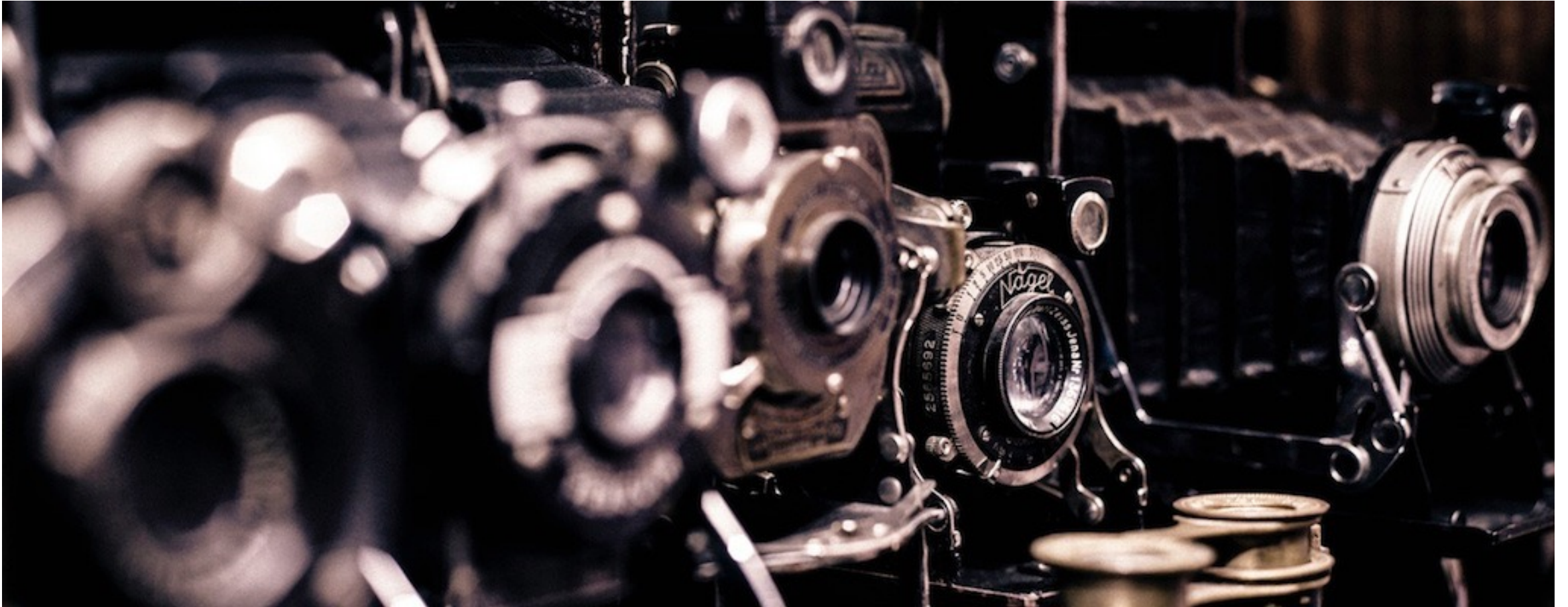


Geometric camera models (cont.)



Overview of today's lecture

- Review of camera matrix.
- Perspective.
- Other camera models.
- Pose estimation.

Slide credits

Most of these slides were adapted from:

- Kris Kitani (15-463, Fall 2016).

Some slides inspired from:

- Fredo Durand (MIT).

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{KR}[\mathbf{I} | -\mathbf{C}]$$

The diagram shows the equation $\mathbf{P} = \mathbf{KR}[\mathbf{I} | -\mathbf{C}]$ with four arrows pointing upwards from question marks to the terms \mathbf{P} , \mathbf{KR} , $[\mathbf{I} | -\mathbf{C}]$, and \mathbf{C} . The arrows originate from question marks located below the terms: one under \mathbf{P} , one under \mathbf{KR} , one under the vertical bar in $[\mathbf{I} | -\mathbf{C}]$, and one under \mathbf{C} .

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{KR}[\mathbf{I} | -\mathbf{C}]$$

The diagram shows the equation $\mathbf{P} = \mathbf{KR}[\mathbf{I} | -\mathbf{C}]$ with four arrows pointing from labels below to the terms \mathbf{P} , \mathbf{K} , \mathbf{I} , and \mathbf{C} . The label for \mathbf{P} is "3x3" and "intrinsic". The labels for \mathbf{K} , \mathbf{I} , and \mathbf{C} are each a question mark.

