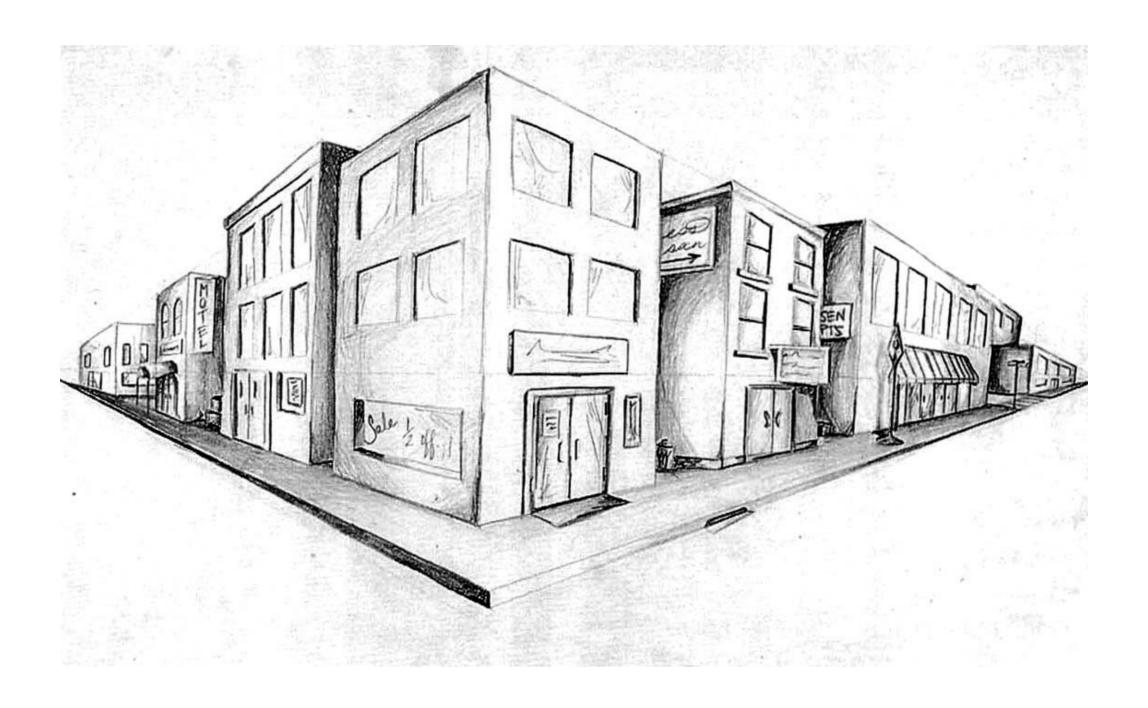
Detecting corners



16-385 Computer Vision Spring 2024, Lecture 5

Overview of today's lecture

- Why detect corners?
- Visualizing quadratics.
- Harris corner detector.
- Multi-scale detection.
- Multi-scale blob detection.

Slide credits

Most of these slides were adapted from:

Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

- Fredo Durand (MIT).
- James Hays (Georgia Tech).

Why detect corners?

Why detect corners?

Image alignment (homography, fundamental matrix)

3D reconstruction

Motion tracking

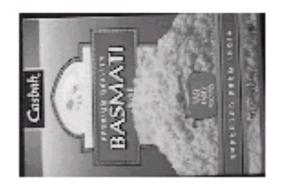
Object recognition

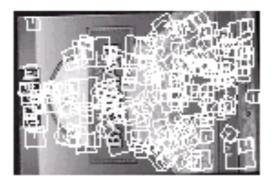
Indexing and database retrieval

Robot navigation

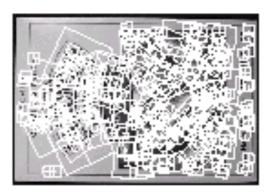
Planar object instance recognition

Database of planar objects

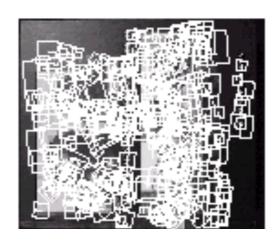












Instance recognition





3D object recognition

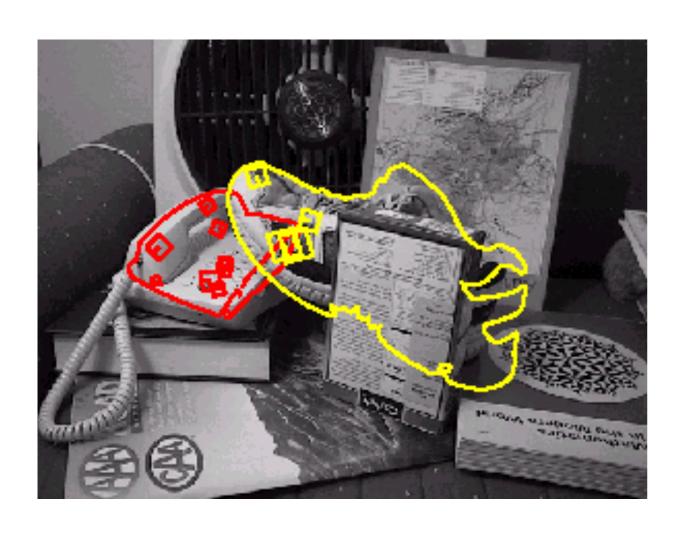
Database of 3D objects

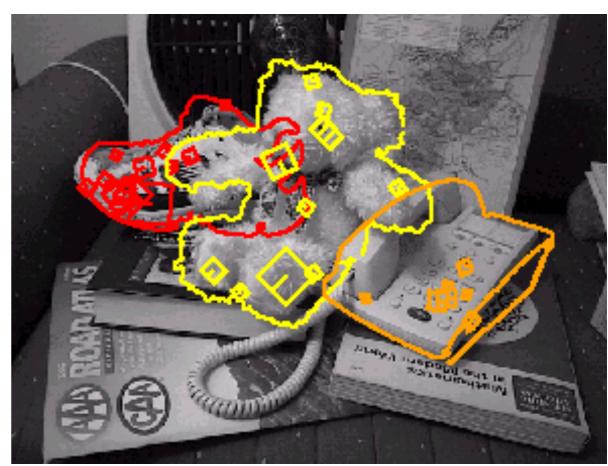


3D objects recognition



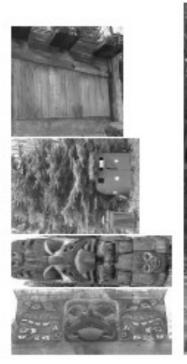




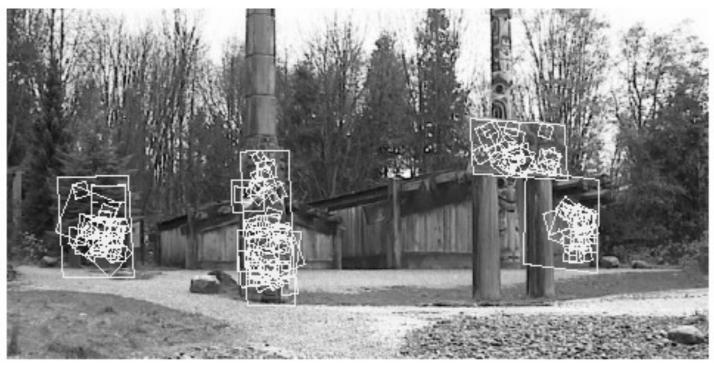


Recognition under occlusion

Location Recognition







Robot Localization

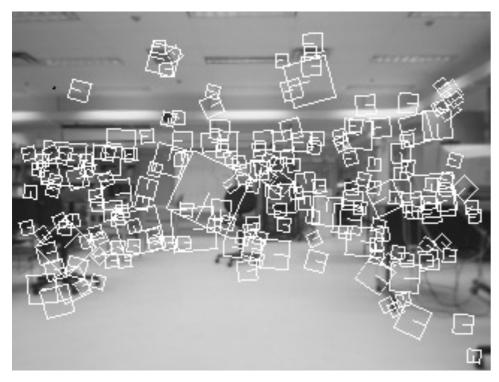


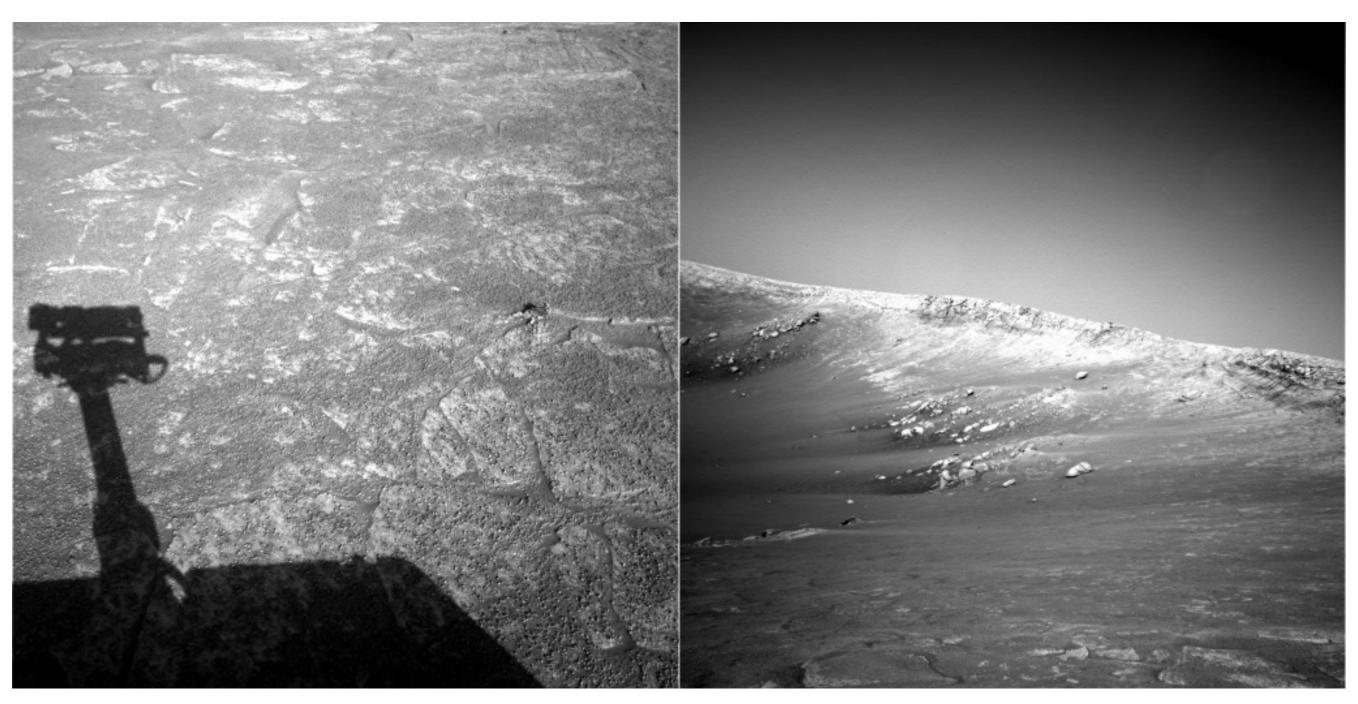




Image matching

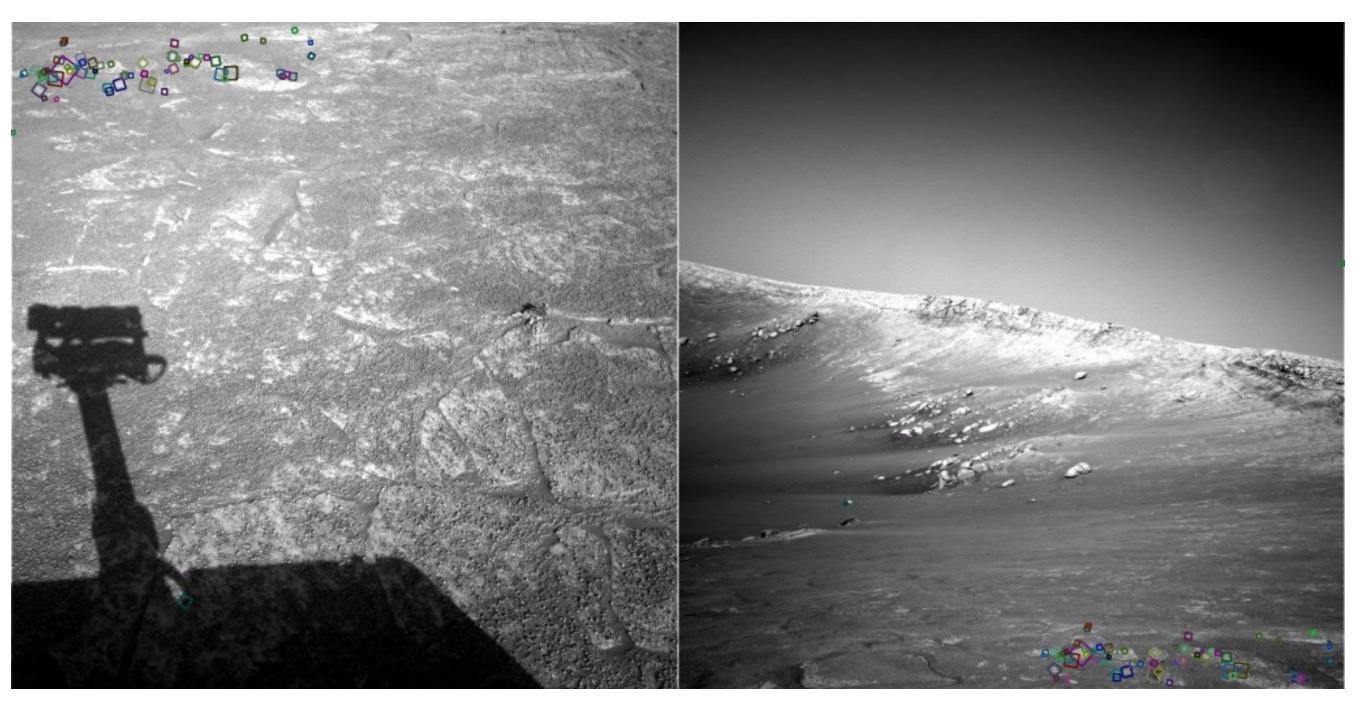




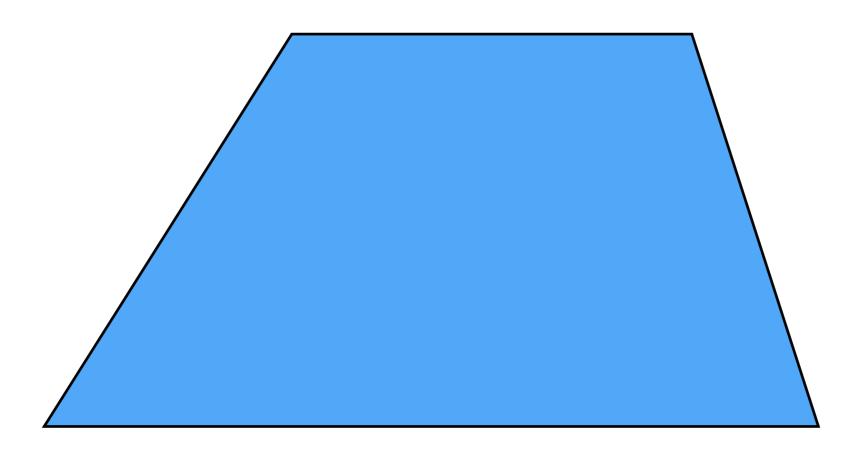


NASA Mars Rover images

Where are the corresponding points?

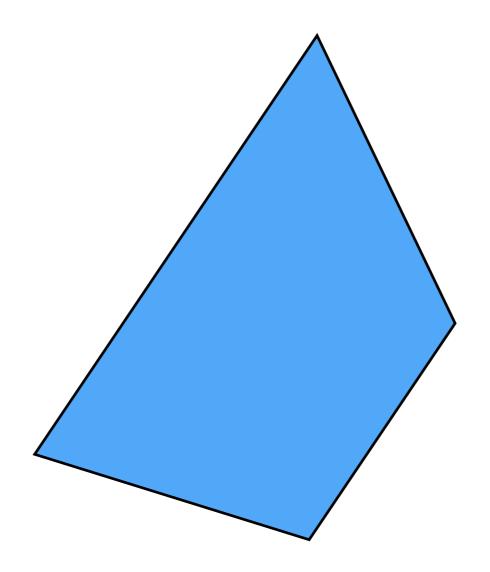


NASA Mars Rover images



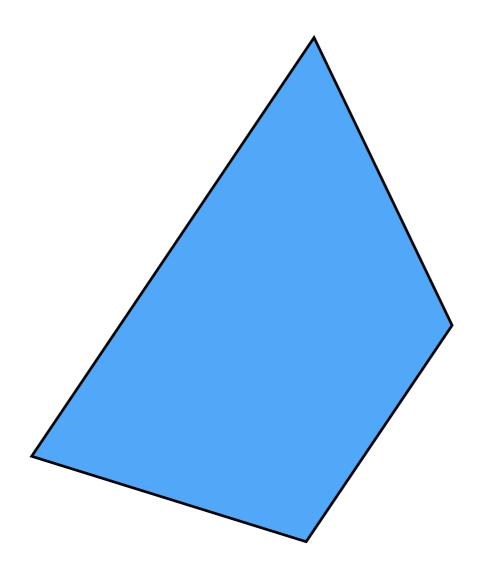
Pick a point in the image. Find it again in the next image.

What type of feature would you select?



Pick a point in the image. Find it again in the next image.

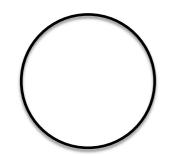
What type of feature would you select?



Pick a point in the image. Find it again in the next image.

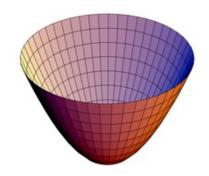
What type of feature would you select? a corner

Visualizing quadratics



Equation of a circle

$$1 = x^2 + y^2$$



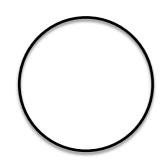
Equation of a 'bowl' (paraboloid)

$$f(x,y) = x^2 + y^2$$

If you slice the bowl at

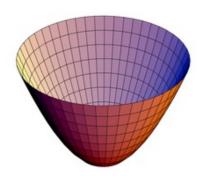
$$f(x,y) = 1$$

what do you get?



Equation of a circle

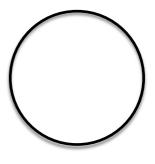
$$1 = x^2 + y^2$$



Equation of a 'bowl' (paraboloid)

$$f(x,y) = x^2 + y^2$$

If you slice the bowl at f(x,y) = 1 what do you get?

