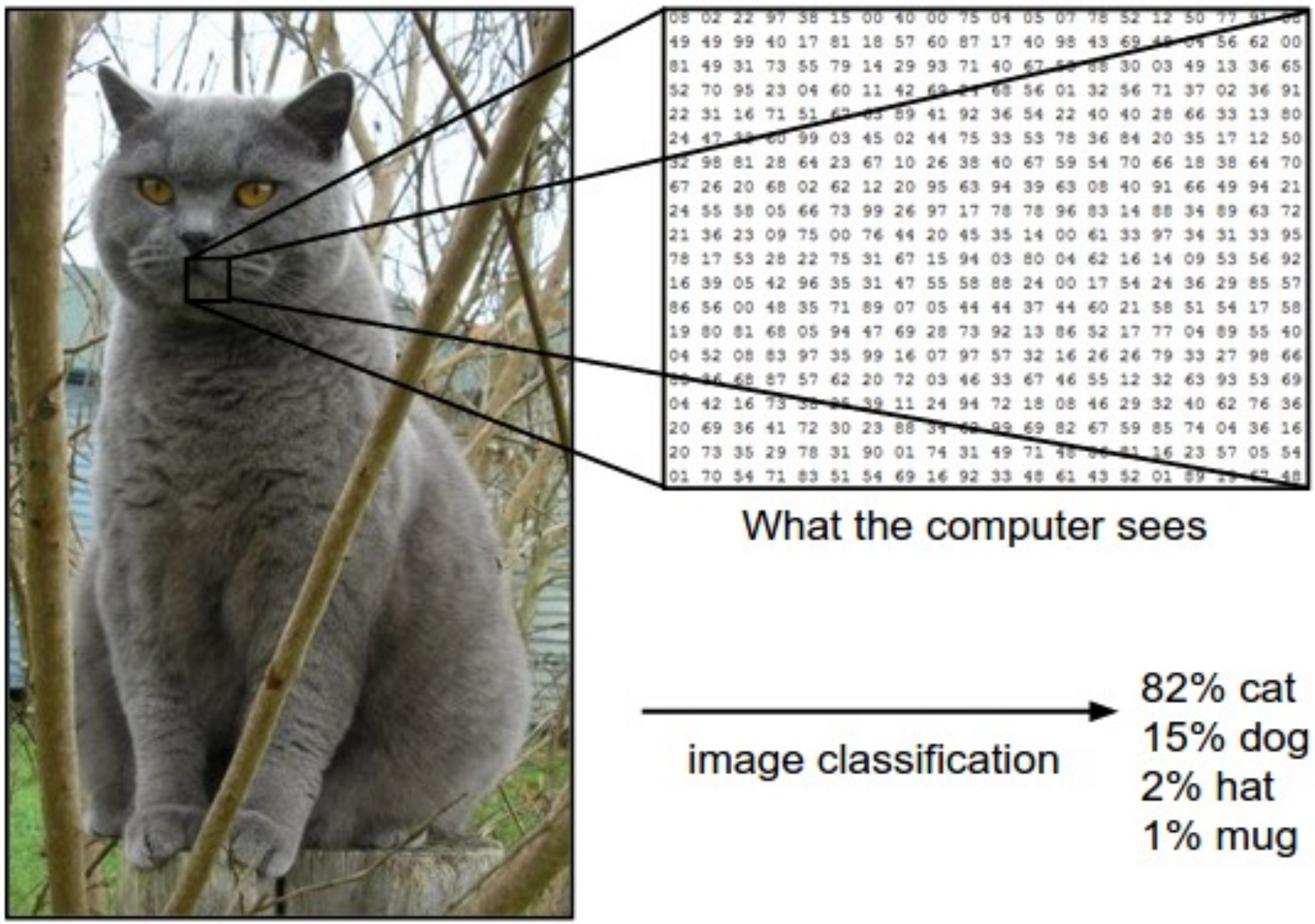

Svetlana Lazebnik

Image Classification

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

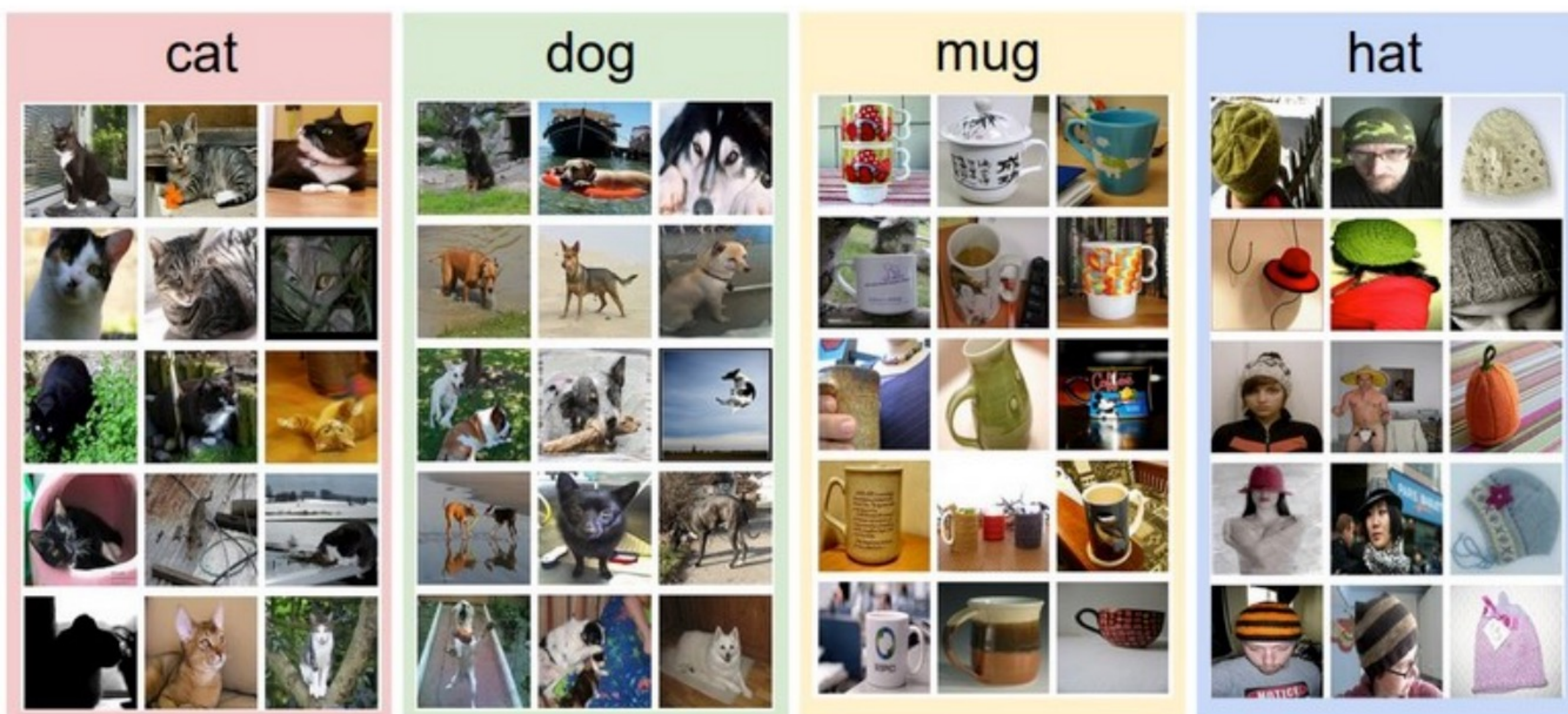
Image Classification: Problem



Data-driven approach

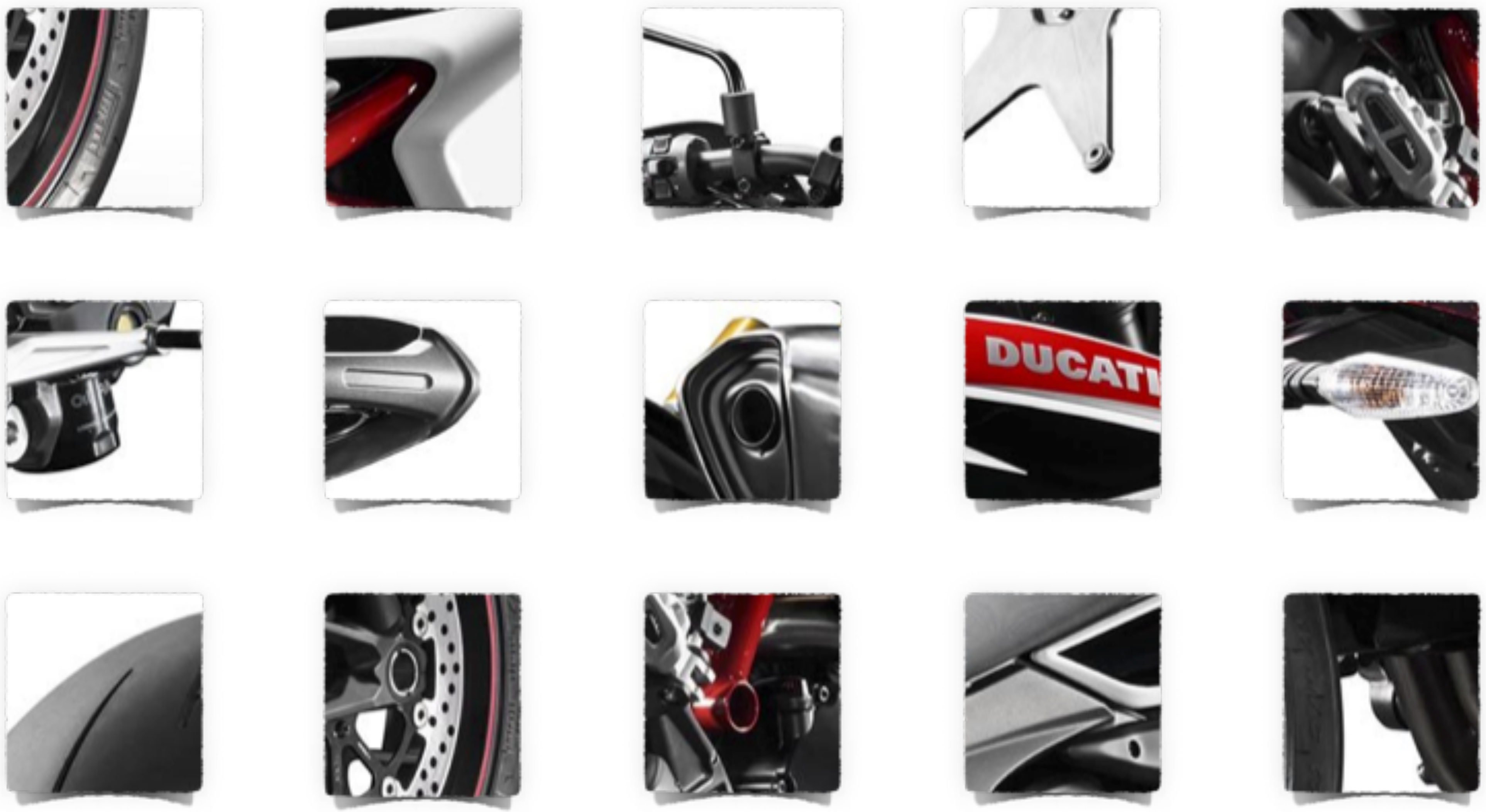
- Collect a database of images with labels
- Use ML to train an image classifier
- Evaluate the classifier on test images

Example training set



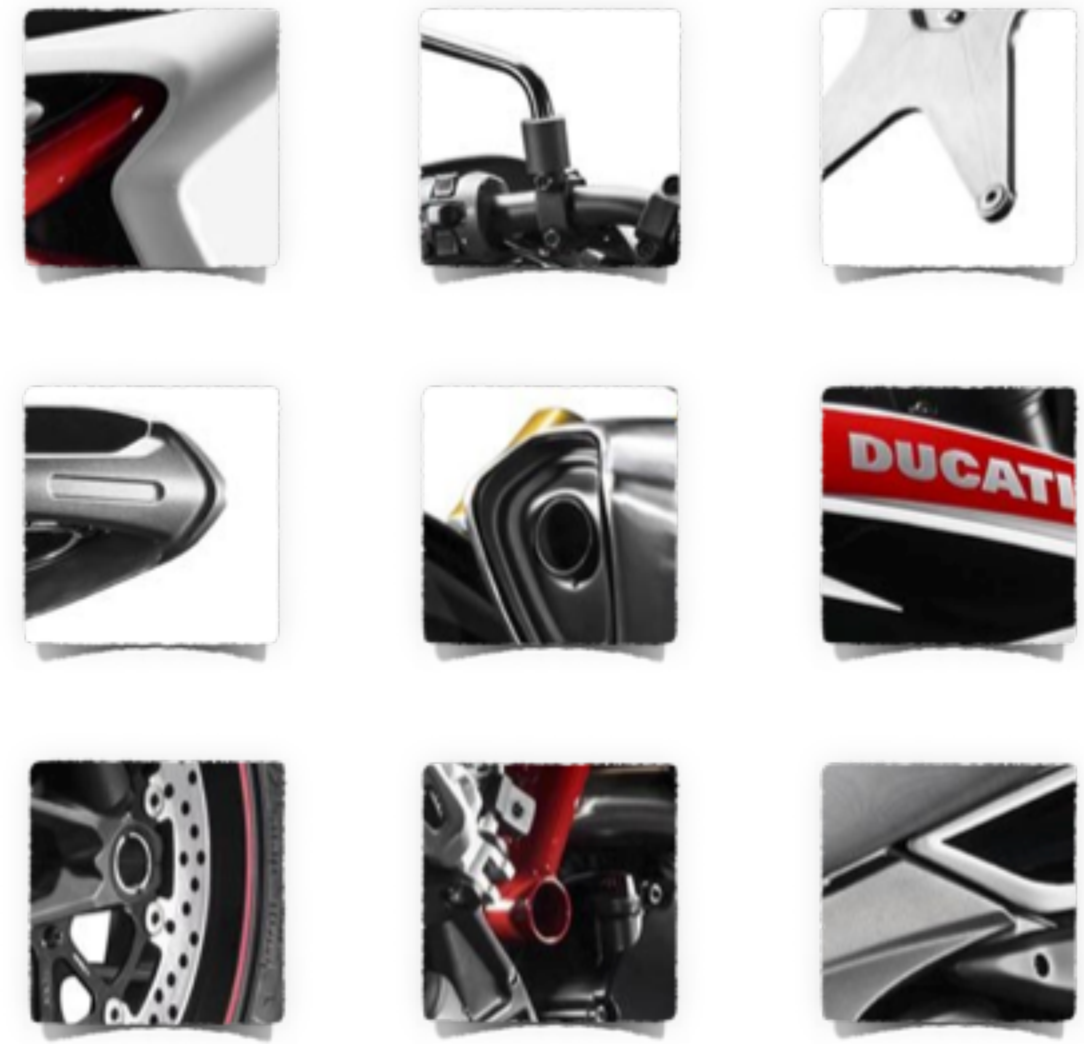
Bag of words

What object do these parts belong to?



Some local features
are very informative

An object as

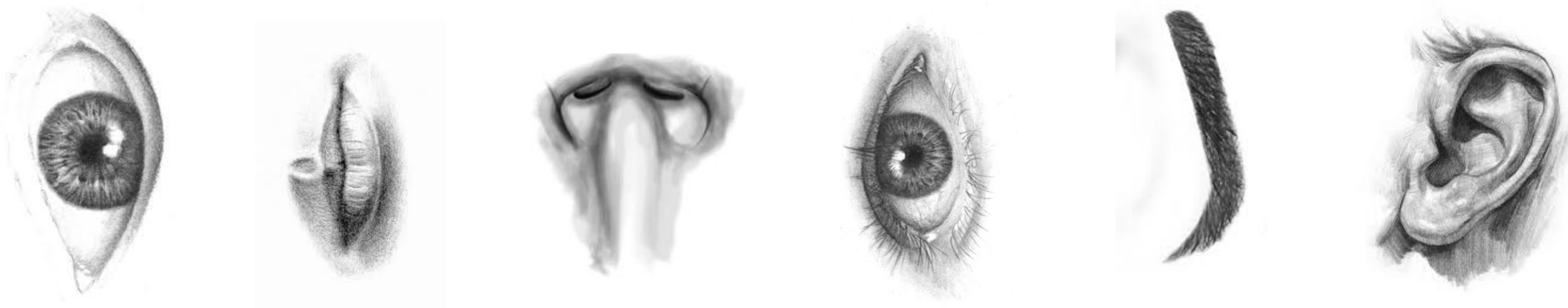


a collection of local features

(bag-of-features)

- deals well with occlusion
- scale invariant
- rotation invariant

(not so) crazy assumption



spatial information of local features
can be ignored for object recognition

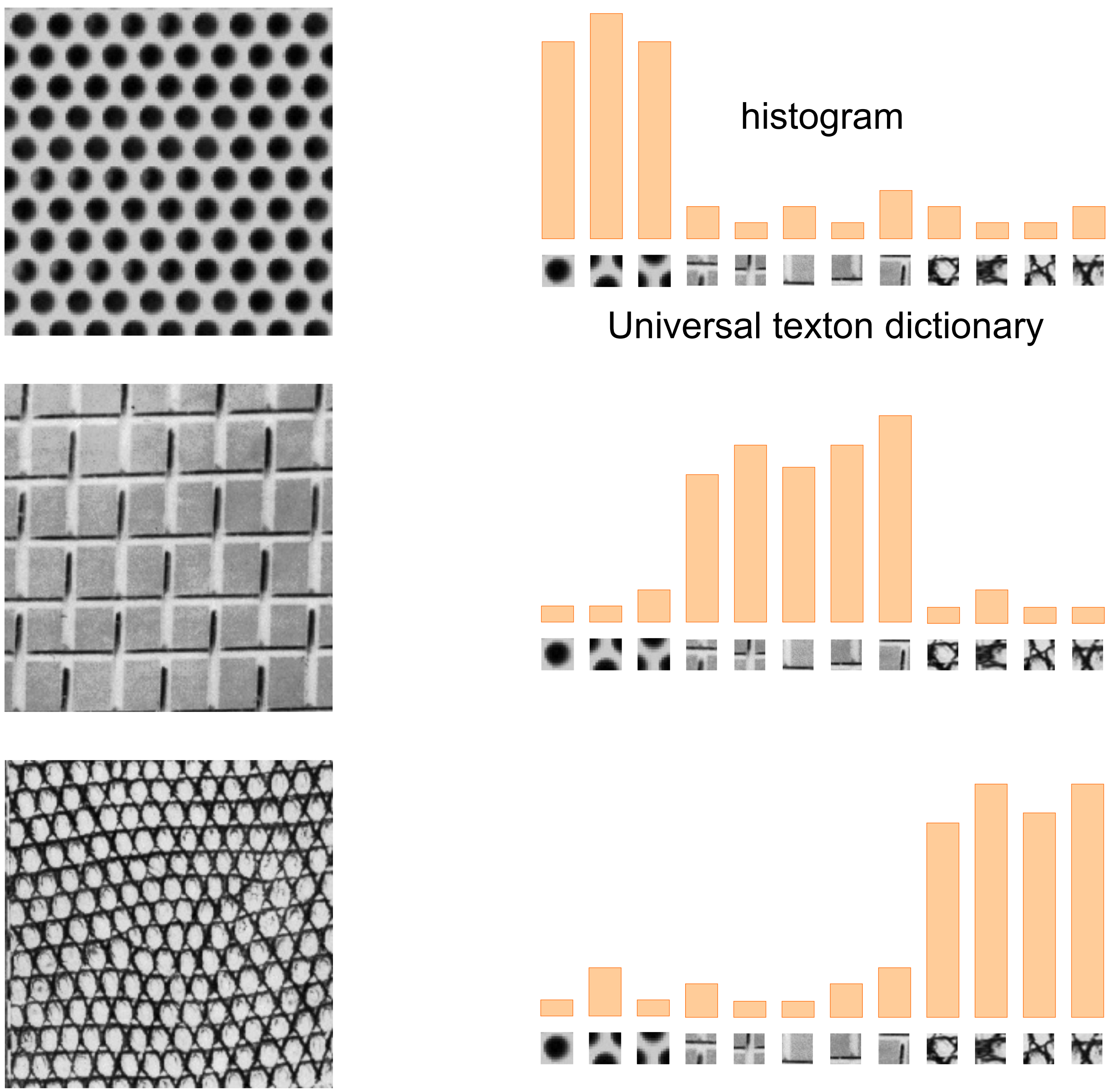
Bag-of-features

represent a data item (document, texture, image)
as a histogram over features

an old idea

(e.g., texture recognition and information retrieval)

Texture recognition



Vector Space Model

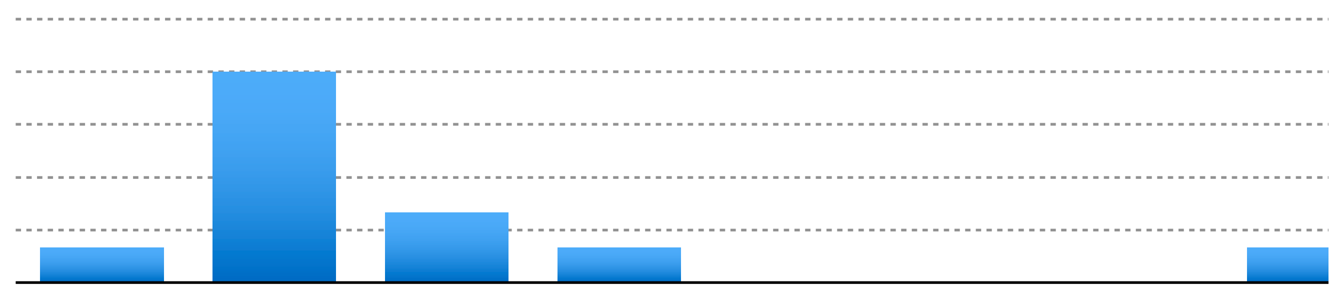
G. Salton. ‘Mathematics and Information Retrieval’ Journal of Documentation,1979


The Newspa

Sunday, December 22, 2013

DARPA Selects Carnegie Me

The Tartan Rescue Team from Carnegie Mellon University's National Robotics Engineering Center ranked third among teams competing in the Defense Advanced Research Projects Agency (DARPA) Robotics Challenge Trials this weekend in Homestead, Fla., and was selected by the agency as one of eight teams eligible for DARPA funding to prepare for next December's finals. The team's four-limbed CMU Highly Intelligent Mobile Platform, or CHIMP, robot scored 18 out of a possible 32 points during the two-day trials. It demonstrated its ability to perform such tasks as removing debris, cutting a hole through a wall and closing a series of valves.



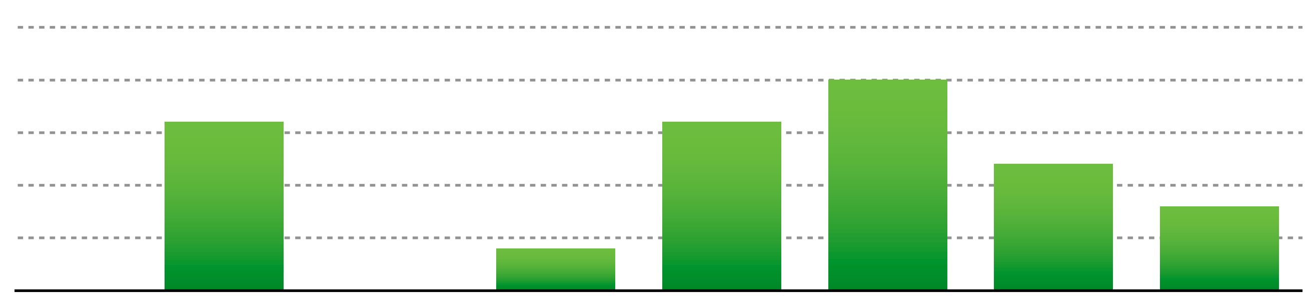
1	6	2	1	0	0	0	1
Tartan	robot	CHIMP	CMU	bio	soft	ankle	sensor


Tartan Tim

Monday, January 20, 2014

Bio-Inspired Robotic Device

PITTSBURGH—A soft, wearable device that mimics the muscles, tendons and ligaments of the lower leg could aid in the rehabilitation of patients with ankle-foot disorders such as drop foot, said Yong-Lae Park, an assistant professor of robotics at Carnegie Mellon University. Park, working with collaborators at Harvard University, the University of Southern California, MIT and BioSensics, developed an active orthotic device using soft plastics and composite materials, instead of a rigid exoskeleton. The soft materials, combined with pneumatic artificial muscles (PAMs), lightweight sensors and advanced control software, made it possible for the robotic device to achieve natural motions in the ankle.



0	4	0	1	4	5	3	2
Tartan	robot	CHIMP	CMU	bio	soft	ankle	sensor

http://www.fodey.com/generators/newspaper/snippet.asp

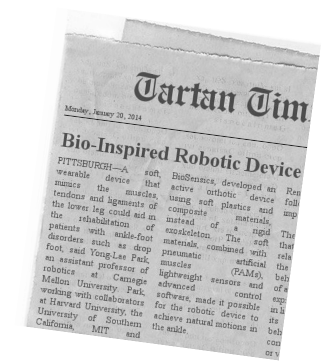
A document (datapoint) is a vector of counts over each word (feature)

$$\boldsymbol{v}_d = [n(w_{1,d}) \quad n(w_{2,d}) \quad \cdots \quad n(w_{T,d})]$$

just a histogram over words

$n(\cdot)$ counts the number of occurrences

What is the similarity between two documents?



A document (datapoint) is a vector of counts over each word (feature)

$$\boldsymbol{v}_d = [n(w_{1,d}) \quad n(w_{2,d}) \quad \cdots \quad n(w_{T,d})]$$

$n(\cdot)$ counts the number of occurrences

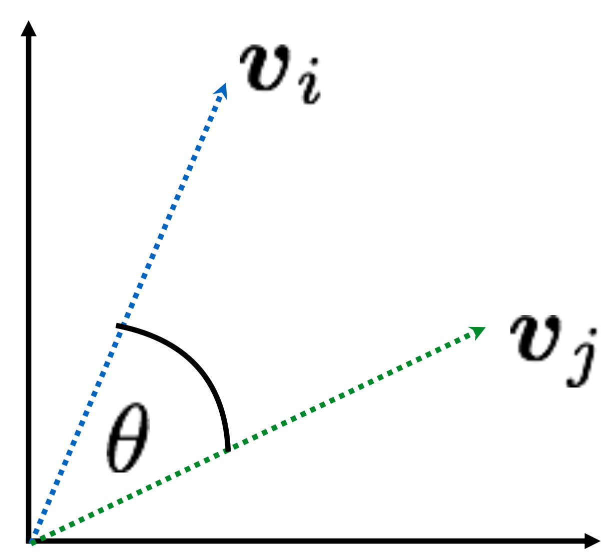
just a histogram over words

What is the similarity between two documents?

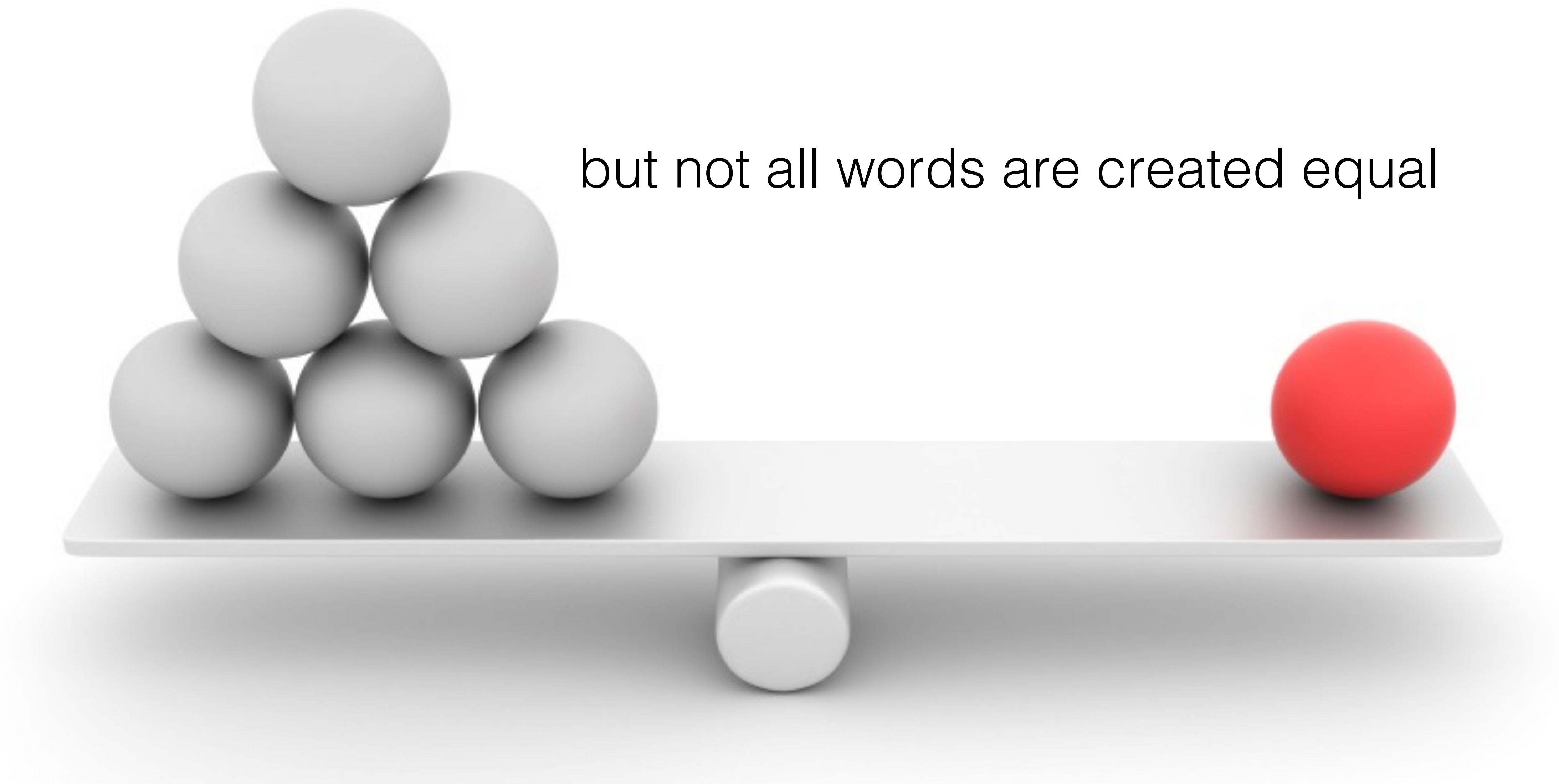
Use any distance you want but the cosine distance is fast.

$$d(\boldsymbol{v}_i, \boldsymbol{v}_j) = \cos\theta$$

$$= \frac{\boldsymbol{v}_i \cdot \boldsymbol{v}_j}{\|\boldsymbol{v}_i\|\|\boldsymbol{v}_j\|}$$



but not all words are created equal



TF-IDF

Term **F**requency **I**nverse **D**ocument **F**requency

$$\boldsymbol{v}_d = \begin{bmatrix} n(w_{1,d}) & n(w_{2,d}) & \cdots & n(w_{T,d}) \end{bmatrix}$$

weigh each word by a heuristic

$$\boldsymbol{v}_d = \begin{bmatrix} n(w_{1,d})\alpha_1 & n(w_{2,d})\alpha_2 & \cdots & n(w_{T,d})\alpha_T \end{bmatrix}$$

term frequency

inverse document frequency

$$n(w_{i,d})\alpha_i = n(w_{i,d}) \log \left\{ \frac{D}{\sum_{d'} \mathbf{1}[w_i \in d']} \right\}$$

(down-weights **common** terms)

Standard BOW pipeline

(for image classification)

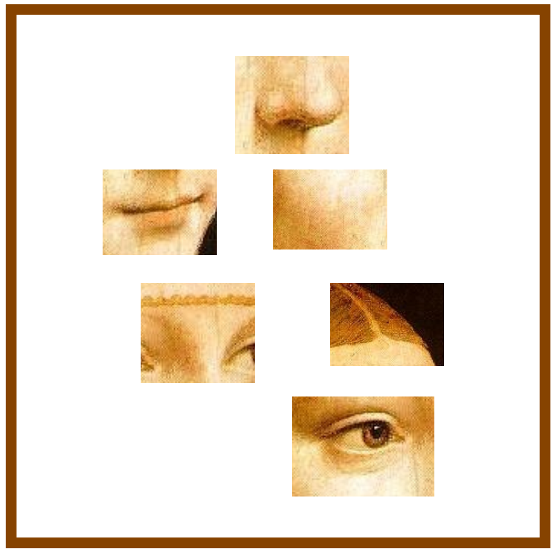
Dictionary Learning:
Learn Visual Words using clustering

Encode:
build Bags-of-Words (BOW) vectors
for each image

Classify:
Train and test data using BOWs

Dictionary Learning:
Learn Visual Words using clustering

1. extract features (e.g., SIFT) from images

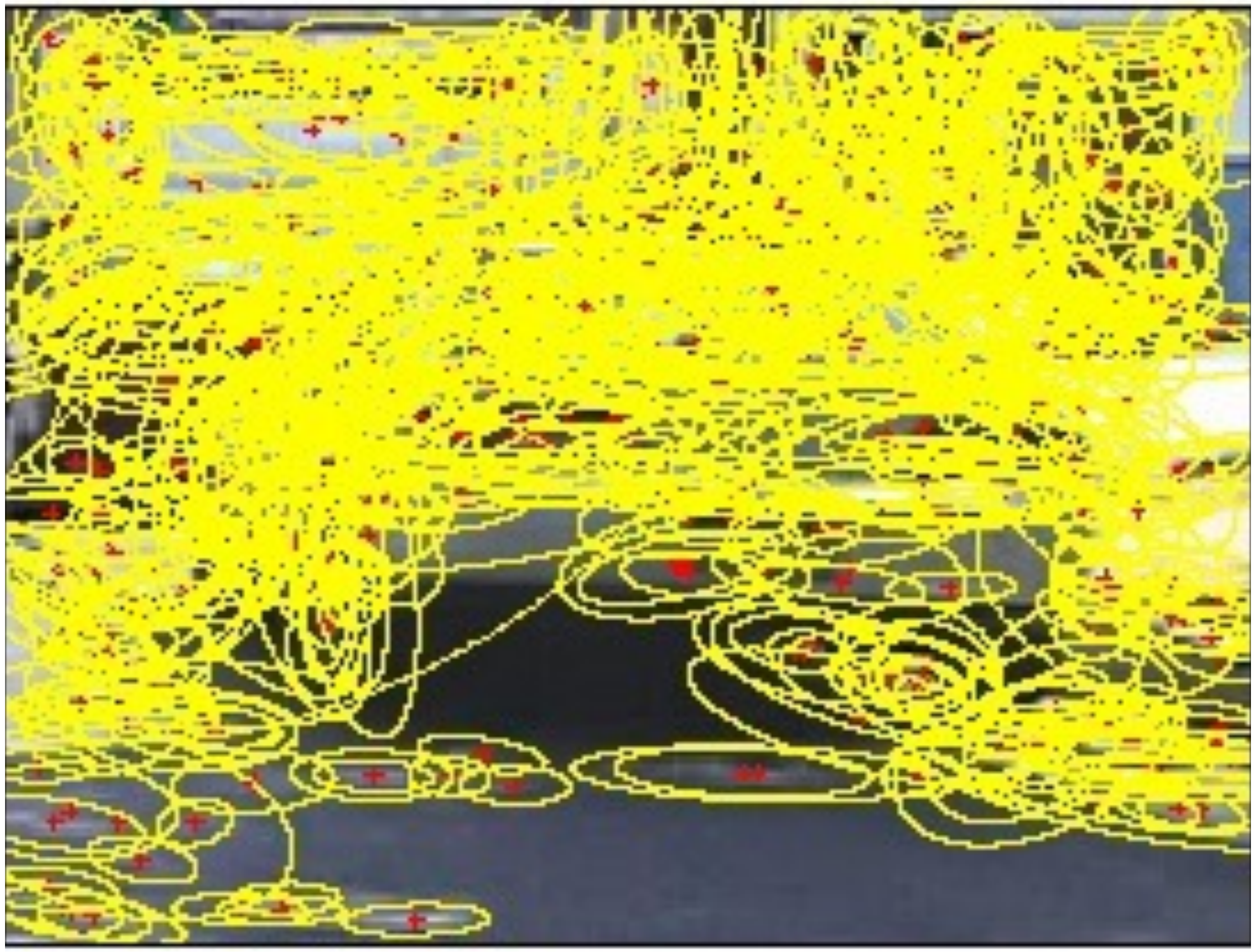


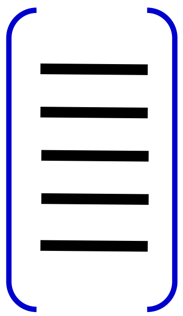
Dictionary Learning:
Learn Visual Words using clustering

2. Learn visual dictionary (e.g., K-means clustering)

What kinds of features can we extract?