3x3
intrinsic

?

?

?

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{KR}[\mathbf{I} | \mathbf{C}]$$

3x3 3x3 ? ?
intrinsics 3D rotation

Recap

What is the size and meaning of each term in the camera matrix?


$$\mathbf{P} = \mathbf{KR}[\mathbf{I} | \mathbf{C}]$$

The diagram shows the equation $\mathbf{P} = \mathbf{KR}[\mathbf{I} | \mathbf{C}]$ with four arrows pointing from labels below to terms in the equation:

- An arrow points from the label "3x3 intrinsics" to the \mathbf{K} matrix.
- An arrow points from the label "3x3 3D rotation" to the \mathbf{R} matrix.
- An arrow points from the label "3x3 identity" to the \mathbf{I} matrix.
- An arrow points from a question mark "?" to the \mathbf{C} matrix.

Recap

What is the size and meaning of each term in the camera matrix?

$$\mathbf{P} = \mathbf{KR}[\mathbf{I} | -\mathbf{C}]$$


3x3 intrinsics 3x3 3D rotation 3x3 identity 3x1 3D translation

Perspective distortion

Finite projective camera

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\mathbf{R} \mid -\mathbf{RC} \right]$$



What does this matrix look like if the camera and world have the same coordinate system?

Finite projective camera

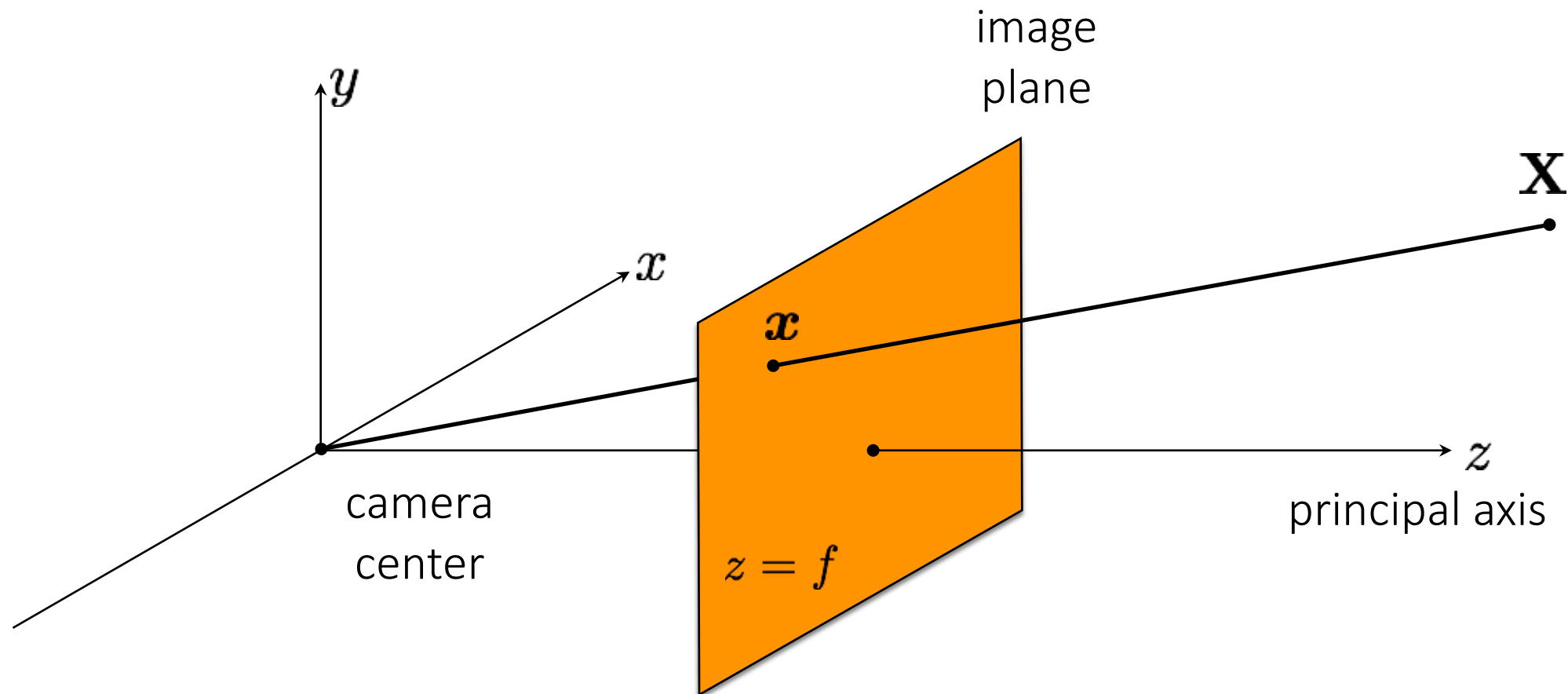
The pinhole camera and all of the more general cameras we have seen so far have “*perspective distortion*”.