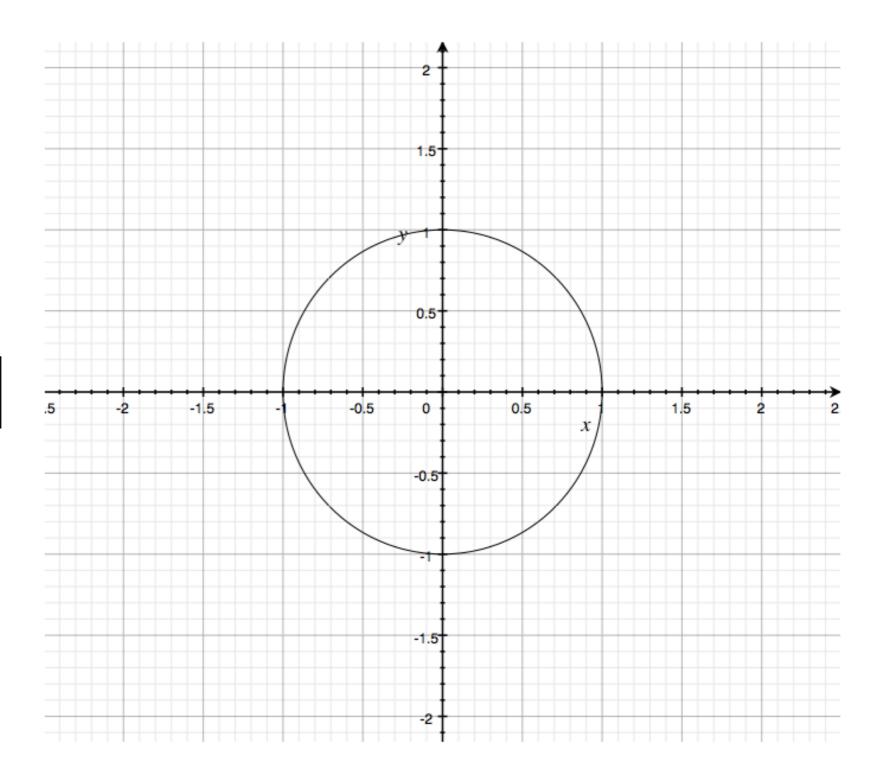


$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x,y) = \left[egin{array}{cccc} x & y \end{array}
ight] \left[egin{array}{cccc} 1 & 0 \ 0 & 1 \end{array}
ight] \left[egin{array}{cccc} x \ y \end{array}
ight]$$

$$f(x,y) = \left[\begin{array}{cc} x & y \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right] \left[\begin{array}{c} x \\ y \end{array}\right]$$
 'sliced at 1'

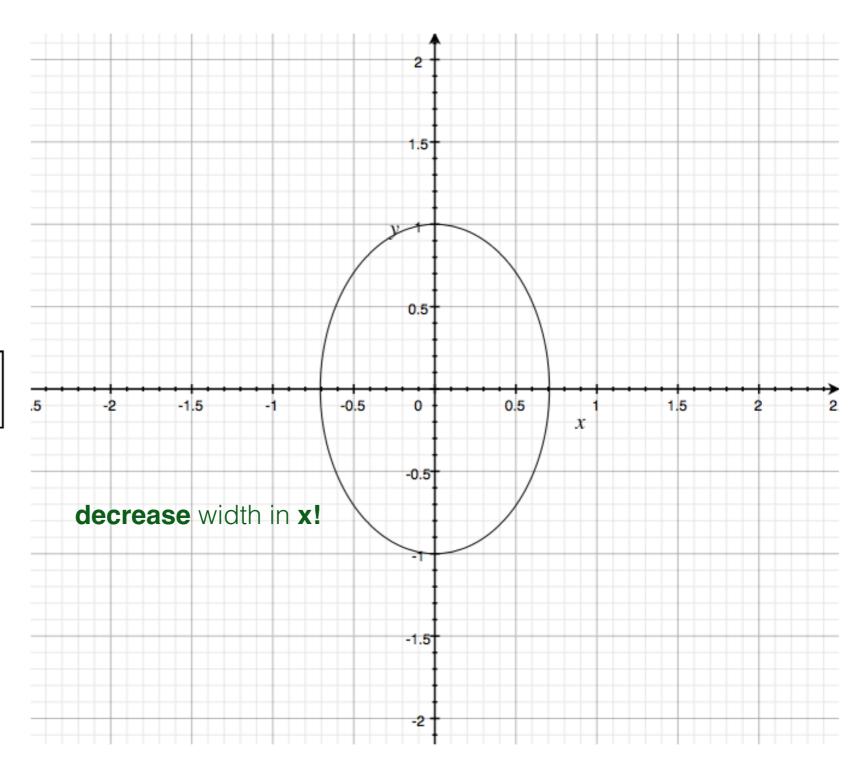


What happens if you **increase** coefficient on **x**?

$$f(x,y) = \left[egin{array}{ccc} x & y \end{array}
ight] \left[egin{array}{ccc} 2 & 0 \ 0 & 1 \end{array}
ight] \left[egin{array}{ccc} x \ y \end{array}
ight]$$

What happens if you **increase** coefficient on **x**?

$$f(x,y) = \left[egin{array}{ccc} x & y \end{array}
ight] \left[egin{array}{ccc} 2 & 0 \ 0 & 1 \end{array}
ight] \left[egin{array}{ccc} x \ y \end{array}
ight] \, rac{1}{5}$$

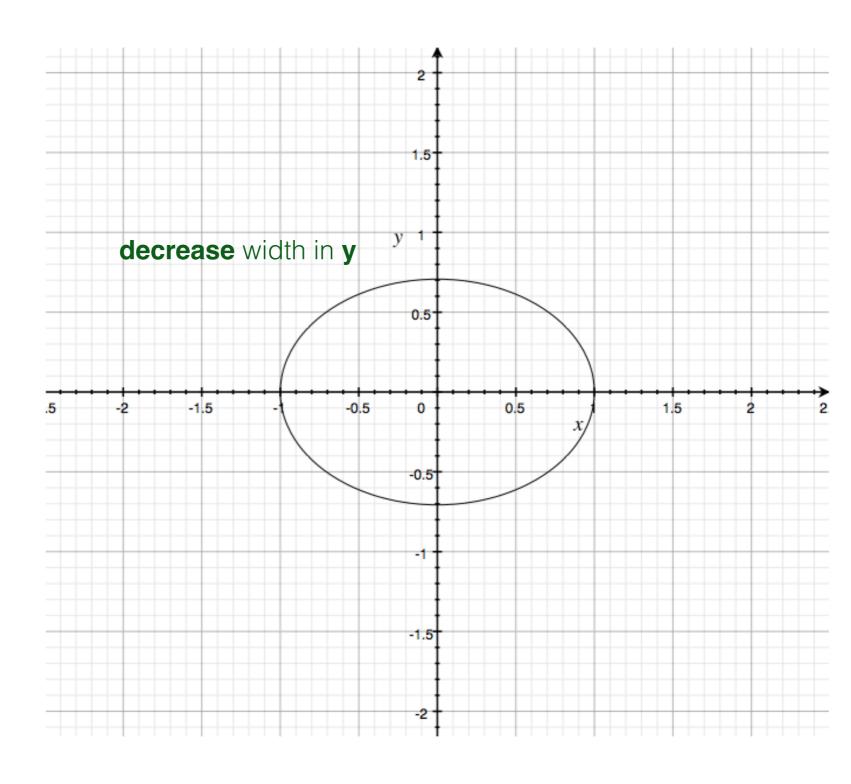


What happens if you **increase** coefficient on **y**?

$$f(x,y) = \left[egin{array}{ccc} x & y \end{array}
ight] \left[egin{array}{ccc} 1 & 0 \ 0 & 2 \end{array}
ight] \left[egin{array}{ccc} x \ y \end{array}
ight]$$

What happens if you **increase** coefficient on **y**?

$$f(x,y) = \left[egin{array}{ccc} x & y \end{array}
ight] \left[egin{array}{ccc} 1 & 0 \ 0 & 2 \end{array}
ight] \left[egin{array}{ccc} x \ y \end{array}
ight]$$



$$f(x,y) = x^2 + y^2$$

can be written in matrix form like this...

$$f(x,y) = \left[\begin{array}{cc} x & y \end{array} \right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \left[\begin{array}{cc} x \\ y \end{array} \right]$$

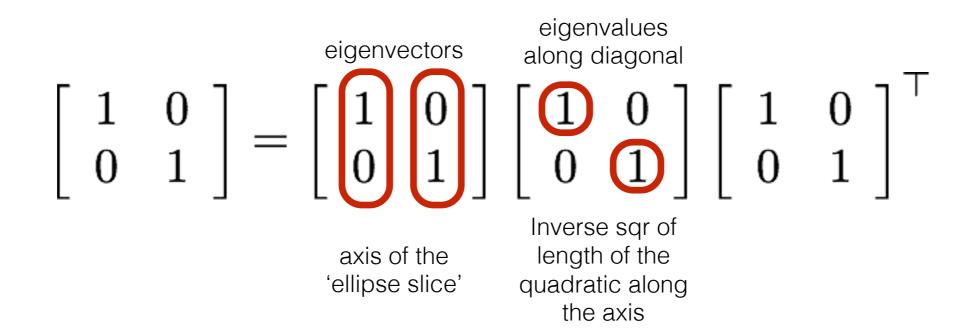
What's the shape? What are the eigenvectors? What are the eigenvalues?

$$f(x,y) = x^2 + y^2$$

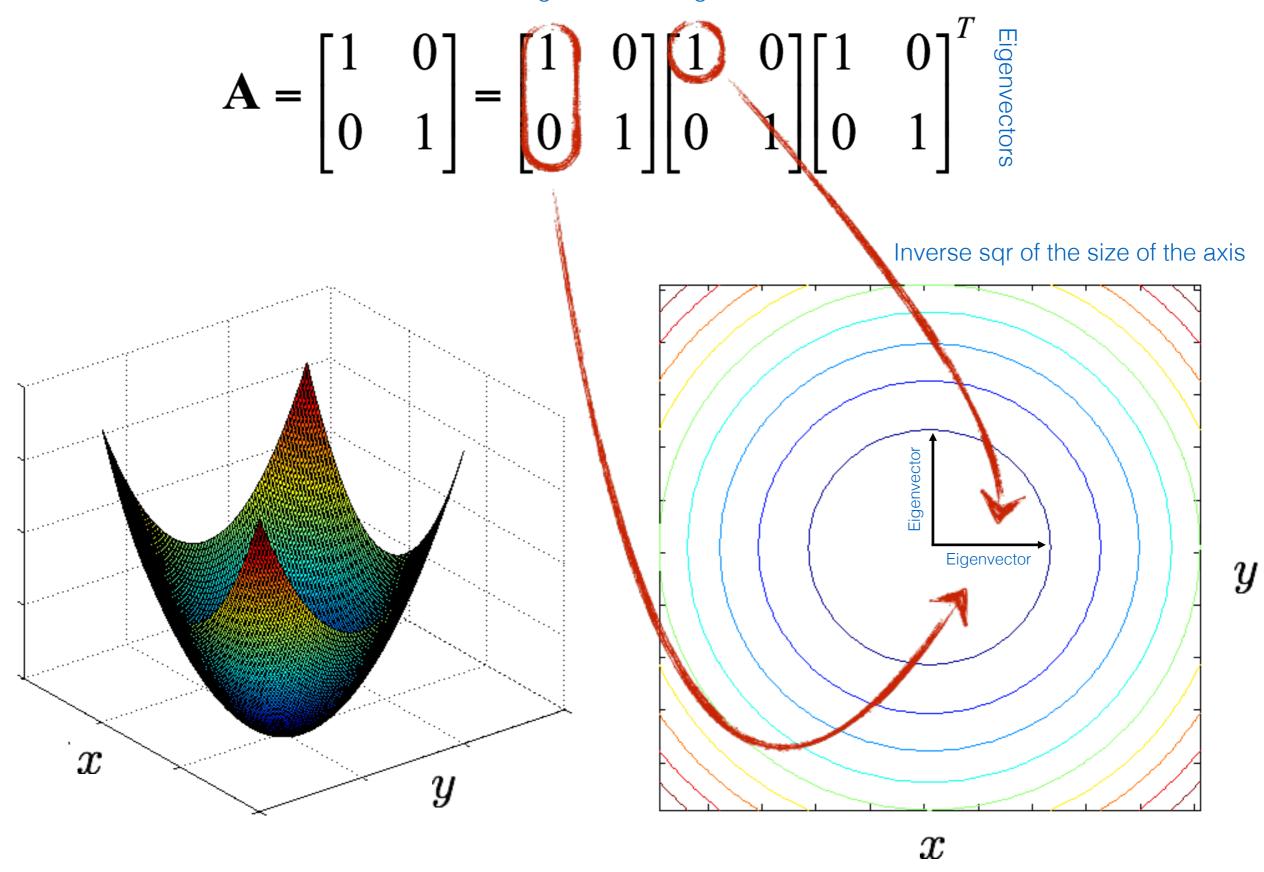
can be written in matrix form like this...

$$f(x,y) = \left[\begin{array}{cc} x & y \end{array} \right] \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \left[\begin{array}{cc} x \\ y \end{array} \right]$$

Result of Singular Value Decomposition (SVD)



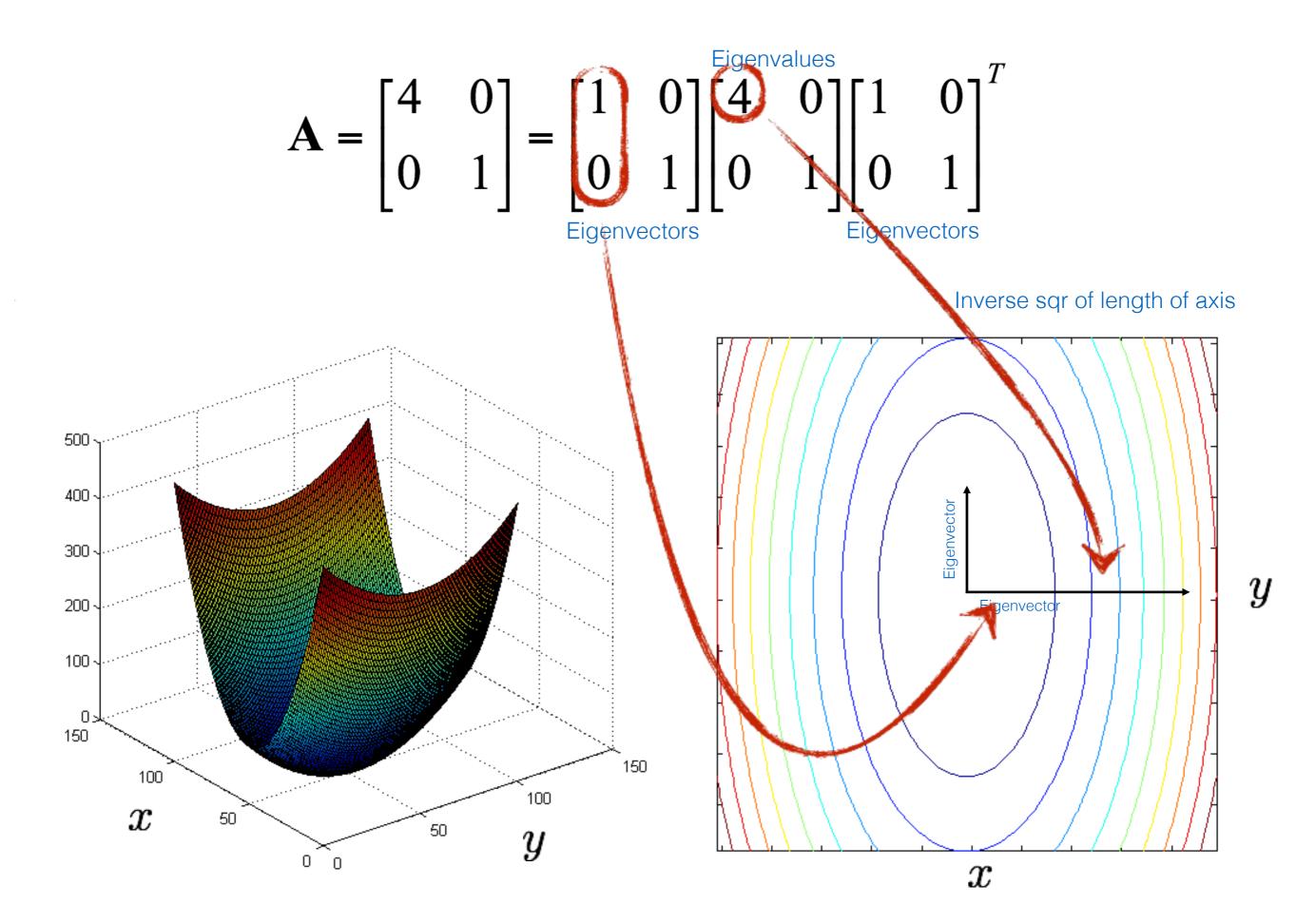
Eigenvectors Eigenvalues



Recall:

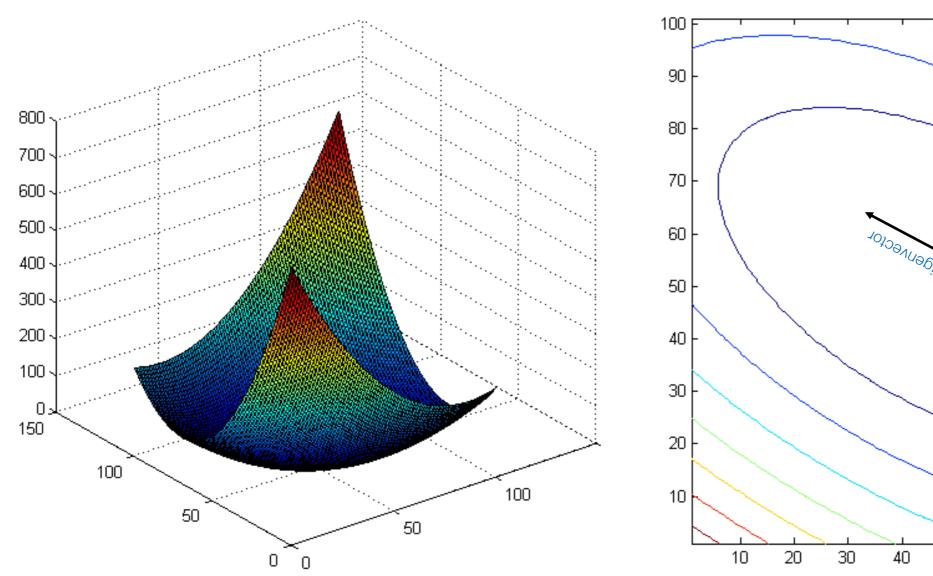
you can smash this bowl in the y direction

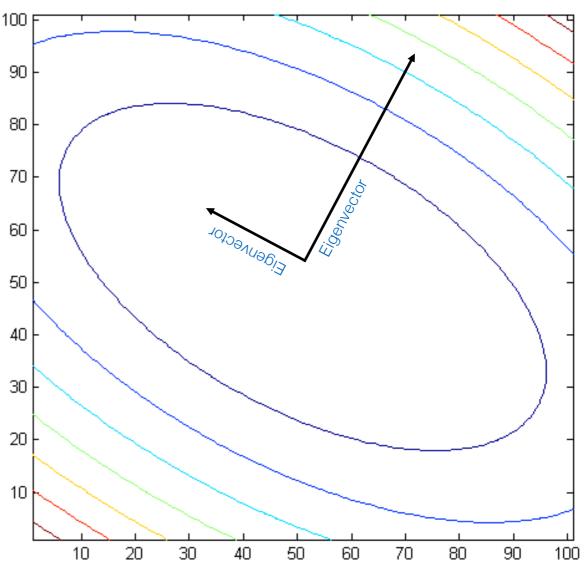
you can smash this bowl in the x direction



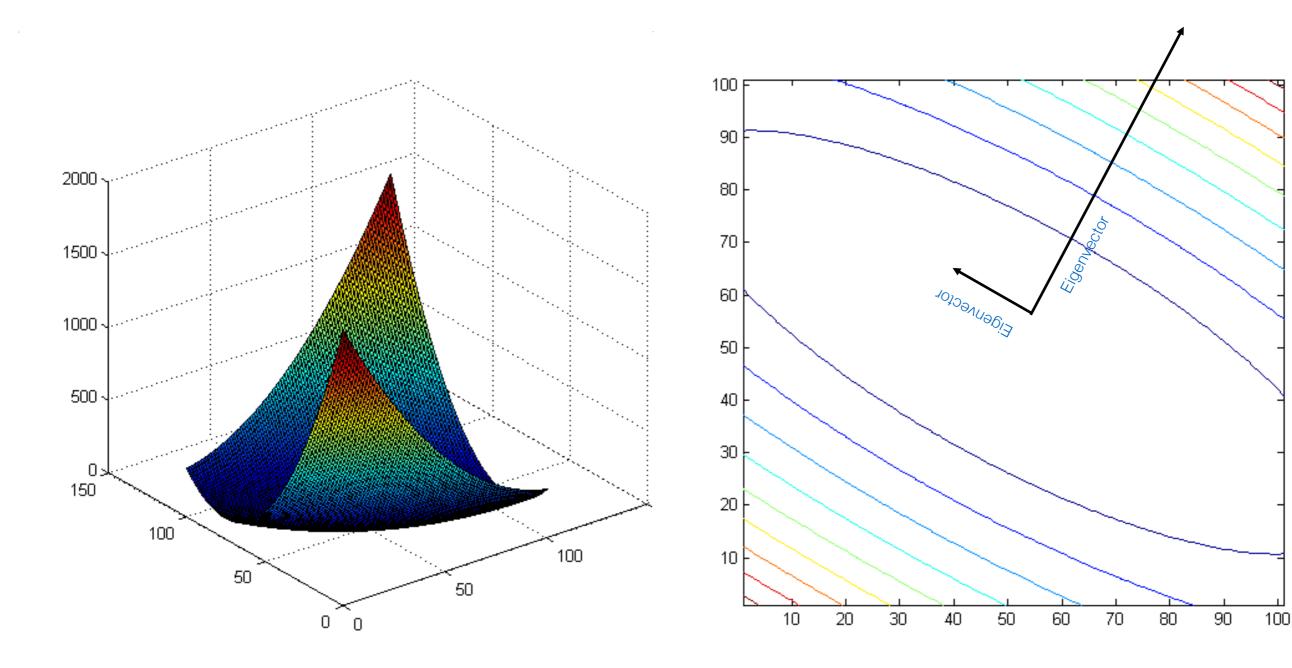
Eigenvalues

$$\mathbf{A} = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^{T}$$
Eigenvectors





$$\mathbf{A} = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^{T}$$
Eigenvectors



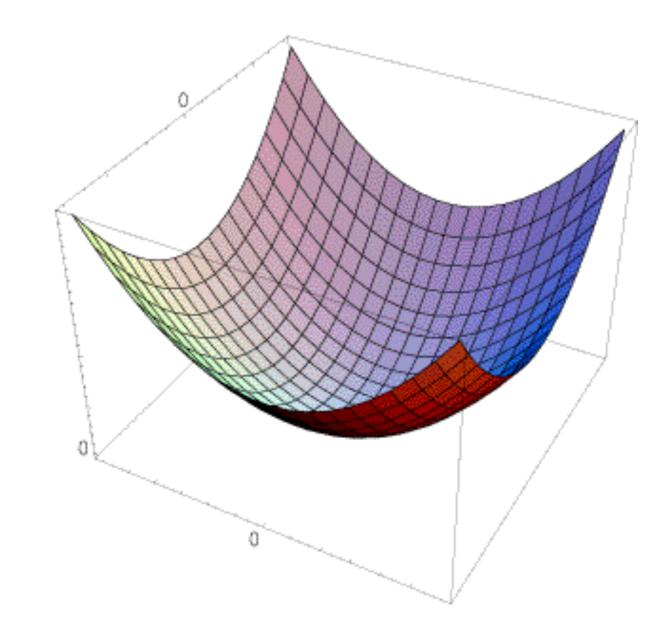
We will need this to understand the...