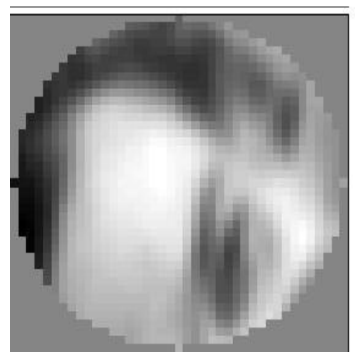
- Regular grid
  - Vogel & Schiele, 2003
  - Fei-Fei & Perona, 2005
- Interest point detector
  - Csurka et al. 2004
  - Fei-Fei & Perona, 2005
  - Sivic et al. 2005
- Other methods
  - Random sampling (Vidal-Naquet & Ullman, 2002)
  - Segmentation-based patches (Barnard et al. 2003)



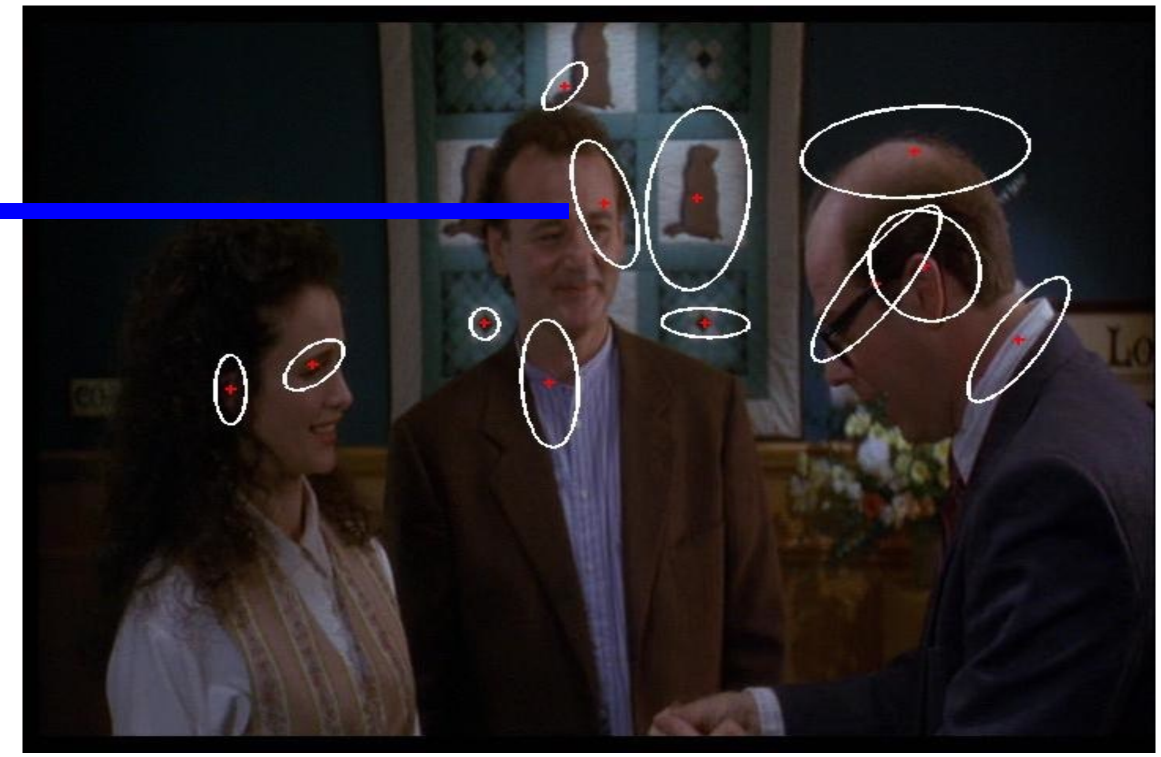


Compute SIFT descriptor

[Lowe'99]



Normalize patch

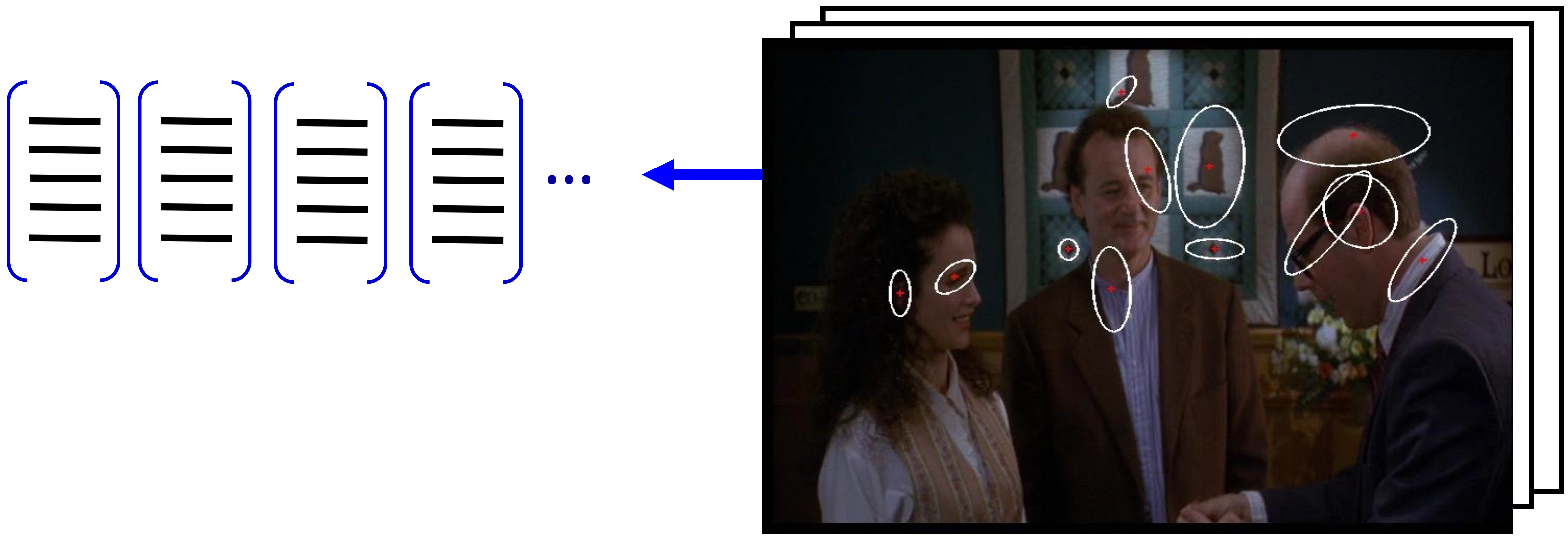


Detect patches

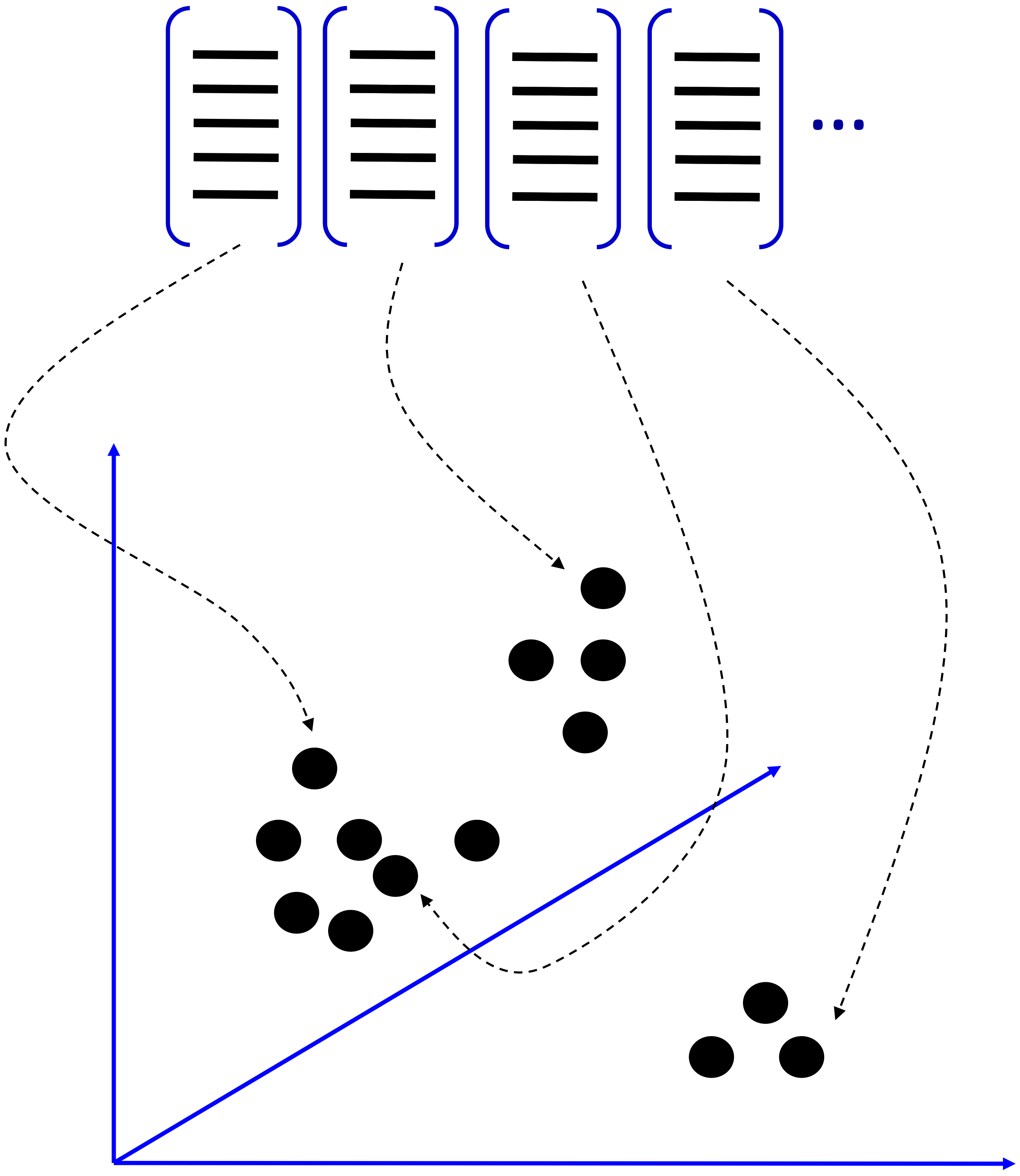
[Mikojaczyk and Schmid '02]

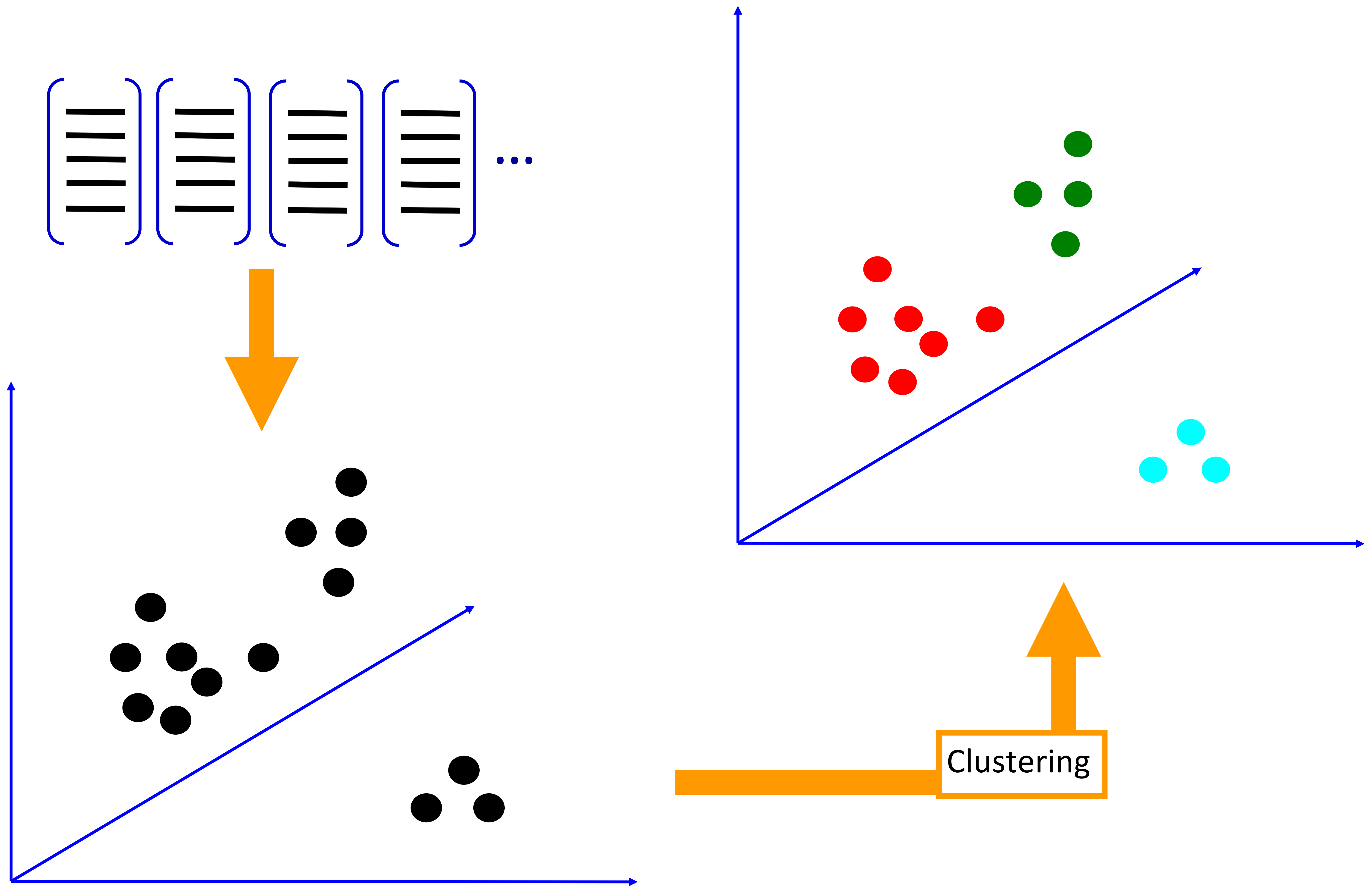
[Mata, Chum, Urban & Pajdla, '02]

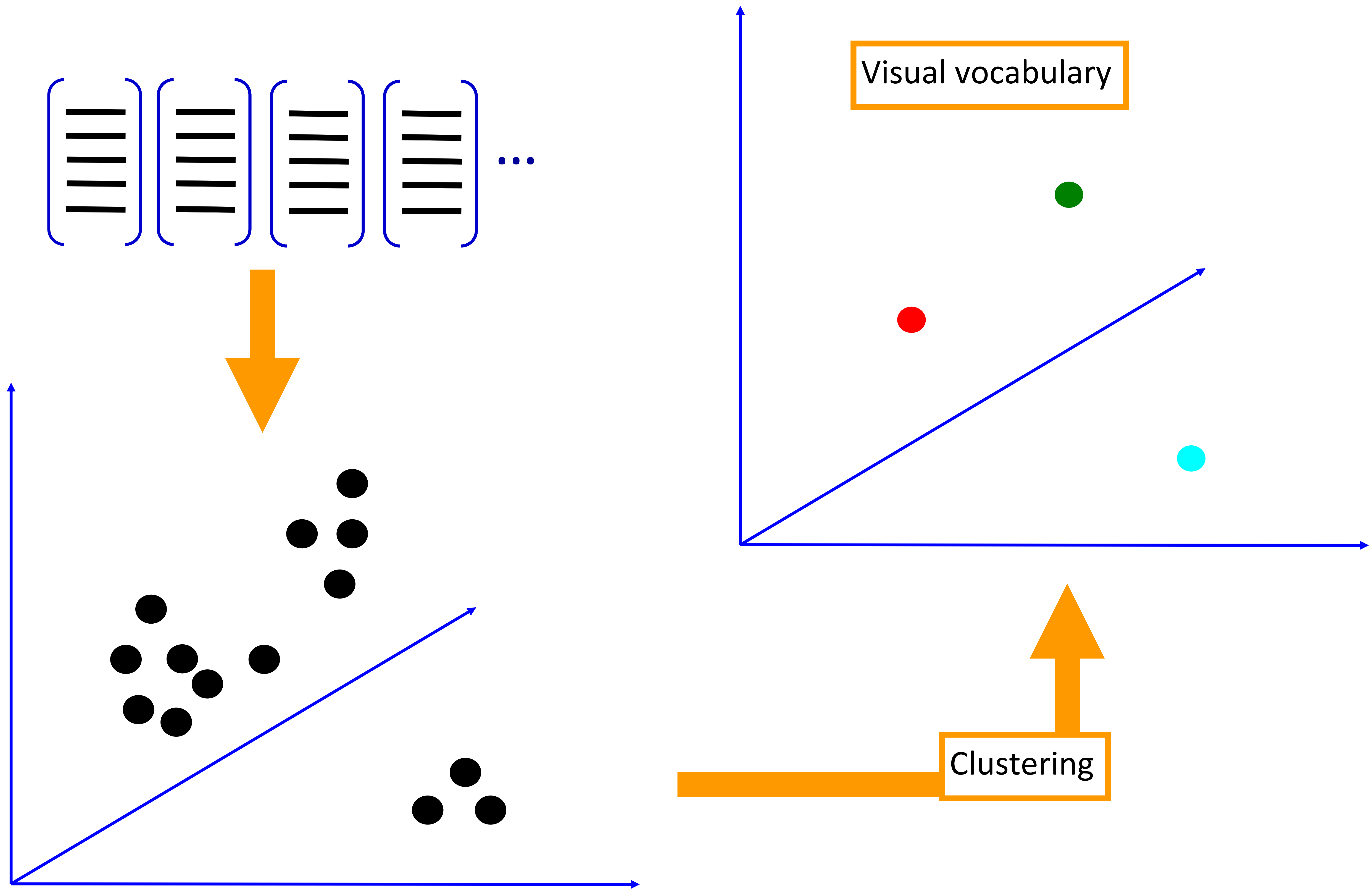
[Sivic & Zisserman, '03]



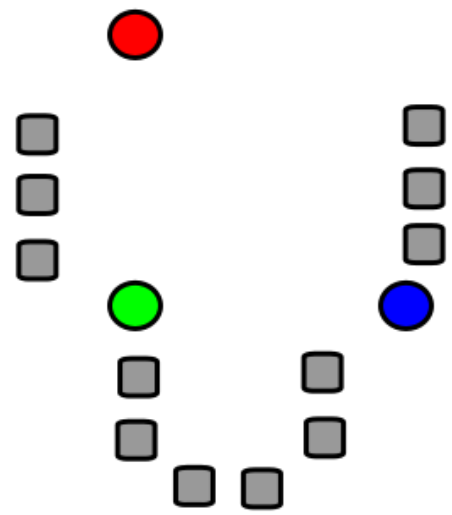
How do we learn the dictionary?



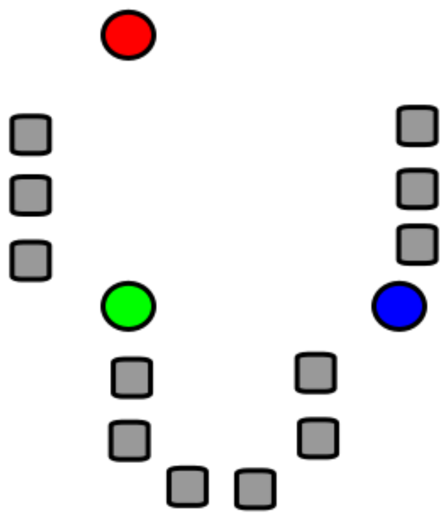

Clustering


Visual vocabulary
Clustering

K-means clustering



1. Select initial
centroids at random



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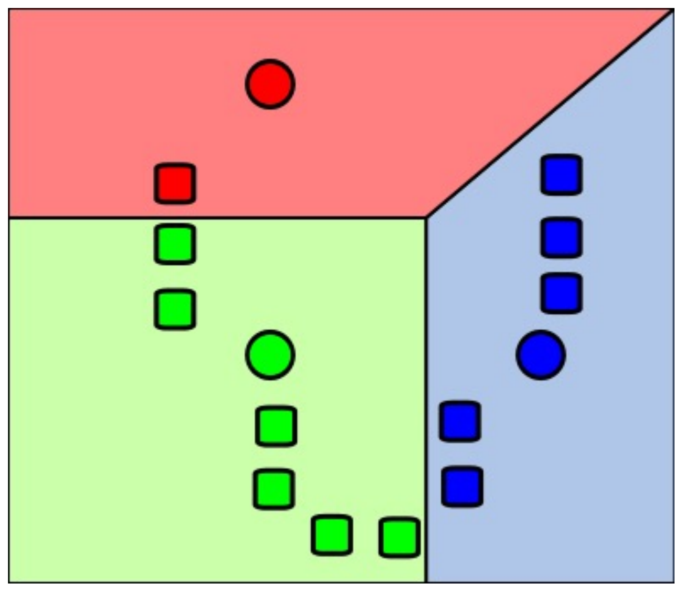
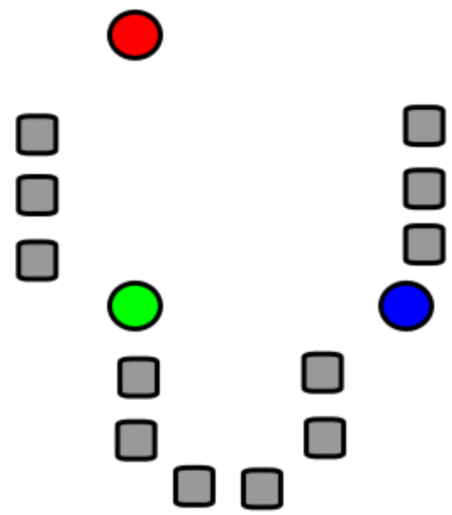


2. Assign each object to the cluster with the nearest centroid.



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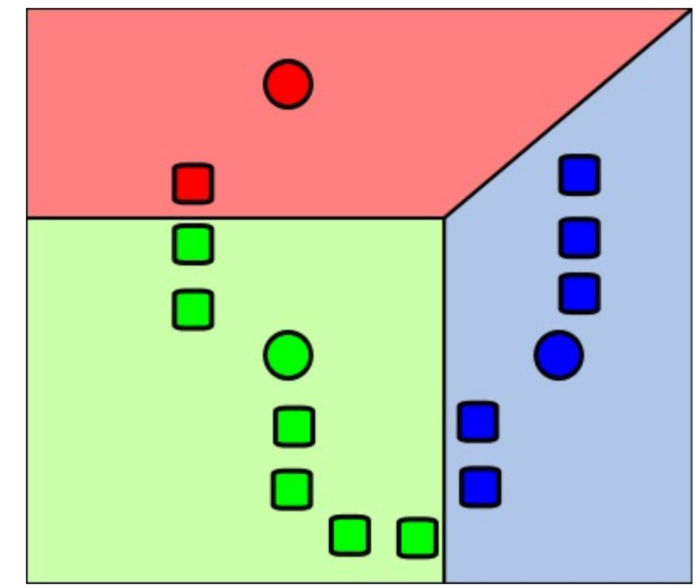

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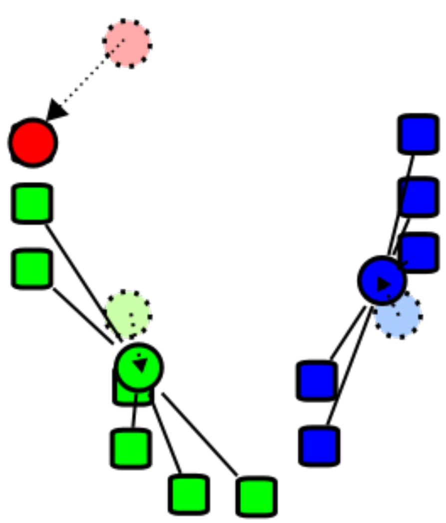
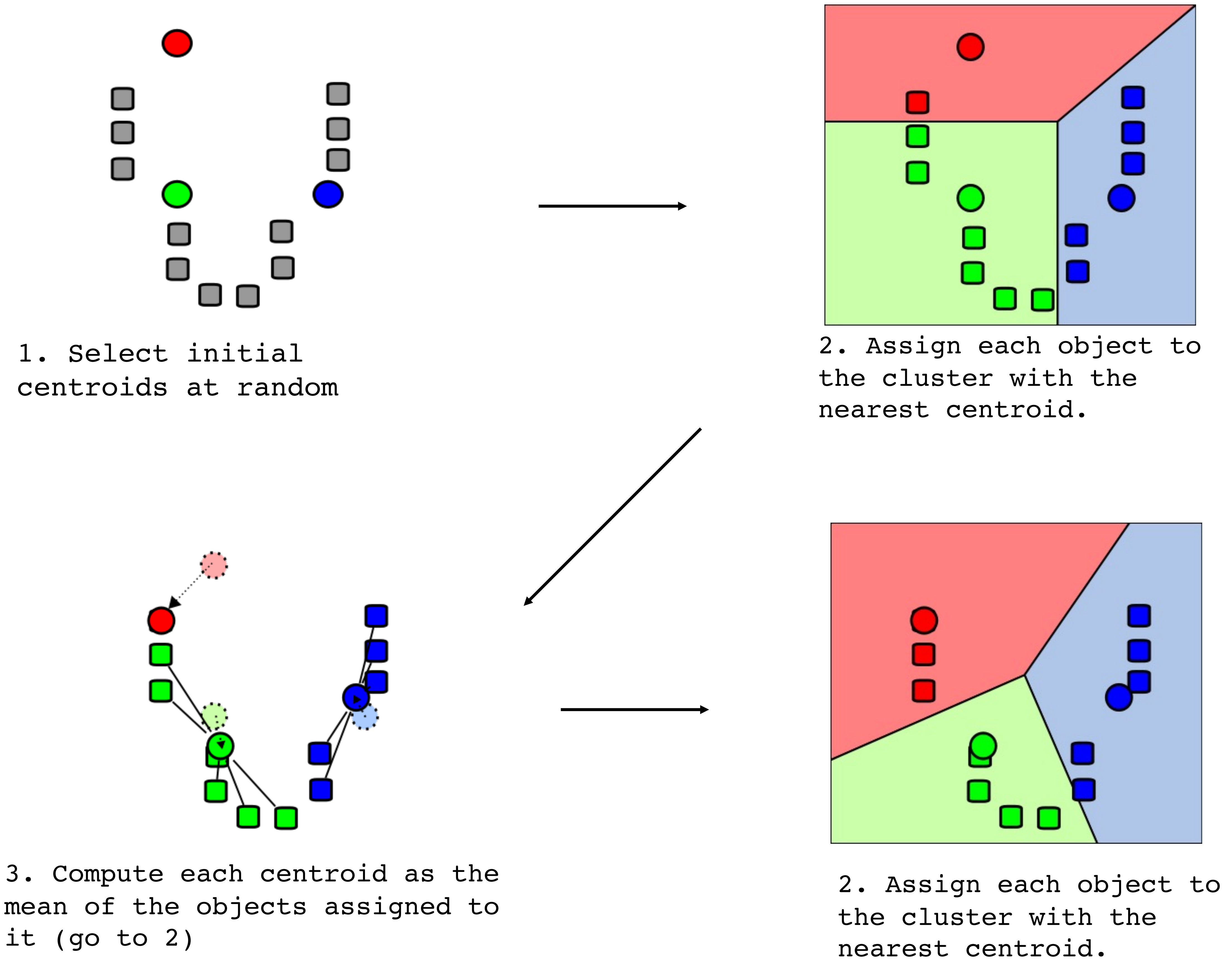
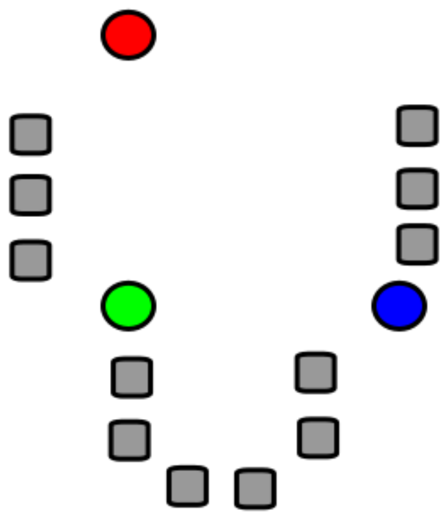


3. Compute each centroid as the mean of the objects assigned to it (go to 2)


1. Select initial centroids at random
2. Assign each object to the cluster with the nearest centroid.
3. Compute each centroid as the mean of the objects assigned to it (go to 2)
2. Assign each object to the cluster with the nearest centroid.