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Perspective projection from
(homogeneous) 3D to 2D coordinates

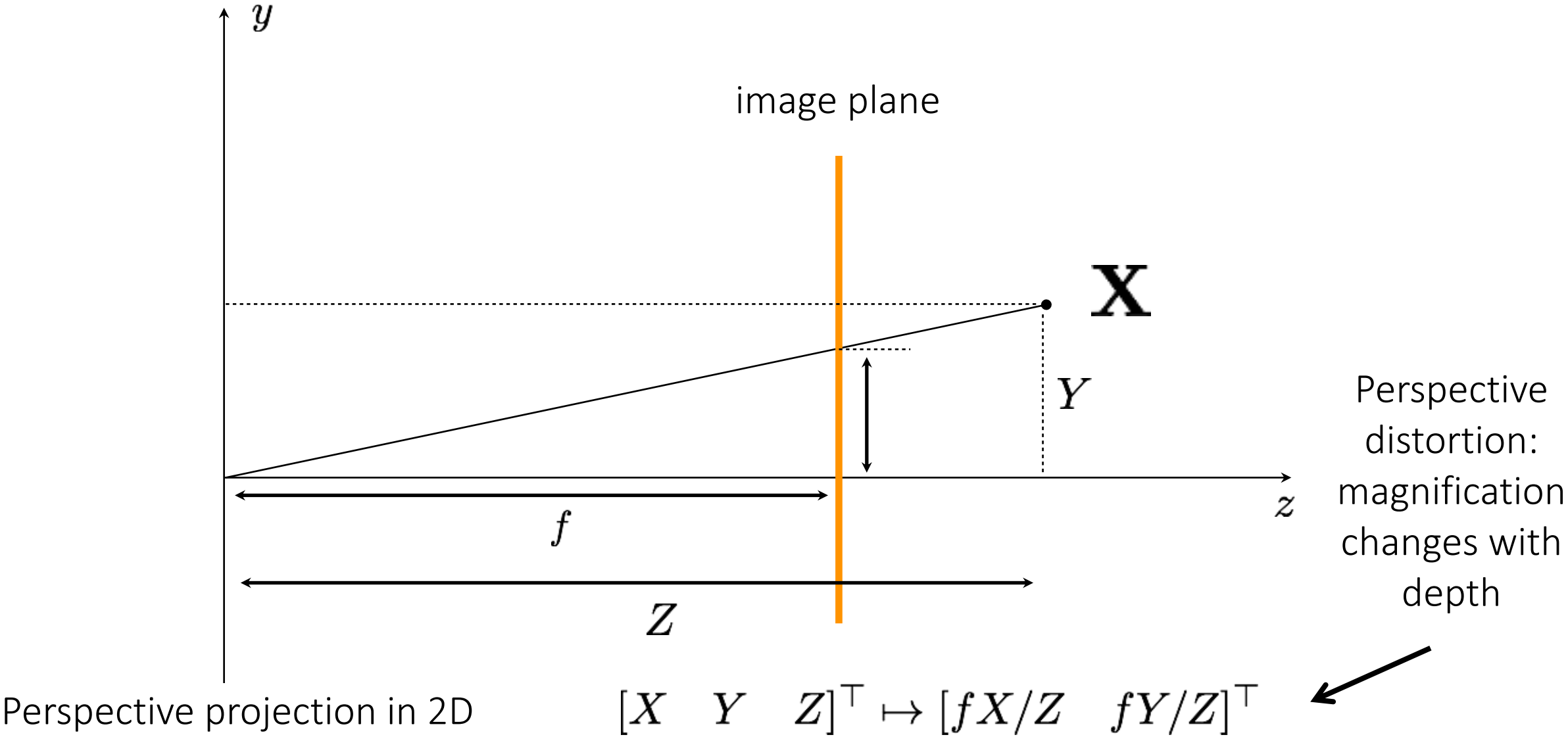
The (rearranged) pinhole camera



Perspective projection in 3D