Error function for Harris Corners

The surface E(u,v) is locally approximated by a quadratic form

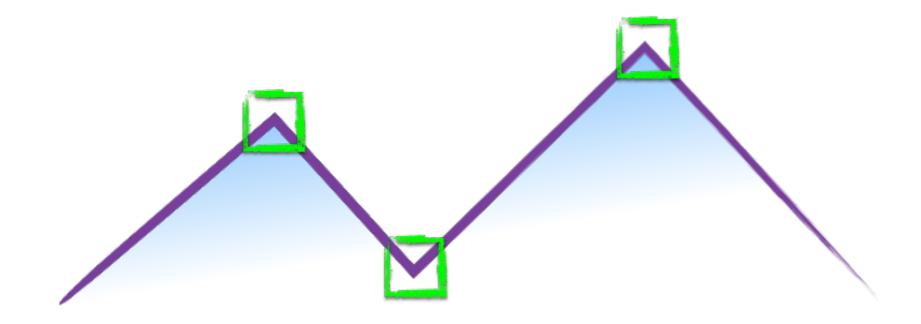
$$E(u,v) \approx [u \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



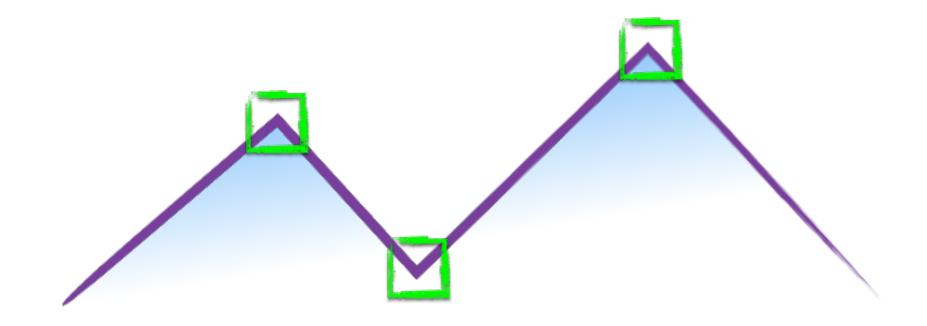
Harris corner detector

How do you find a corner?



How do you find a corner?

[Moravec 1980]

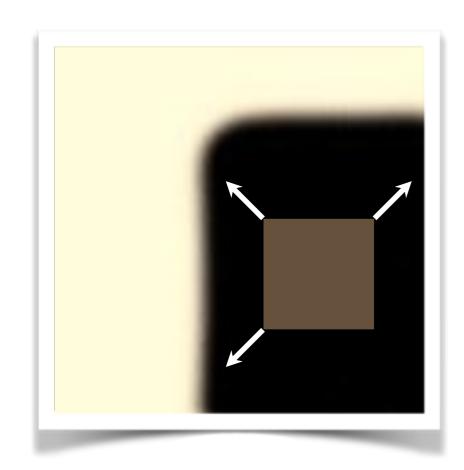


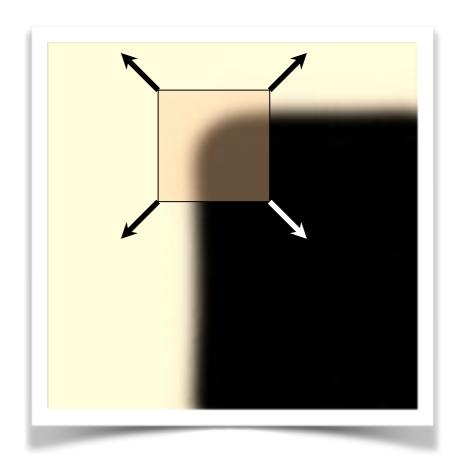
Easily recognized by looking through a small window

Shifting the window should give large change in intensity

Easily recognized by looking through a small window

Shifting the window should give large change in intensity





"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

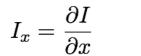
Design a program to detect corners

(hint: use image gradients)

Finding corners

(a.k.a. PCA)

- 1.Compute image gradients over small region
- 2.Subtract mean from each image gradient
- 3.Compute the covariance matrix
- 4.Compute eigenvectors and eigenvalues
- 5.Use threshold on eigenvalues to detect corners





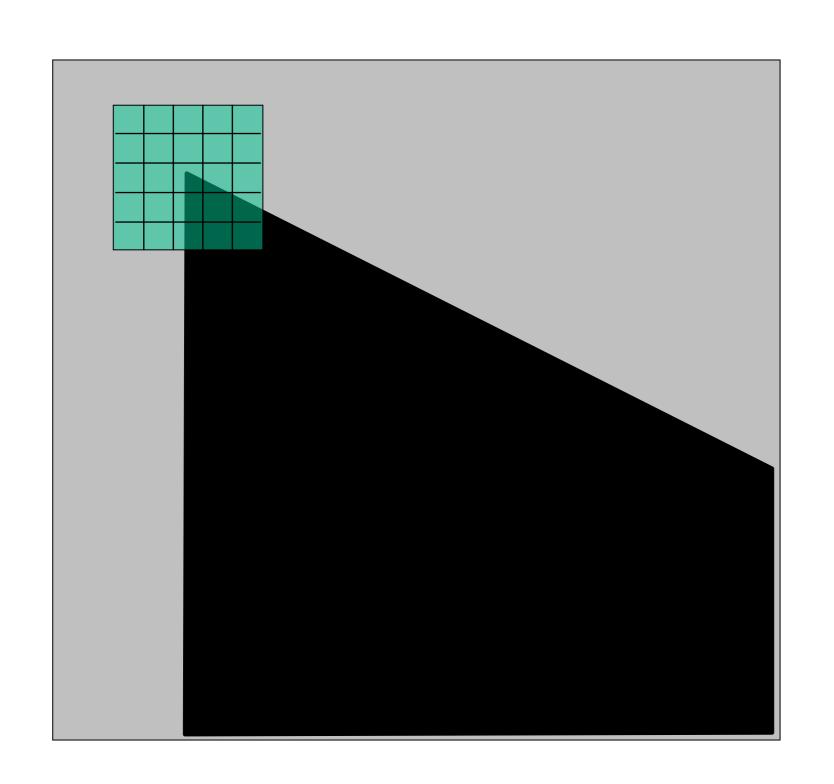
$$I_y = \frac{\partial I}{\partial y}$$



$$\left[\begin{array}{ccc} \sum\limits_{p \in P} I_x I_x & \sum\limits_{p \in P} I_x I_y \\ \sum\limits_{p \in P} I_y I_x & \sum\limits_{p \in P} I_y I_y \end{array}\right]$$

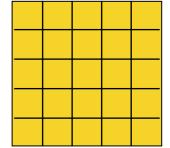
1. Compute image gradients over a small region (not just a single pixel)

1. Compute image gradients over a small region (not just a single pixel)



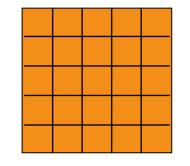
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$

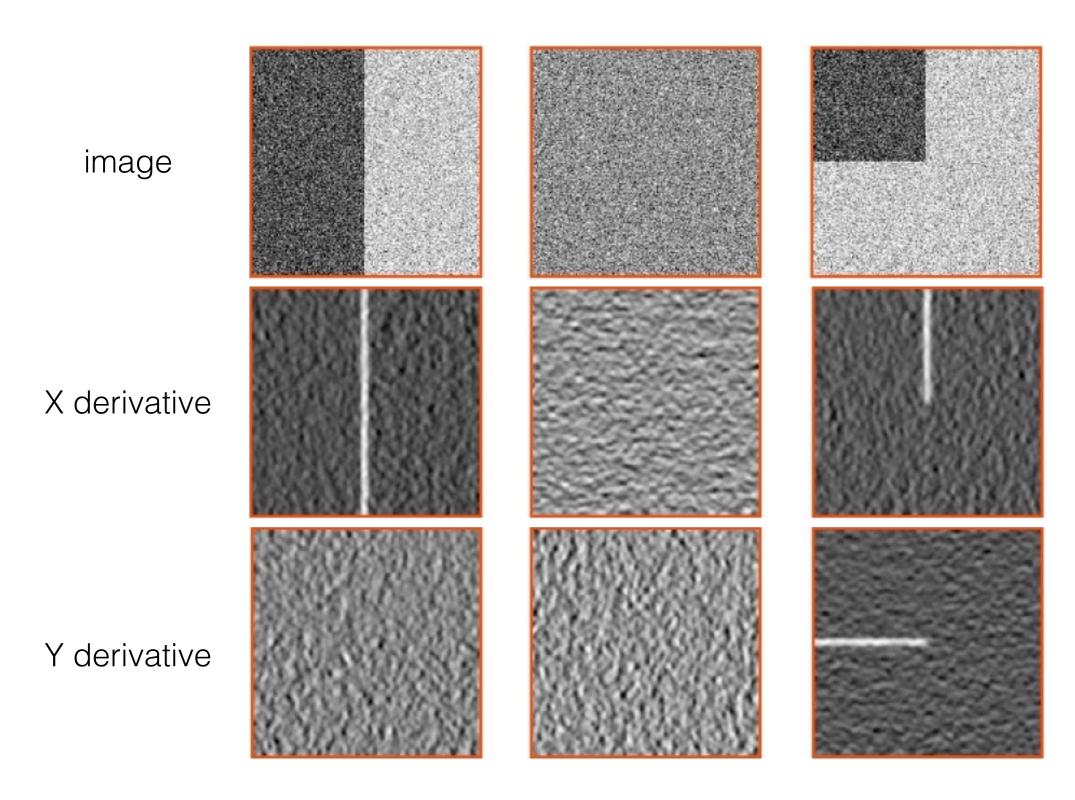


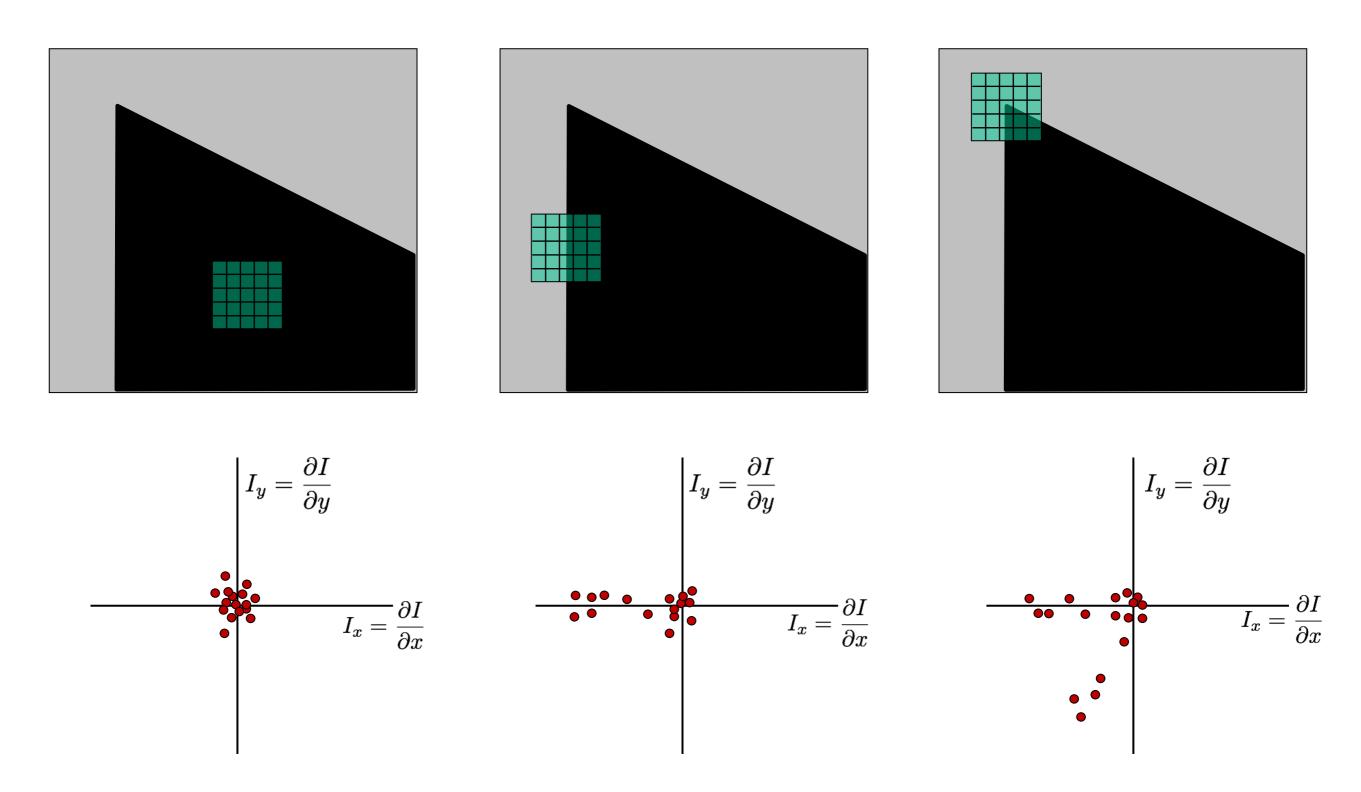
array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$

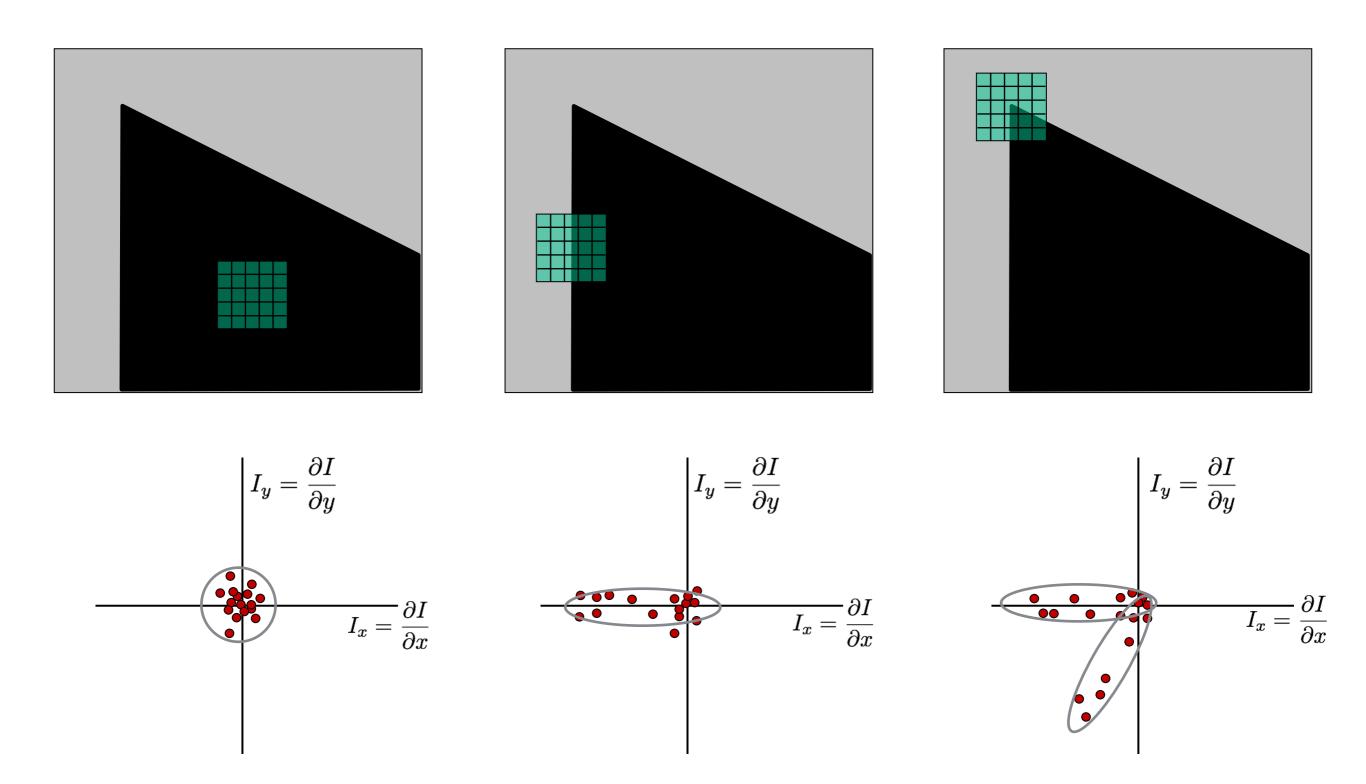


visualization of gradients

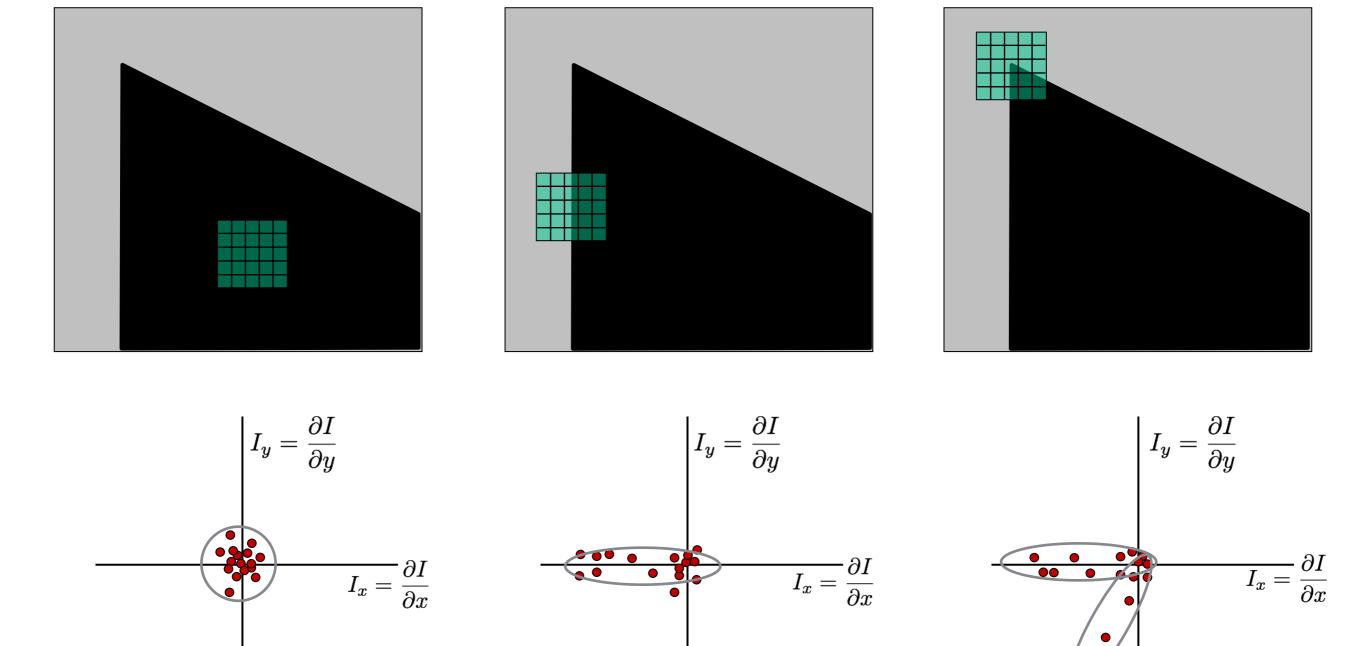




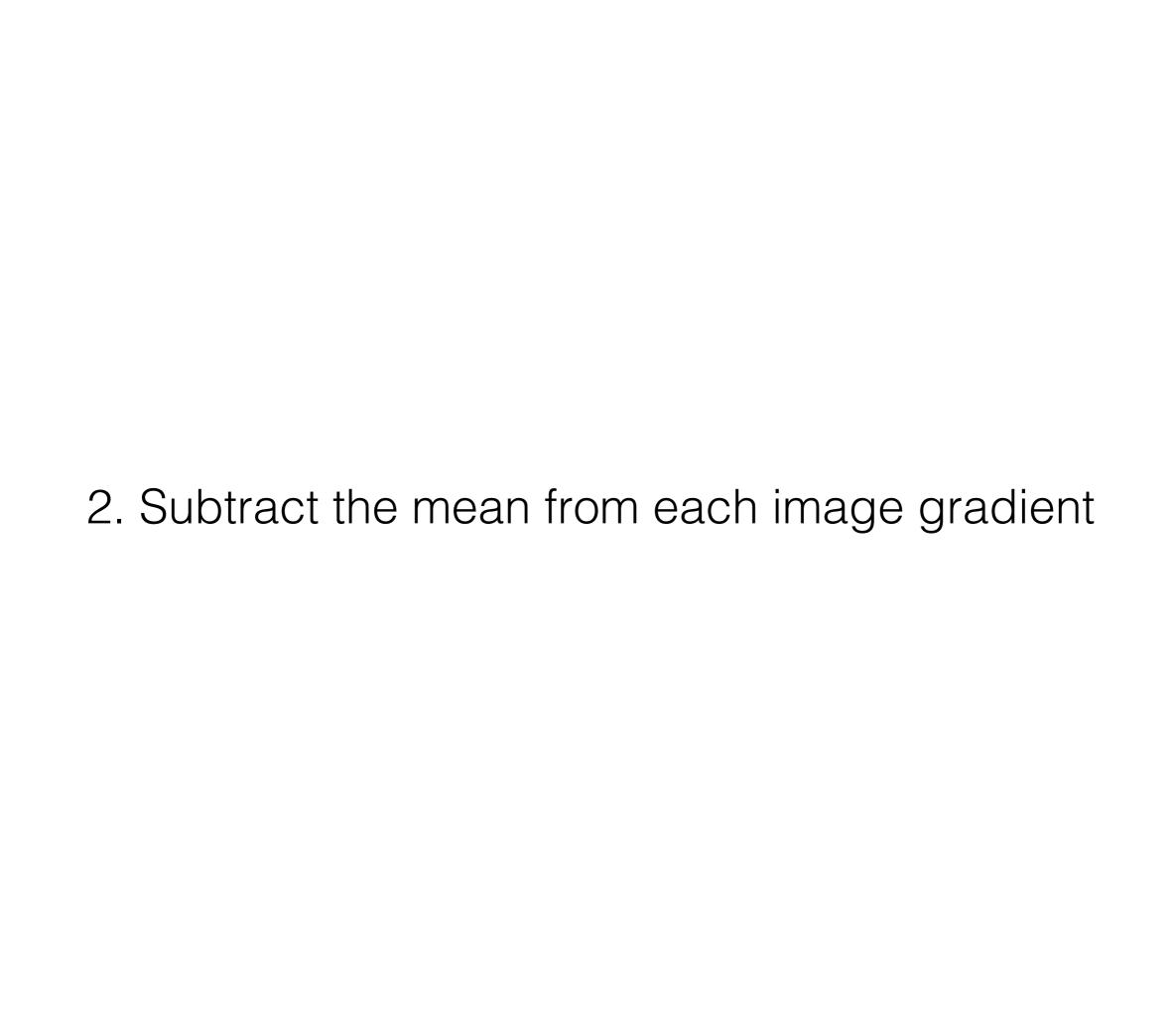
What does the distribution tell you about the region?