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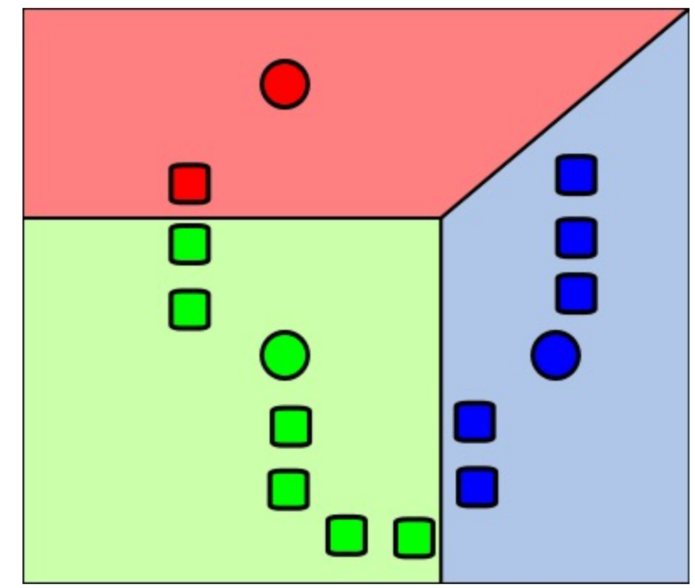

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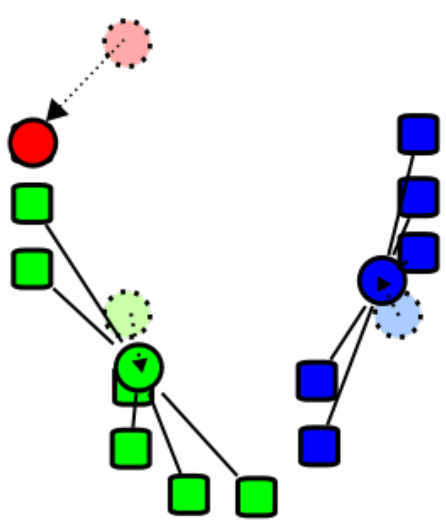

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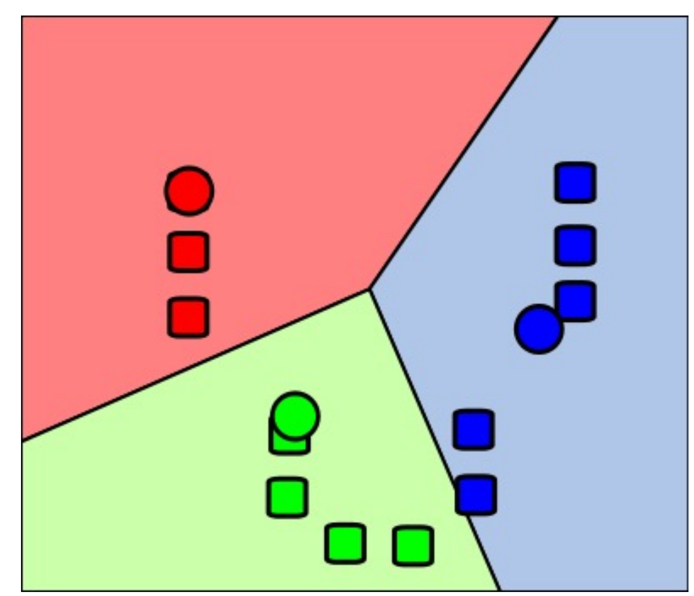


2. Assign each object to the cluster with the nearest centroid.

Repeat previous 2 steps until no change

K-means Clustering

Given k:

1. Select initial centroids at random.

2. Assign each object to the cluster with the nearest centroid.

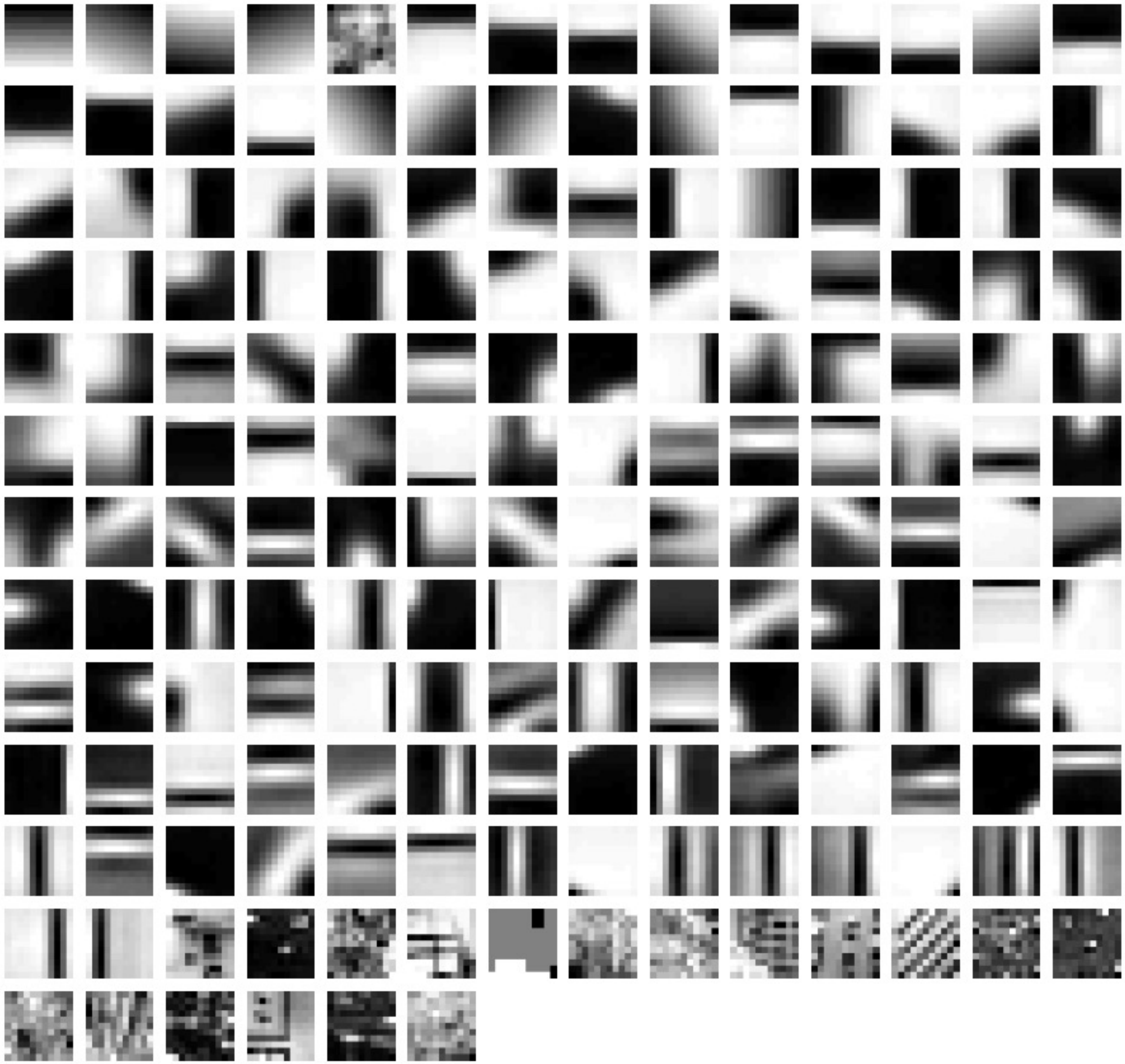
3. Compute each centroid as the mean of the objects assigned to it.

4. Repeat previous 2 steps until no change.

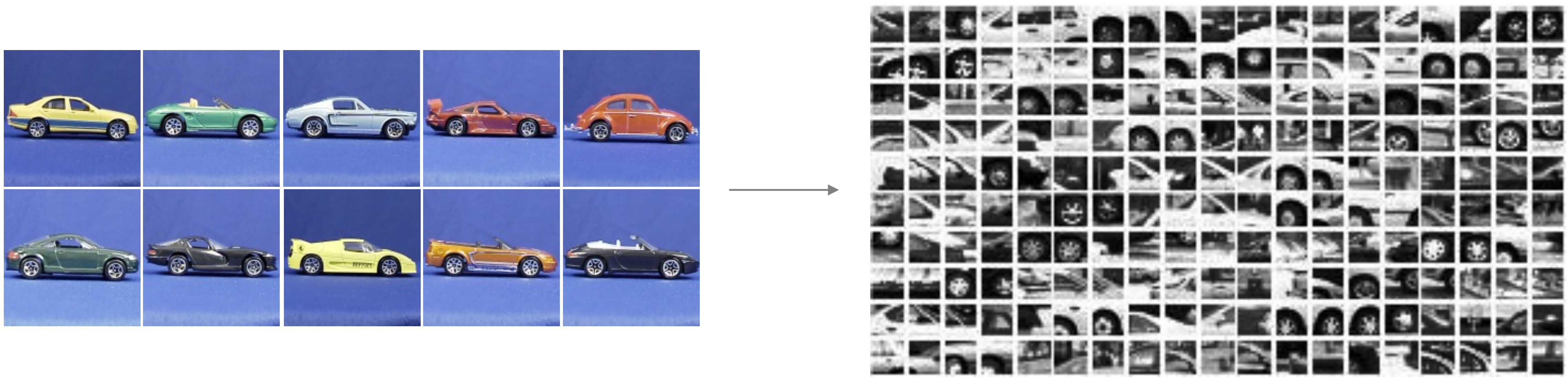
*From what **data** should I learn the dictionary?*

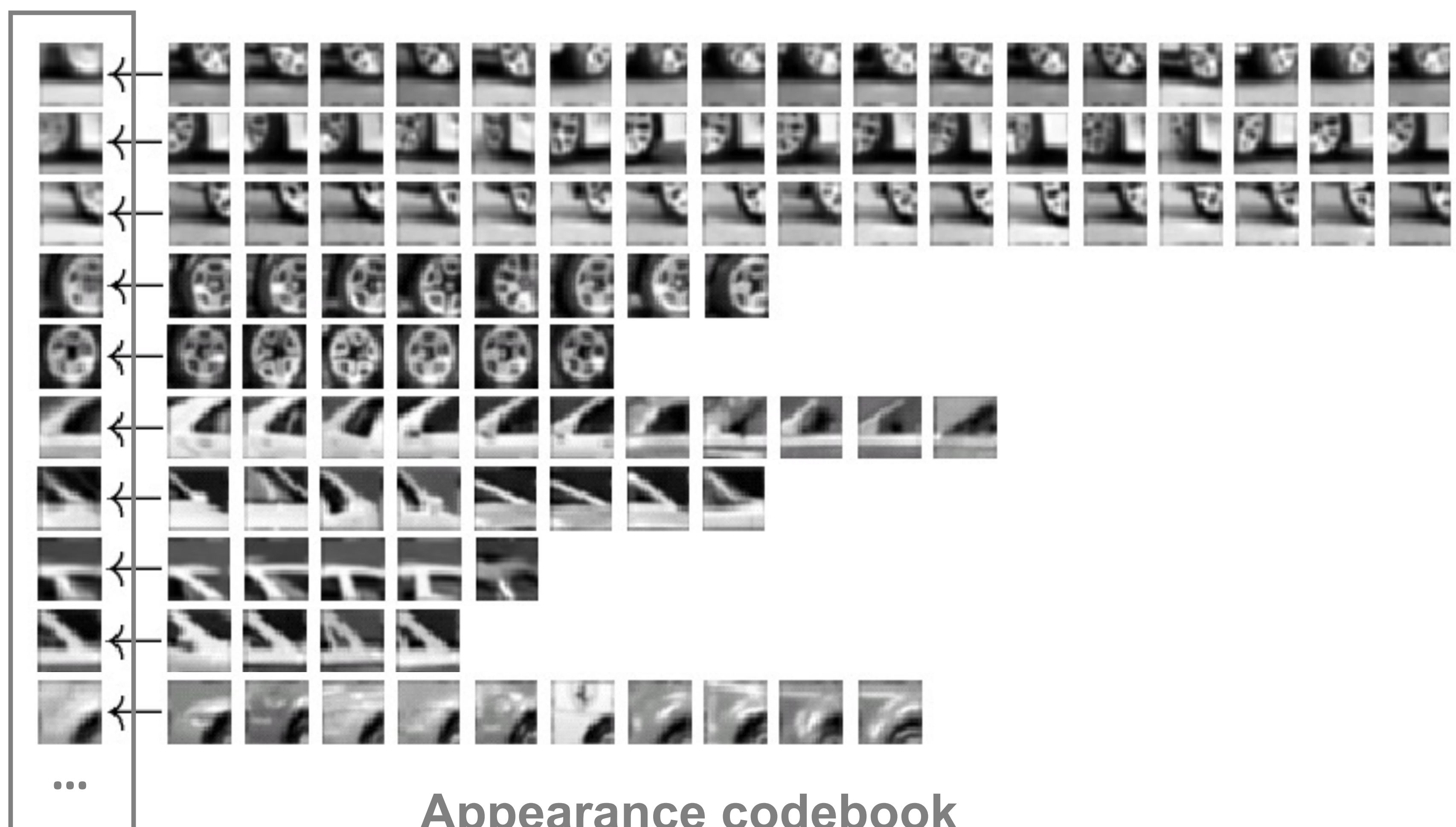
- Dictionary can be learned on separate training set
- Provided the training set is sufficiently representative, the dictionary will be “universal”

Example visual dictionary



Example dictionary

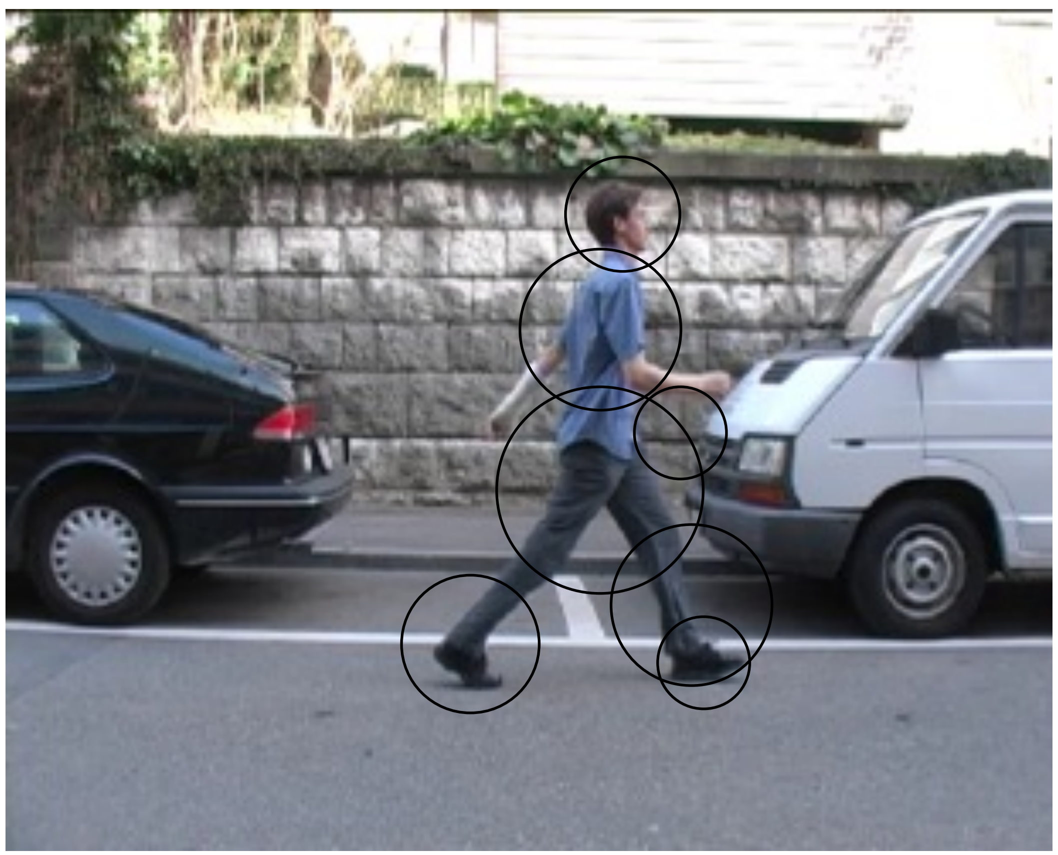




Appearance codebook

Source: B. Leibe

Another dictionary

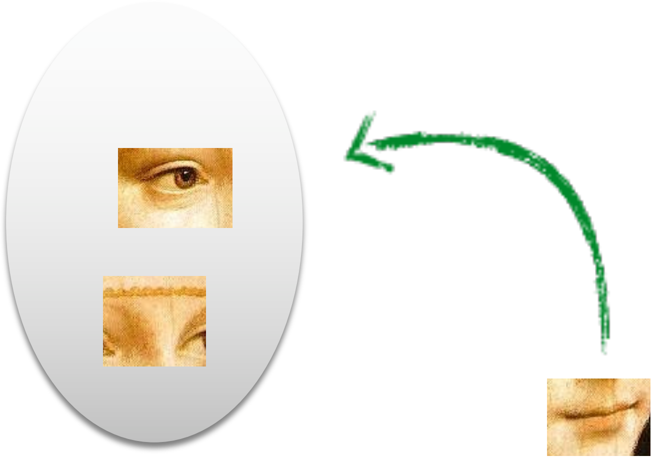


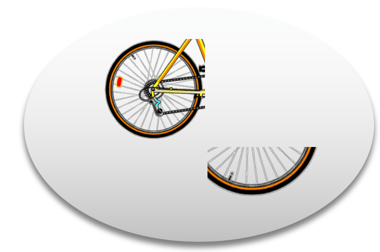
Source: B. Leibe

Dictionary Learning:
Learn Visual Words using clustering

Encode:
build Bags-of-Words (BOW) vectors
for each image

Classify:
Train and test data using BOWs

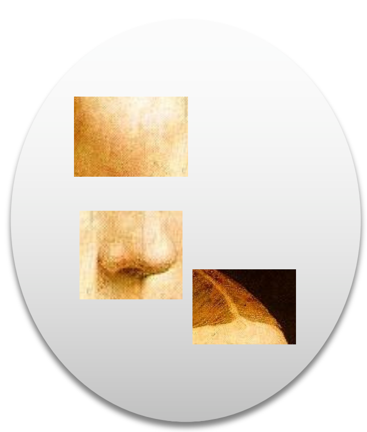




1. Quantization: image features gets associated to a visual word (nearest cluster center)

Encode:

build Bags-of-Words (BOW) vectors for each image

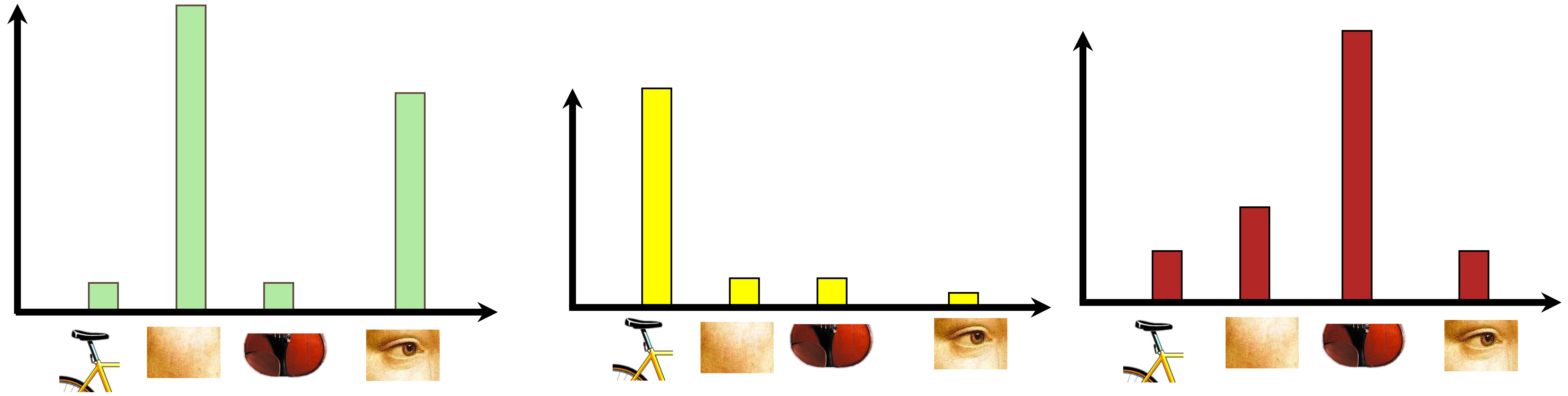


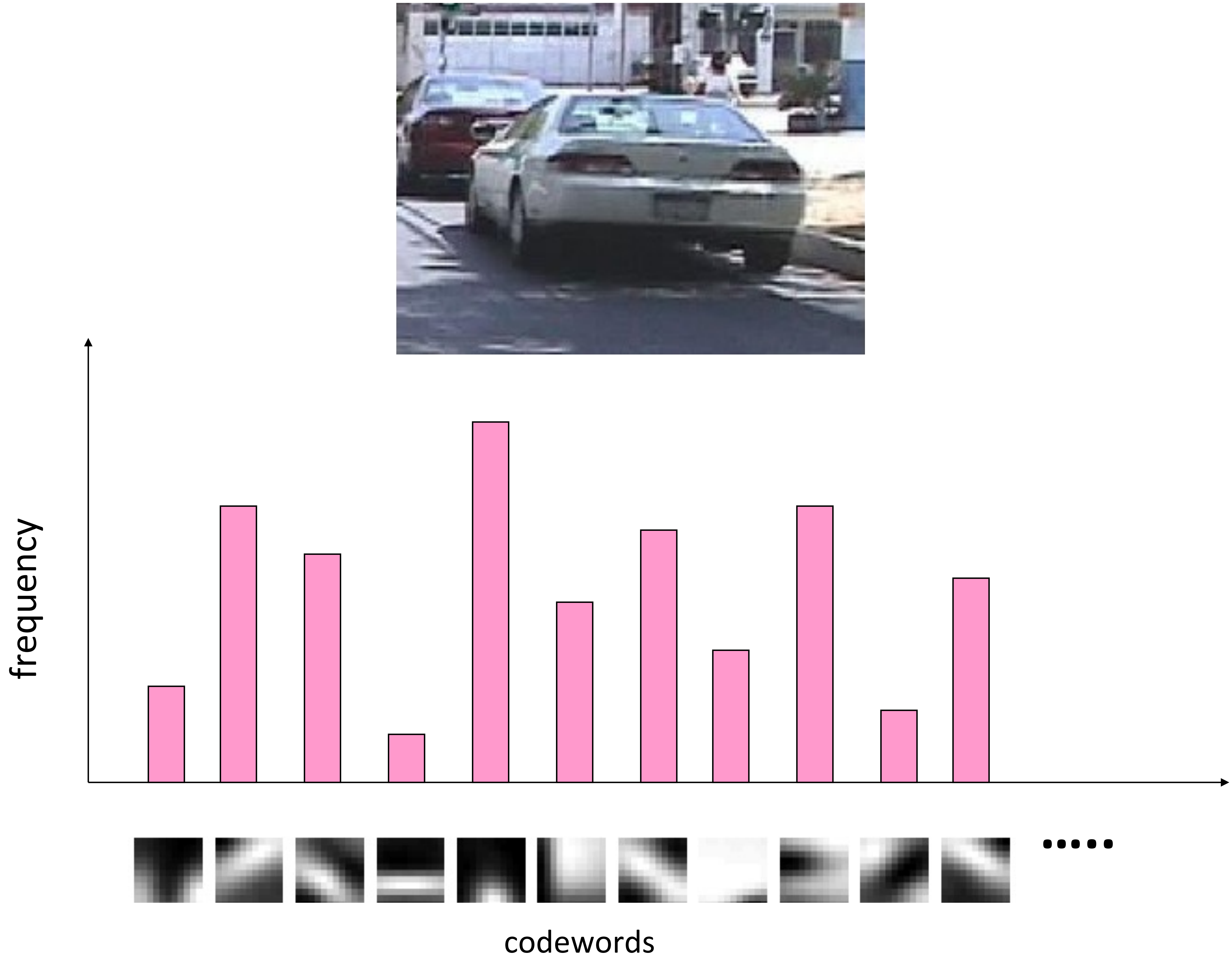
Encode:

build Bags-of-Words (BOW) vectors

for each image

2. Histogram: count the number of visual word occurrences

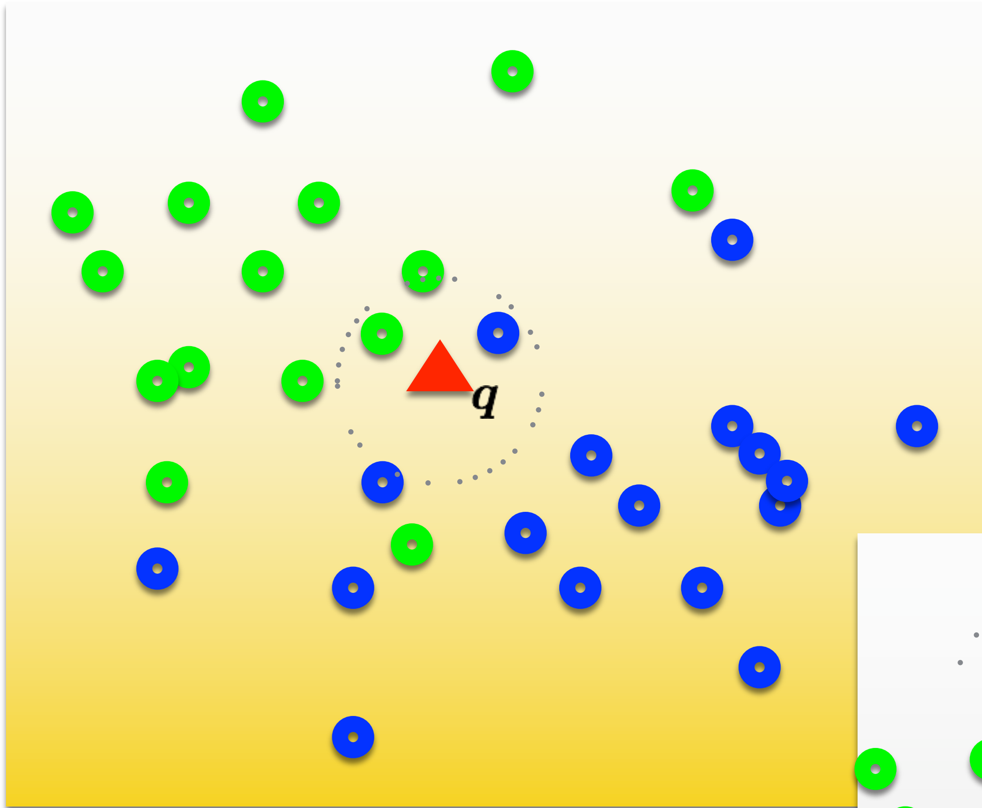



frequency
.....
codewords

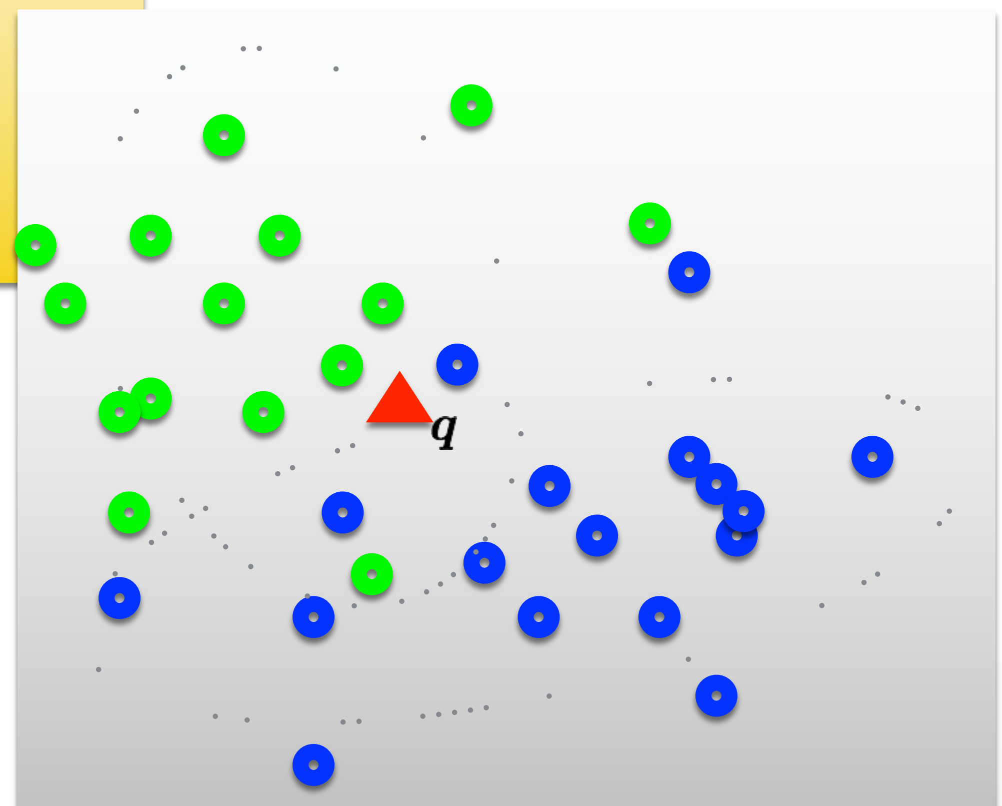
Dictionary Learning:
Learn Visual Words using clustering