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

The 2D view of the (rearranged) pinhole camera



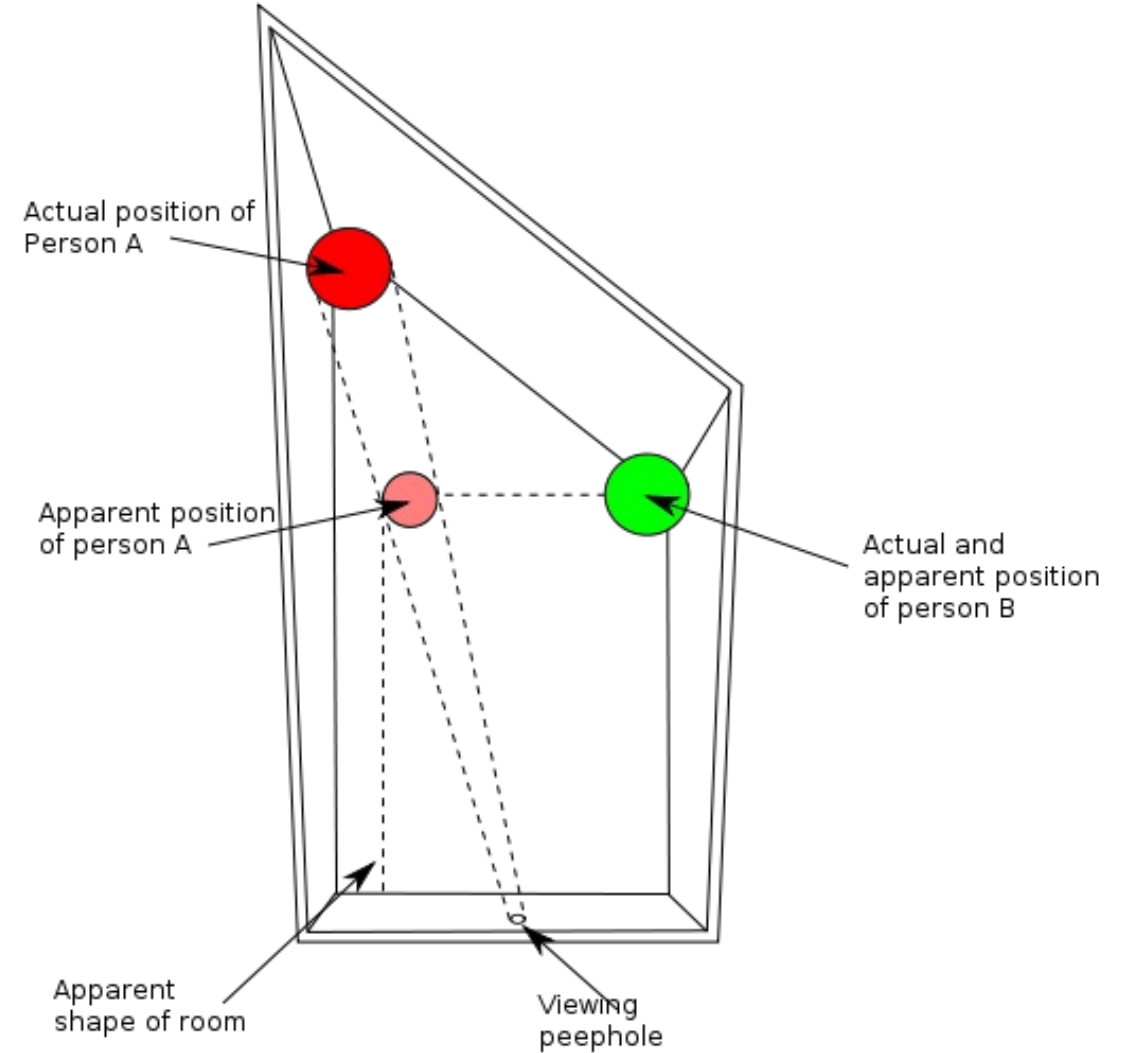
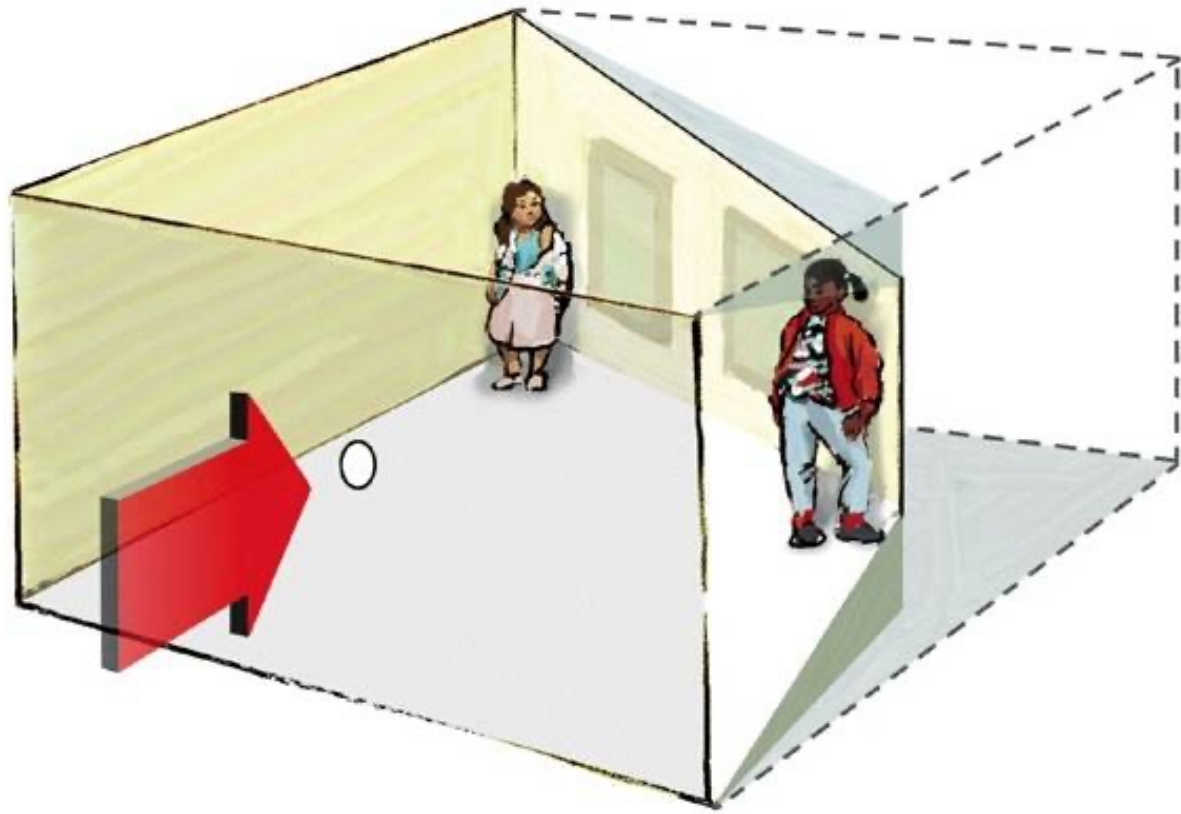
Forced perspective