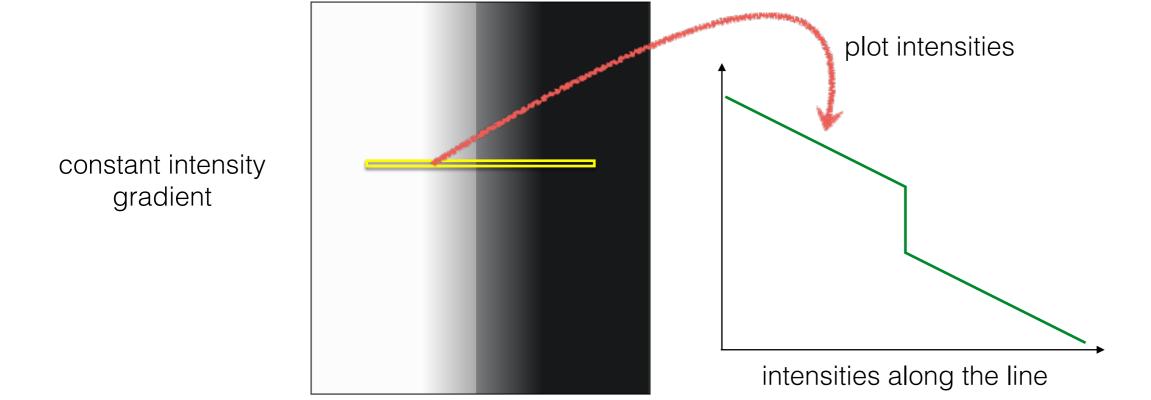
distribution reveals edge orientation and magnitude



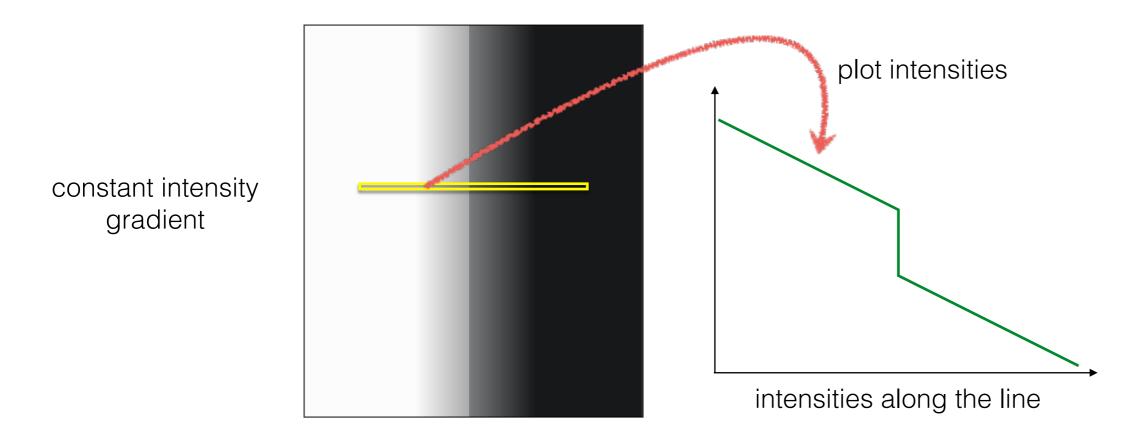
How do you quantify orientation and magnitude?

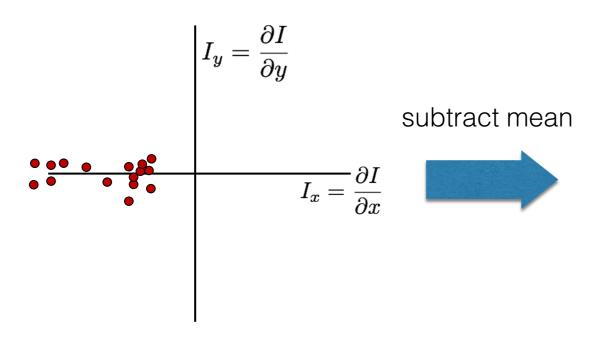


2. Subtract the mean from each image gradient



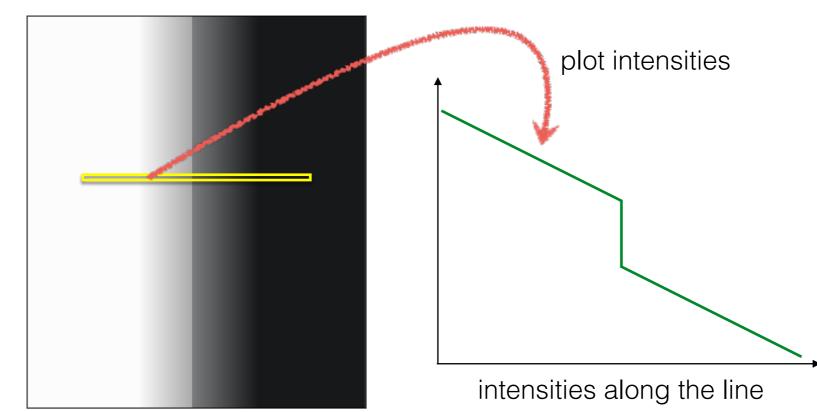
2. Subtract the mean from each image gradient





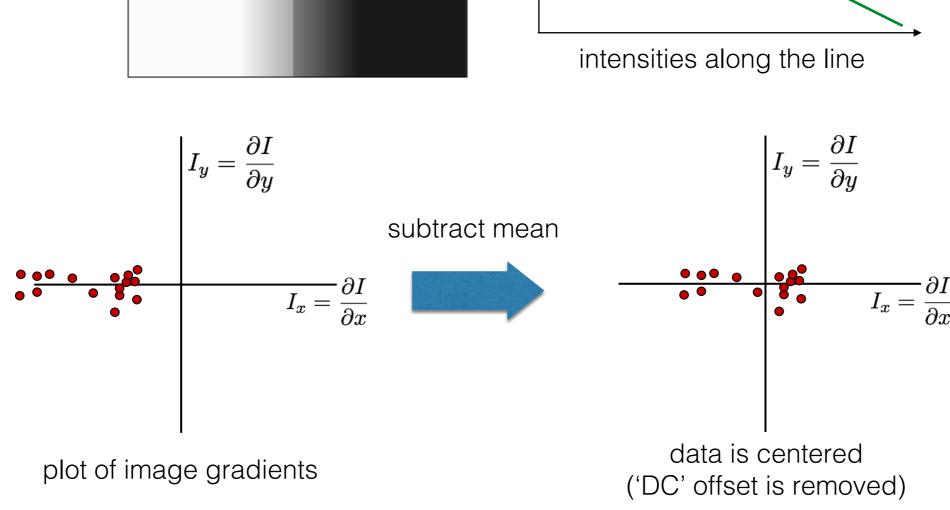
plot of image gradients

2. Subtract the mean from each image gradient



constant intensity

gradient



3. Compute the covariance matrix

3. Compute the covariance matrix

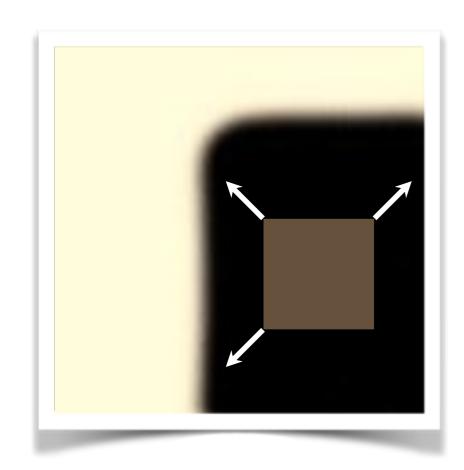
$$\left[\begin{array}{ccc} \sum\limits_{p\in P}I_xI_x & \sum\limits_{p\in P}I_xI_y \\ \sum\limits_{p\in P}I_yI_x & \sum\limits_{p\in P}I_yI_y \end{array}\right]$$

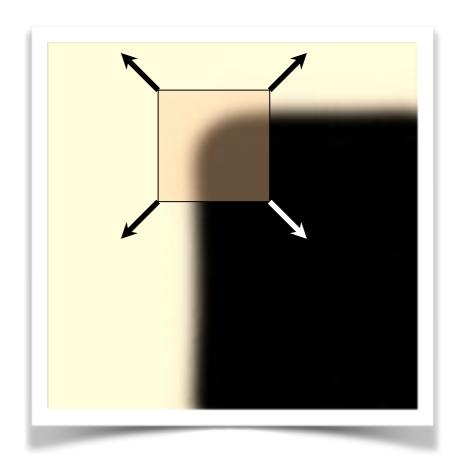
$$I_x = \frac{\partial I}{\partial x}$$
 $I_y = \frac{\partial I}{\partial y}$ $\sum_{p \in P} I_x I_y = \operatorname{Sum}($ \star array of x gradients $I_y = \frac{\partial I}{\partial y}$

Where does this covariance matrix come from?

Easily recognized by looking through a small window

Shifting the window should give large change in intensity





"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

Some mathematical background...

Error function

Change of intensity for the shift [u,v]:

$$E(u,v) = \sum_{x,y} w(x,y) \Big[I(x+u,y+v) - I(x,y) \Big]^2$$
Error Window Shifted Intensity Intensity

Window function w(x,y) = 01 in window, 0 outside Gaussian

Error function approximation

Change of intensity for the shift [u,v]:

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

First-order Taylor expansion of I(x,y) about (0,0) (bilinear approximation for small shifts)

Bilinear approximation

For small shifts [u,v] we have a 'bilinear approximation':

Change in appearance for a shift [u,v]

$$E(u,v) \cong \begin{bmatrix} u,v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

'second moment' matrix 'structure tensor'

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

By computing the gradient covariance matrix...

$$\left[\begin{array}{ccc} \sum\limits_{p\in P}I_xI_x & \sum\limits_{p\in P}I_xI_y \\ \sum\limits_{p\in P}I_yI_x & \sum\limits_{p\in P}I_yI_y \end{array}\right]$$

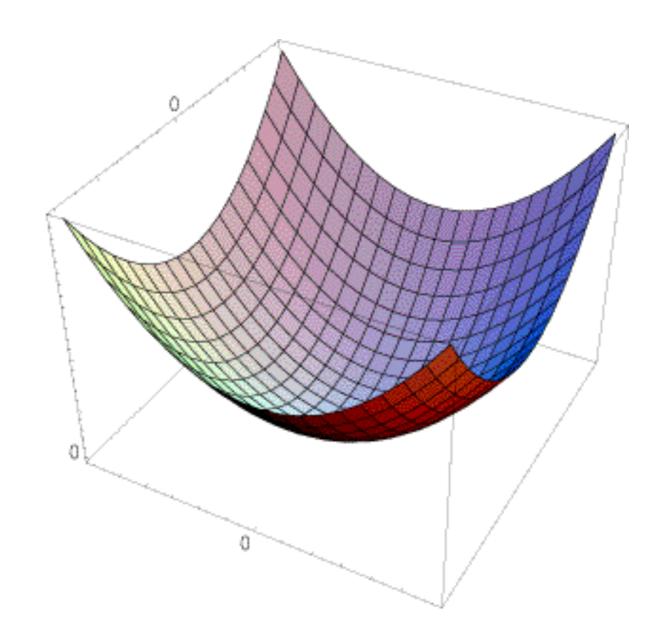
we are fitting a quadratic to the gradients over a small image region

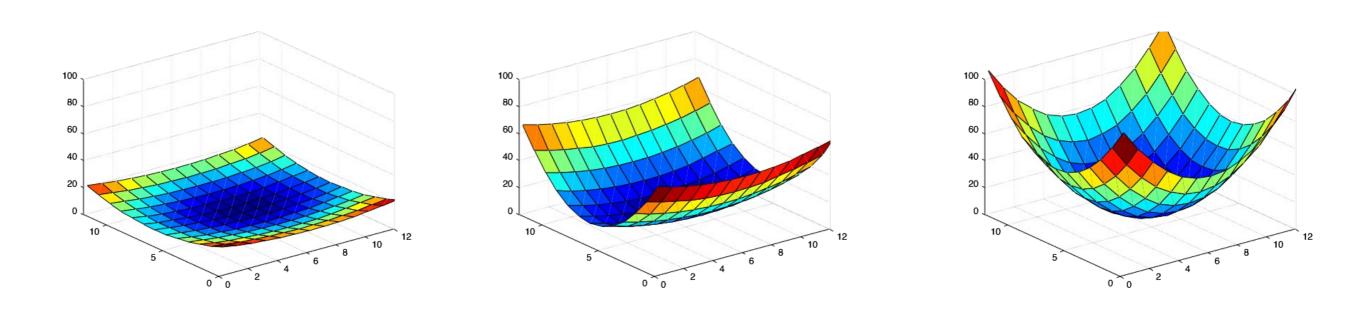
Visualization of a quadratic

The surface E(u,v) is locally approximated by a quadratic form

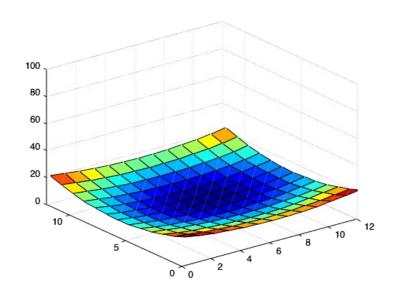
$$E(u,v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

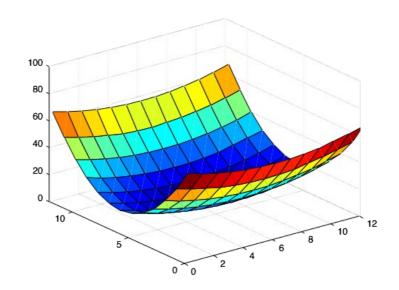
$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

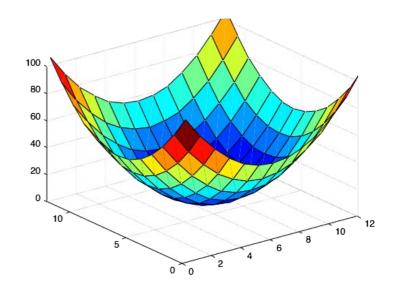




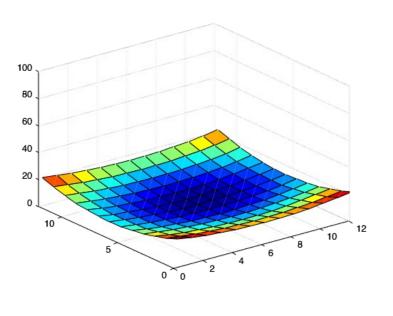
What kind of image patch do these surfaces represent?



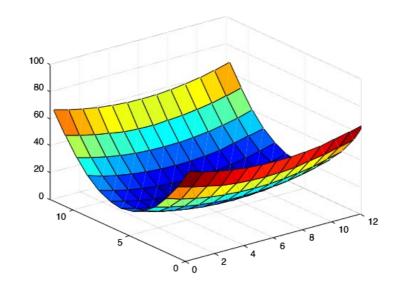




flat

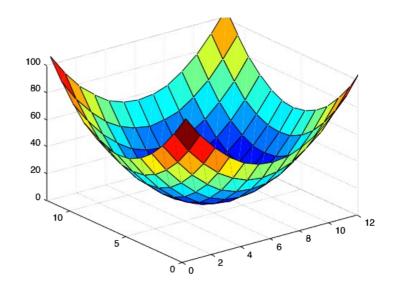


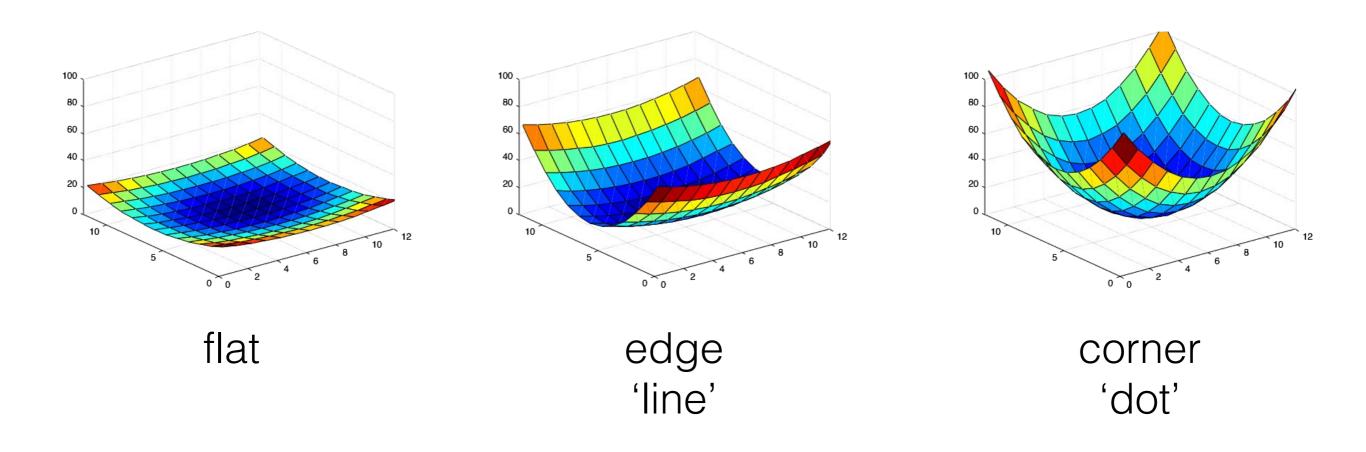
flat



edge

'line'

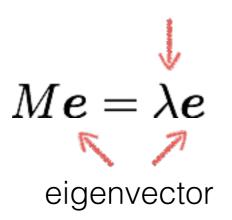




eigenvalue



eigenvalue



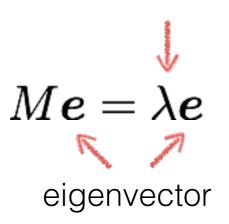
$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of

 $M - \lambda I$

(returns a polynomial)

eigenvalue



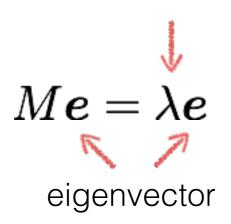
$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of (returns a polynomial)

$$M - \lambda I$$

2. Find the roots of polynomial $\det(M-\lambda I)=0$

eigenvalue



$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of (returns a polynomial)

$$M - \lambda I$$

2. Find the roots of polynomial (returns eigenvalues)

$$\det(M - \lambda I) = 0$$

3. For each eigenvalue, solve (returns eigenvectors)

$$(M - \lambda I)\mathbf{e} = 0$$

eig(M)

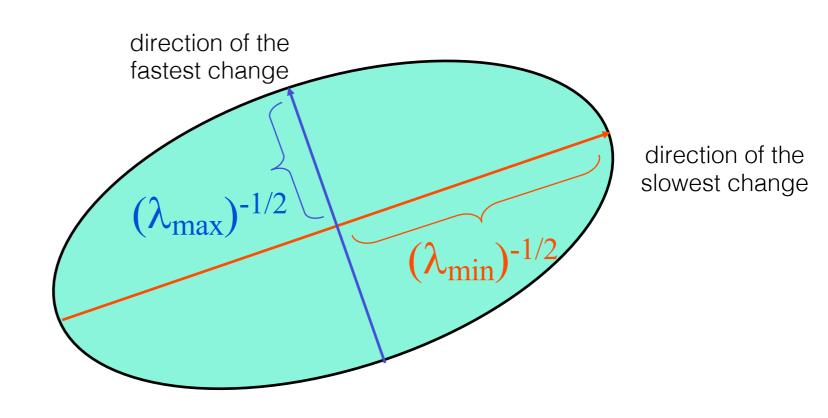
Visualization as an ellipse

Since M is symmetric, we have $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R

Ellipse equation:

$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



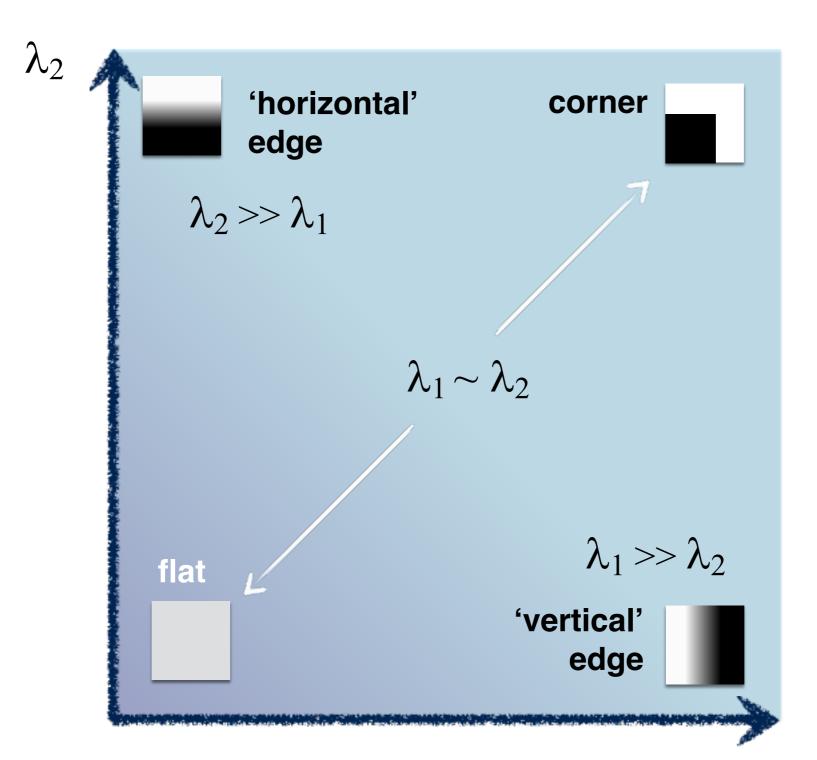
$$\lambda_2 >> \lambda_1$$

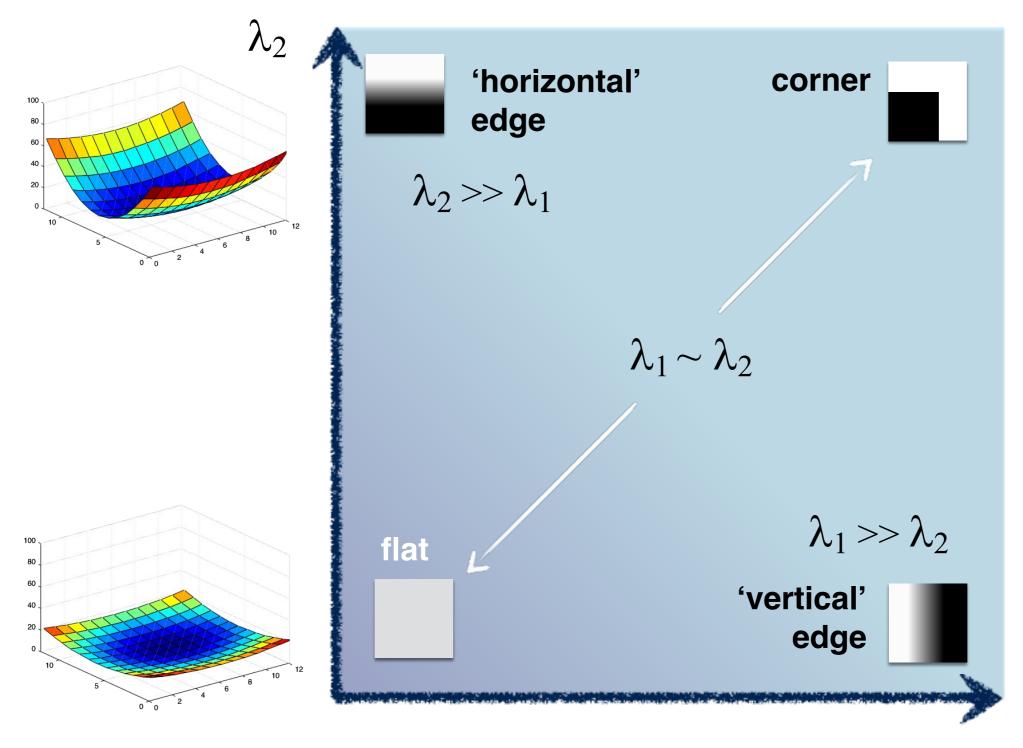
What kind of image patch does each region represent?

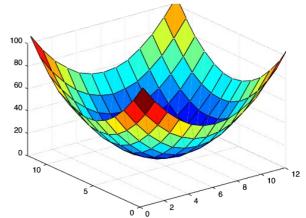
$$\lambda_1 \sim 0$$
 $\lambda_2 \sim 0$

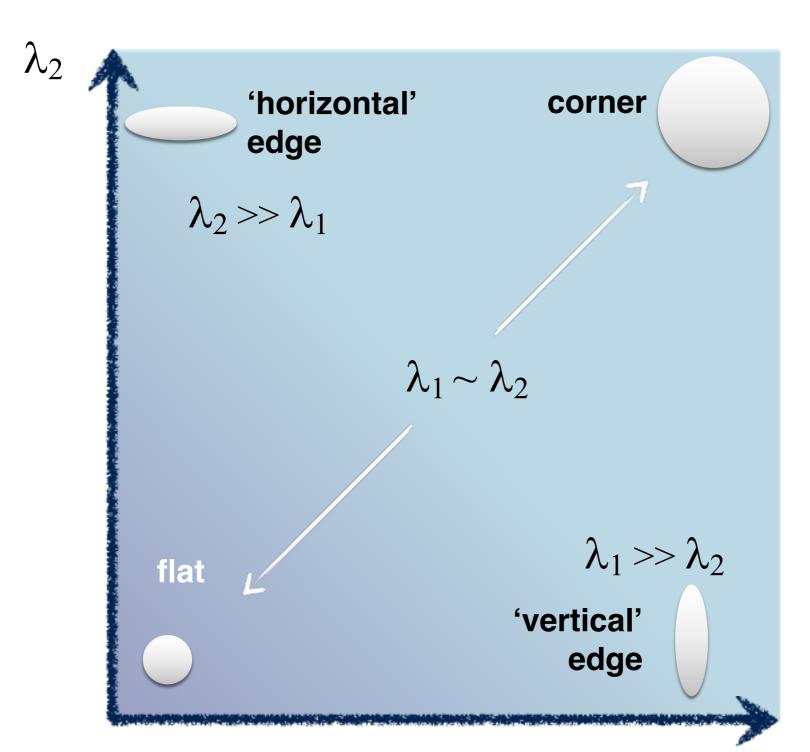
$$\lambda_2 \sim 0$$

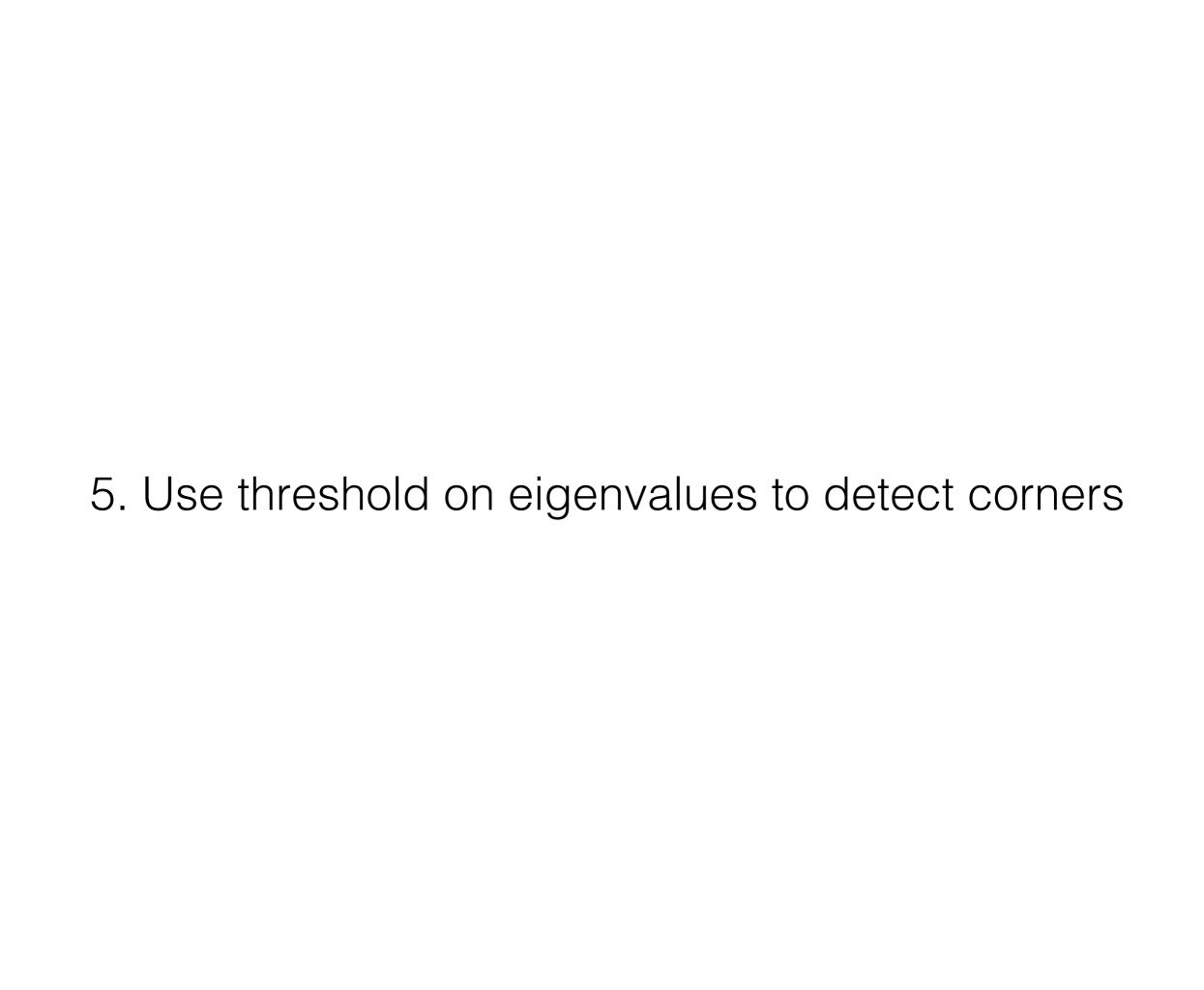
$$\lambda_1 >> \lambda_2$$



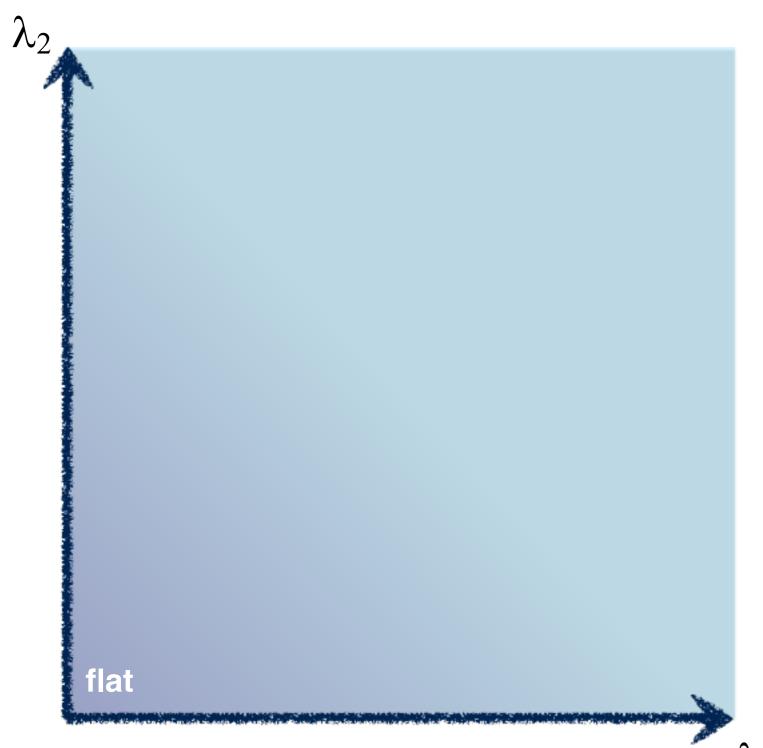








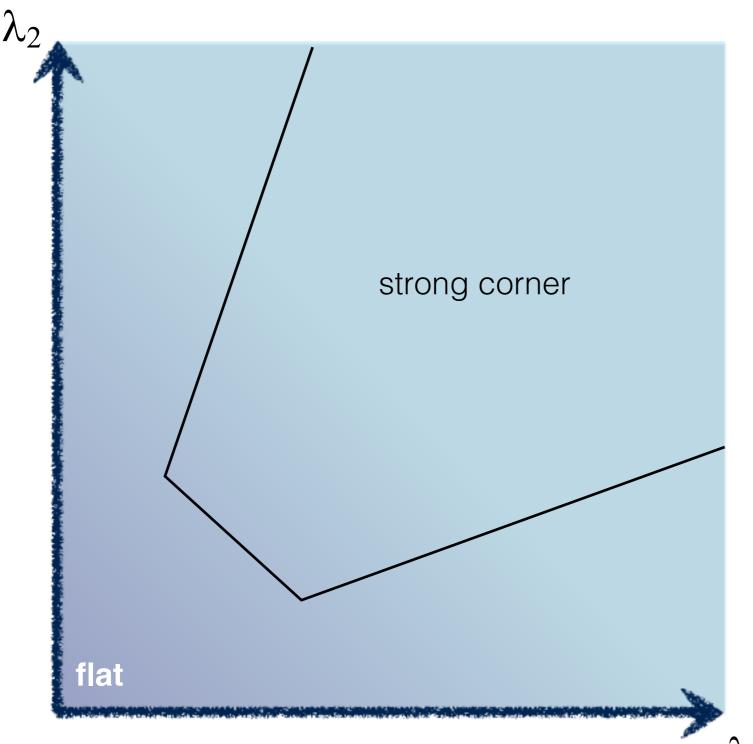
5. Use threshold on eigenvalues to detect corners



Think of a function to score 'cornerness'

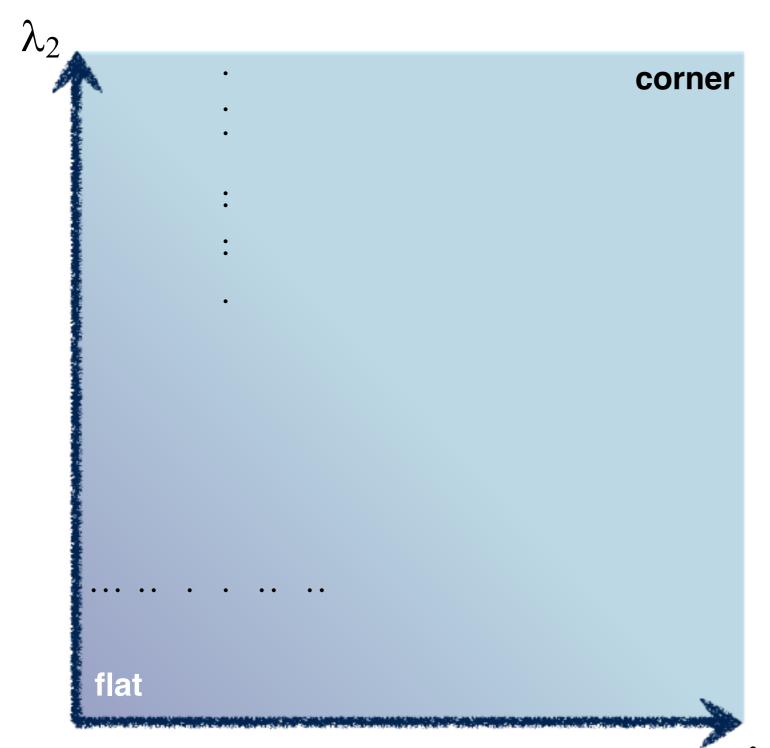
 λ

5. Use threshold on eigenvalues to detect corners



Think of a function to score 'cornerness'

5. Use threshold on eigenvalues to detect corners (a function of)

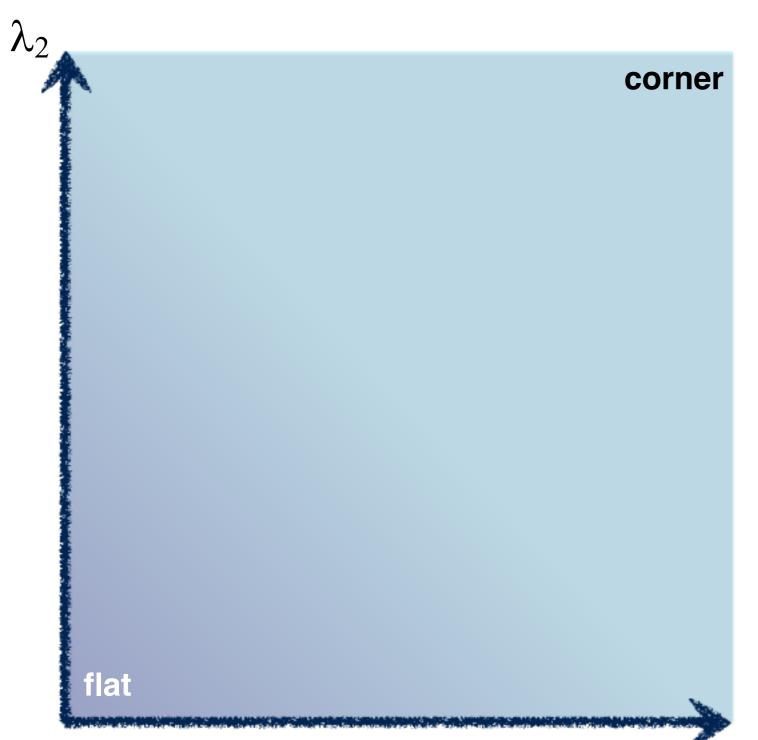


Use the smallest eigenvalue as the response function

$$R = \min(\lambda_1, \lambda_2)$$

 λ_1

5. Use threshold on eigenvalues to detect corners (a function of)



$$R = \lambda_1 \lambda_2 - \kappa (\lambda_1 + \lambda_2)^2$$

Can compute this more efficiently...

5. Use threshold on eigenvalues to detect corners (a function of)

 λ_2

corner

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$

$$R \ll 0$$

flat

$$\det M = \lambda_1 \lambda_2$$

$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

$$det \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc$$

$$trace\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d$$

Harris & Stephens (1988)

$$R = \det(M) - \kappa \operatorname{trace}^2(M)$$

Kanade & Tomasi (1994)

$$R = \min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$R = \frac{\det(M)}{\operatorname{trace}(M) + \epsilon}$$

Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." 1988.

1. Compute x and y derivatives of image

$$I_x = G_{\sigma}^x * I \qquad I_y = G_{\sigma}^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2}$$
 $S_{y^2} = G_{\sigma'} * I_{y^2}$ $S_{xy} = G_{\sigma'} * I_{xy}$

Harris Detector

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." 1988.

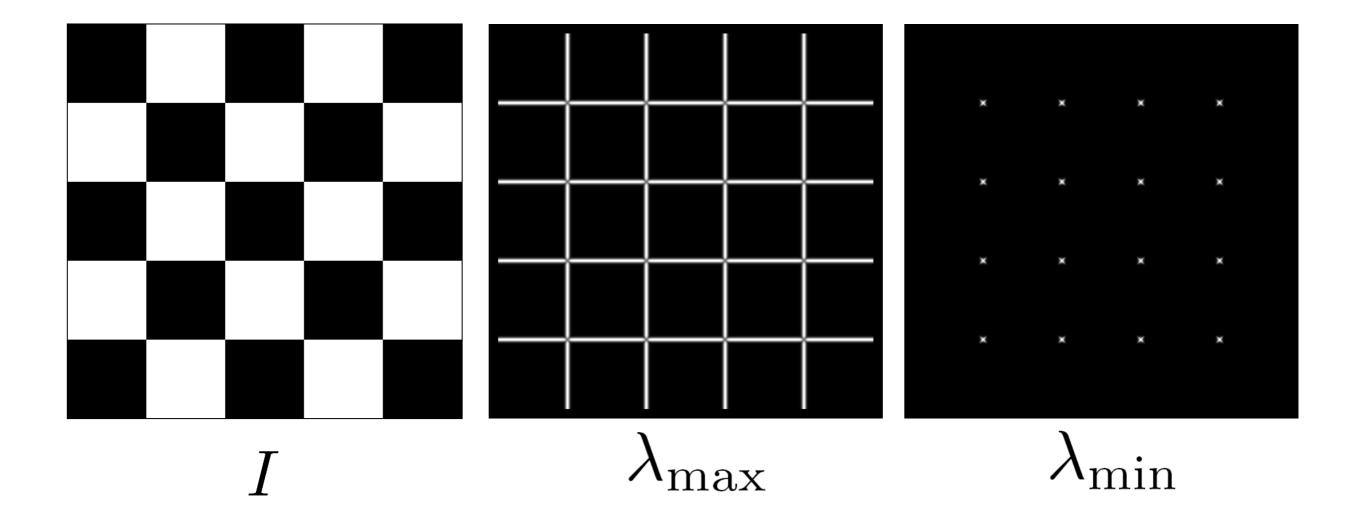
4. Define the matrix at each pixel

$$M(x,y) = \begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = \det M - k (\operatorname{trace} M)^2$$

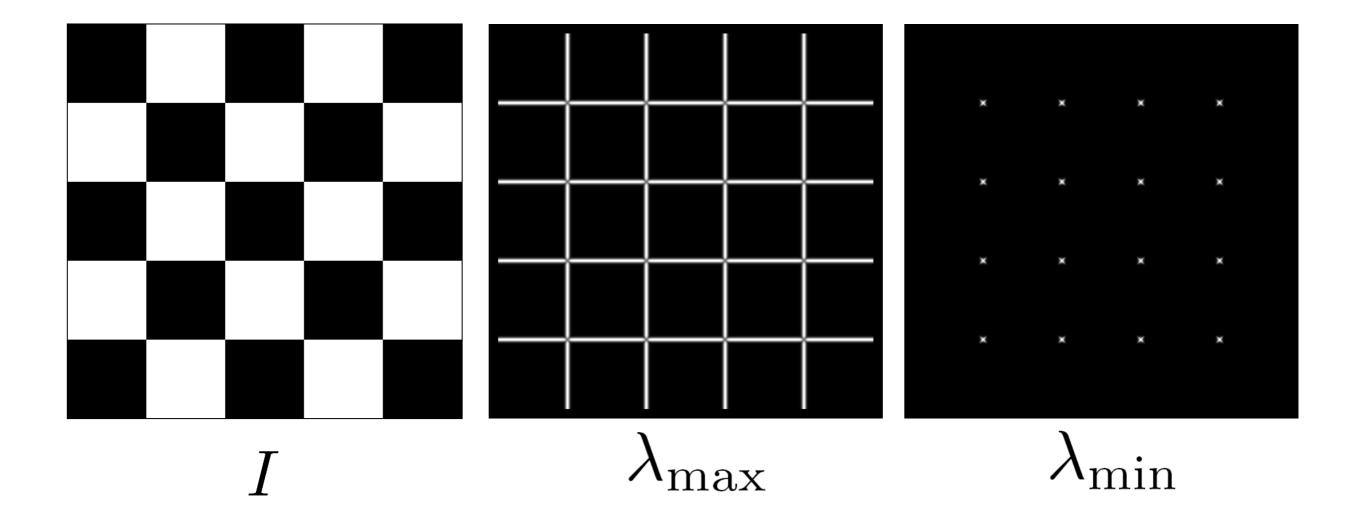
6. Threshold on value of R; compute non-max suppression.