Encode:
build Bags-of-Words (BOW) vectors
for each image

Classify:
Train and test data using BOWs

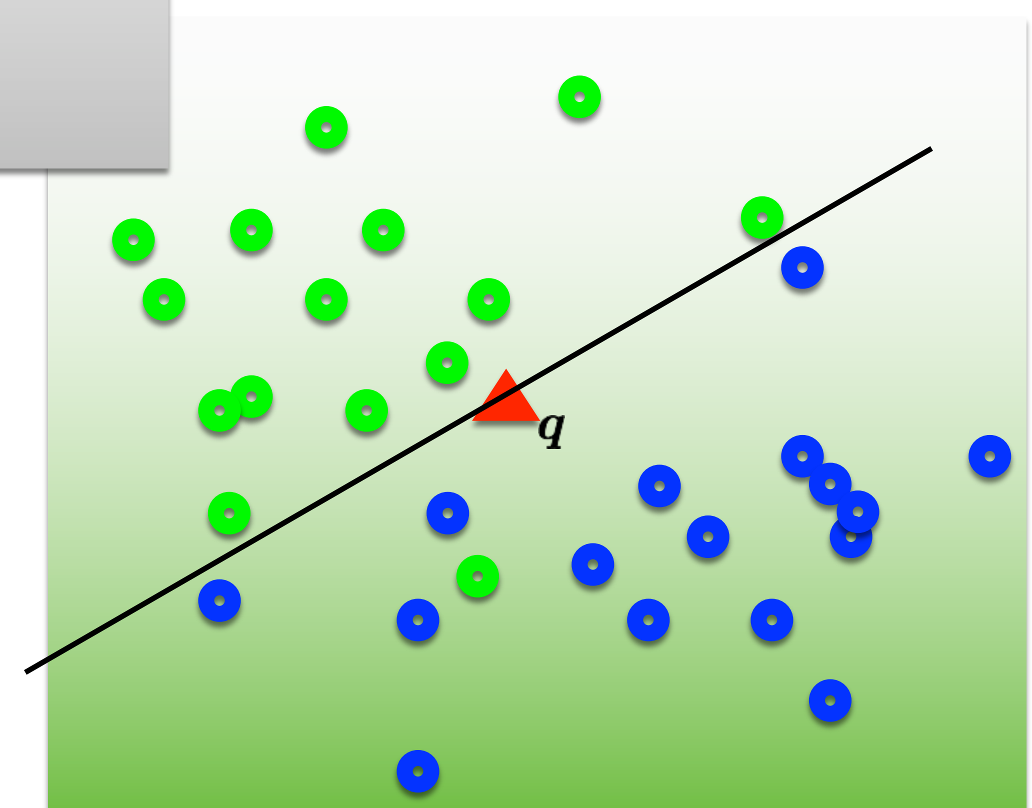


K nearest neighbors



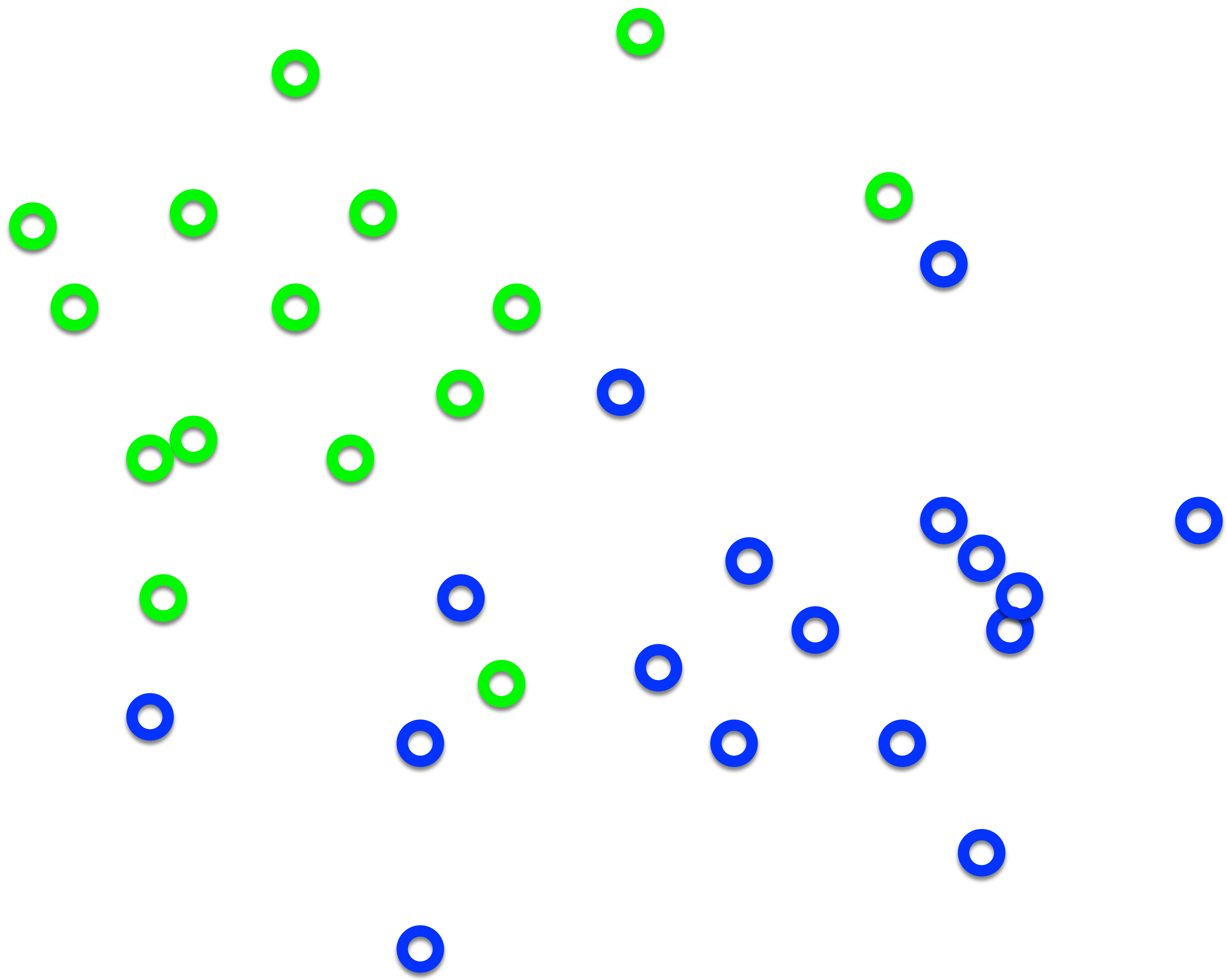
Naïve Bayes

Support Vector Machine

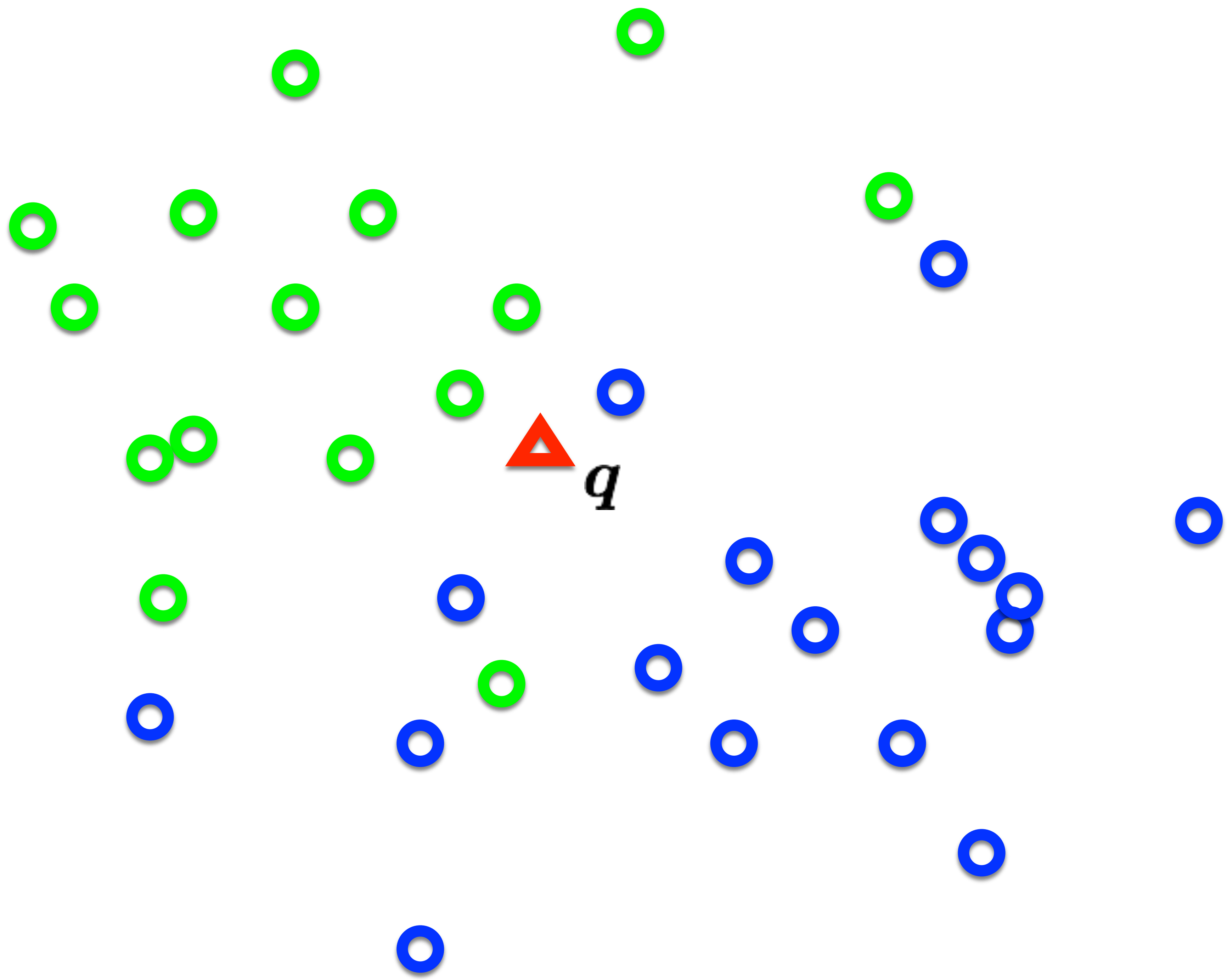


K nearest neighbors

Distribution of data from two classes

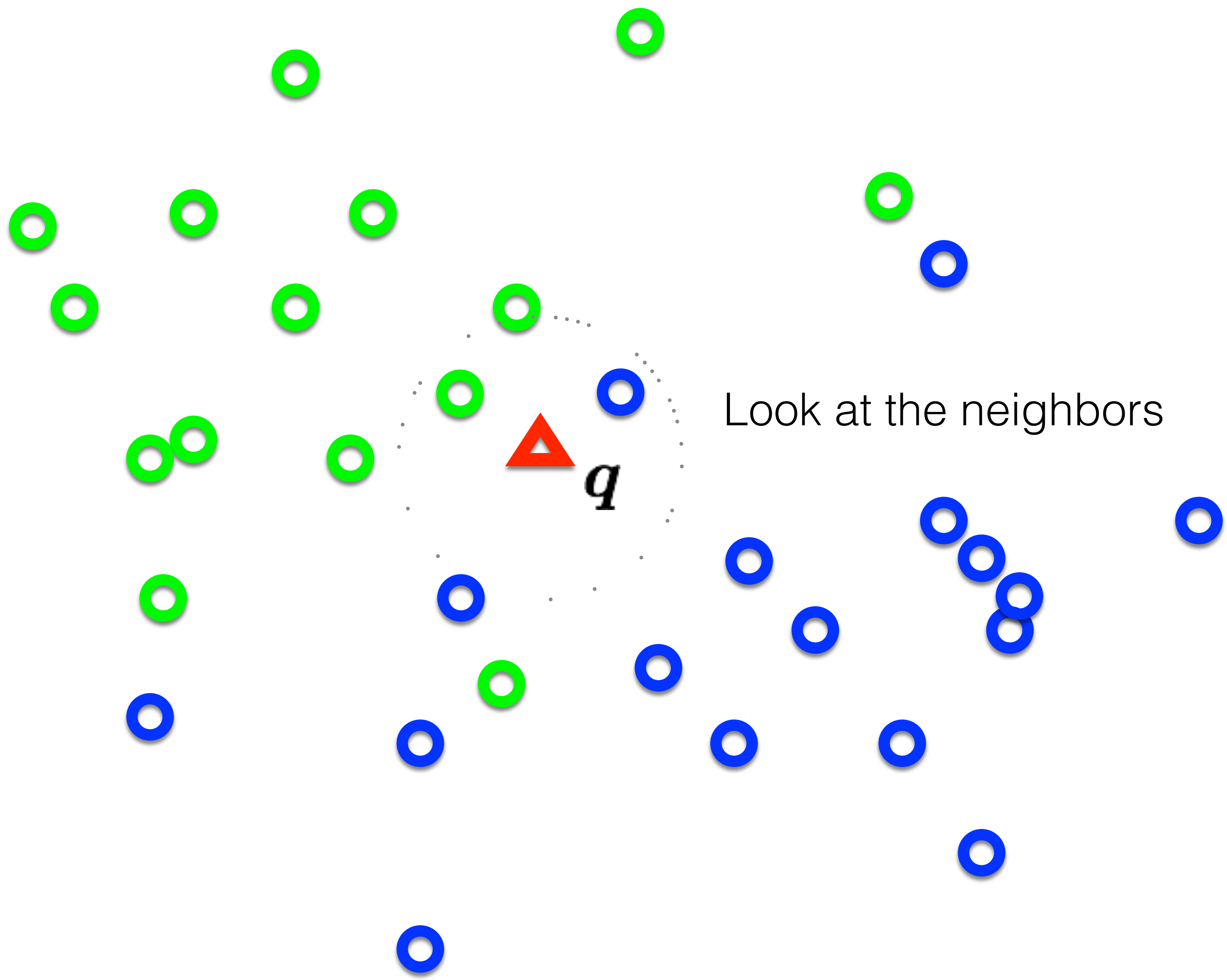


Distribution of data from two classes

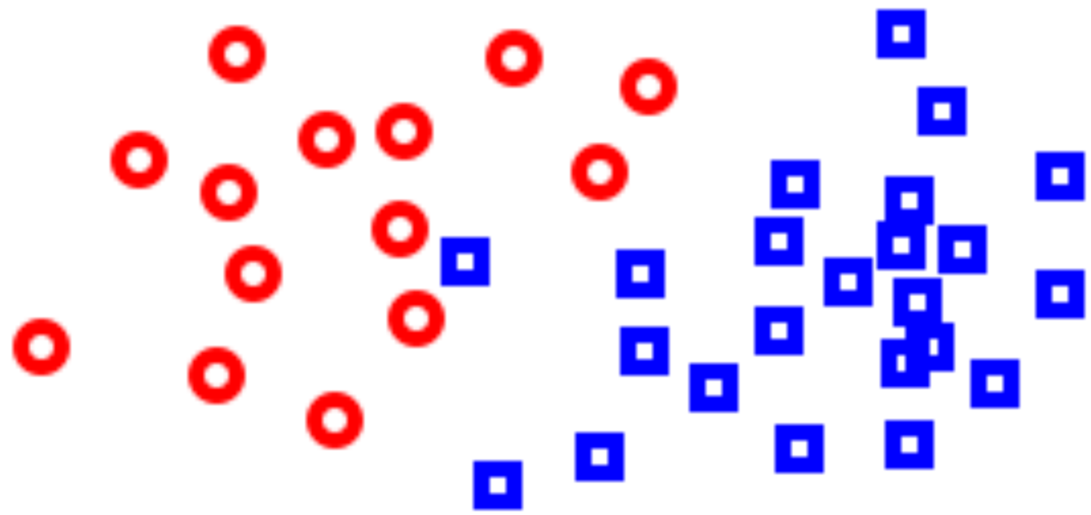


Which class does q belong too?

Distribution of data from two classes



K-Nearest Neighbor (KNN) Classifier

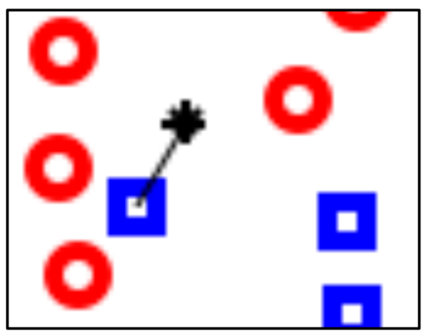


<u>Non-parametric</u> pattern classification approach

Consider a two class problem where each sample consists of two measurements (x,y).

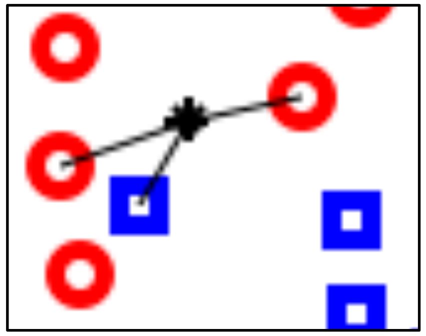
For a given query point q, assign the class of the nearest neighbor

k = 1

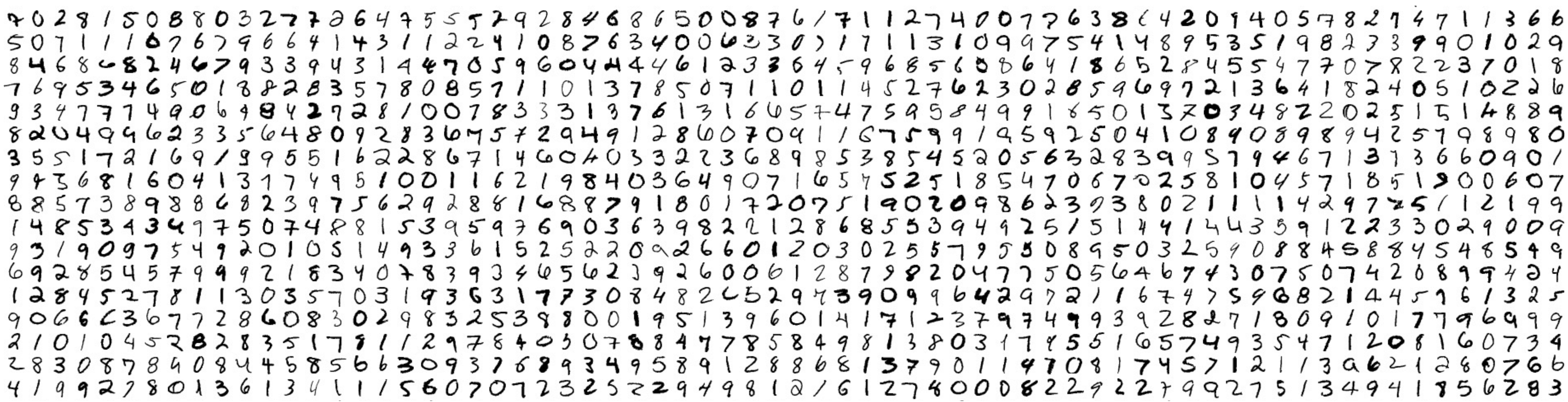


Compute the k nearest neighbors and assign the class by <u>majority vote</u>.

k = 3



Nearest Neighbor is competitive



MNIST Digit Recognition

- Handwritten digits
- 28x28 pixel images: d = 784
- 60,000 training samples
- 10,000 test samples

Yann LeCunn

	Test Error Rate (%)
Linear classifier (1-layer NN)	12.0
K-nearest-neighbors, Euclidean	5.0
K-nearest-neighbors, Euclidean, deskewed	2.4
K-NN, Tangent Distance, 16x16	1.1
K-NN, shape context matching	0.67
1000 RBF + linear classifier	3.6
SVM deg 4 polynomial	1.1
2-layer NN, 300 hidden units	4.7
2-layer NN, 300 HU, [deskewing]	1.6
LeNet-5, [distortions]	0.8
Boosted LeNet-4, [distortions]	0.7

What is the best distance metric between data points?

- Typically Euclidean distance
- Locality sensitive distance metrics
- Important to normalize.
  Dimensions have different scales

How many K?

- Typically k=1 is good
- Cross-validation (try different k!)

Distance metrics

$$D(\boldsymbol{x},\boldsymbol{y}) = \sqrt{(x_1-y_1)^2+\cdots+(x_N-y_N)^2}$$

Euclidean

$$D(\boldsymbol{x},\boldsymbol{y}) = \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{\|\boldsymbol{x}\|\|\boldsymbol{y}\|} = \frac{x_1y_1+\cdots+x_Ny_N}{\sqrt{\sum_n x_n^2}\sqrt{\sum_n y_n^2}}$$

Cosine

$$D(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{2}\sum_n \frac{(x_n-y_n)^2}{(x_n+y_n)}$$

Chi-squared

Choice of distance metric

- Hyperparameter

L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p \left(I_1^p - I_2^p\right)^2}$$

- Two most commonly used special cases of p-norm

$$||x||_p = \left(|x_1|^p + \cdots + |x_n|^p\right)^{\frac{1}{p}} \qquad p \geq 1, x \in \mathbb{R}^n$$

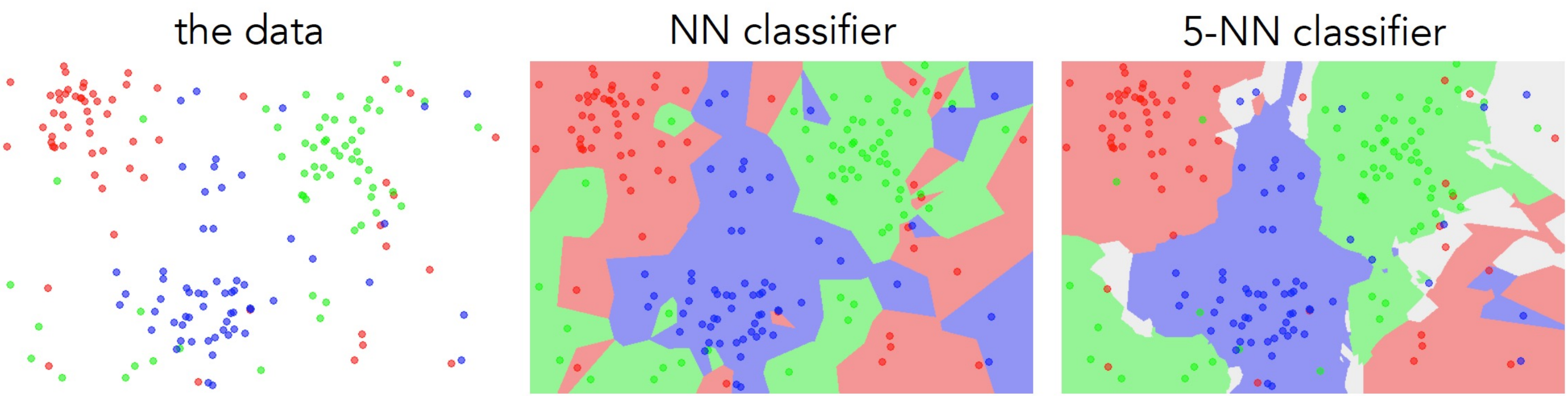
CIFAR-10 and NN results

Example dataset: **CIFAR-10**
10 labels
50,000 training images
10,000 test images.

For every test image (first column), examples of nearest neighbors in rows

k-nearest neighbor

- Find the k closest points from training data
- Labels of the k points “vote” to classify



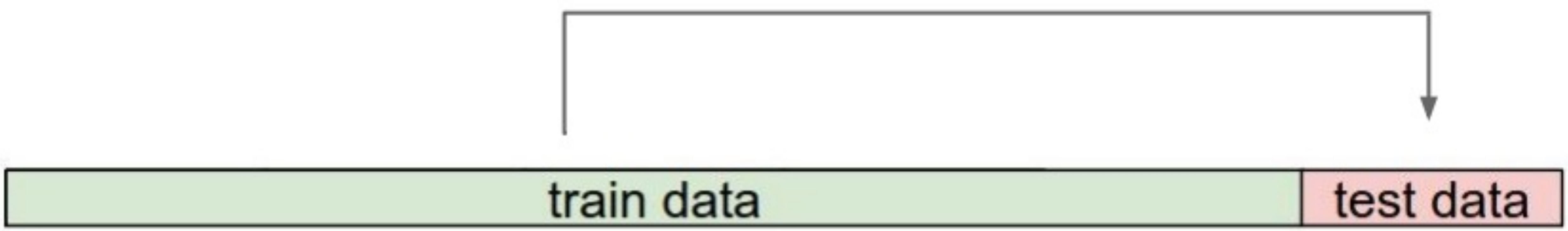
Hyperparameters

- What is the best distance to use?
- What is the best value of k to use?

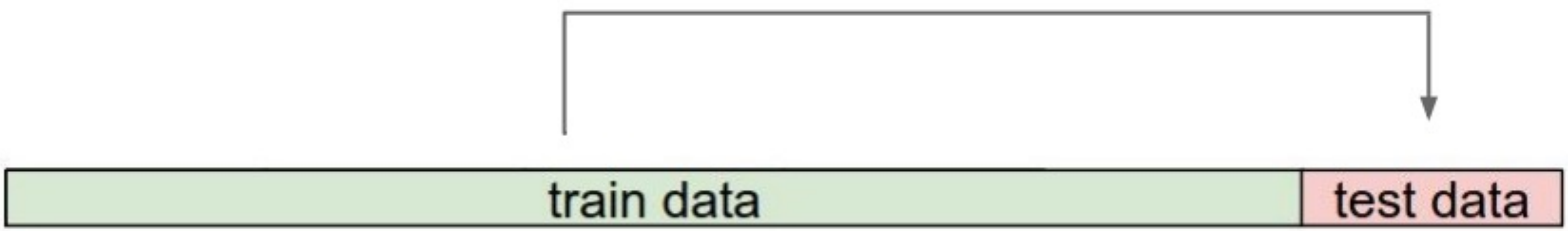
- i.e., how do we set the hyperparameters?

- Very problem-dependent
- Must try them all and see what works best

Try out what hyperparameters work best on test set.

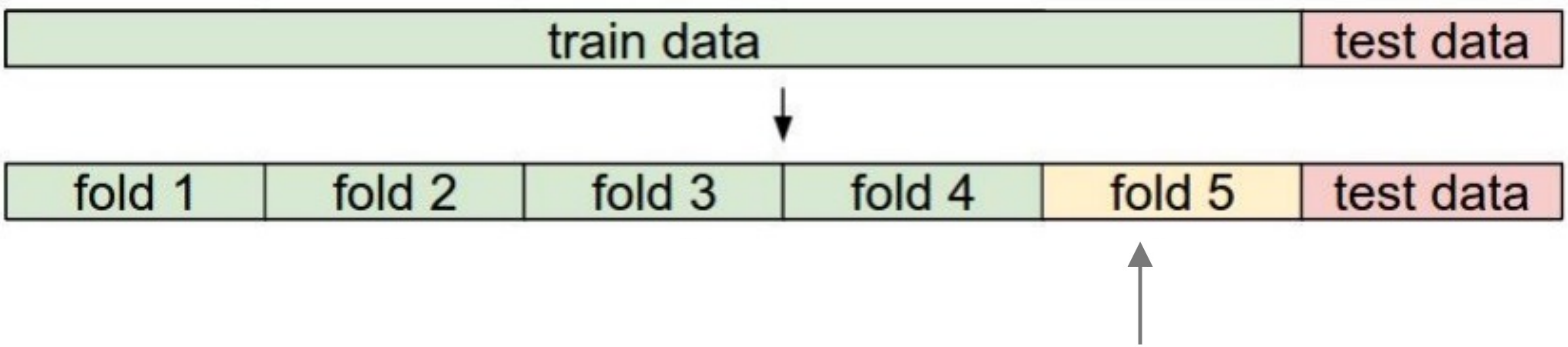


Try out what hyperparameters work best on test set.



VERY BAD IDEA! The test set is a proxy for the generalization performance!
Use only VERY SPARINGLY, at the end.

Validation

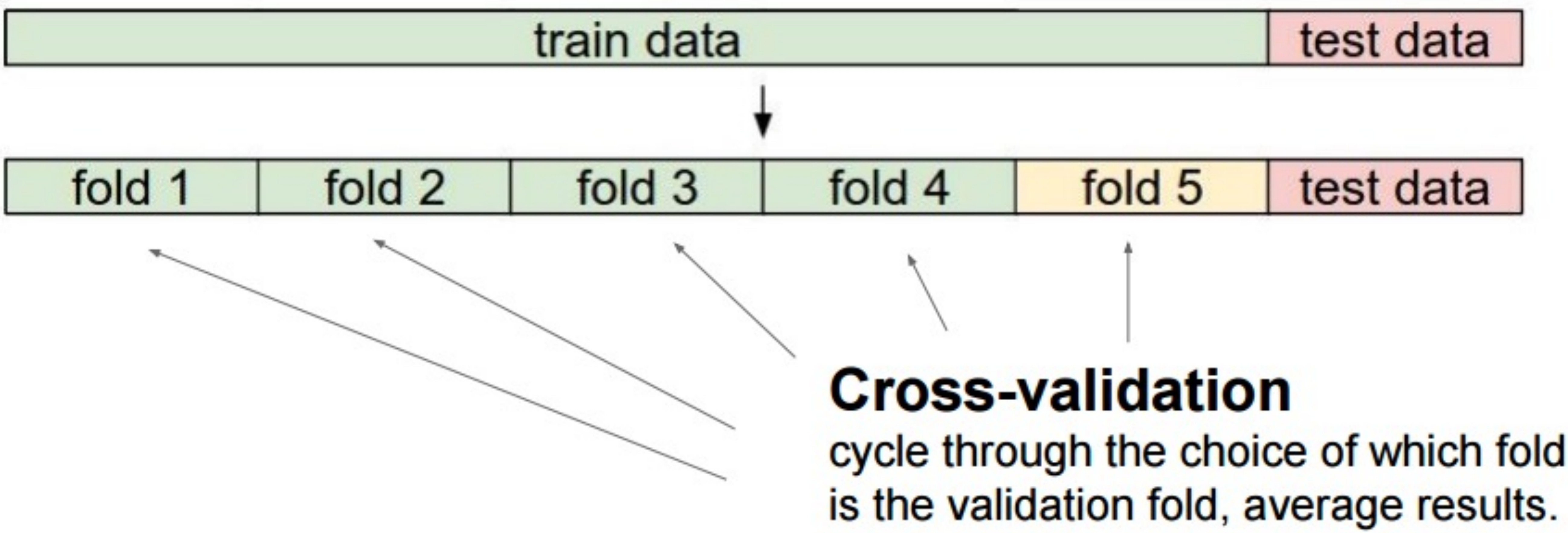


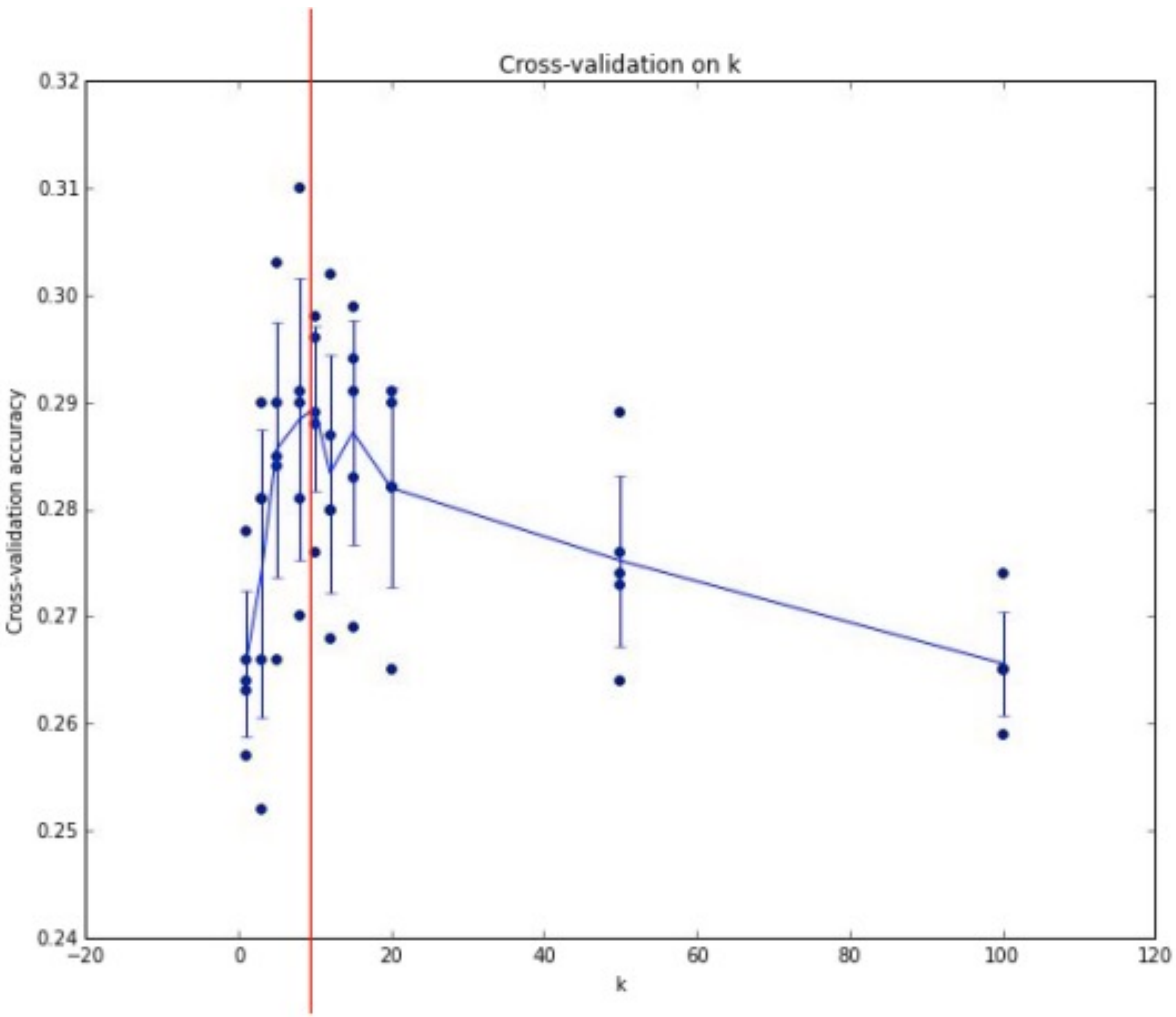
Validation data

use to tune hyperparameters

evaluate on test set ONCE at the end

Cross-validation





Example of
5-fold cross-validation
for the value of **k.**

Each point: single
outcome.

The line goes
through the mean, bars
indicated standard
deviation

(Seems that k ~= 7 works best
for this data)

How to pick hyperparameters?

- Methodology
  - Train, validate, test

- Train for original model
- Validate to find hyperparameters
- Test to understand generalizability

Pros

- simple yet effective

Cons

- search is expensive (can be sped-up)
- storage requirements
- difficulties with high-dimensional data

kNN -- Complexity and Storage

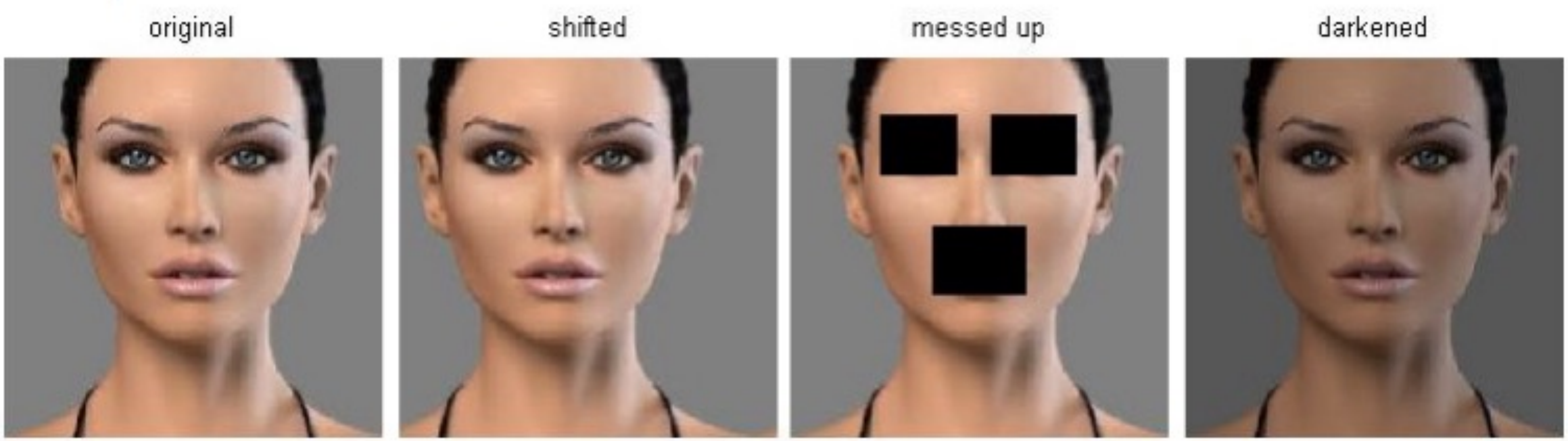
- N training images, M test images

- Training: O(1)
- Testing: O(MN)

- Hmm…
  - Normally need the opposite
  - Slow training (ok), fast testing (necessary)

k-Nearest Neighbor on images **never used.**

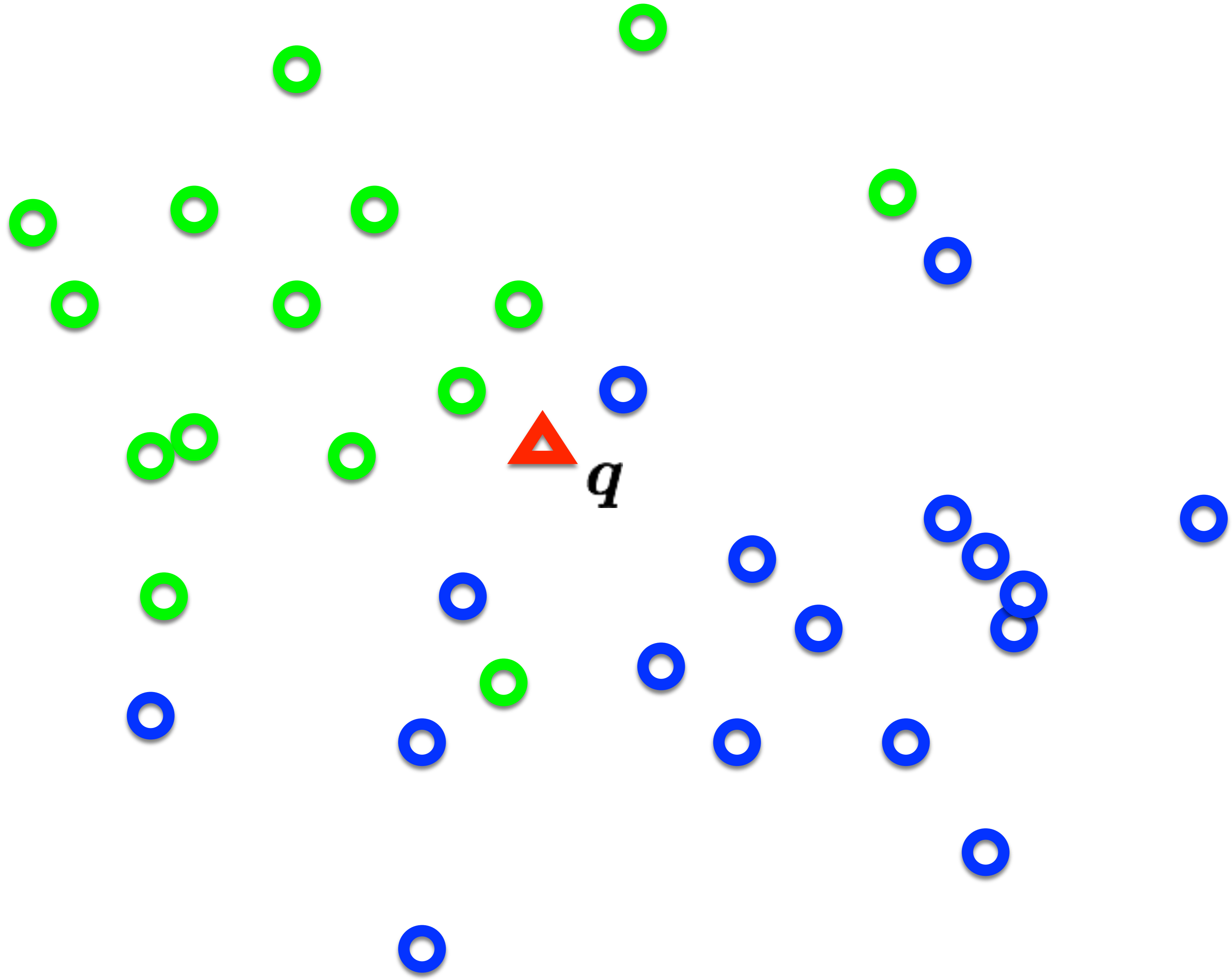
- terrible performance at test time
- distance metrics on level of whole images can be very unintuitive



(all 3 images have same L2 distance to the one on the left)

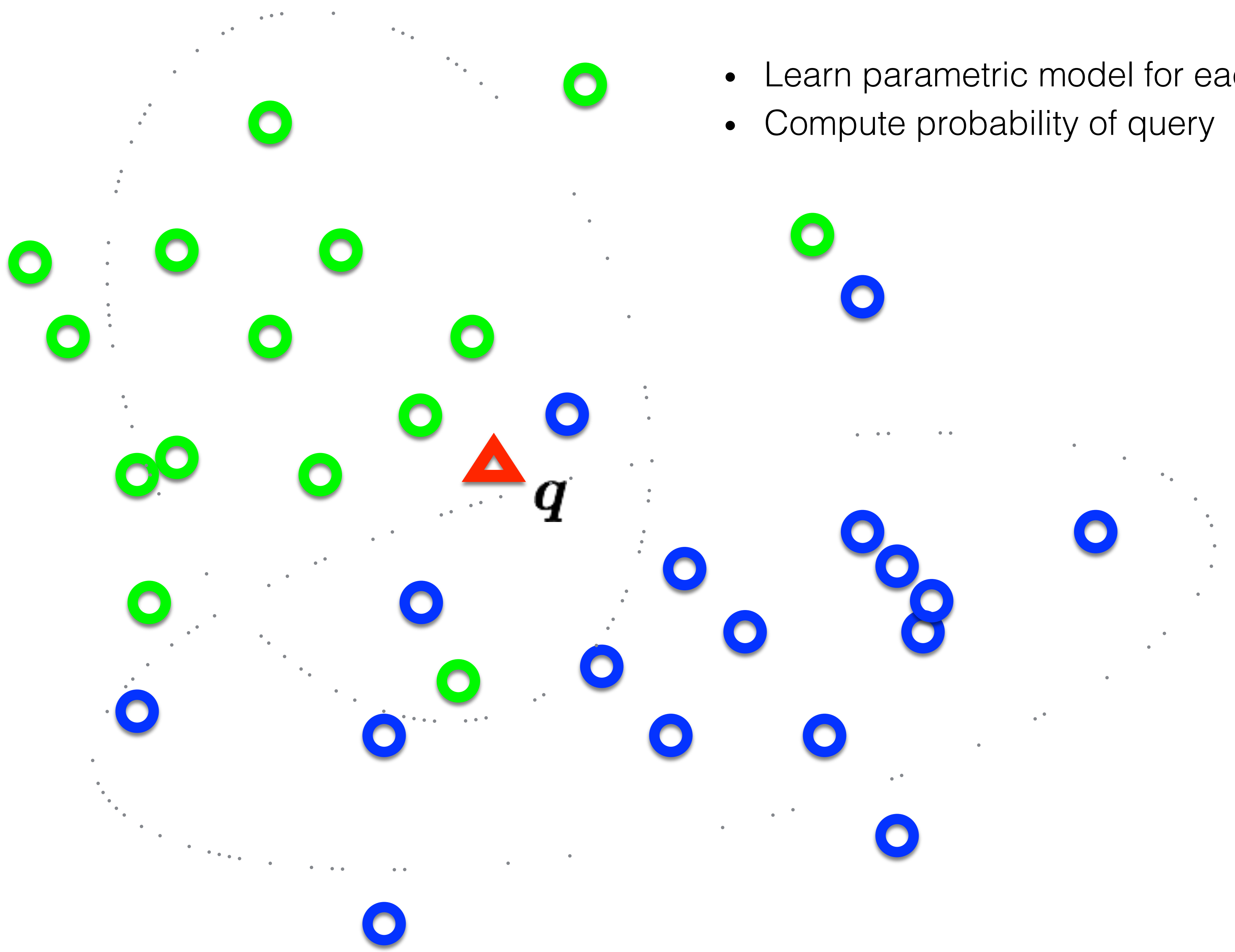
Naïve Bayes

Distribution of data from two classes



Which class does q belong too?