The Ames room illusion



The Ames room illusion



The arrow illusion



Forced Perspective Display



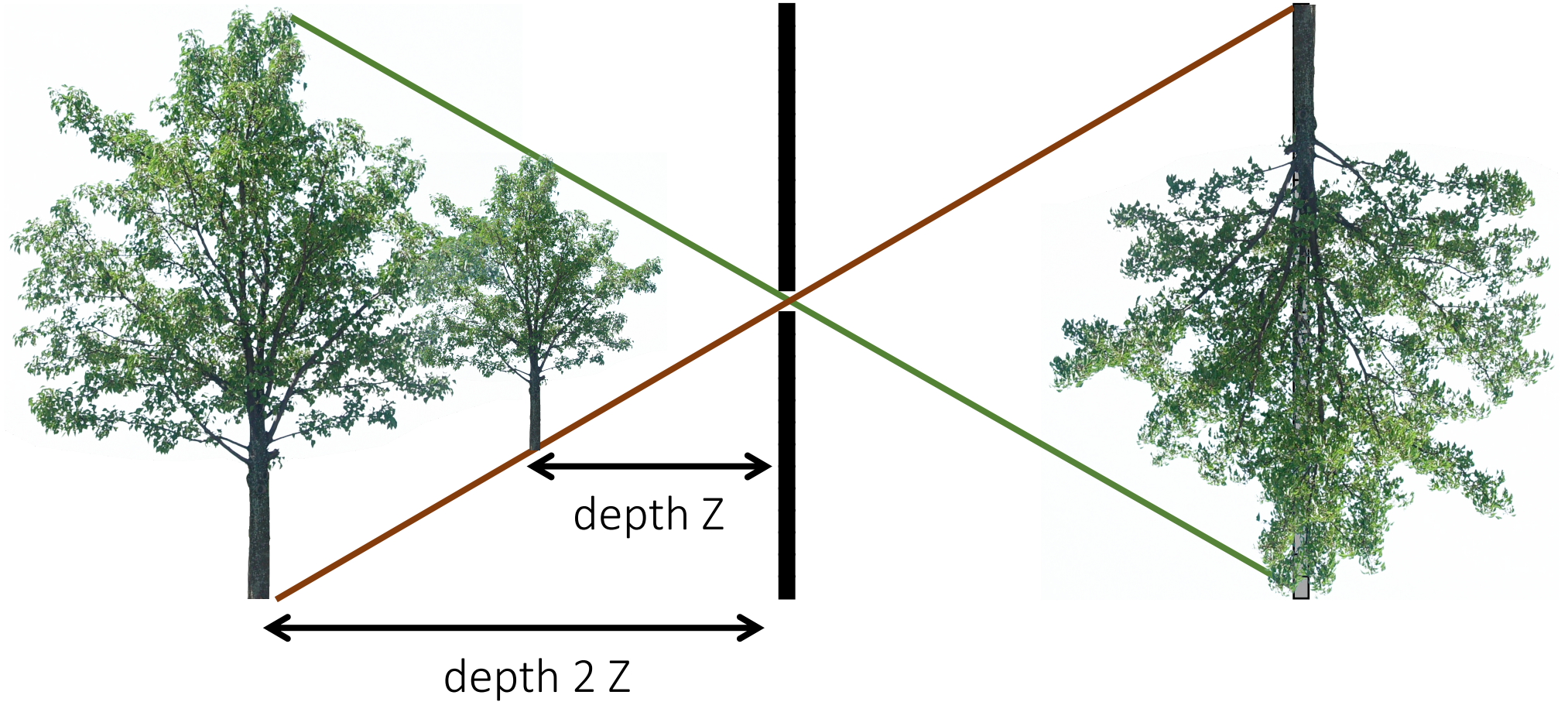