Yet another option:

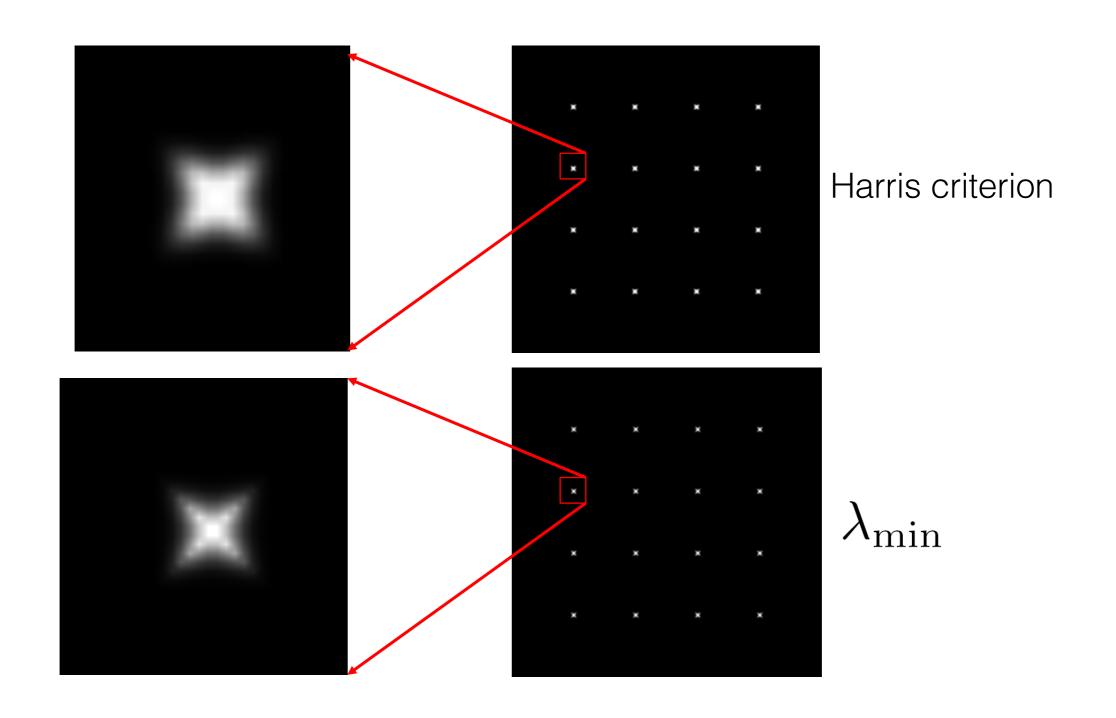
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

How do you write this equivalently using determinant and trace?



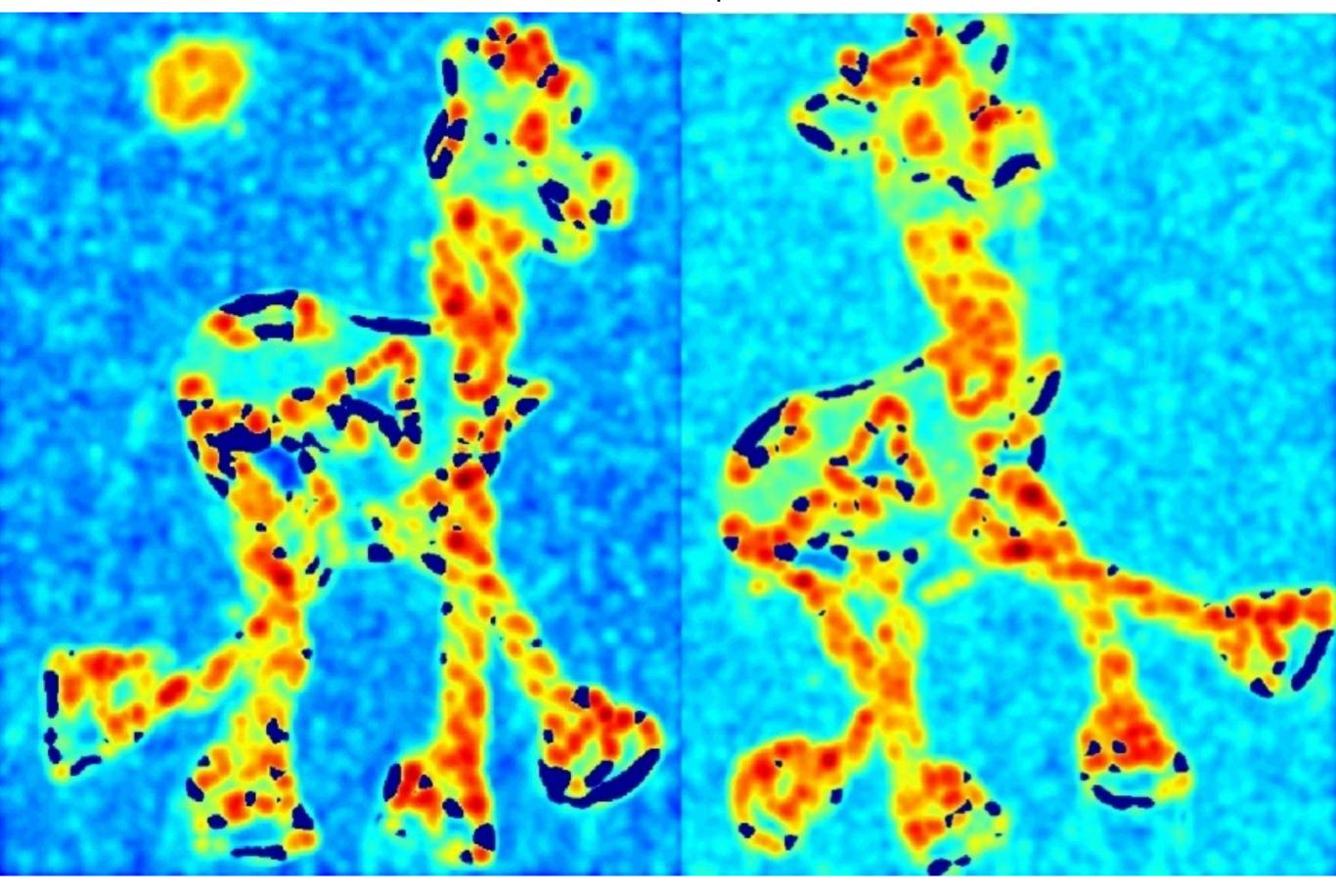
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{determinant(H)}{trace(H)}$$

Different criteria

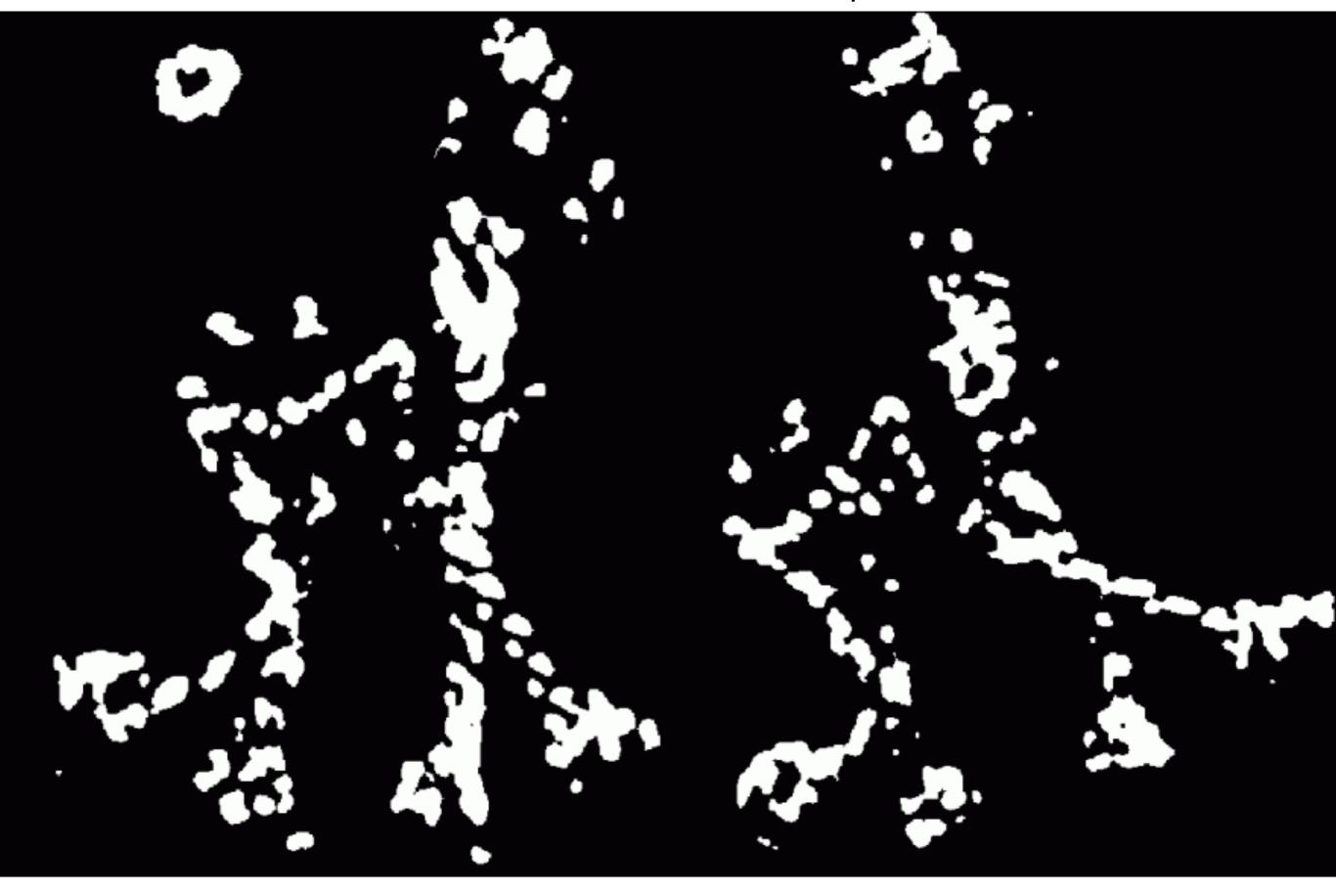




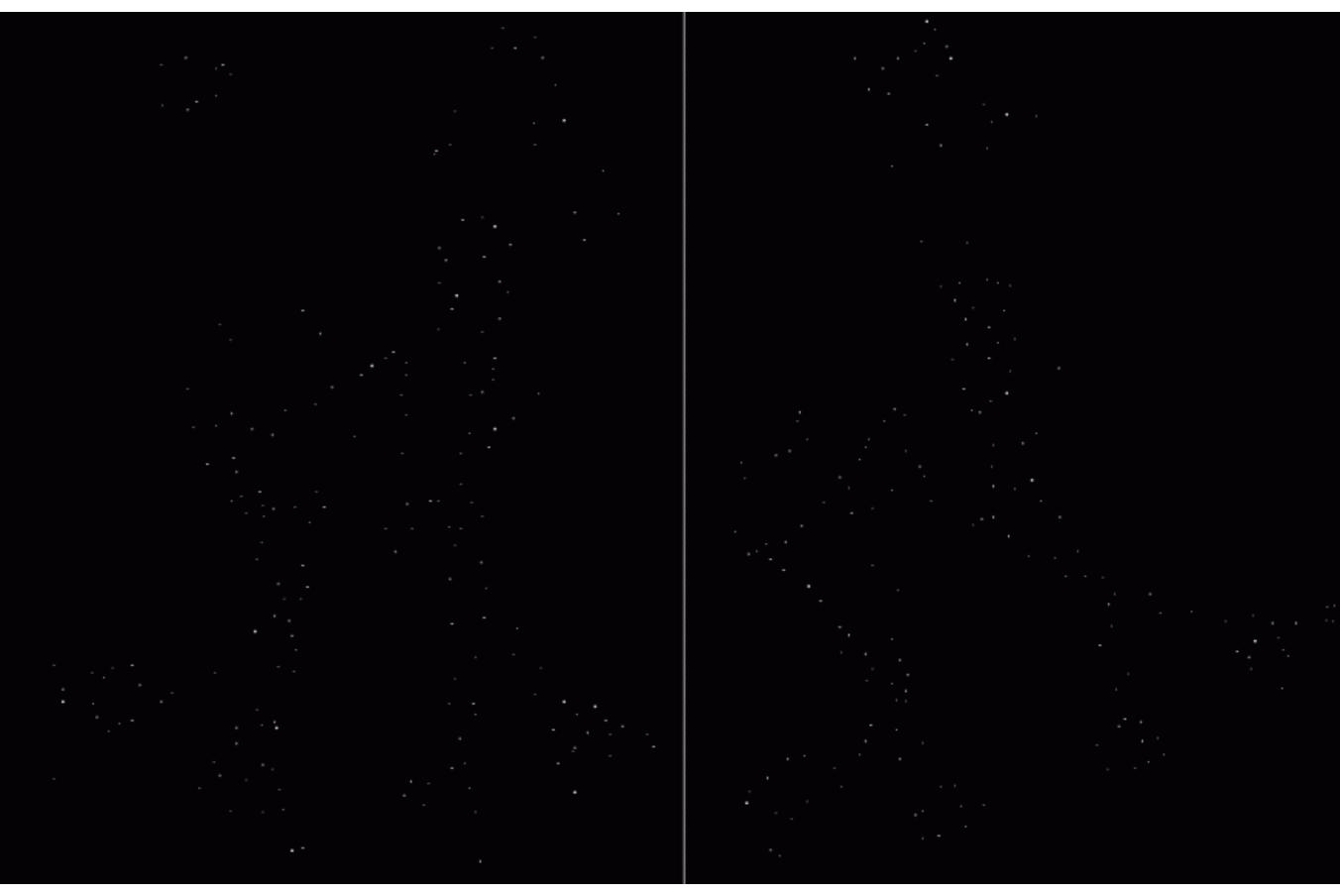
Corner response



Thresholded corner response

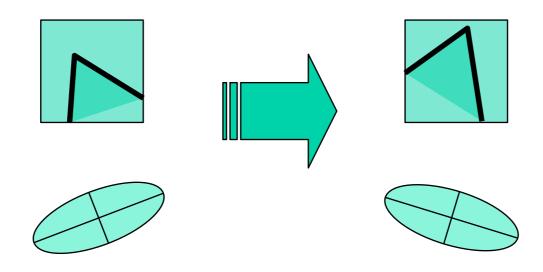


Non-maximal suppression





Harris corner response is invariant to rotation



Ellipse rotates but its shape (eigenvalues) remains the same

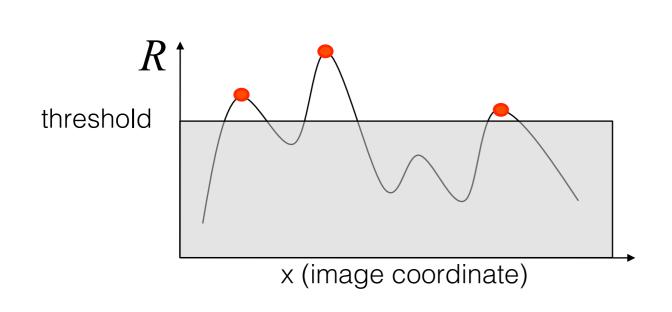
Corner response R is invariant to image rotation

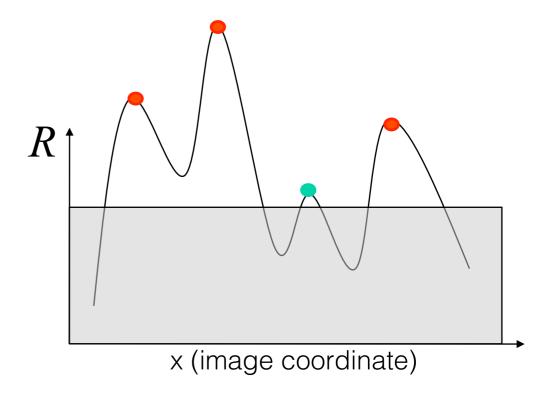
Harris corner response is invariant to intensity changes

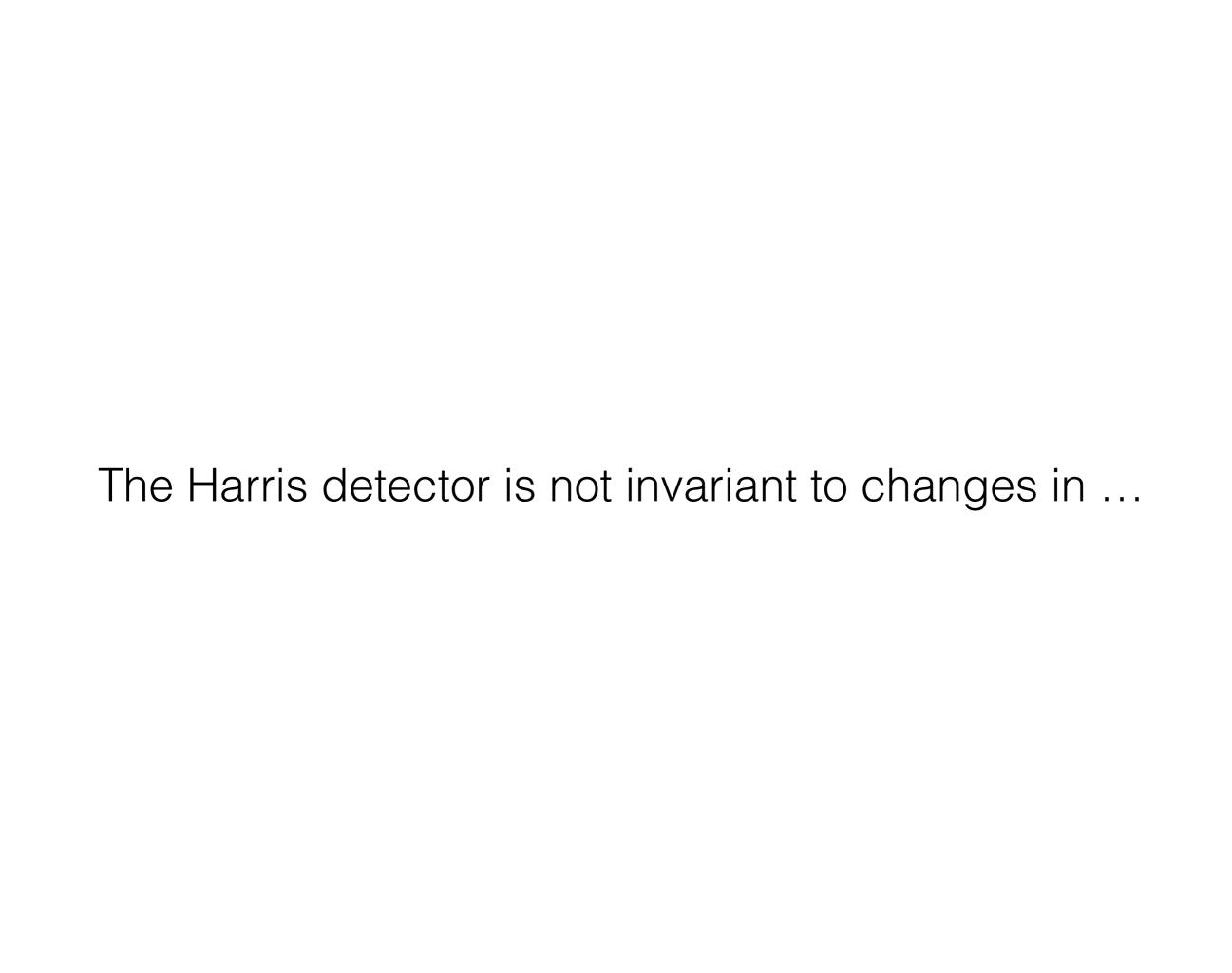
Partial invariance to affine intensity change

Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

Intensity scale: $I \rightarrow a I$





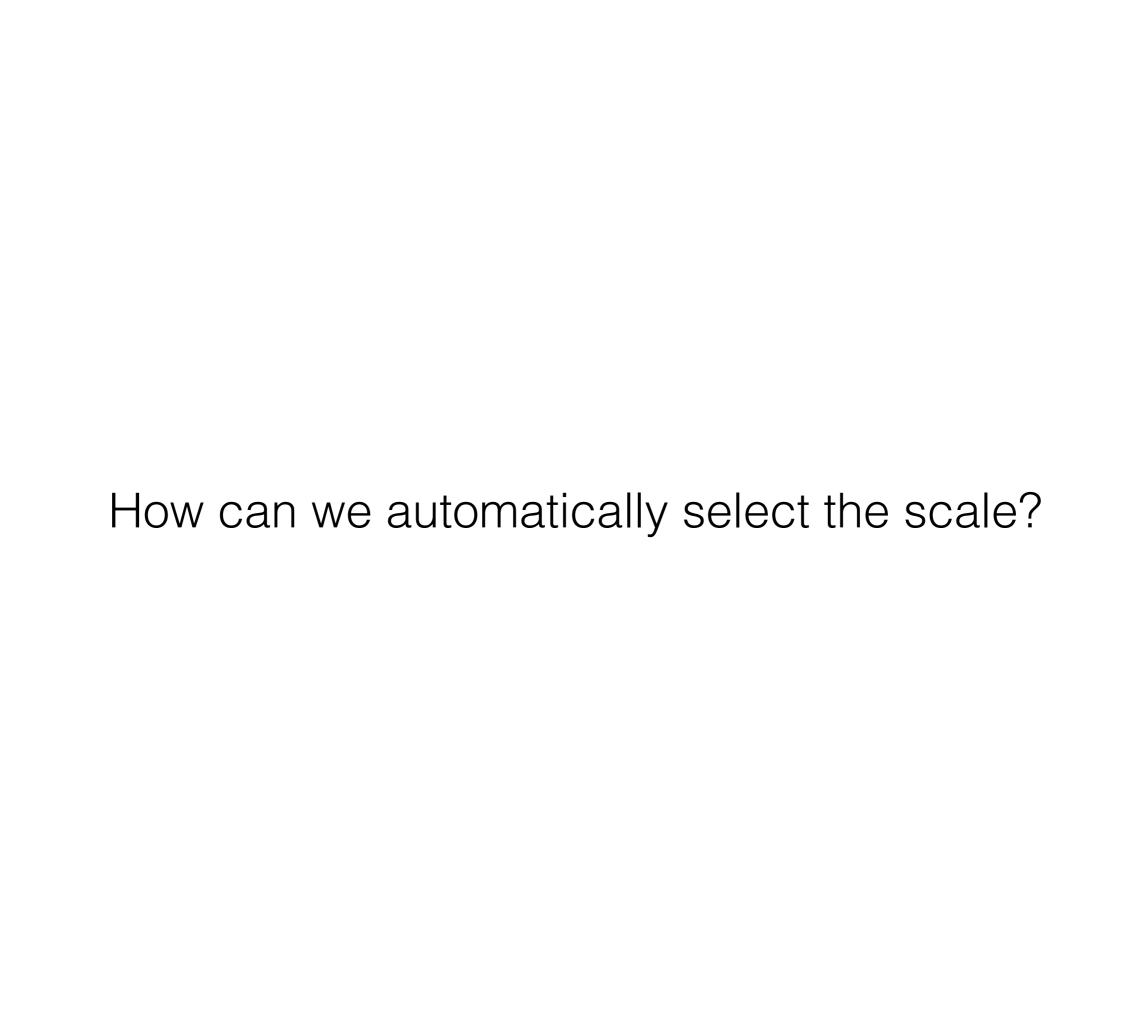


The Harris corner detector is <u>not</u> invariant to scale



Multi-scale detection



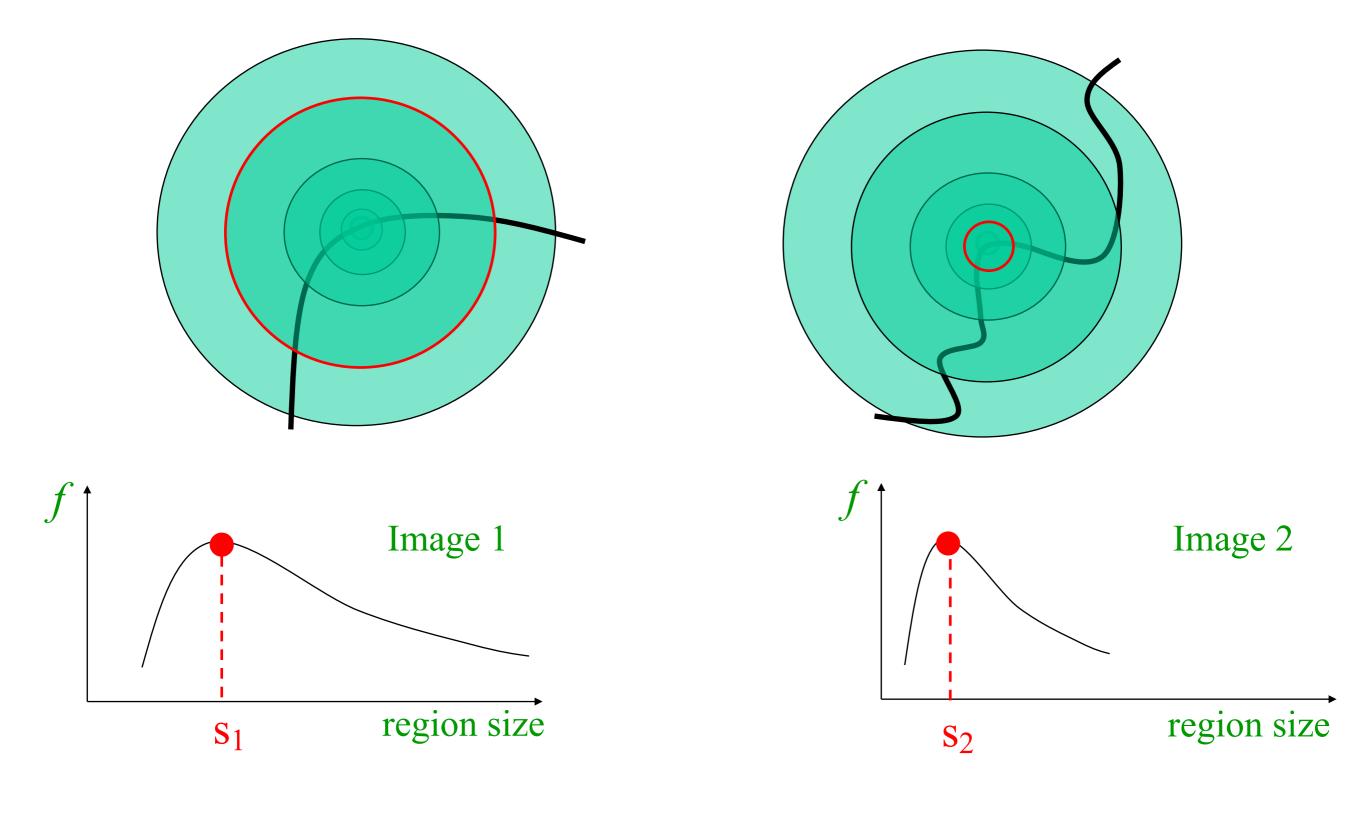


Multi-scale blob detection

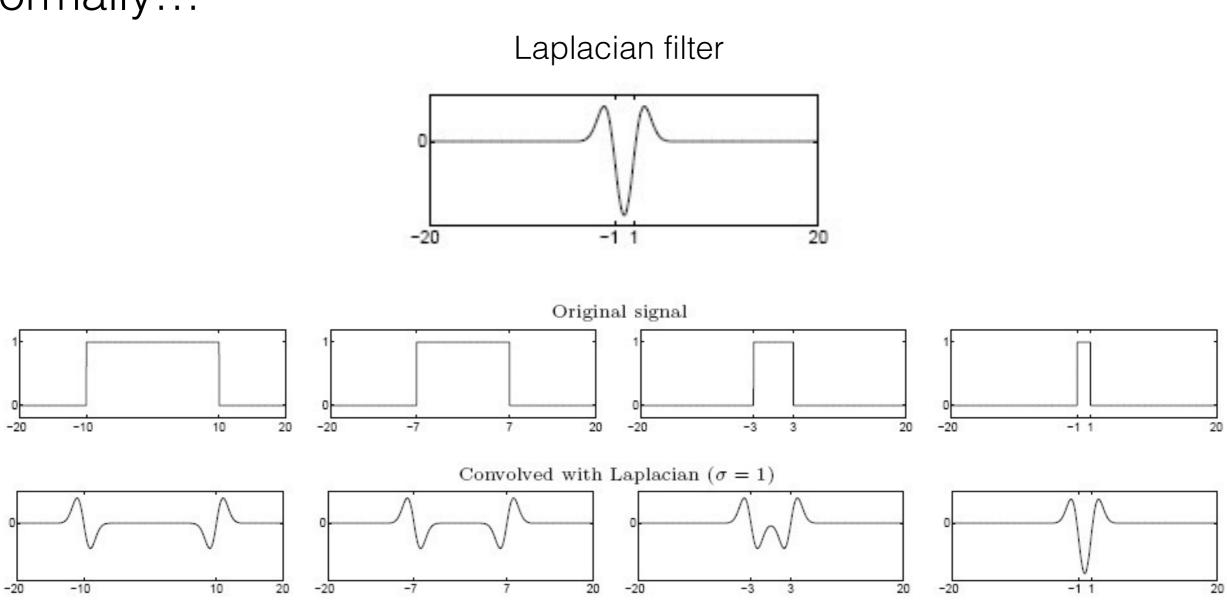


Intuitively...

Find local maxima in both position and scale

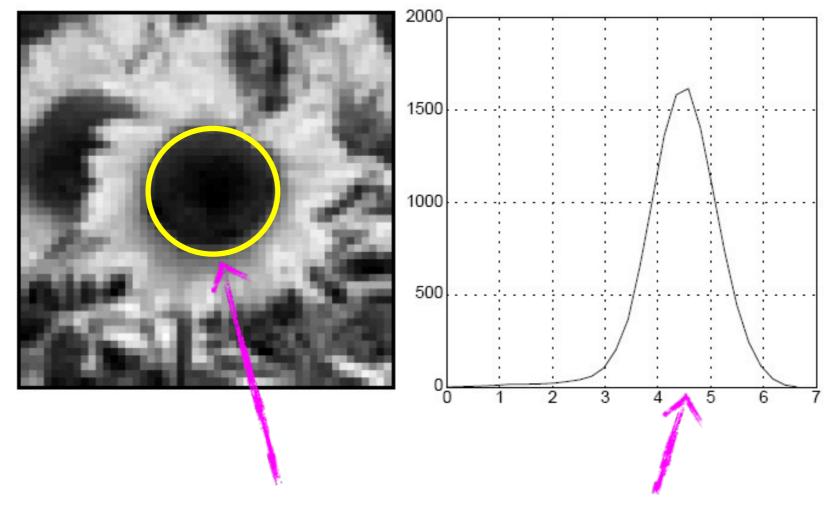


Formally...



Highest response when the signal has the same **characteristic scale** as the filter

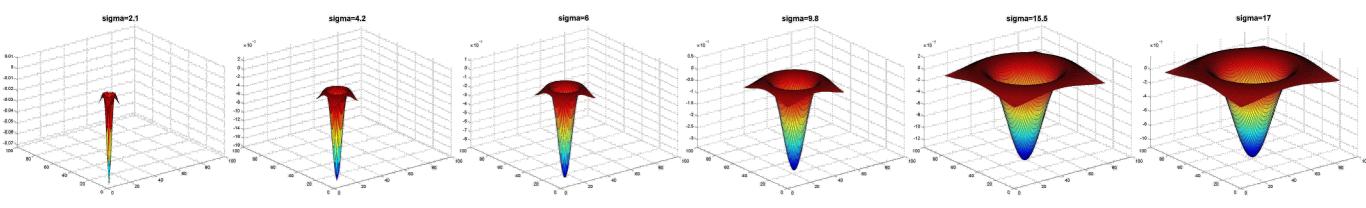
characteristic scale - the scale that produces peak filter response



characteristic scale

we need to search over characteristic scales

What happens if you apply different Laplacian filters?

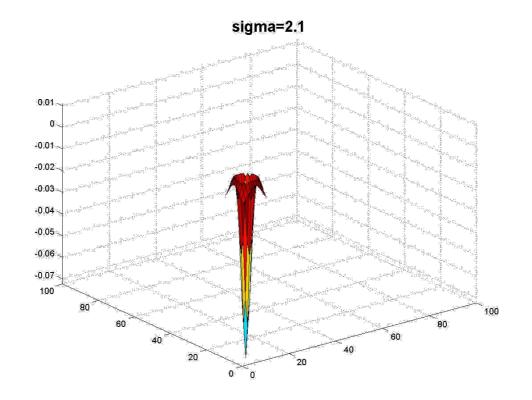


Full size

3/4 size



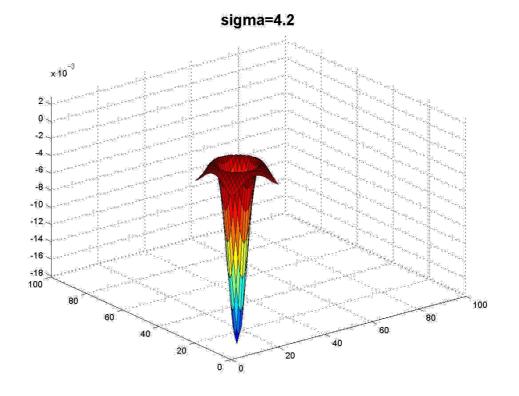






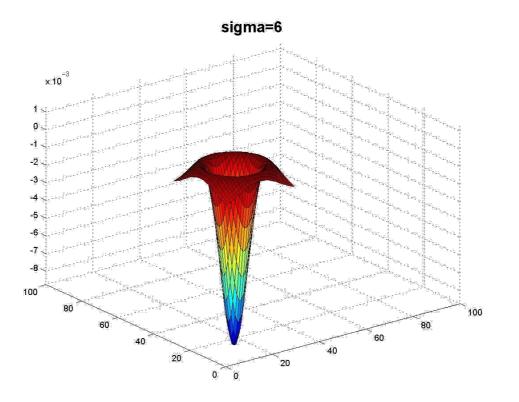


jet color scale blue: low, red: high



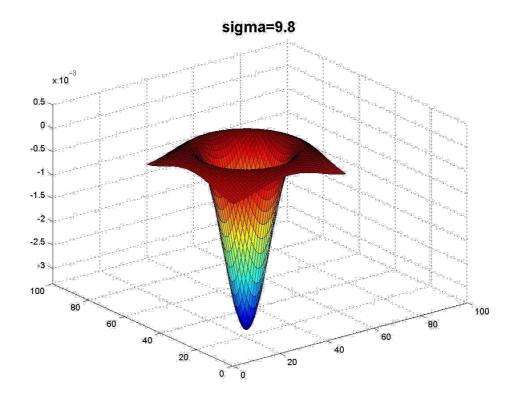






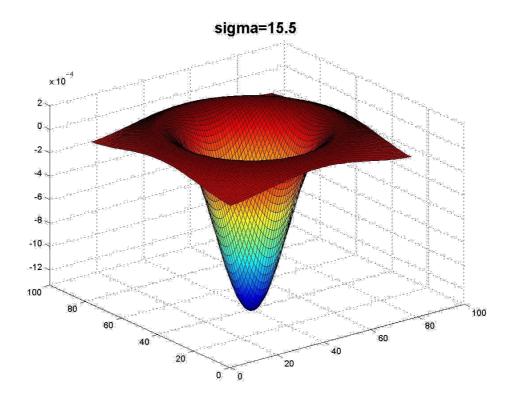






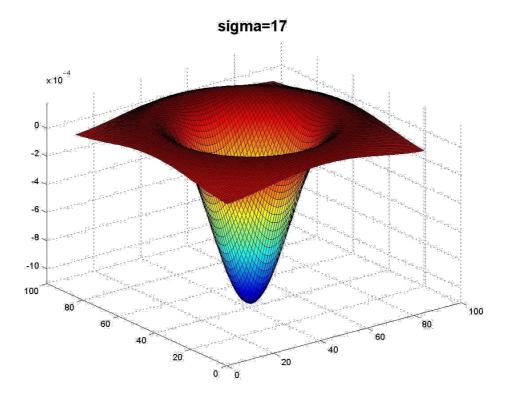














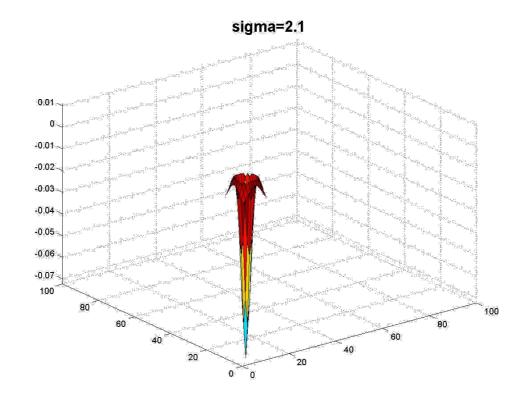


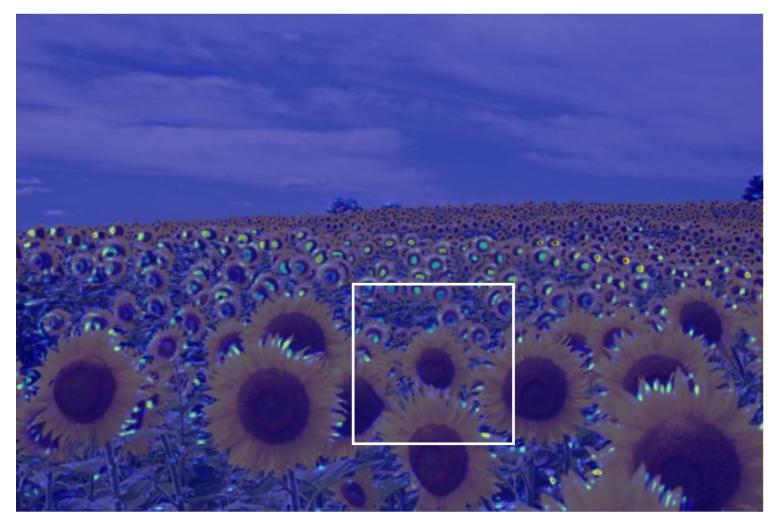
What happened when you applied different Laplacian filters?