Distribution of data from two classes



- Learn parametric model for each class
- Compute probability of query

This is called the posterior.

the probability of a class z given the observed features X

$$p(\boldsymbol{z}|\boldsymbol{X})$$

For classification, z is a discrete random variable
(e.g., car, person, building)

X is a set of observed features
(e.g., features from a single image)

(it's a function that returns a single probability value)

This is called the posterior:

the probability of a class z given the observed features X

$$p(\boldsymbol{z}|\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$$

For classification, z is a discrete random variable (e.g., car, person, building)

Each x is an observed feature (e.g., visual words)

(it's a function that returns a single probability value)

Recall:

The posterior can be decomposed according to **Bayes' Rule**

likelihood prior

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

posterior

In our context…

$$p(\boldsymbol{z}|\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) = \frac{p(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N|\boldsymbol{z})p(\boldsymbol{z})}{p(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)}$$

The naive Bayes' classifier is solving this optimization

$$\hat{z} = \arg\max_{z \in \boldsymbol{z}} p(z|\boldsymbol{X})$$

MAP (maximum a posteriori) estimate

$$\hat{z} = \arg\max_{z \in \boldsymbol{z}} \frac{p(\boldsymbol{X}|z)p(z)}{p(\boldsymbol{X})}$$

Bayes' Rule

$$\hat{z} = \arg\max_{z \in \boldsymbol{z}} p(\boldsymbol{X}|z)p(z)$$

Remove constants

To optimize this…we need to compute this

Compute the likelihood…

A naive Bayes' classifier assumes all features are
conditionally independent

$$\begin{aligned} p(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N | \boldsymbol{z}) &= p(\boldsymbol{x}_1 | \boldsymbol{z}) p(\boldsymbol{x}_2, \ldots, \boldsymbol{x}_N | \boldsymbol{z}) \\ &= p(\boldsymbol{x}_1 | \boldsymbol{z}) p(\boldsymbol{x}_2 | \boldsymbol{z}) p(\boldsymbol{x}_3, \ldots, \boldsymbol{x}_N | \boldsymbol{z}) \\ &= p(\boldsymbol{x}_1 | \boldsymbol{z}) p(\boldsymbol{x}_2 | \boldsymbol{z}) \cdots p(\boldsymbol{x}_N | \boldsymbol{z}) \end{aligned}$$

To compute the MAP estimate

Given (1) a set of known parameters

$$p(\boldsymbol{z}) \quad p(\boldsymbol{x}|\boldsymbol{z})$$

(2) observations

$$\{x_1, x_2, \ldots, x_N\}$$

Compute which z has the largest probability

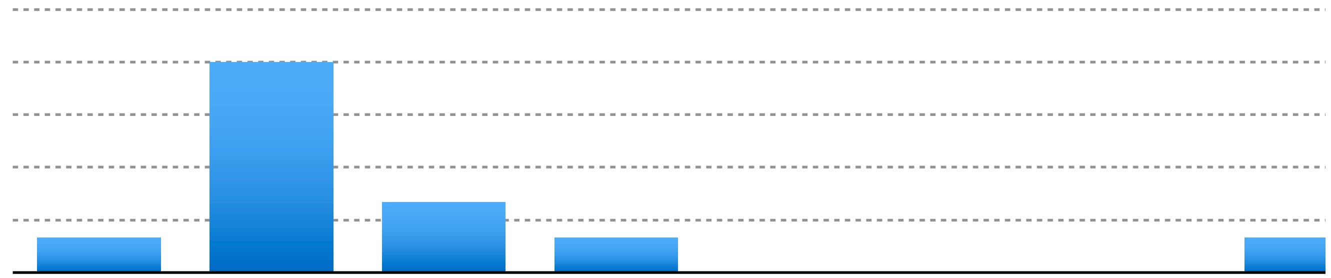
$$\hat{z} = \arg\max_{z \in \boldsymbol{z}} p(z) \prod_n p(x_n|z)$$

The Newspa

Sunday, December 22, 2013

DARPA Selects Carnegie Me

The Tartan Rescue Team from Carnegie Mellon University's National Robotics Engineering Center ranked third among teams competing in the Defense Advanced Research Projects Agency (DARPA) Robotics Challenge Trials this weekend in Homestead, Fla., and was selected by the agency as one of eight teams eligible for DARPA funding to prepare for next December's finals. The team's four-limbed CMU Highly Intelligent Mobile Platform, or CHIMP, robot scored 18 out of a possible 32 points during the two-day trials. It demonstrated its ability to perform such tasks as removing debris, cutting a hole through a wall and closing a series of valves.



count	1	6	2	1	0	0	0	1
word	Tartan	robot	CHIMP	CMU	bio	soft	ankle	sensor
p(x|z)	0.09	0.55	0.18	0.09	0.0	0.0	0.0	0.09

$$p(X|z) = \prod_v p(x_v|z)^{c(w_v)}$$

$$= (0.09)^1 (0.55)^6 \cdots (0.09)^1$$

Numbers get really small so use log probabilities

$$\log p(X|z = \text{‘grandchallenge’}) = -2.42 - 3.68 - 3.43 - 2.42 - 0.07 - 0.07 - 0.07 - 2.42 = -14.58$$

$$\log p(X|z = \text{‘softrobot’}) = -7.63 - 9.37 - 15.18 - 2.97 - 0.02 - 0.01 - 0.02 - 2.27 = -37.48$$

* typically add pseudo-counts (0.001)
** this is an example for computing the likelihood, need to multiply times **prior** to get posterior

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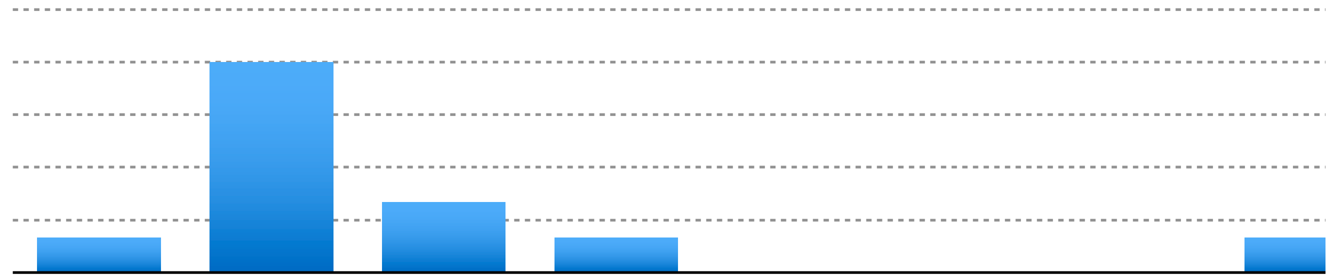


count	1	6	2	1	0	0	0	1
word	Tartan	robot	CHIMP	CMU	bio	soft	ankle	sensor
p(x|z)	0.09	0.55	0.18	0.09	0.0	0.0	0.0	0.09

log p(X|z=grand challenge) = **- 14.58**

log p(X|z=bio inspired) = - 37.48

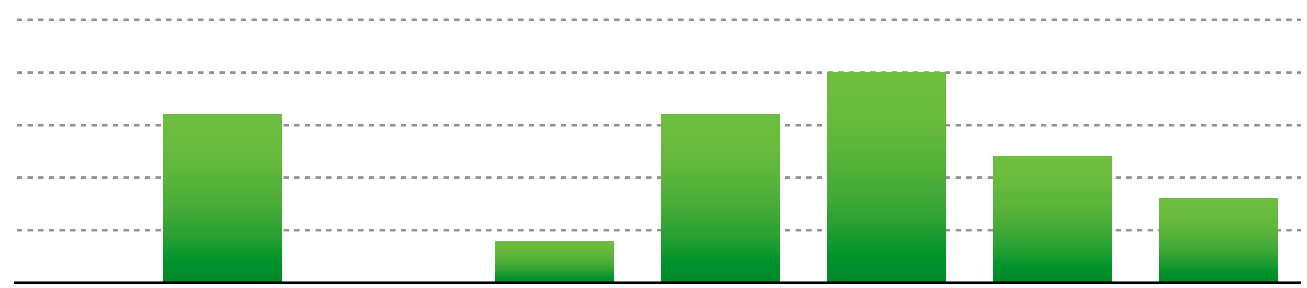
Tartan Tim

Monday, January 20, 2014

Bio-Inspired Robotic Device

PITTSBURGH—A soft, wearable device that mimics the muscles, tendons and ligaments of the lower leg could aid in the rehabilitation of patients with ankle-foot disorders such as drop foot, said Yong-Lae Park, an assistant professor of robotics at Carnegie Mellon University. Park, working with collaborators at Harvard University, the University of Southern California, MIT and BioSensics, developed an active orthotic device using soft plastics and composite materials, instead of a rigid exoskeleton. The soft materials, combined with pneumatic artificial muscles (PAMs), lightweight sensors and advanced control software, made it possible for the robotic device to achieve natural motions in the ankle.

http://www.fodey.com/generators/newspaper/snippet.asp



count	0	4	0	1	4	5	3	2
word	Tartan	robot	CHIMP	CMU	bio	soft	ankle	sensor
p(x|z)	0.0	0.21	0.0	0.05	0.21	0.26	0.16	0.11

log p(X|z=grand challenge) = - 94.06

log p(X|z=bio inspired) = **- 32.41**

* typically add pseudo-counts (0.001)

** this is an example for computing the likelihood, need to multiply times prior to get posterior

Support Vector Machine

Image Classification

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

Score function

class scores

Linear Classifier

define a **score function**

data (histogram)

$$f(x_i, W, b) = Wx_i + b$$

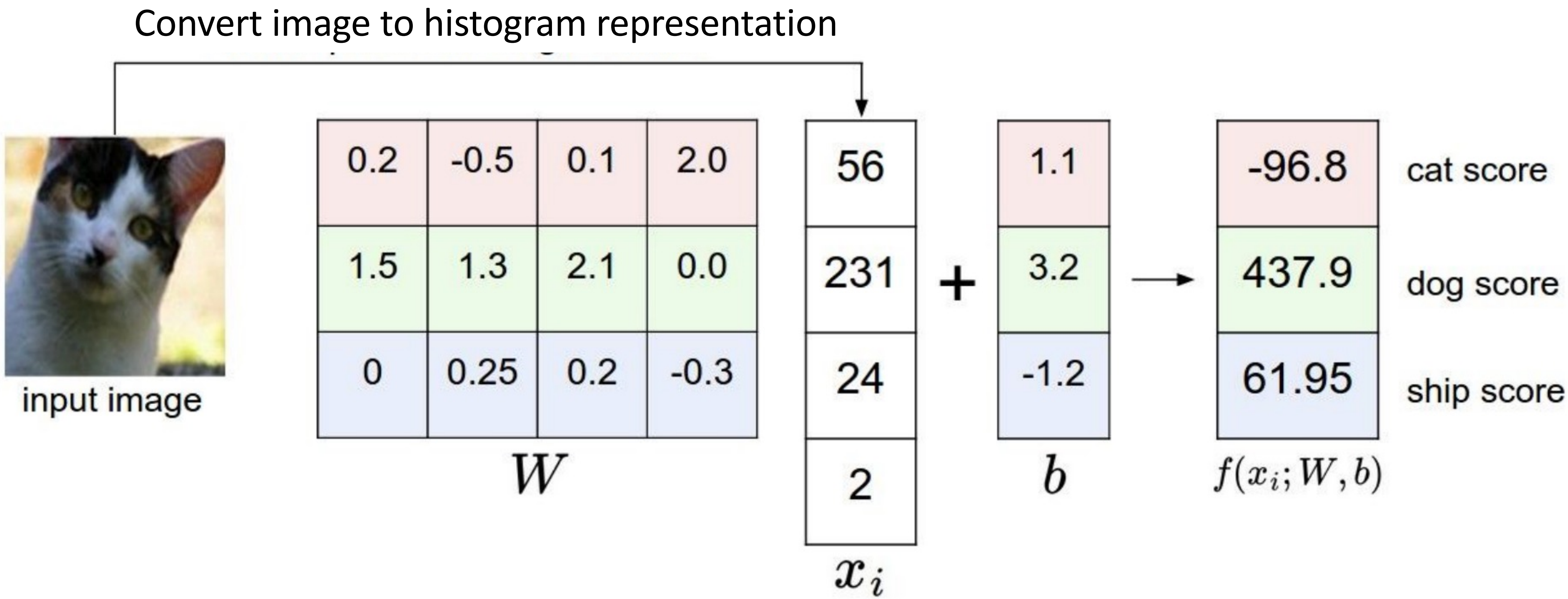
class scores

"weights"

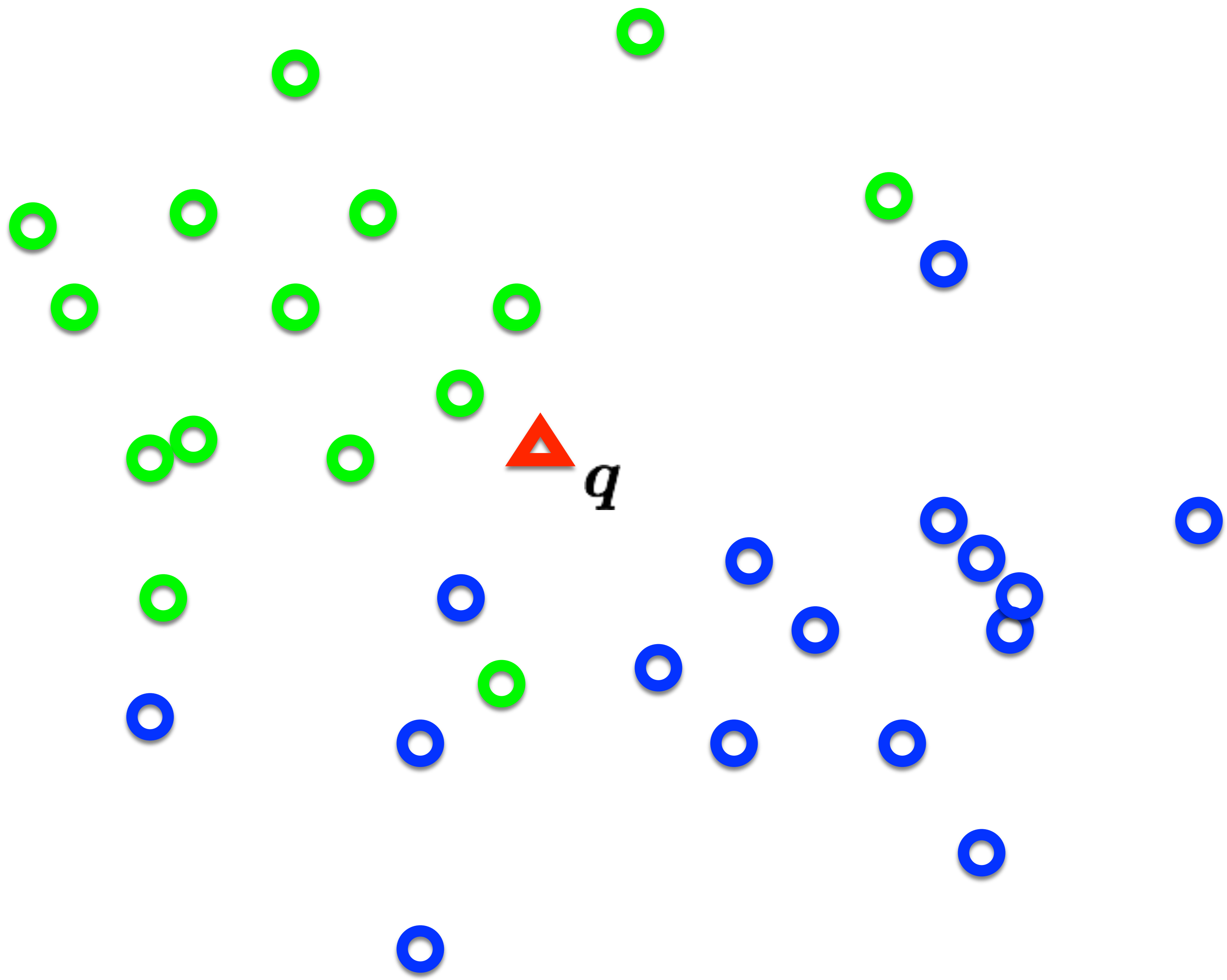
"bias vector"

"parameters"

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Distribution of data from two classes



Which class does q belong too?

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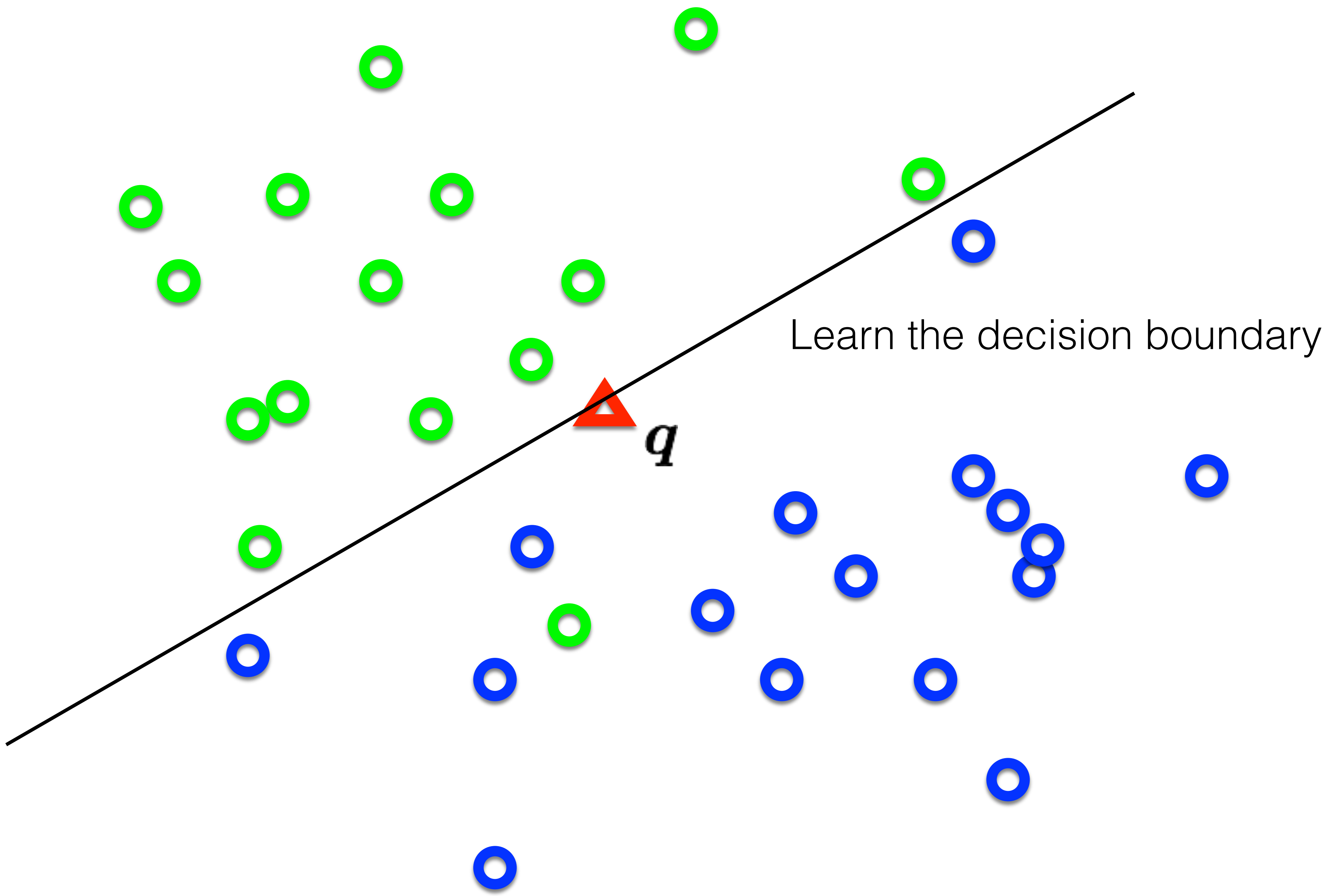
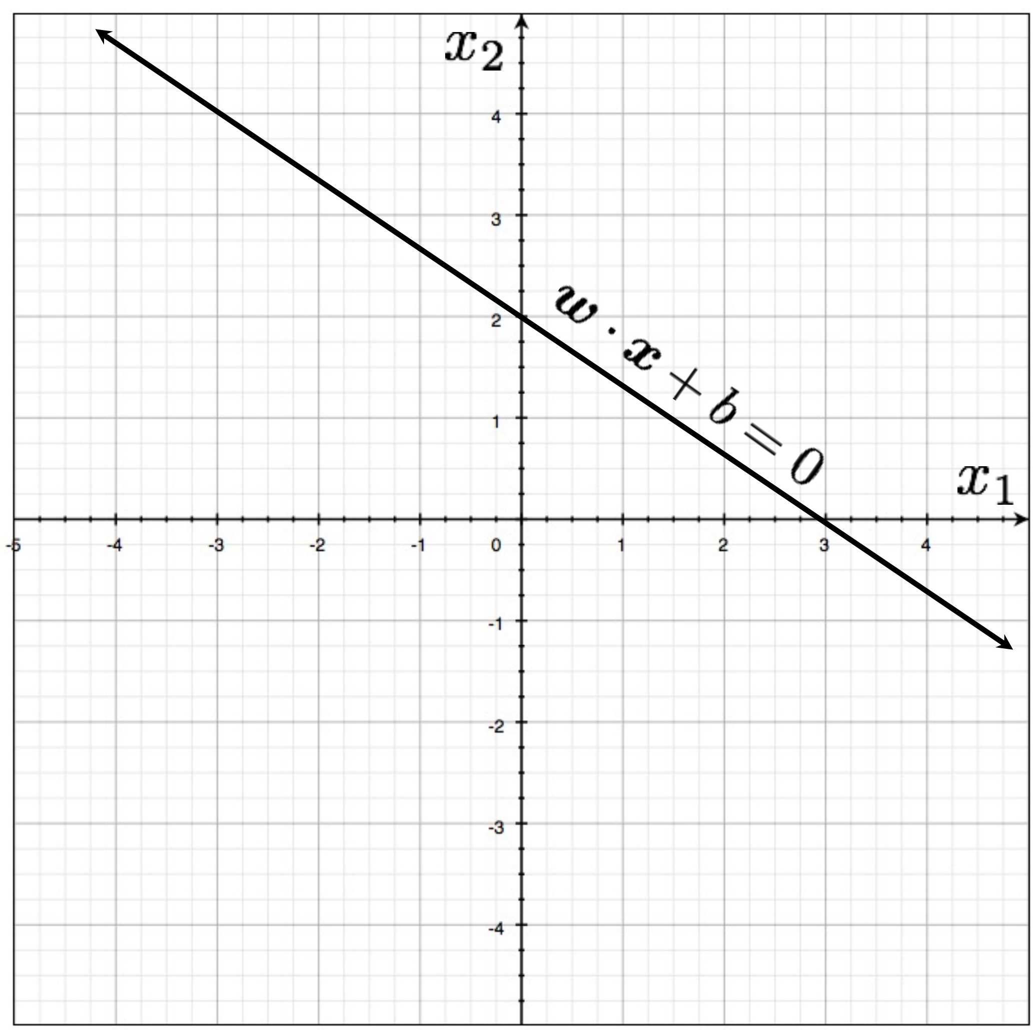


First we need to understand hyperplanes…

Hyperplanes (lines) in 2D

$$w_1x_1 + w_2x_2 + b = 0$$



a line can be written as dot product plus a bias

$$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$$

$$\boldsymbol{w} \in \mathcal{R}^2$$

another version, add a weight 1 and push the bias inside

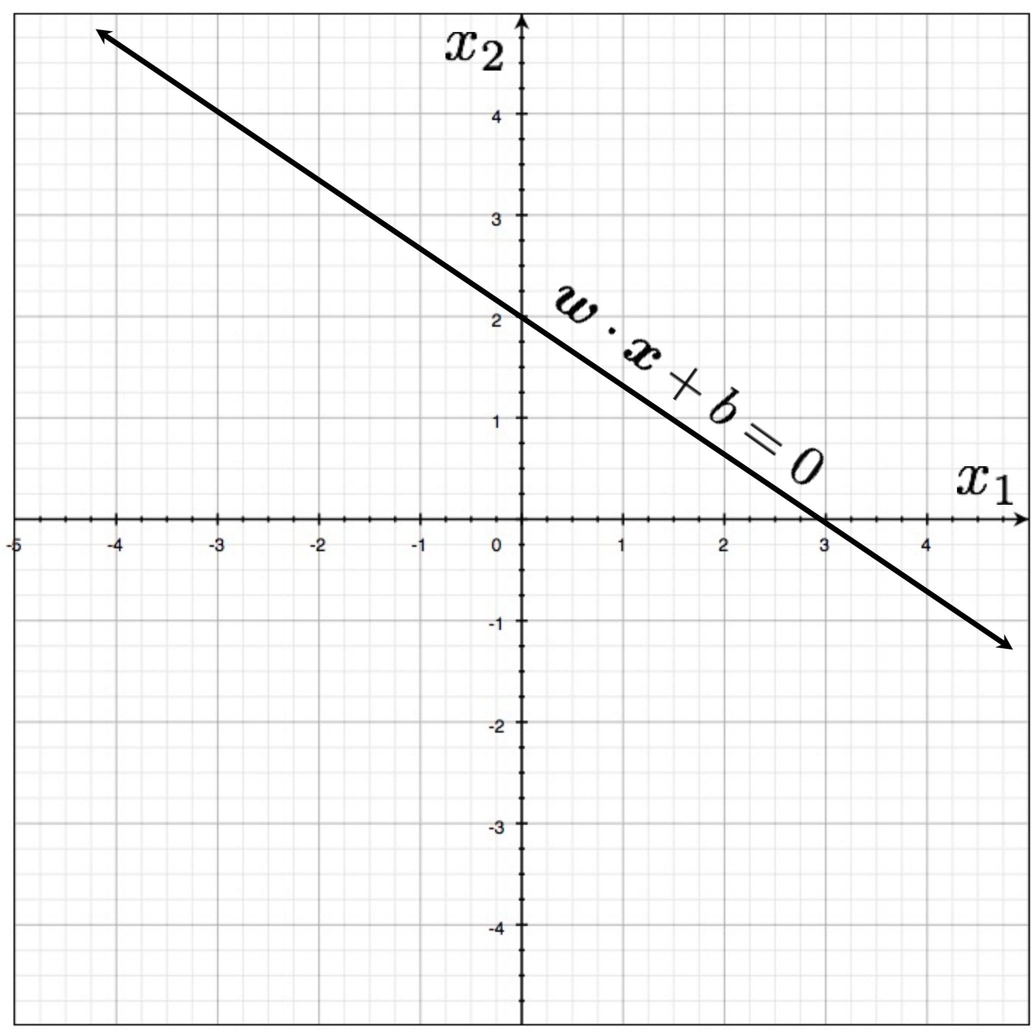
$$\boldsymbol{w} \cdot \boldsymbol{x} = 0$$

$$\boldsymbol{w} \in \mathcal{R}^3$$

Hyperplanes (lines) in 2D

$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$ (offset/bias outside) $\boldsymbol{w} \cdot \boldsymbol{x} = 0$ (offset/bias inside)

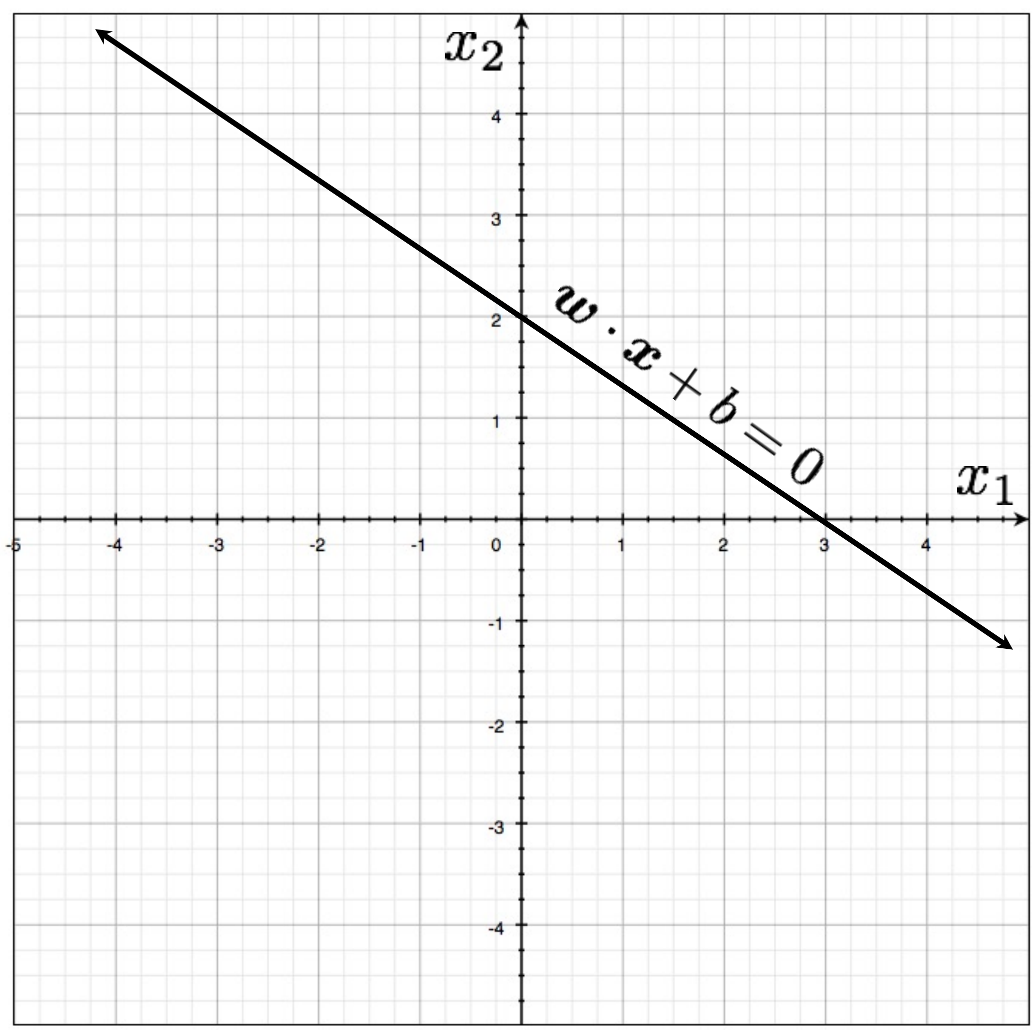
$$w_1 x_1 + w_2 x_2 + b = 0$$



Hyperplanes (lines) in 2D

$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$ (offset/bias outside) $\boldsymbol{w} \cdot \boldsymbol{x} = 0$ (offset/bias inside)

$$w_1 x_1 + w_2 x_2 + b = 0$$



Important property:

Free to choose any normalization of w

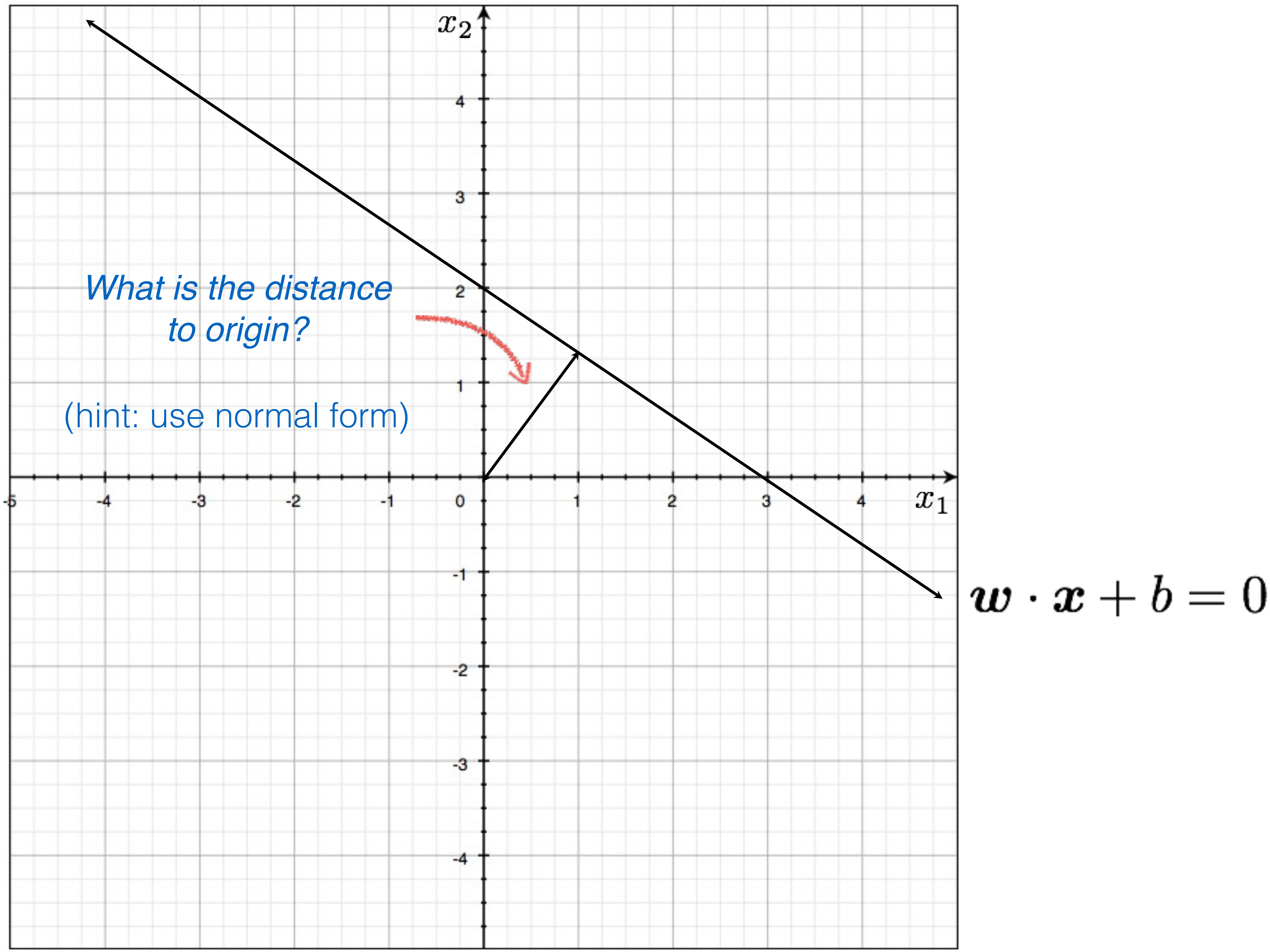
The line

$$w_1 x_1 + w_2 x_2 + b = 0$$

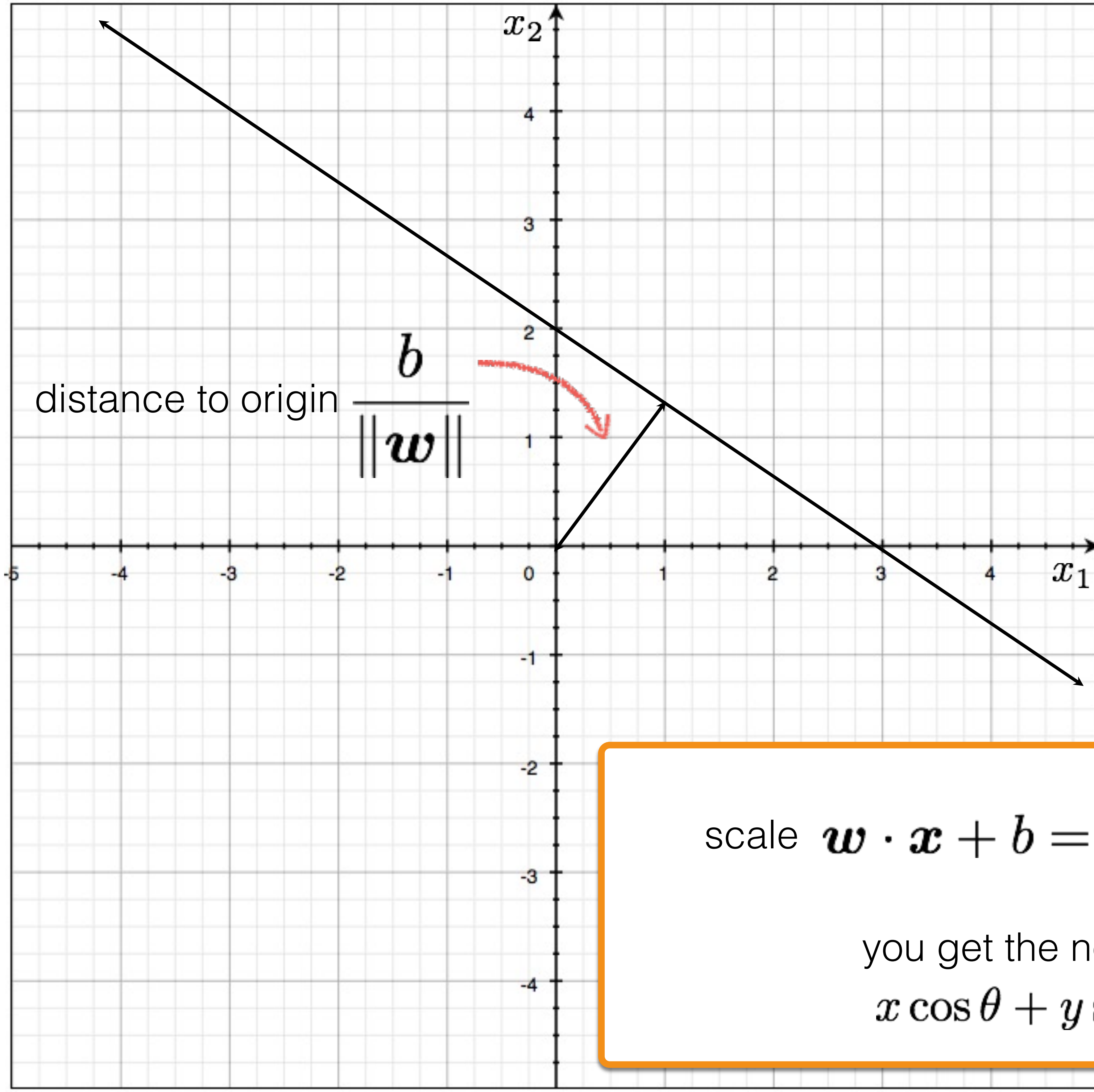
and the line

$$\lambda(w_1 x_1 + w_2 x_2 + b) = 0$$

define the same line



x_2
x_1
What is the distance to origin?
(hint: use normal form)
$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$



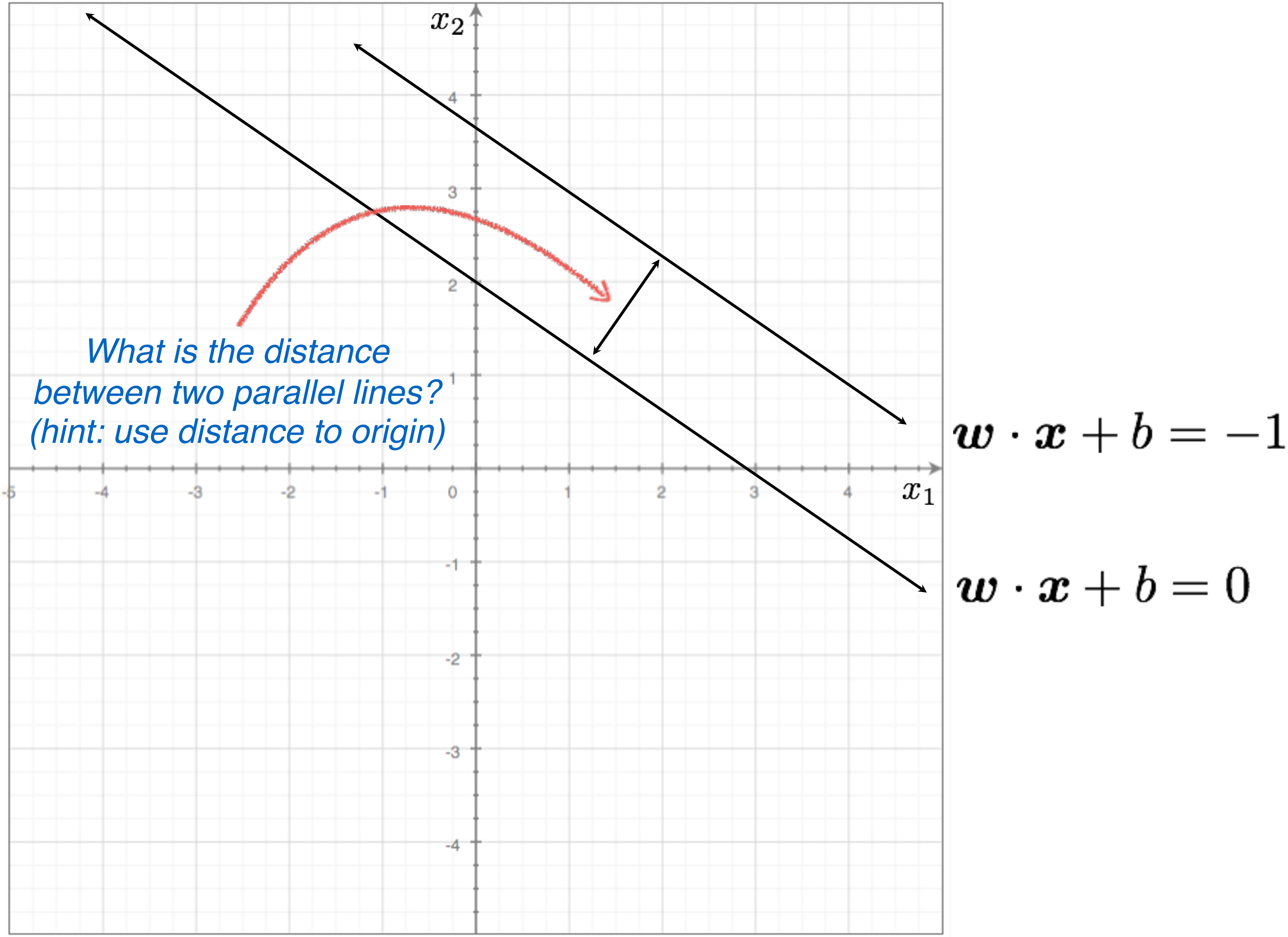
distance to origin $\dfrac{b}{\|\boldsymbol{w}\|}$

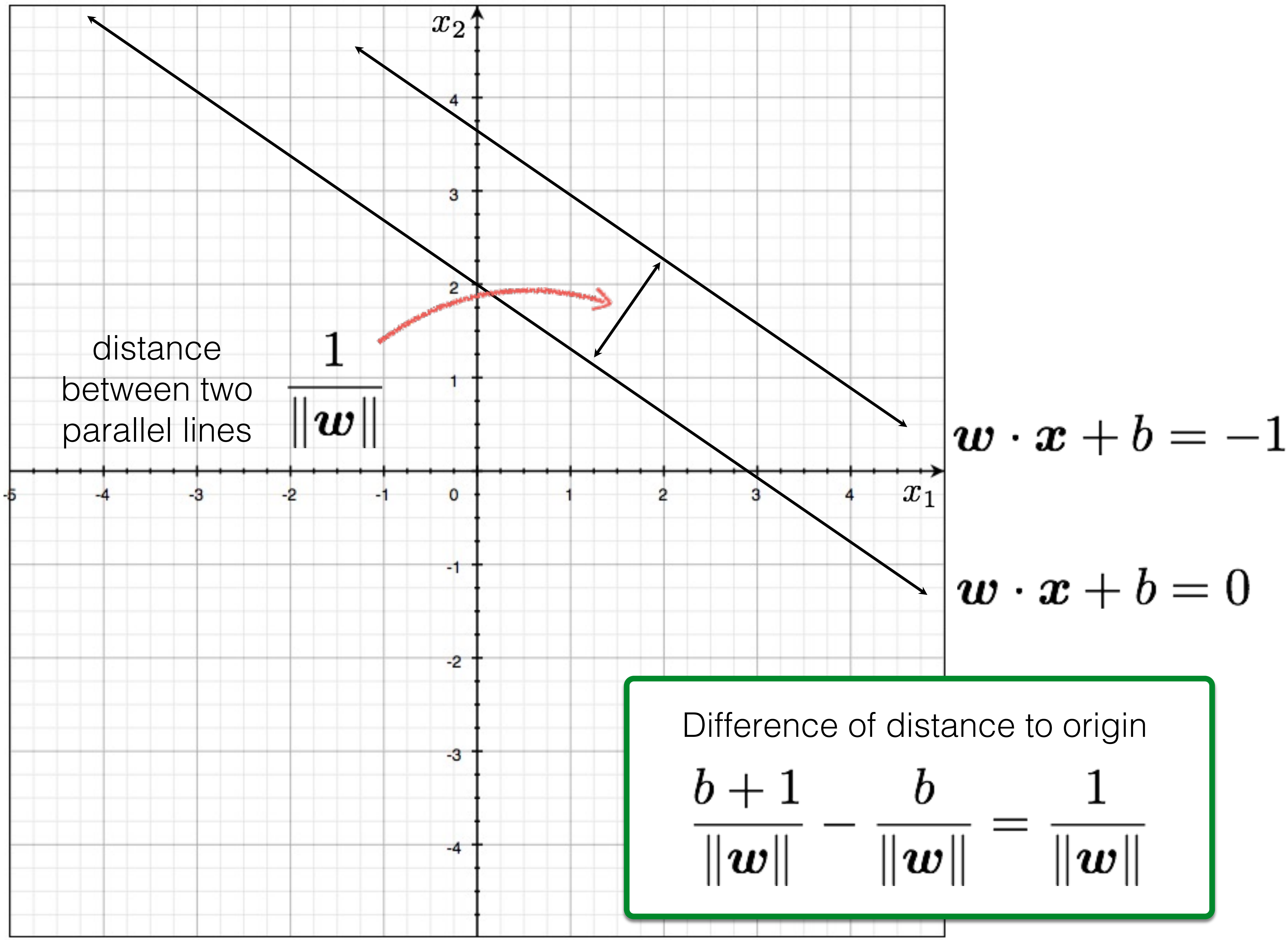
$$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$$

scale $\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$ by $\dfrac{1}{\|\boldsymbol{w}\|}$

you get the normal form

$$x \cos\theta + y \sin\theta = \rho$$

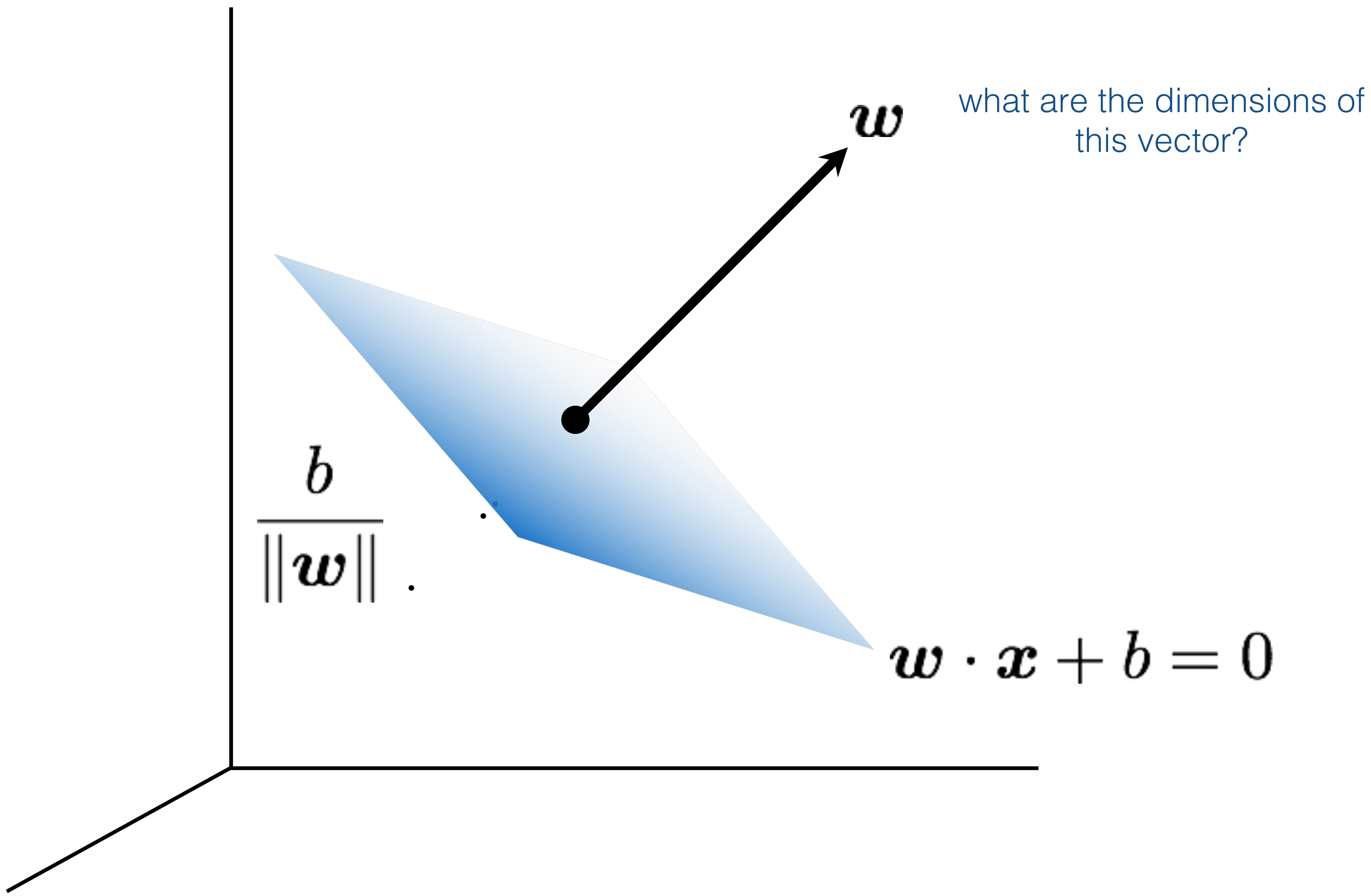

x_2
x_1
What is the distance
between two parallel lines?
(hint: use distance to origin)
$\boldsymbol{w} \cdot \boldsymbol{x} + b = -1$
$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$



x_2
x_1
4
3
2
1
0
-1
-2
-3
-4
-5
distance between two parallel lines
$\frac{1}{\|\boldsymbol{w}\|}$
$\boldsymbol{w} \cdot \boldsymbol{x} + b = -1$
$\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$
Difference of distance to origin
$\frac{b+1}{\|\boldsymbol{w}\|} - \frac{b}{\|\boldsymbol{w}\|} = \frac{1}{\|\boldsymbol{w}\|}$