Forced Perspective Display



Magnification depends on depth

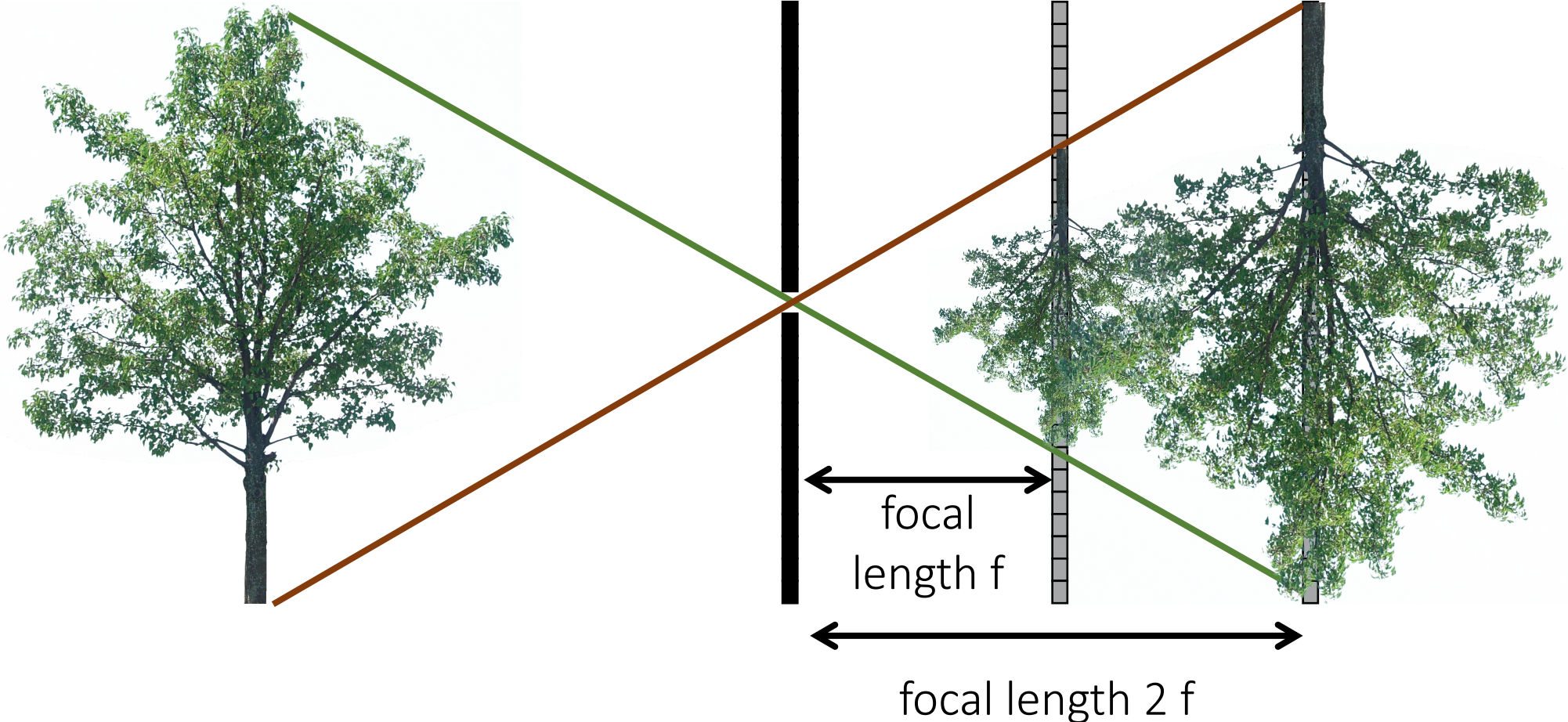
What happens as we change the focal length?