Full size 3/4 size

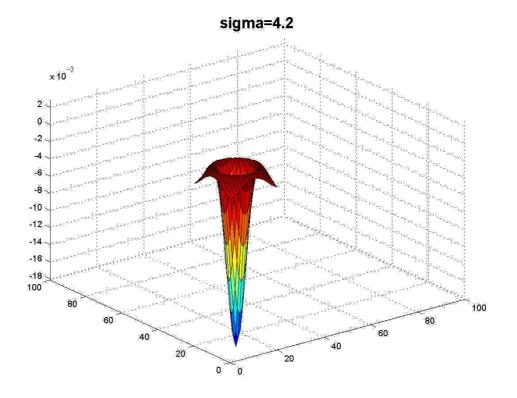




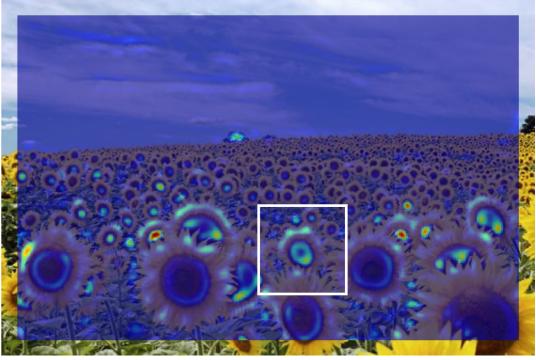


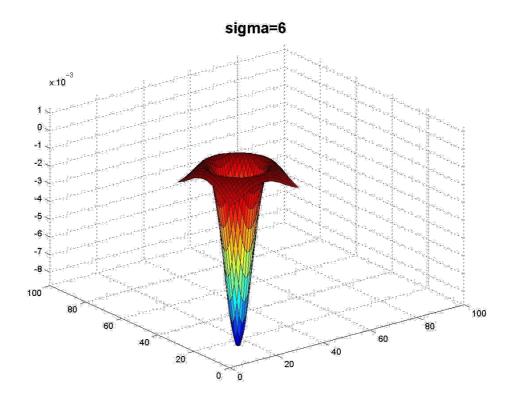


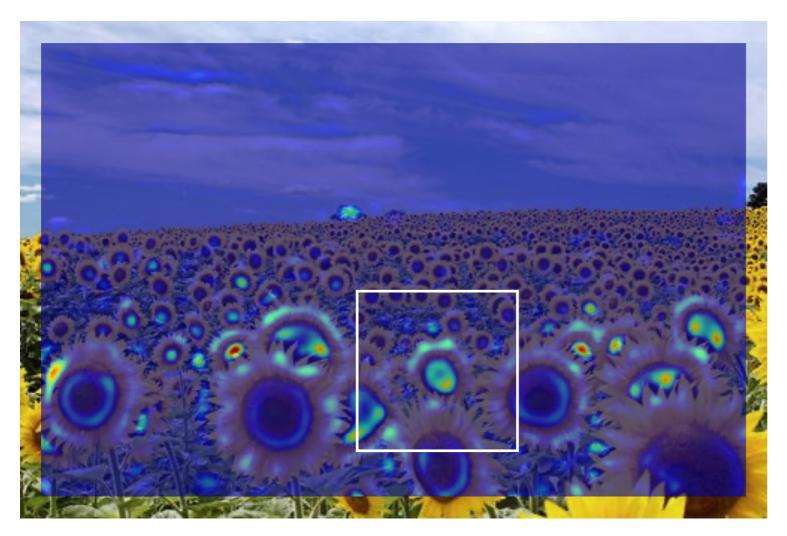




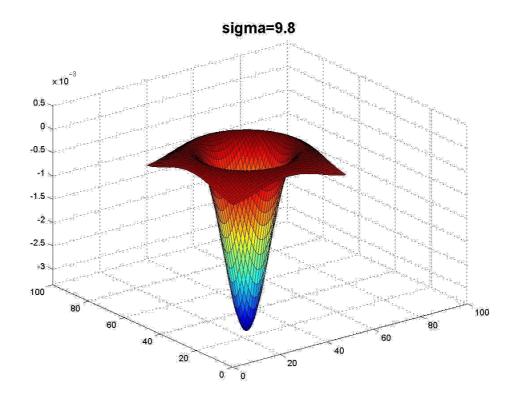


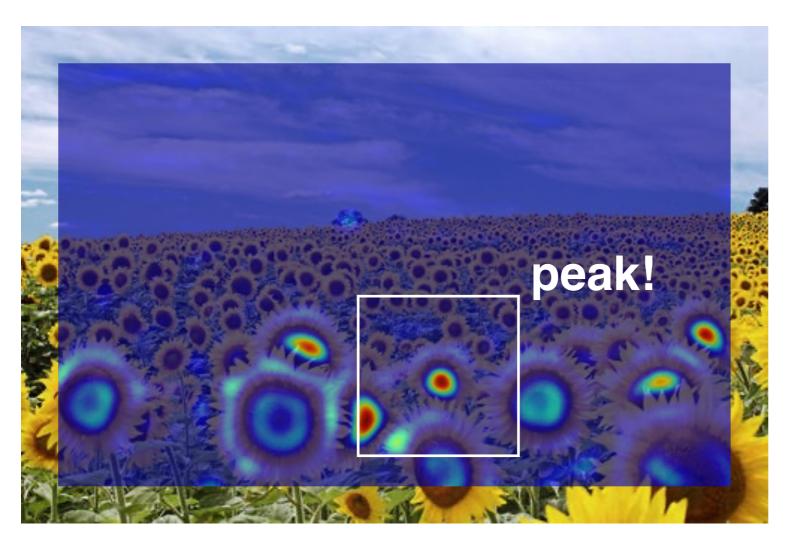


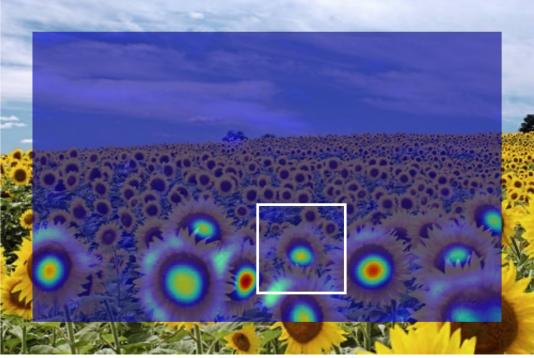


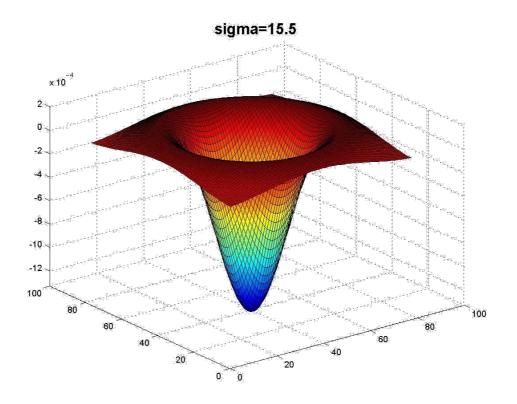


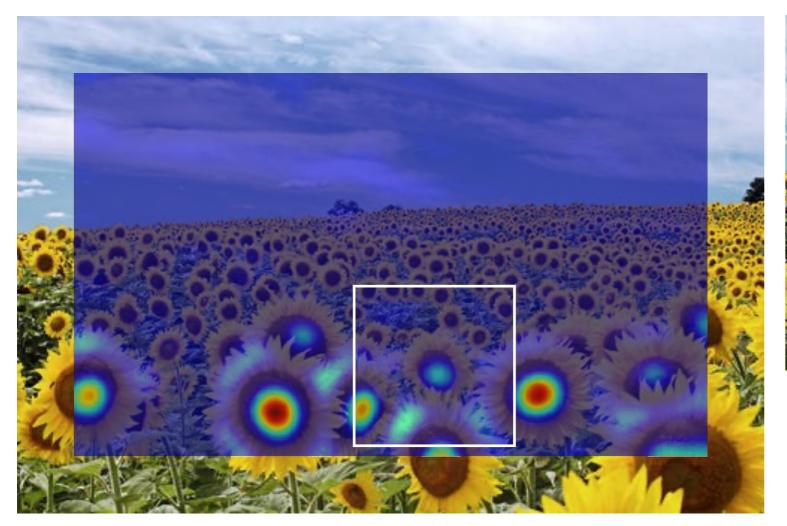


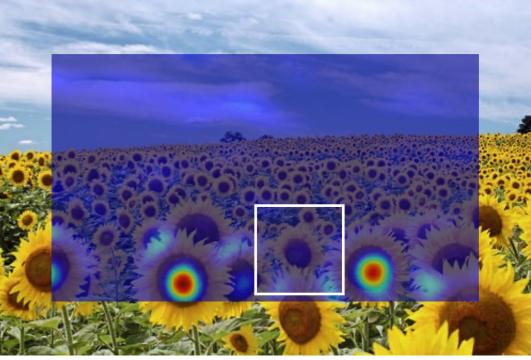


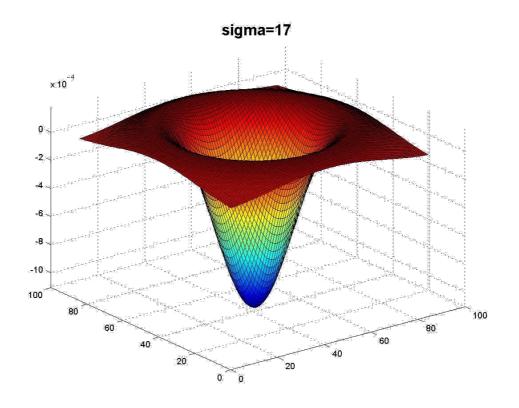


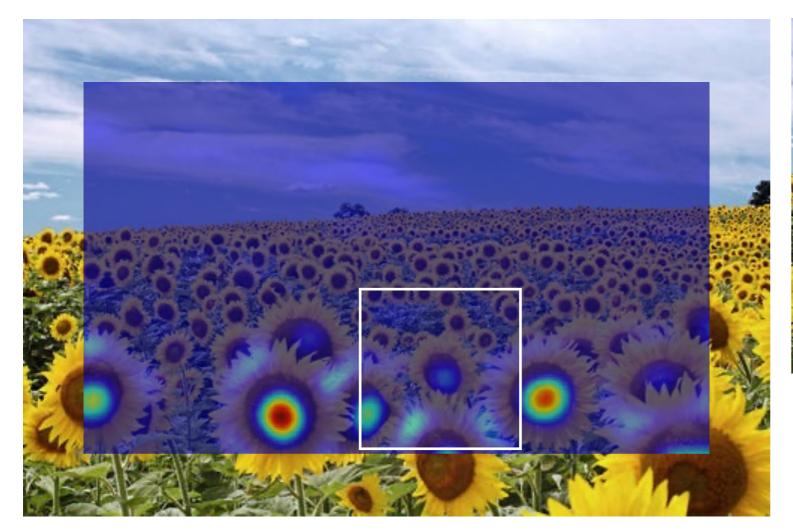


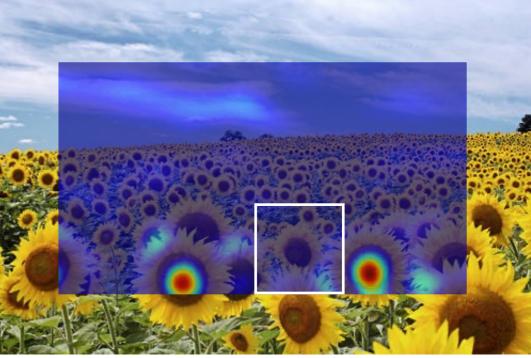


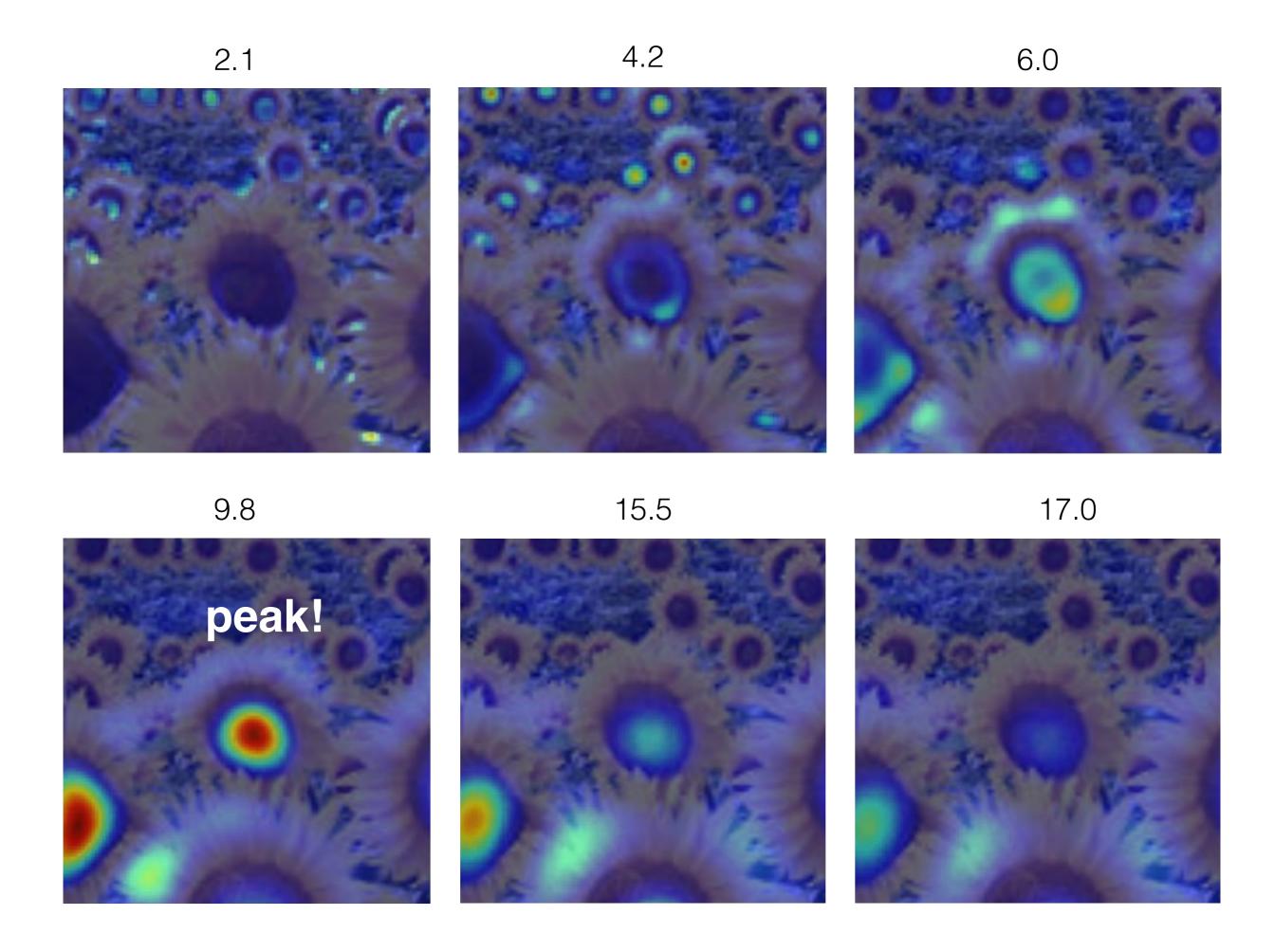




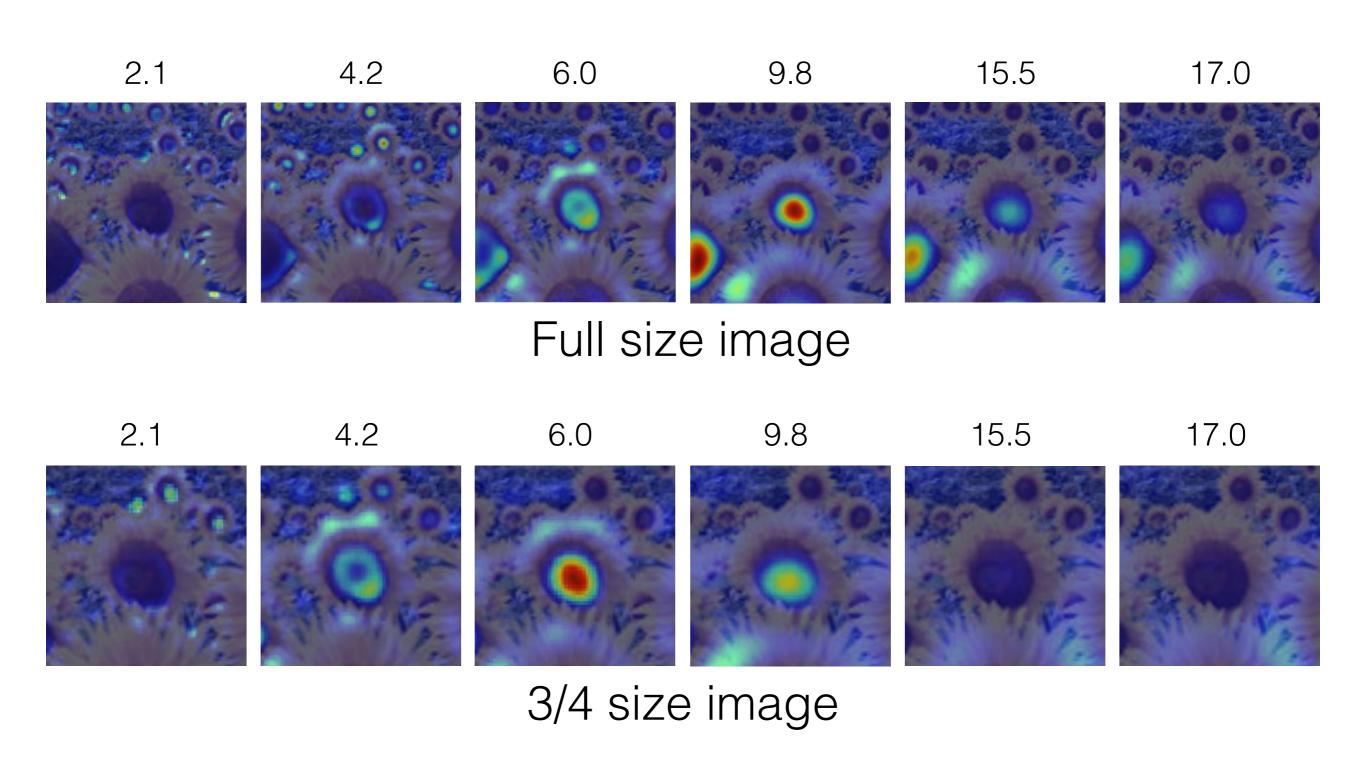




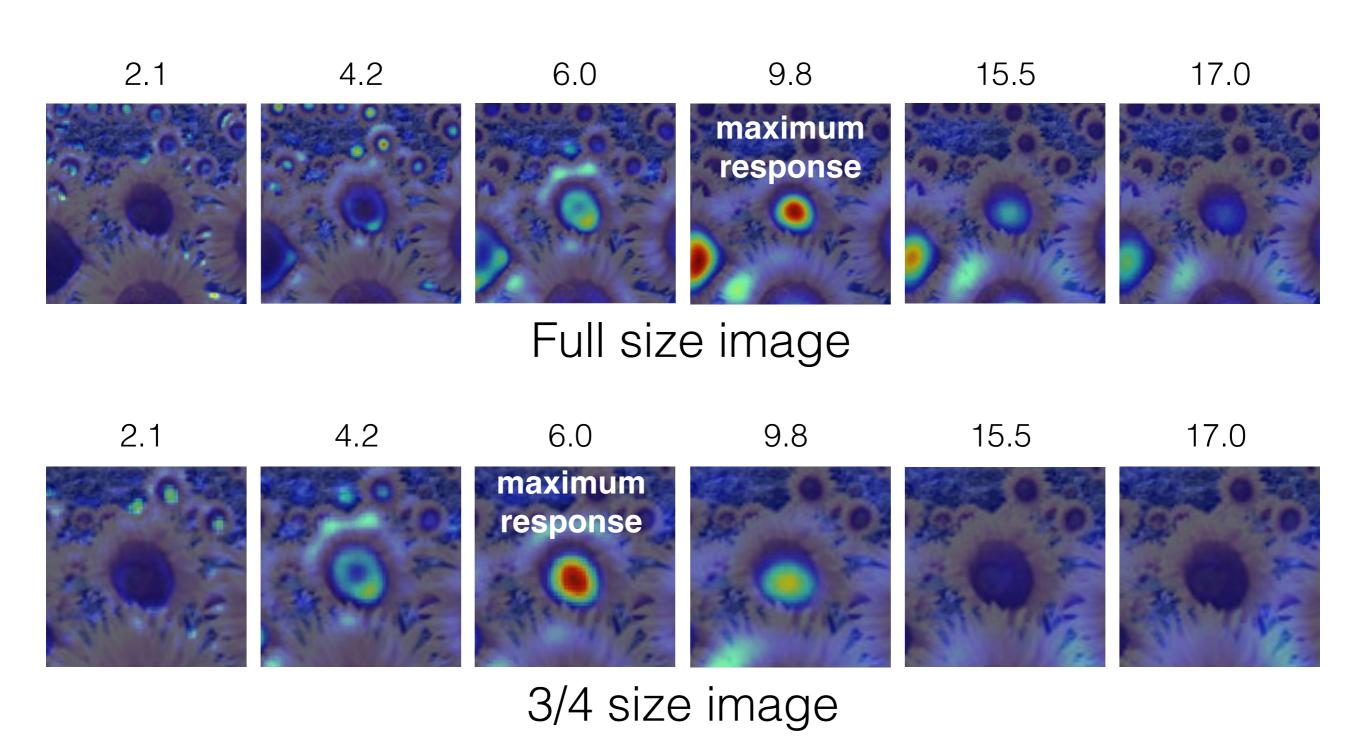


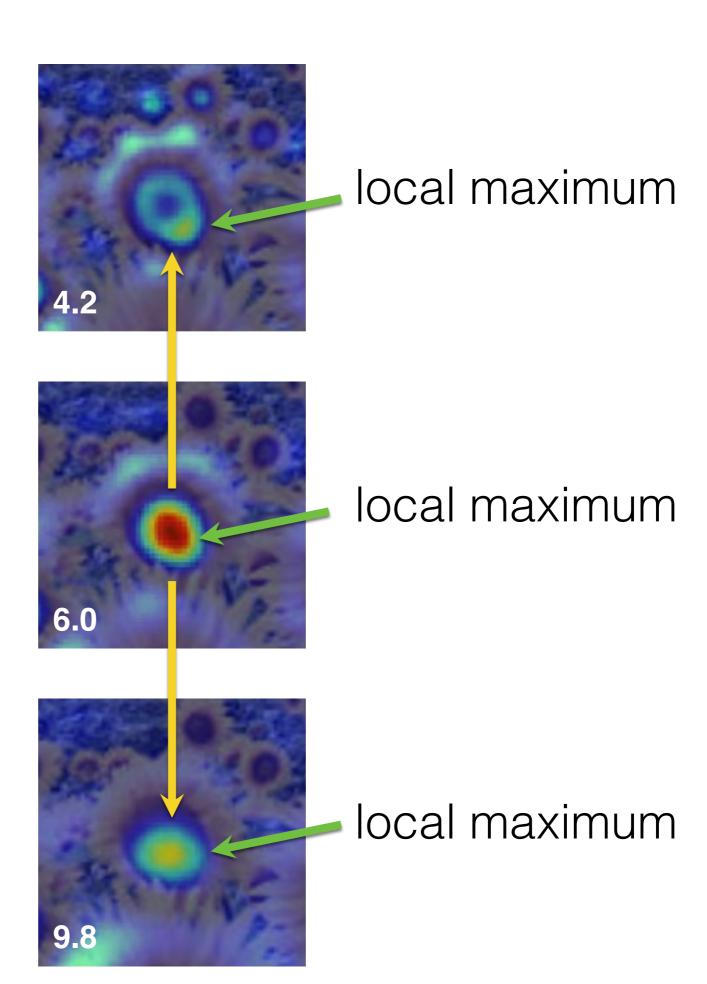


optimal scale



optimal scale





cross-scale maximum

How would you implement scale selection?

Implementation

For each level of the Gaussian pyramid compute feature response (e.g. Harris, Laplacian)

For each level of the Gaussian pyramid

if local maximum and cross-scale

save scale and location of feature (x,y,s)