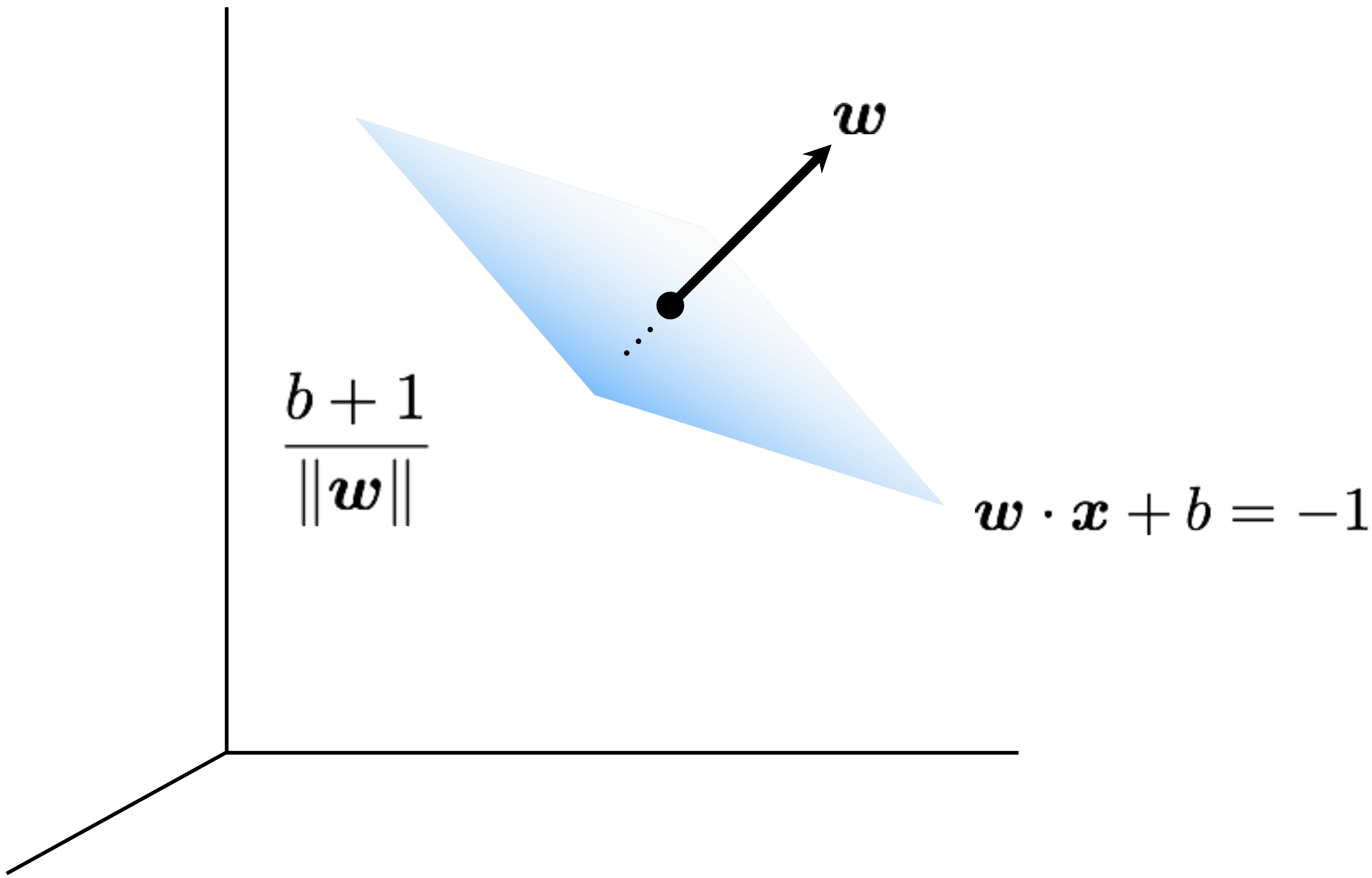
Now we can go to 3D …

Hyperplanes (planes) in 3D

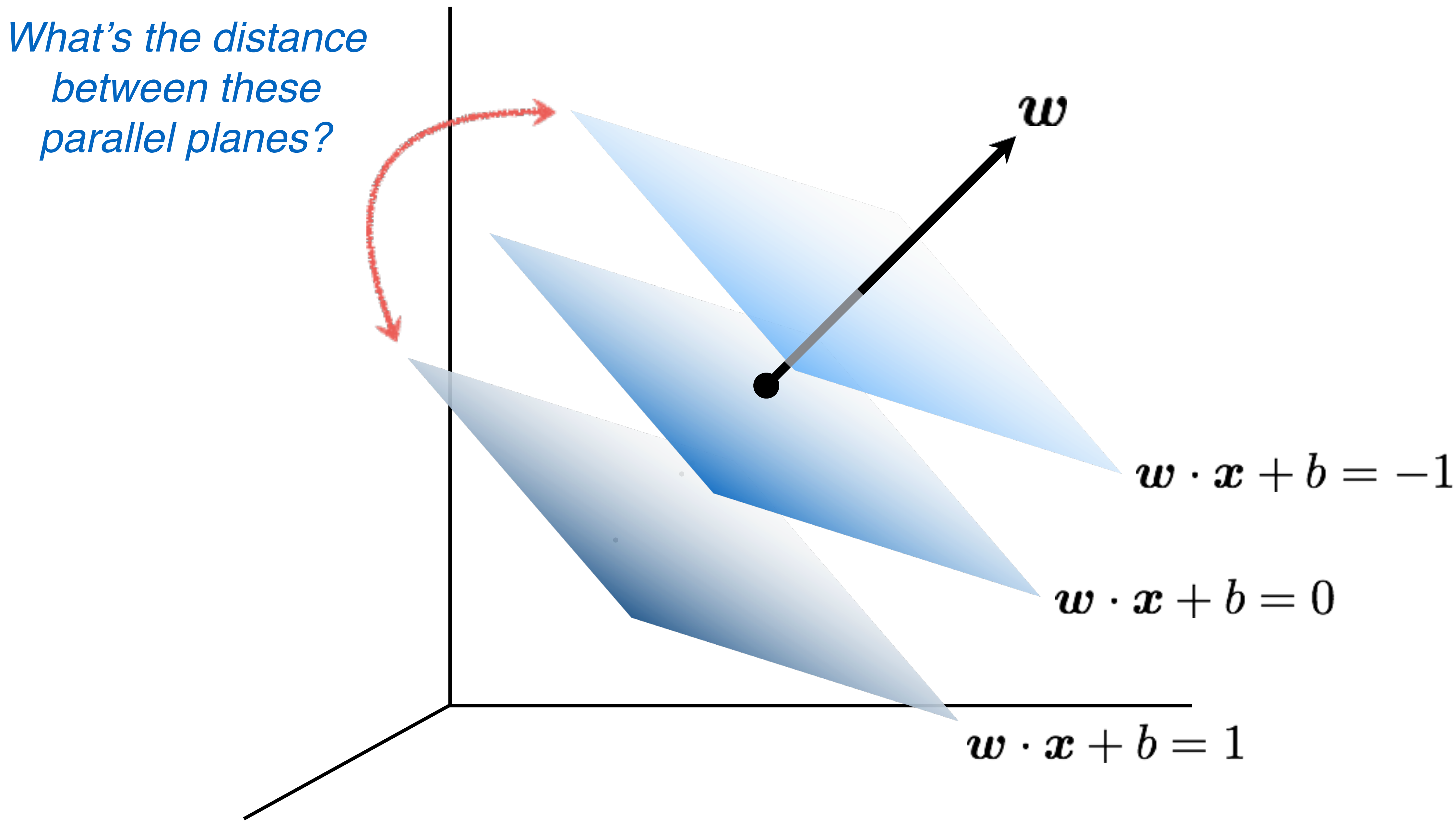


*What happens if you change **b**?*

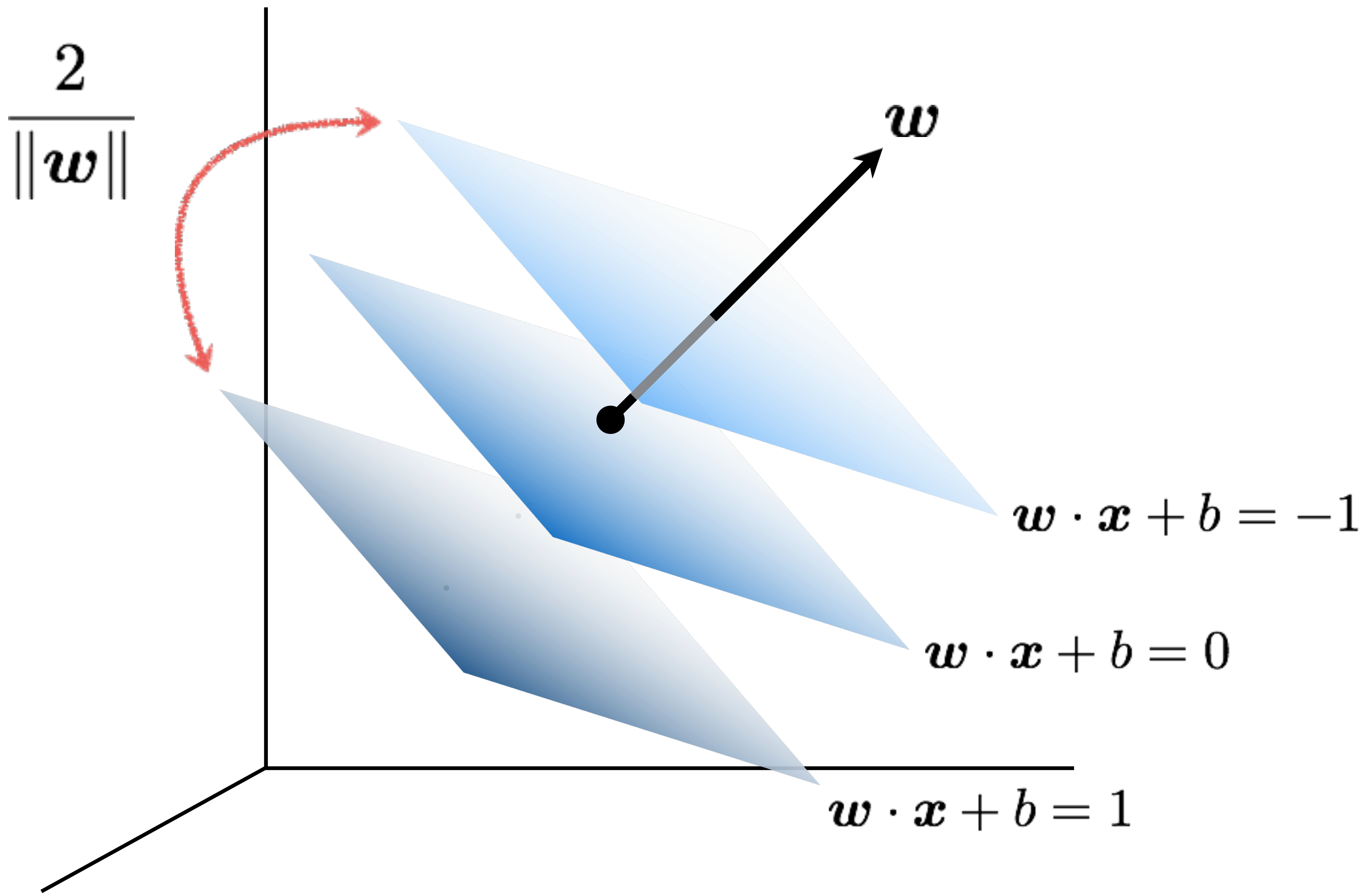
Hyperplanes (planes) in 3D



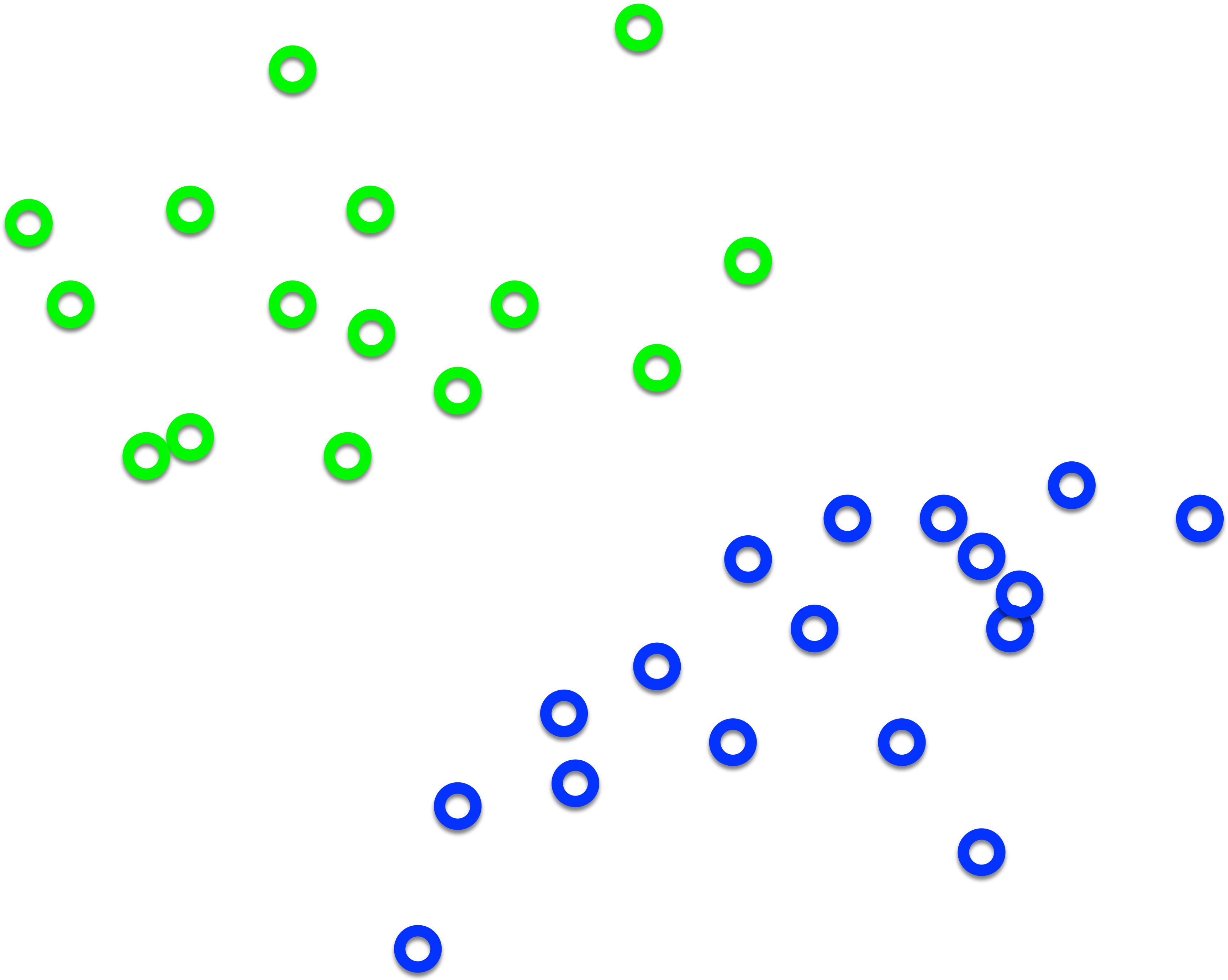
Hyperplanes (planes) in 3D



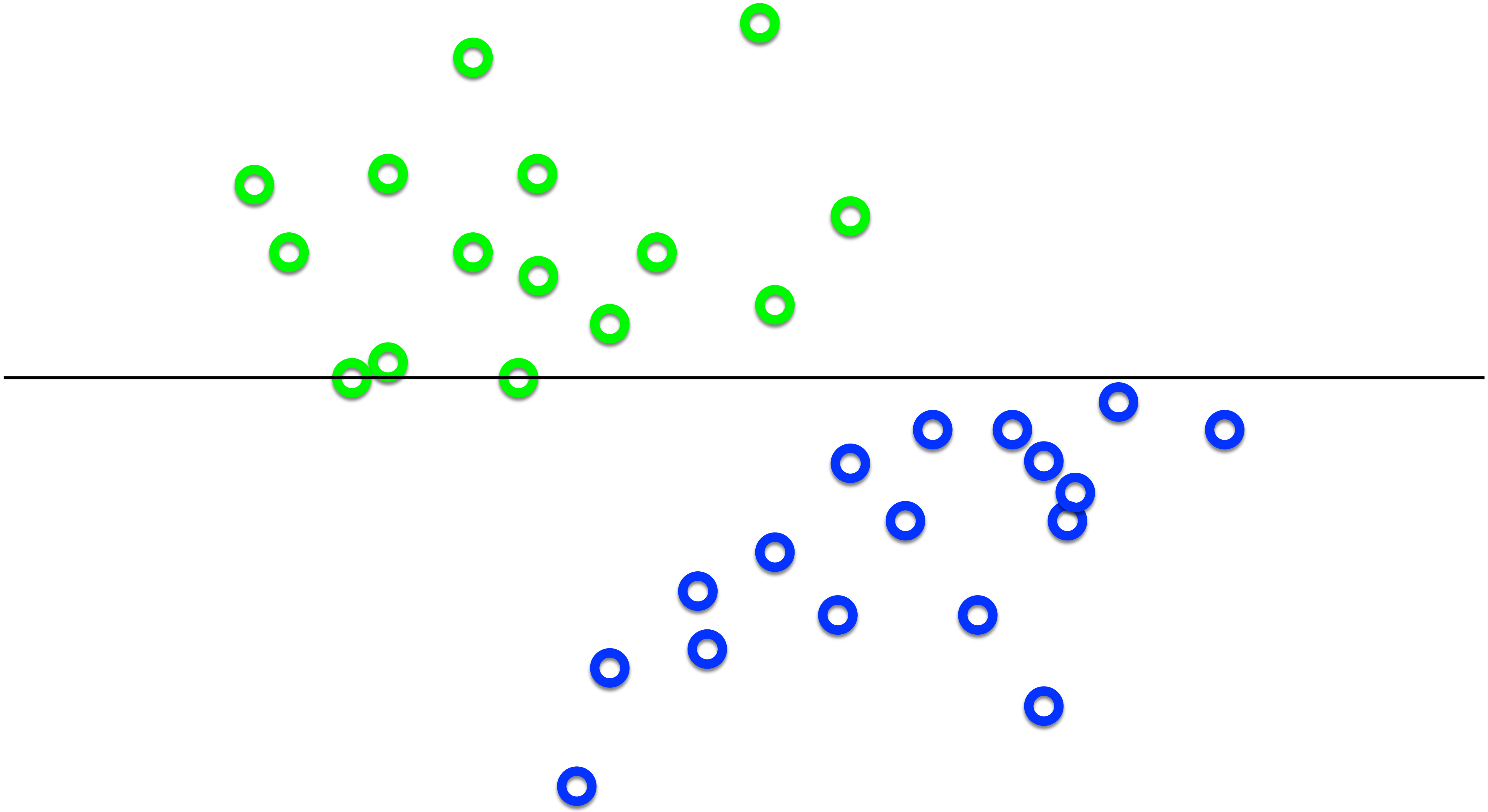
Hyperplanes (planes) in 3D



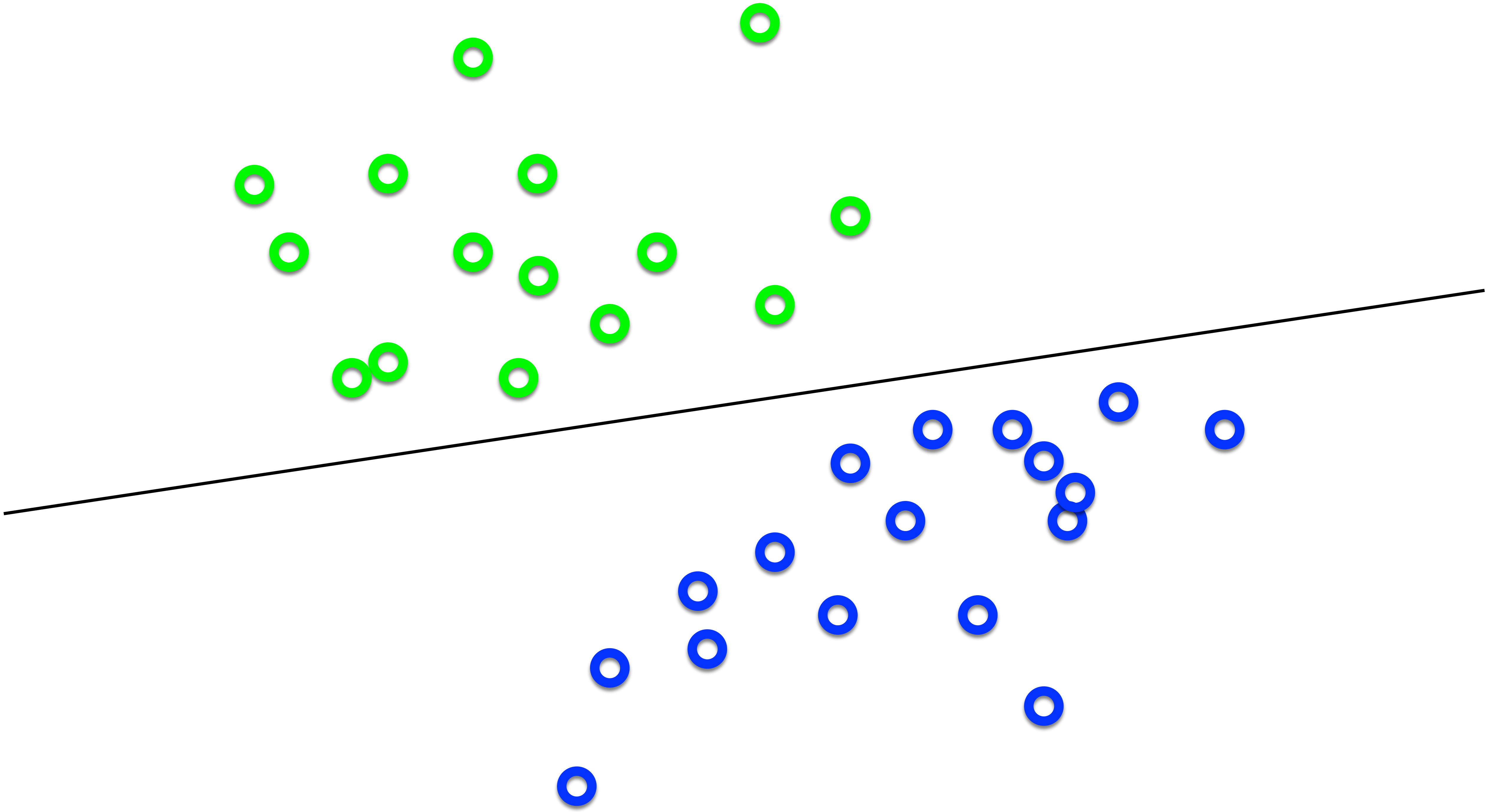
What's the best **w**?



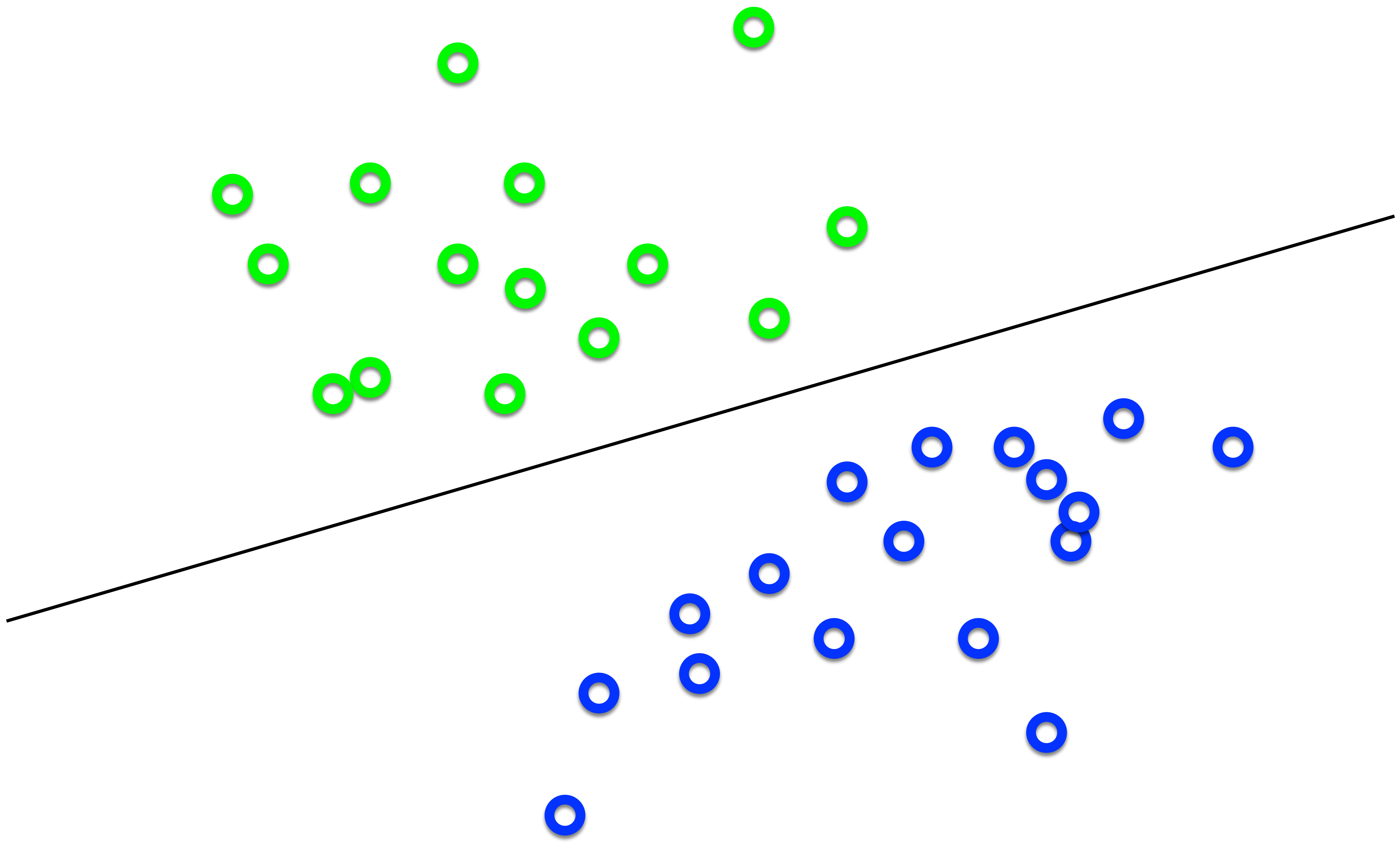
What's the best **w**?



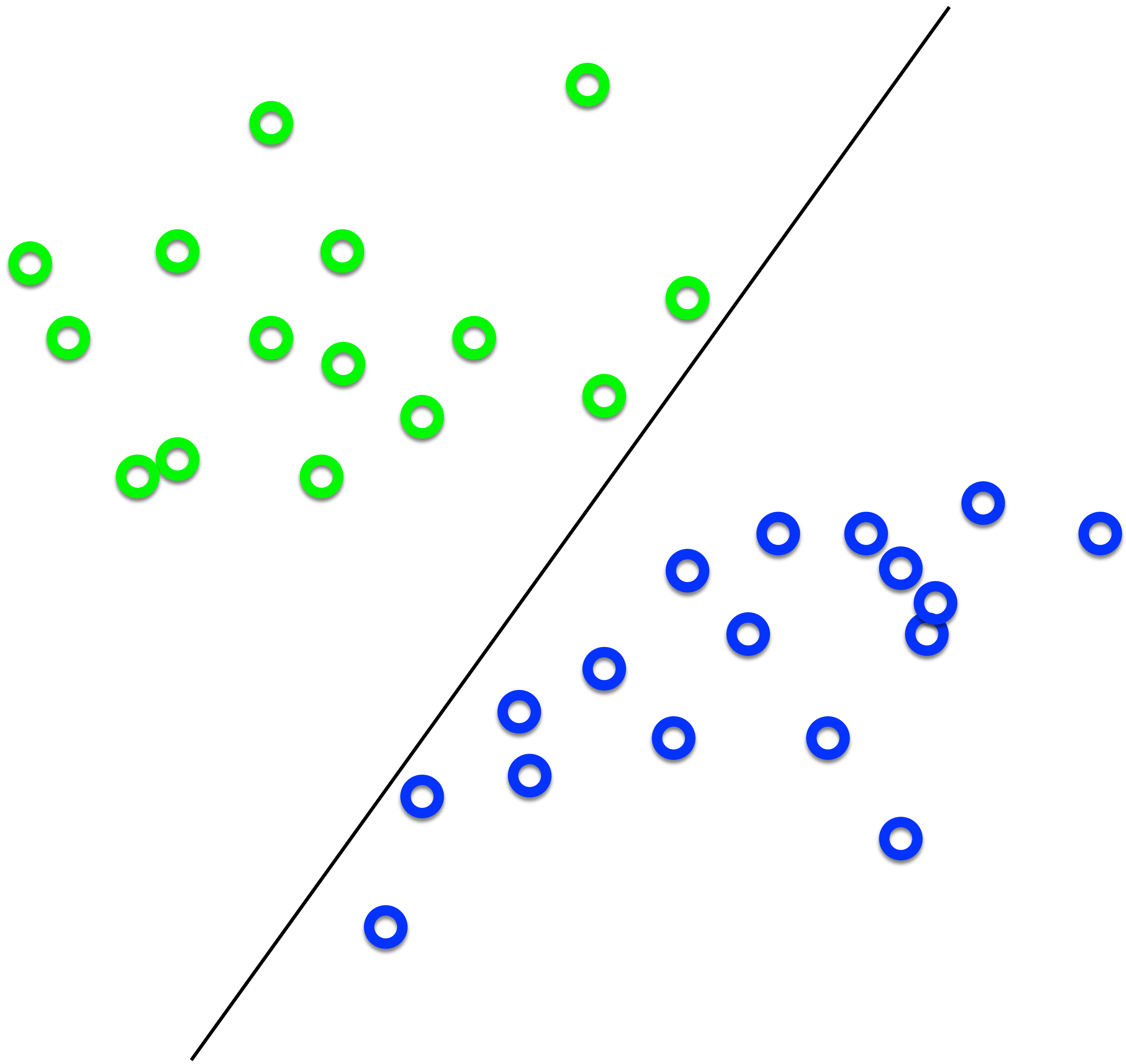
What’s the best **w**?



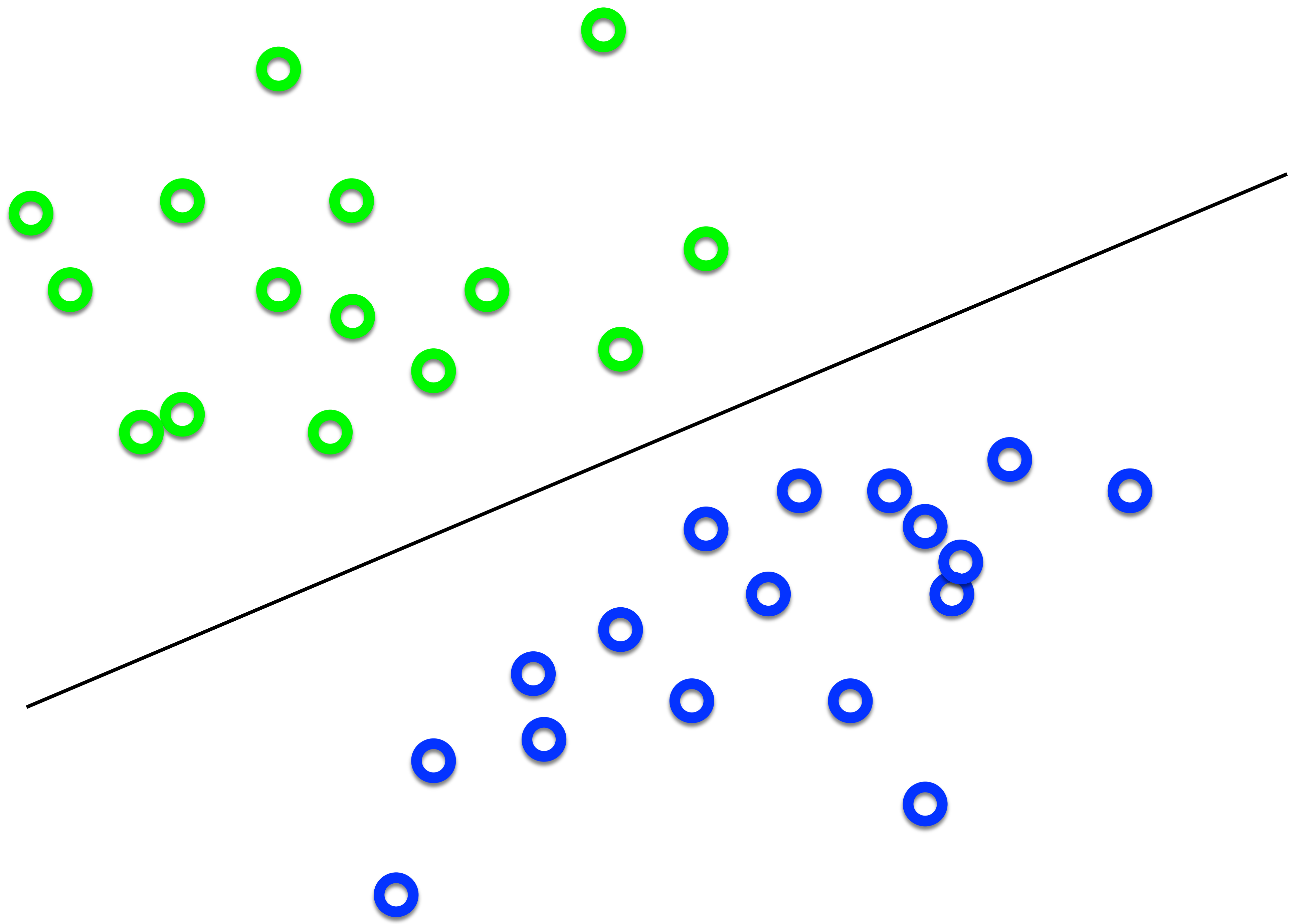
What's the best **w**?



What's the best **w**?

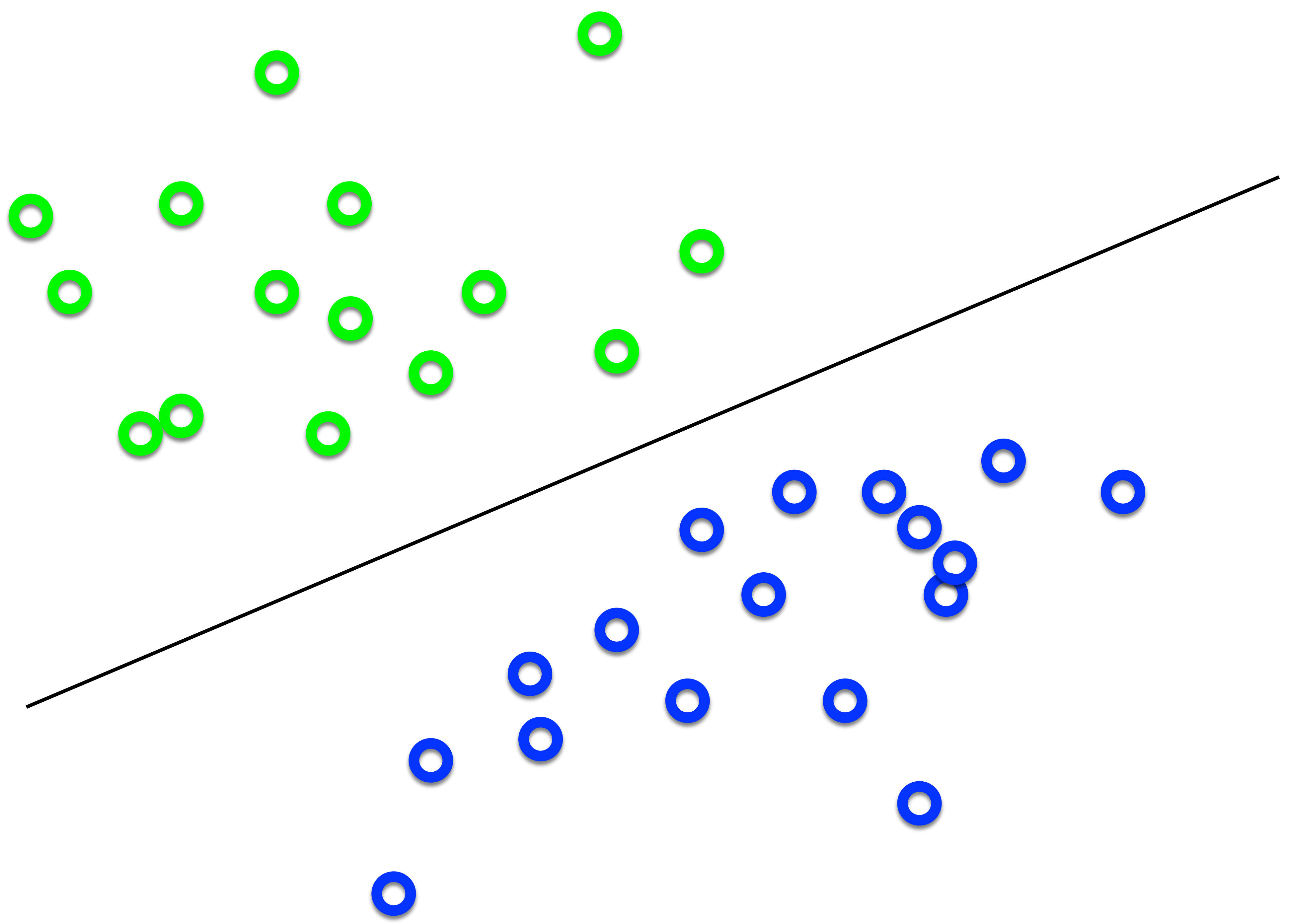


What's the best **w**?



Intuitively, the line that is the farthest from all interior points

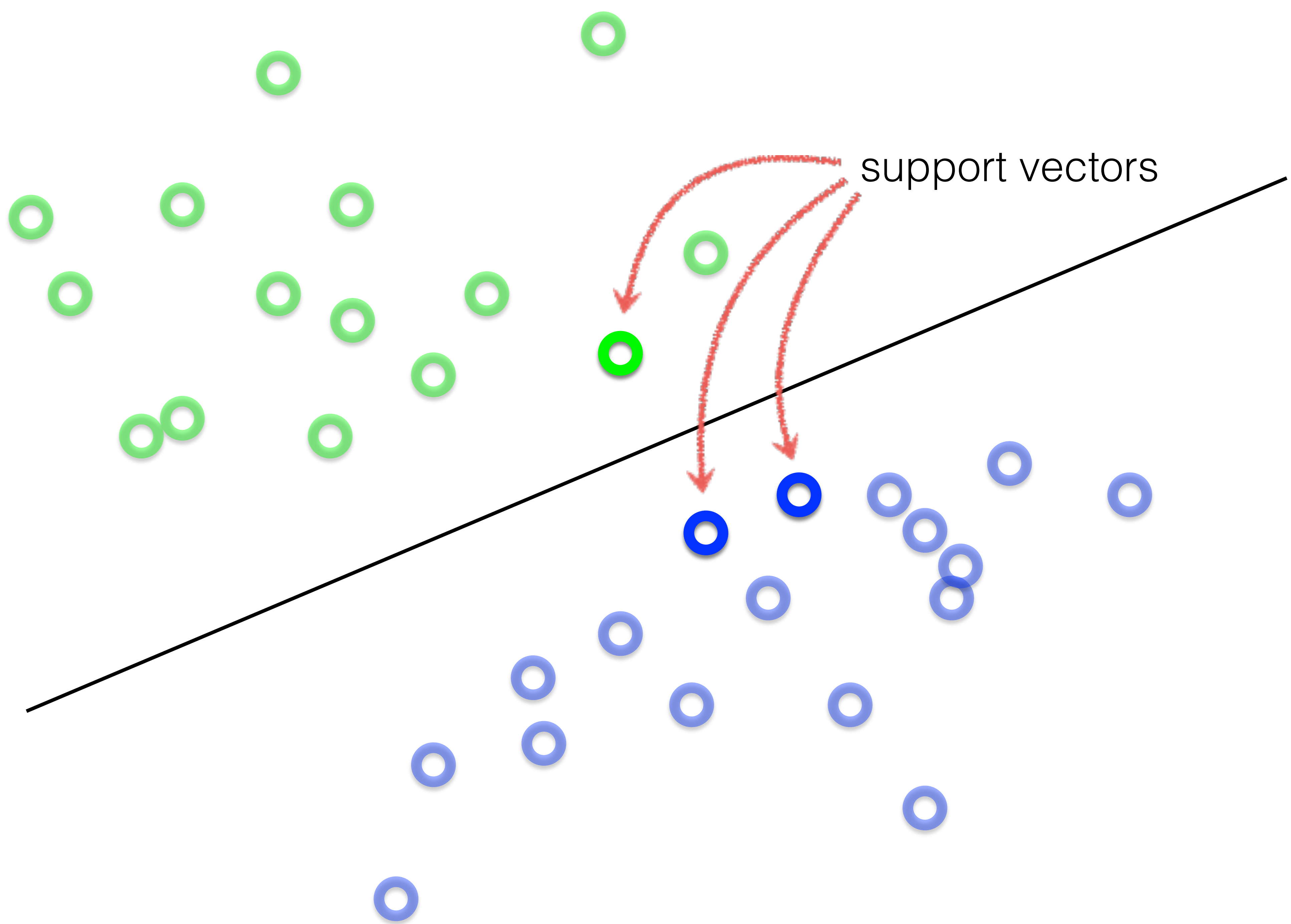
What's the best **w**?



Maximum Margin solution:

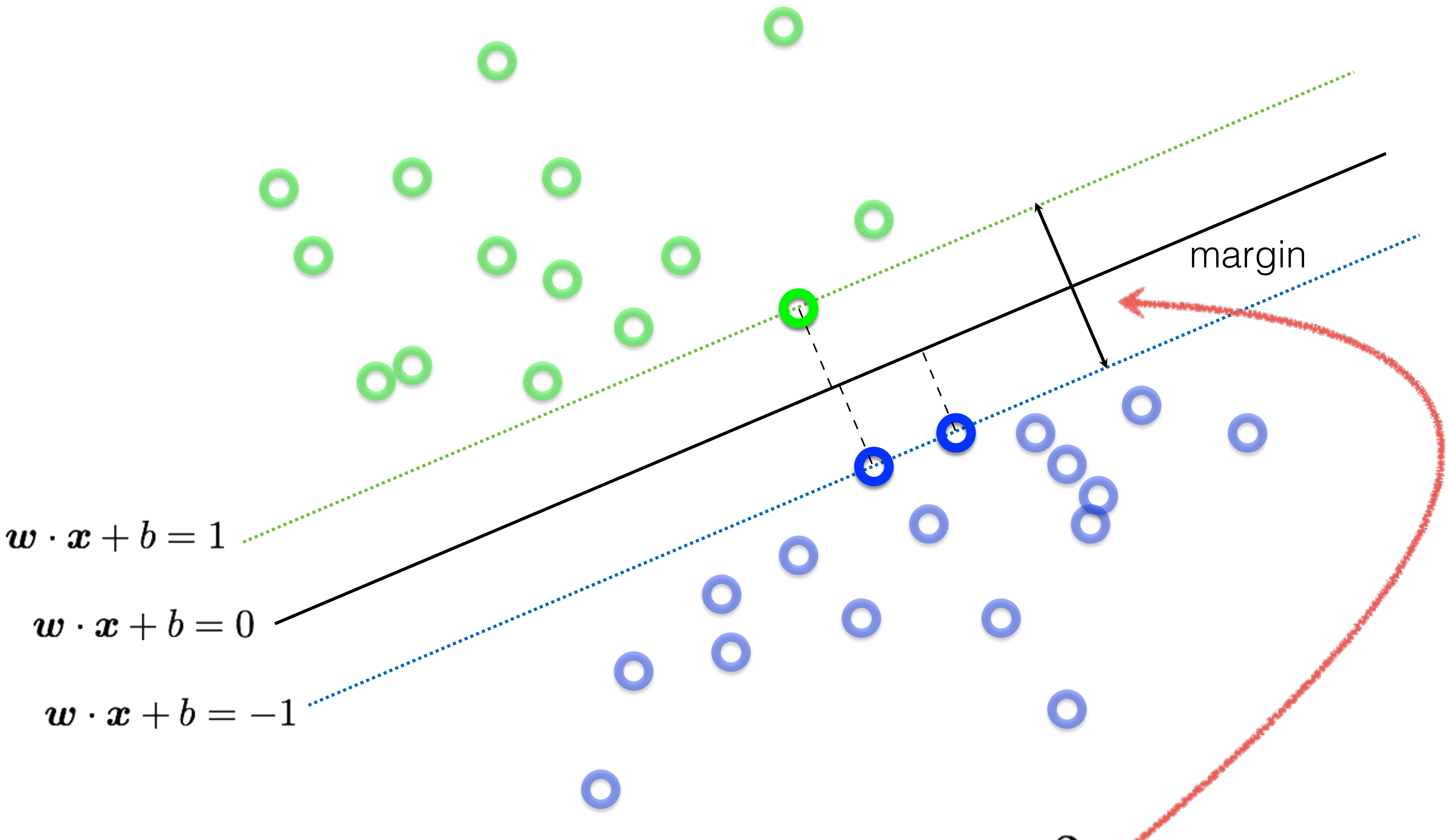
most stable to perturbations of data

What's the best **w**?