real-world
object



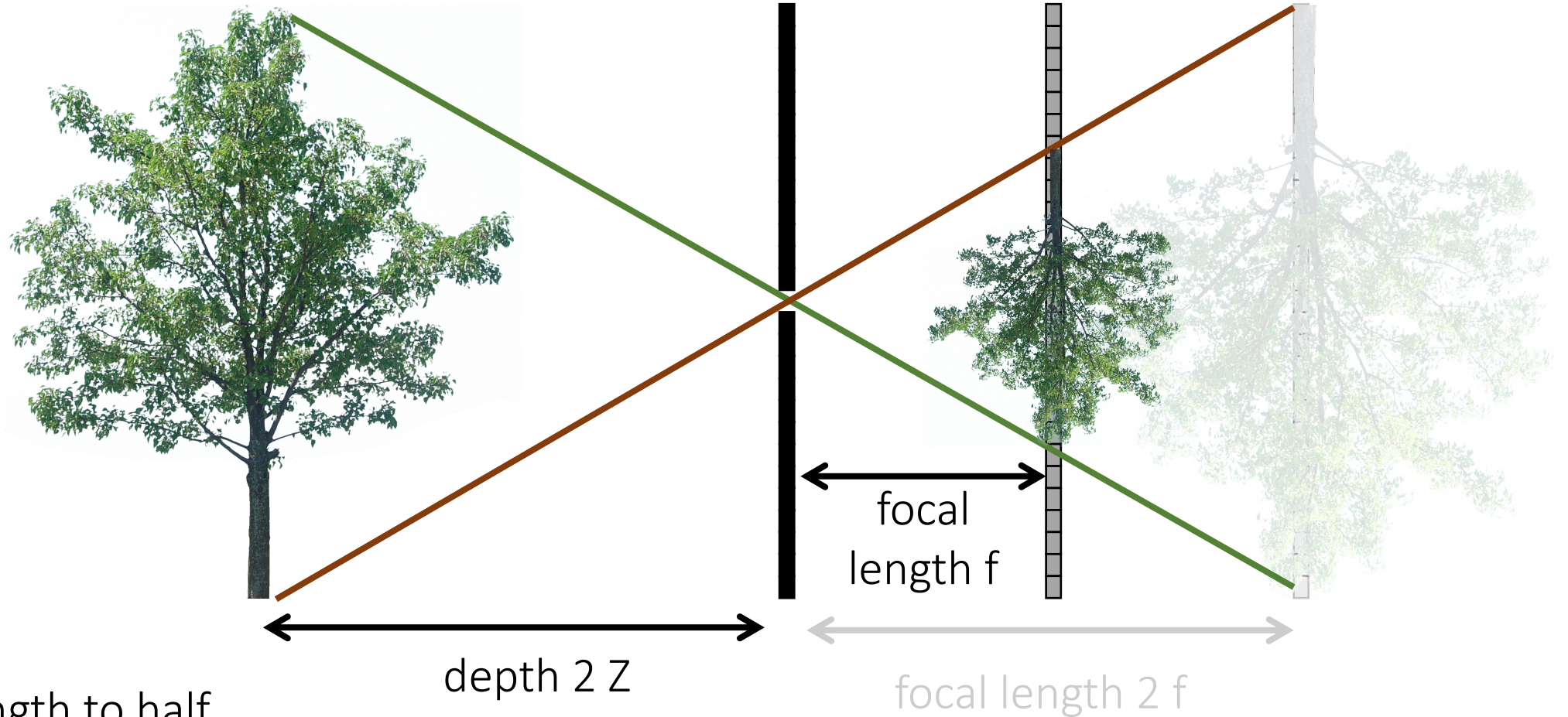
Magnification depends on focal length

real-world
object



What if...