Want a hyperplane that is far away from 'inner points'

Find hyperplane **w** such that …



the gap between parallel hyperplanes $\frac{2}{||\boldsymbol{w}||}$ is maximized

Can be formulated as a maximization problem

$$\max_{\boldsymbol{w}} \frac{2}{\|\boldsymbol{w}\|}$$

$$\text{subject to } \boldsymbol{w} \cdot \boldsymbol{x}_i + b \begin{array}{l} \geq +1 \text{ if } y_i = +1 \\ \leq -1 \text{ if } y_i = -1 \end{array} \text{ for } i = 1, \ldots, N$$

What does this constraint mean?

label of the data point

Why is it +1 and -1?

Can be formulated as a maximization problem

$$\max_{\boldsymbol{w}} \frac{2}{\|\boldsymbol{w}\|}$$

$$\text{subject to } \boldsymbol{w} \cdot \boldsymbol{x}_i + b \begin{array}{l} \geq +1 \quad \text{if } \; y_i = +1 \\ \leq -1 \quad \text{if } \; y_i = -1 \end{array} \text{ for } \; i = 1, \ldots, N$$

Equivalently,

Where did the 2 go?

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|$$

$$\text{subject to } \; y_i(\boldsymbol{w} \cdot \boldsymbol{x}_i + b) \geq 1 \; \text{ for } \; i = 1, \ldots, N$$

What happened to the labels?

'Primal formulation' of a linear SVM

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|$$

Objective Function

$$\text{subject to } y_i(\boldsymbol{w} \cdot \boldsymbol{x}_i + b) \geq 1 \text{ for } i = 1, \ldots, N$$

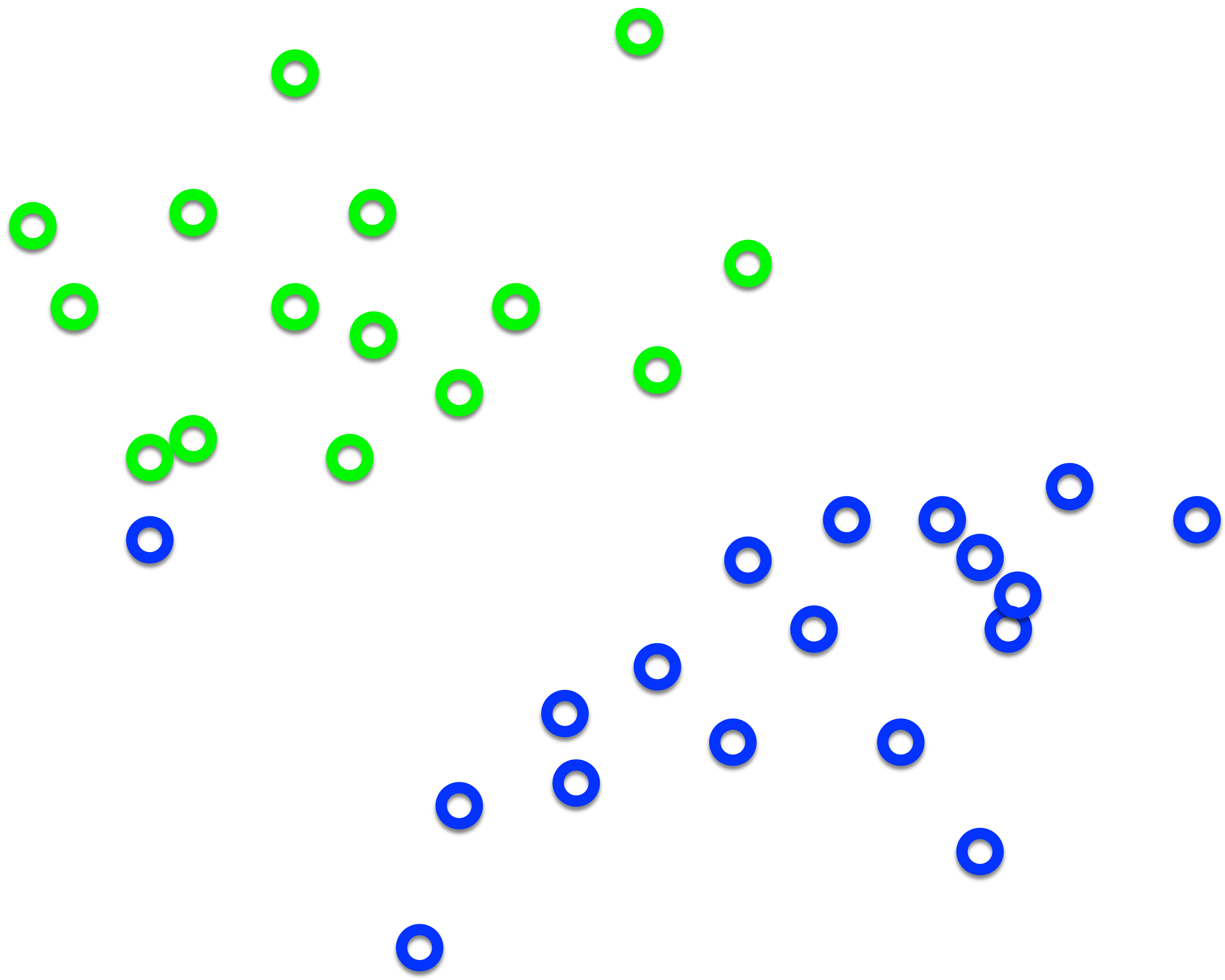
Constraints

This is a convex quadratic programming (QP) problem

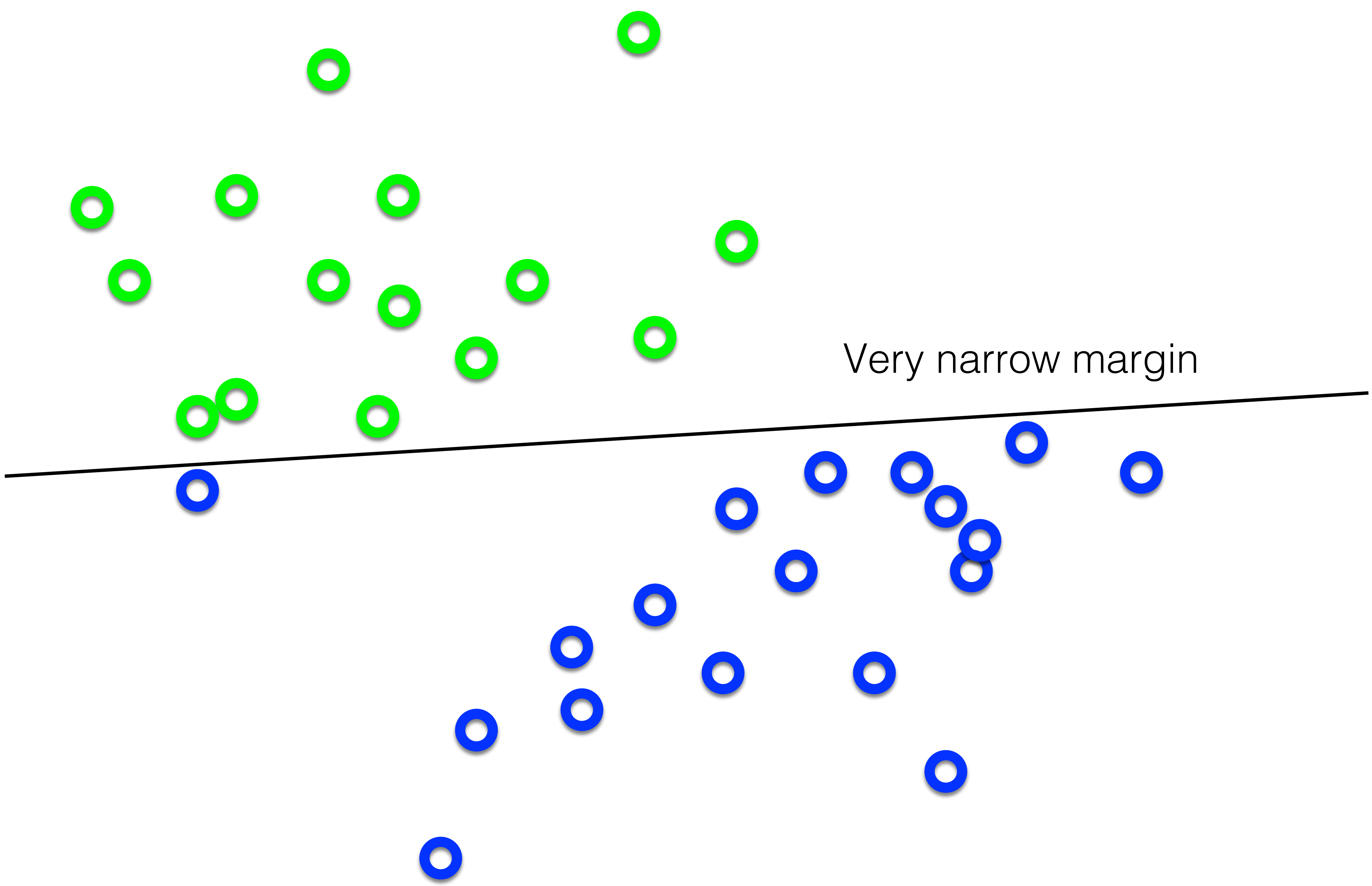
(a unique solution exists)

‘soft’ margin

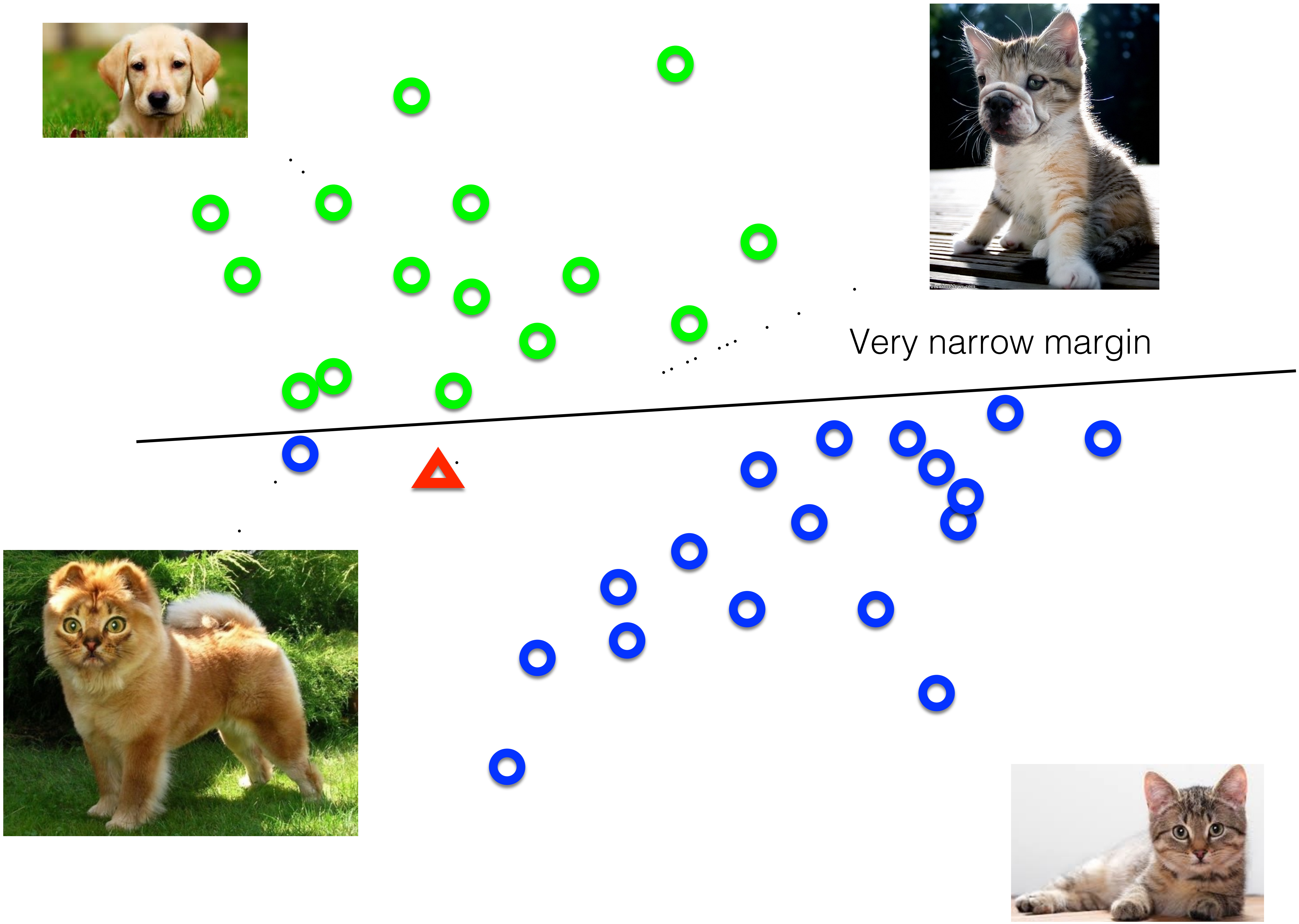
What's the best **w**?



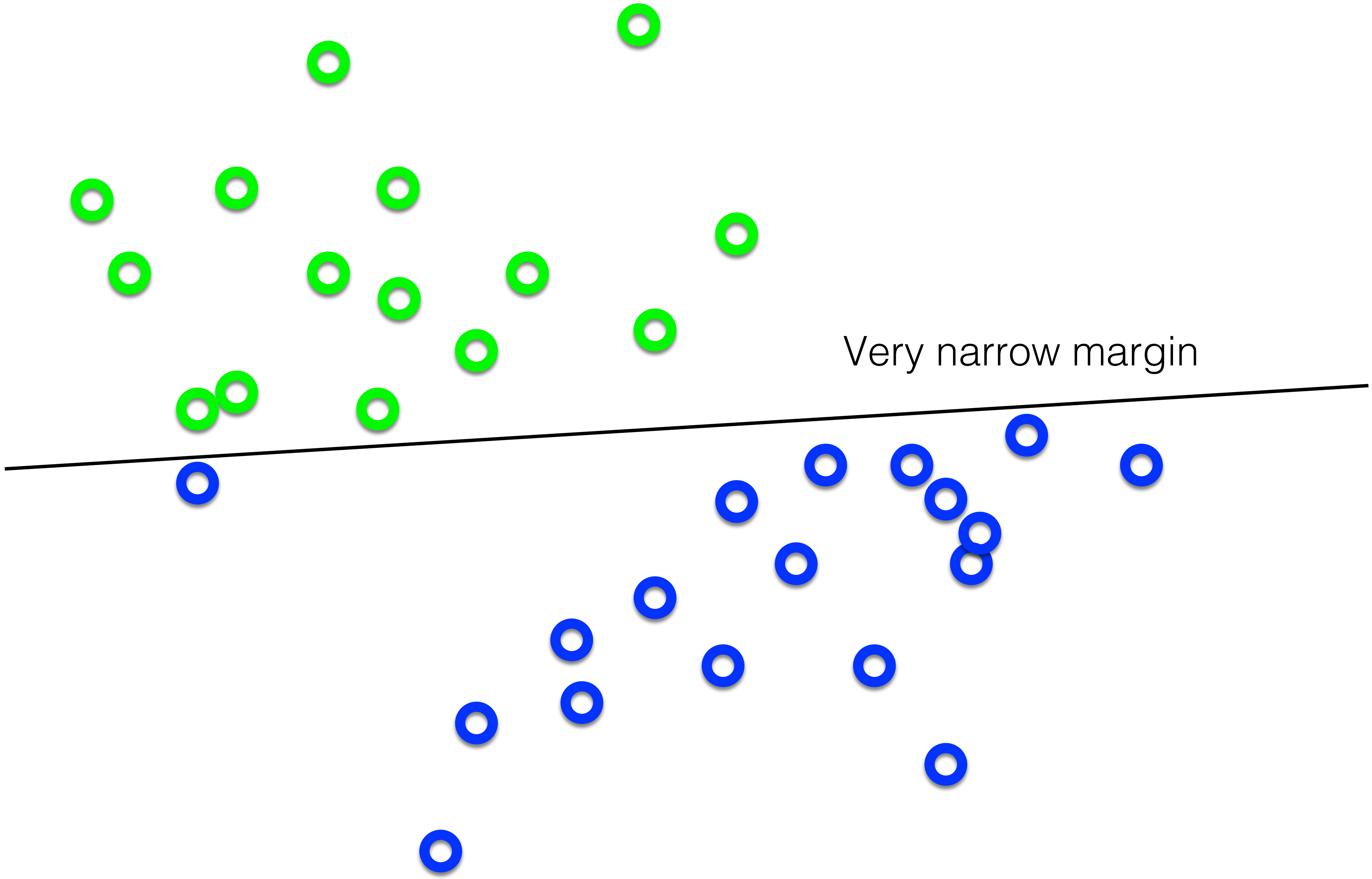
What's the best **w**?



Separating cats and dogs

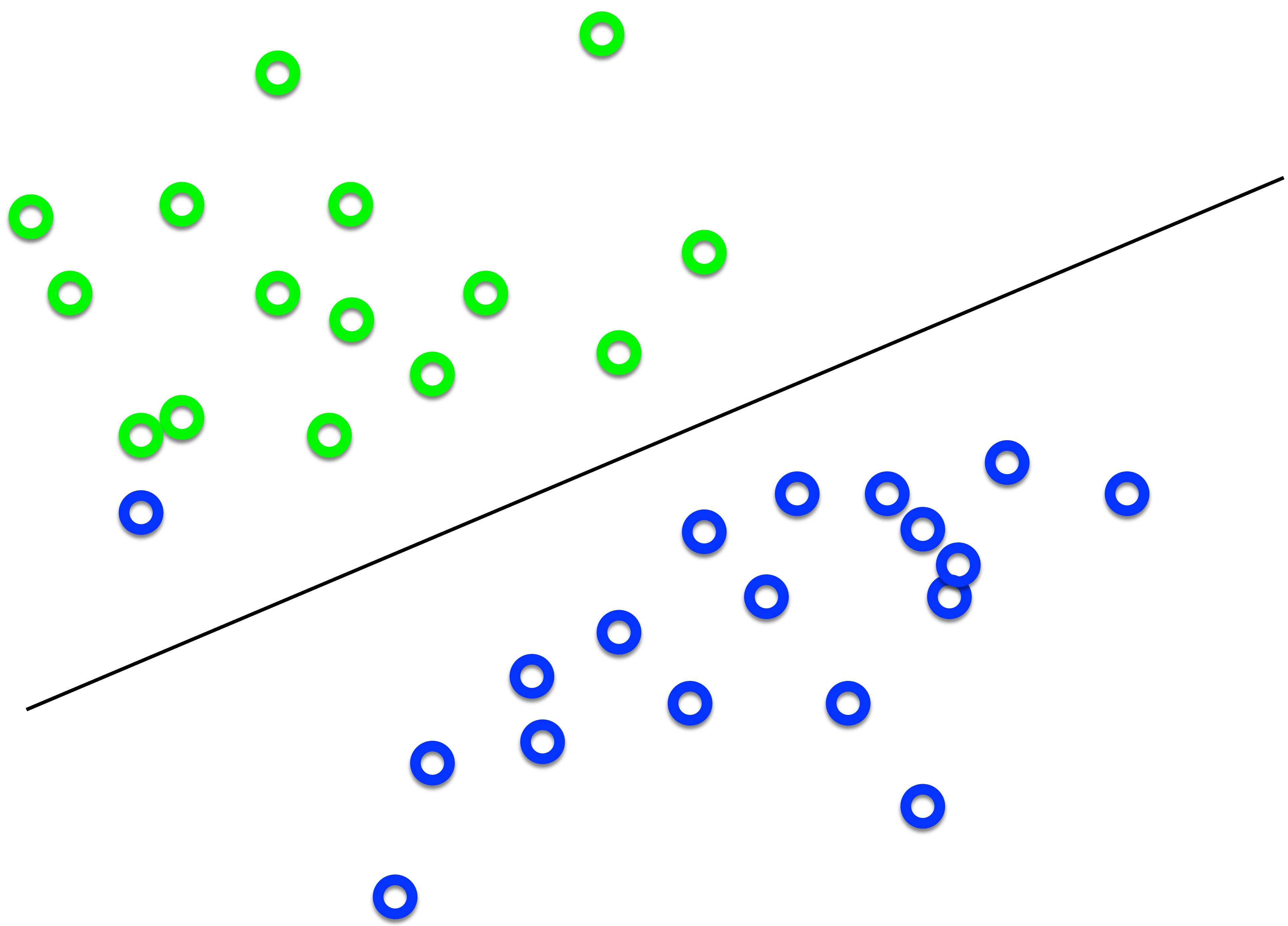


What's the best **w**?



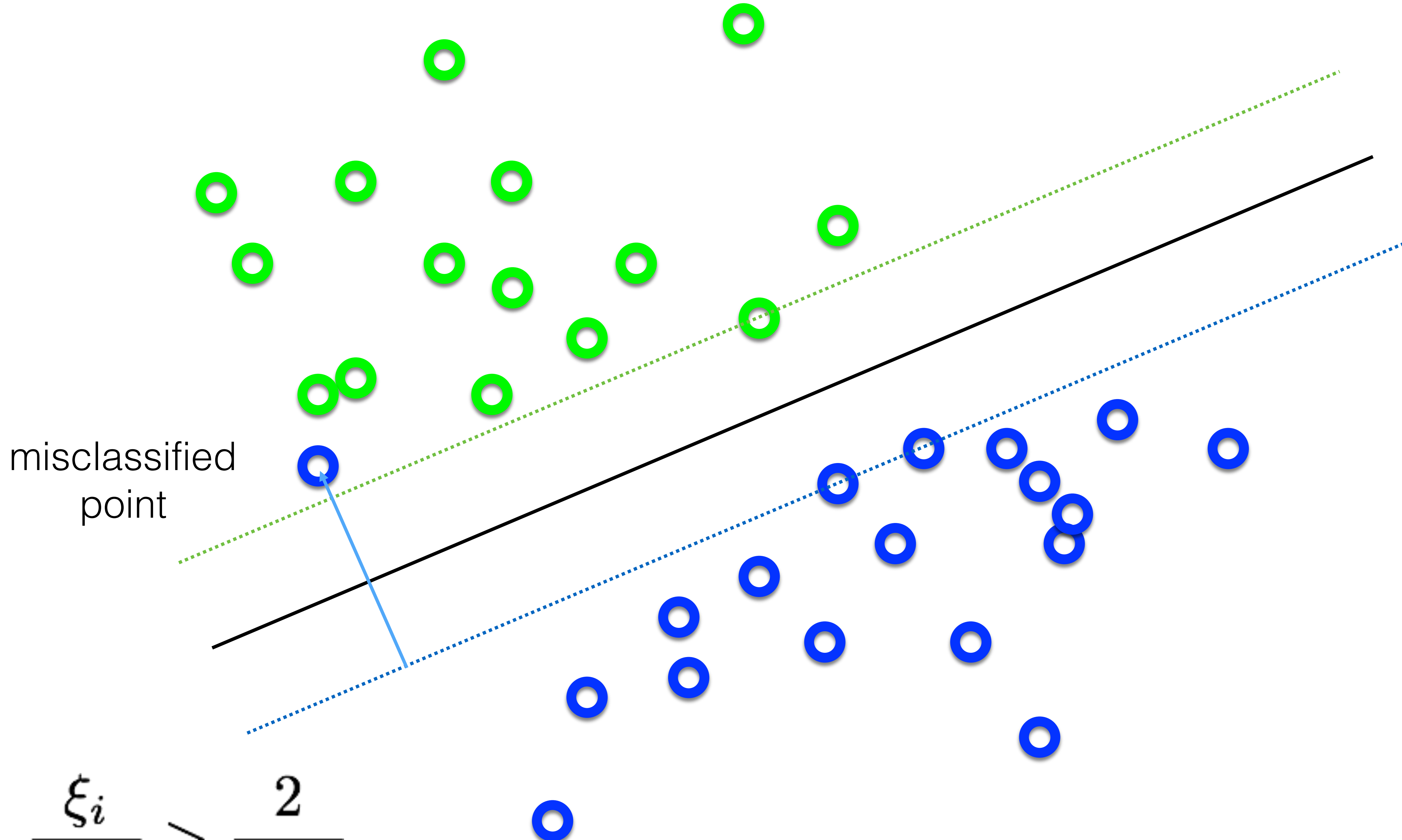
Intuitively, we should allow for some misclassification if we can get more robust classification

What's the best **w**?



Trade-off between the MARGIN and the MISTAKES
(might be a better solution)

Adding slack variables $\xi_i \geq 0$



$$\frac{\xi_i}{\|\boldsymbol{w}\|} > \frac{2}{\|\boldsymbol{w}\|}$$

‘soft’ margin

objective

$$\min_{\boldsymbol{w},\boldsymbol{\xi}} \|\boldsymbol{w}\|^2 + C\sum_i \xi_i$$

subject to

$$y_i(\boldsymbol{w}^\top \boldsymbol{x}_i + b) \geq 1 - \xi_i$$

for $i = 1, \ldots, N$

'soft' margin

objective

$$\min_{\boldsymbol{w},\boldsymbol{\xi}} \|\boldsymbol{w}\|^2 + C\sum_i \xi_i$$

subject to

$$y_i(\boldsymbol{w}^\top \boldsymbol{x}_i + b) \geq 1 - \xi_i$$

for $i = 1, \ldots, N$

The slack variable allows for mistakes,
as long as the inverse margin is minimized.

'soft' margin

objective

$$\min_{\boldsymbol{w},\boldsymbol{\xi}} \|\boldsymbol{w}\|^2 + C\sum_i \xi_i$$

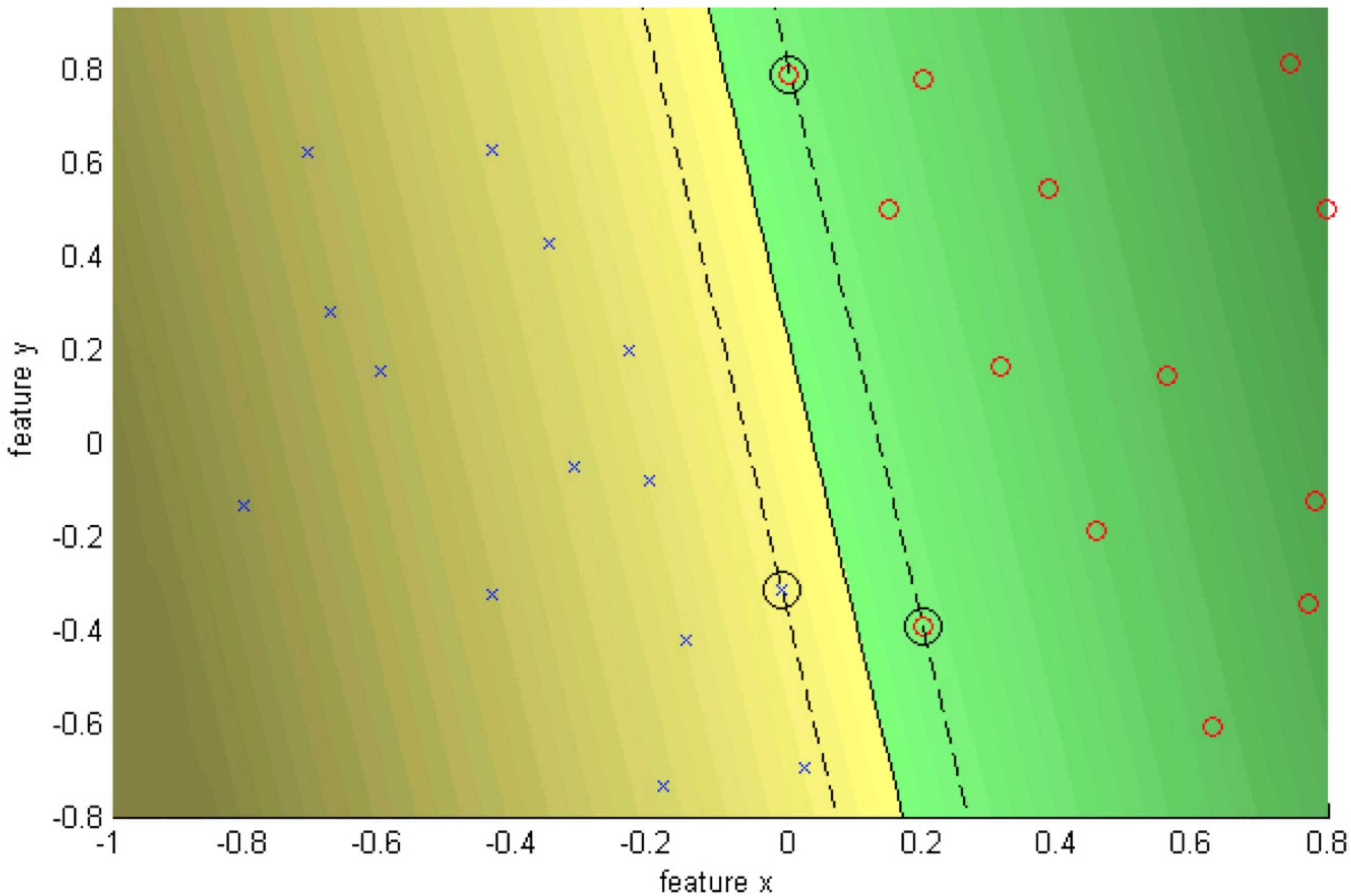
subject to

$$y_i(\boldsymbol{w}^\top \boldsymbol{x}_i + b) \geq 1 - \xi_i$$

$$\text{for} \quad i = 1, \ldots, N$$

- Every constraint can be satisfied if slack is large
- C is a regularization parameter
    - Small C: ignore constraints (larger margin)
    - Big C: constraints (small margin)
- Still QP problem (unique solution)

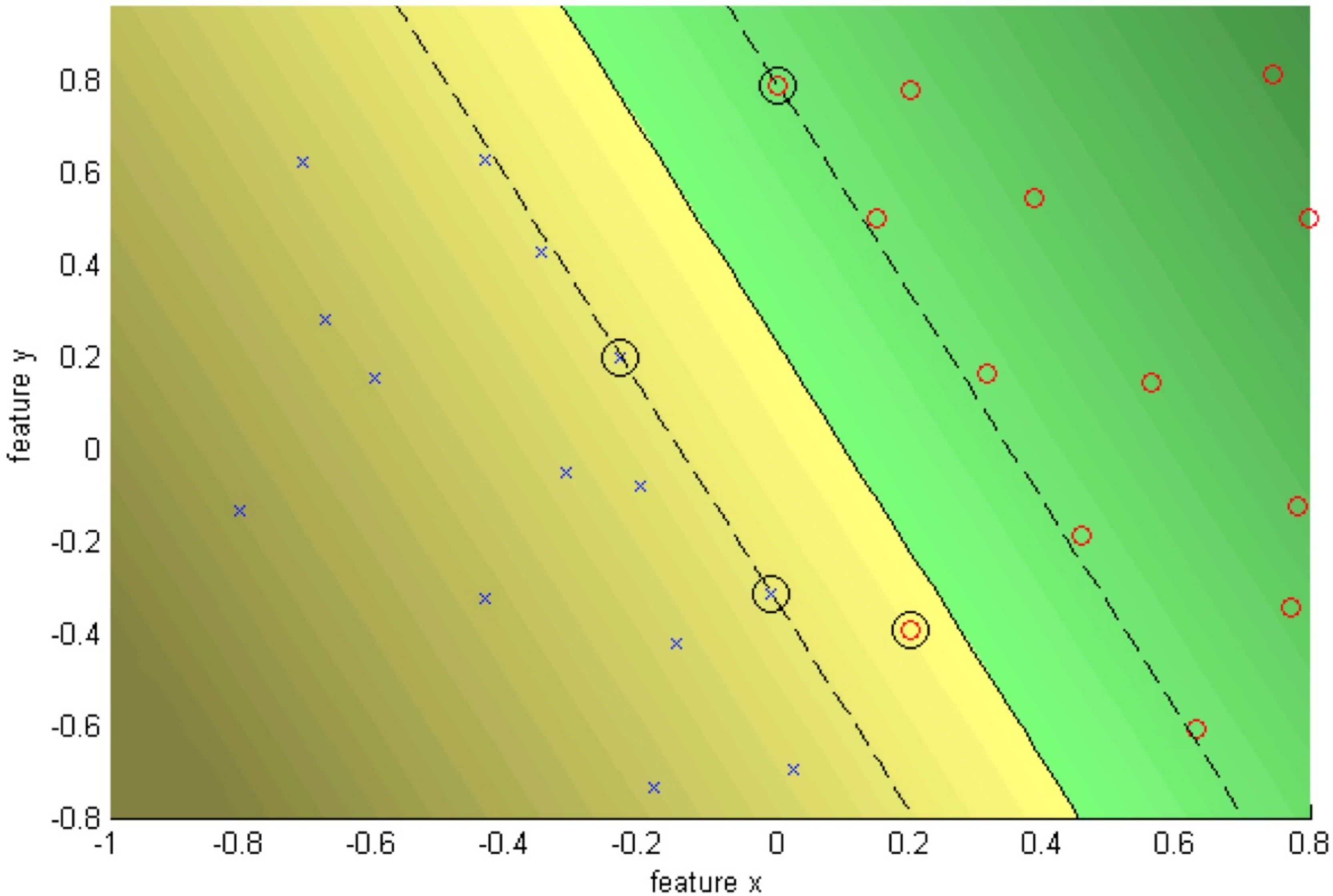
C = Infinity   hard margin



Comment Window

SVM (L1) by Sequential Minimal Optimizer
Kernel: linear (-), C: Inf
Kernel evaluations: 971
Number of Support Vectors: 3
Margin: 0.0966
Training error: 0.00%

C = 10   soft margin



Comment Window

SVM (L1) by Sequential Minimal Optimizer
Kernel: linear (-), C: 10.0000
Kernel evaluations: 2645
Number of Support Vectors: 4
Margin: 0.2265
Training error: 3.70%