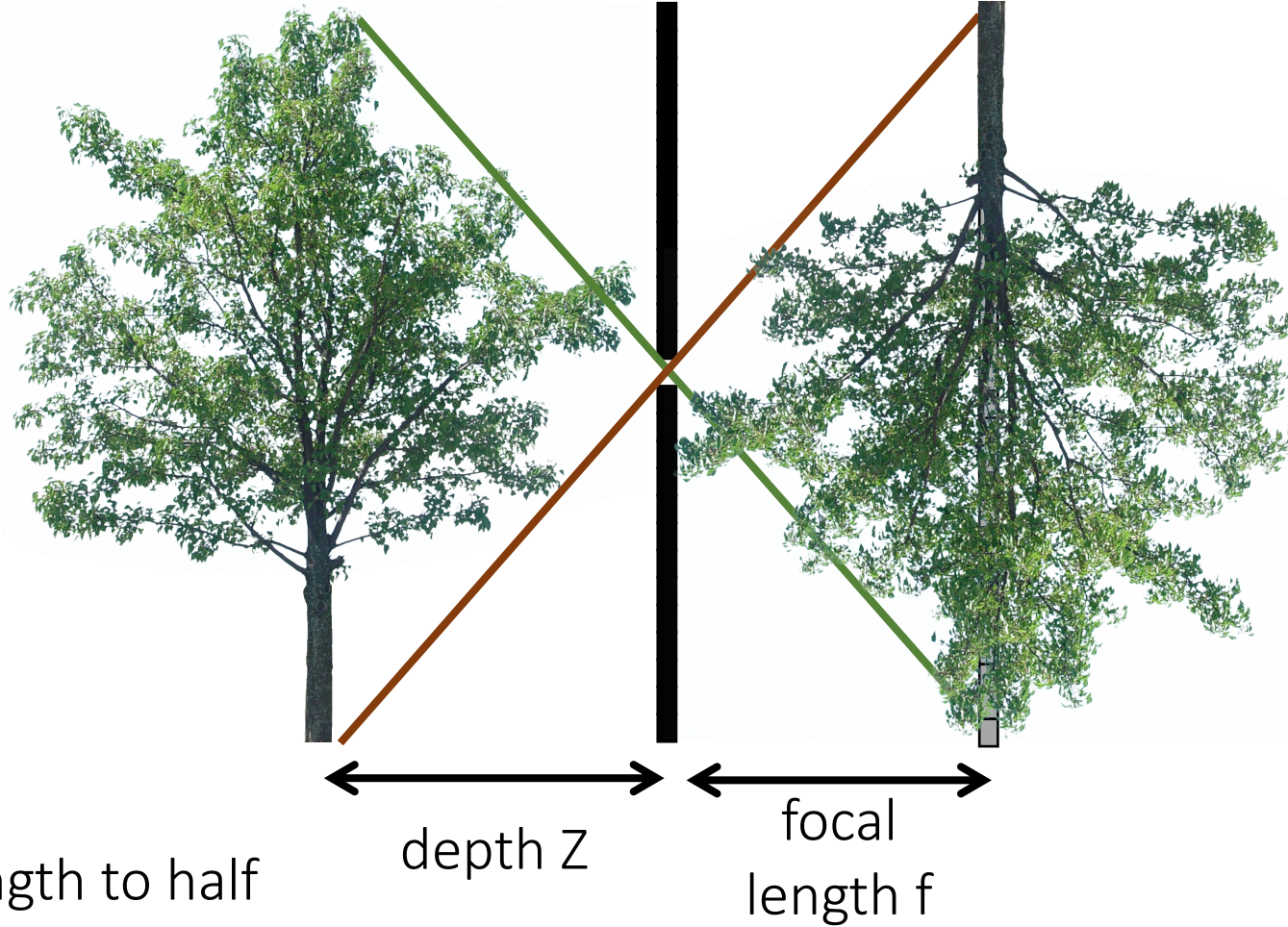
real-world
object



1. Set focal length to half

What if...

real-world
object



Is this the same image as
the one I had at focal
length $2f$ and distance $2Z$?

1. Set focal length to half
2. Set depth to half

Perspective distortion



long focal length

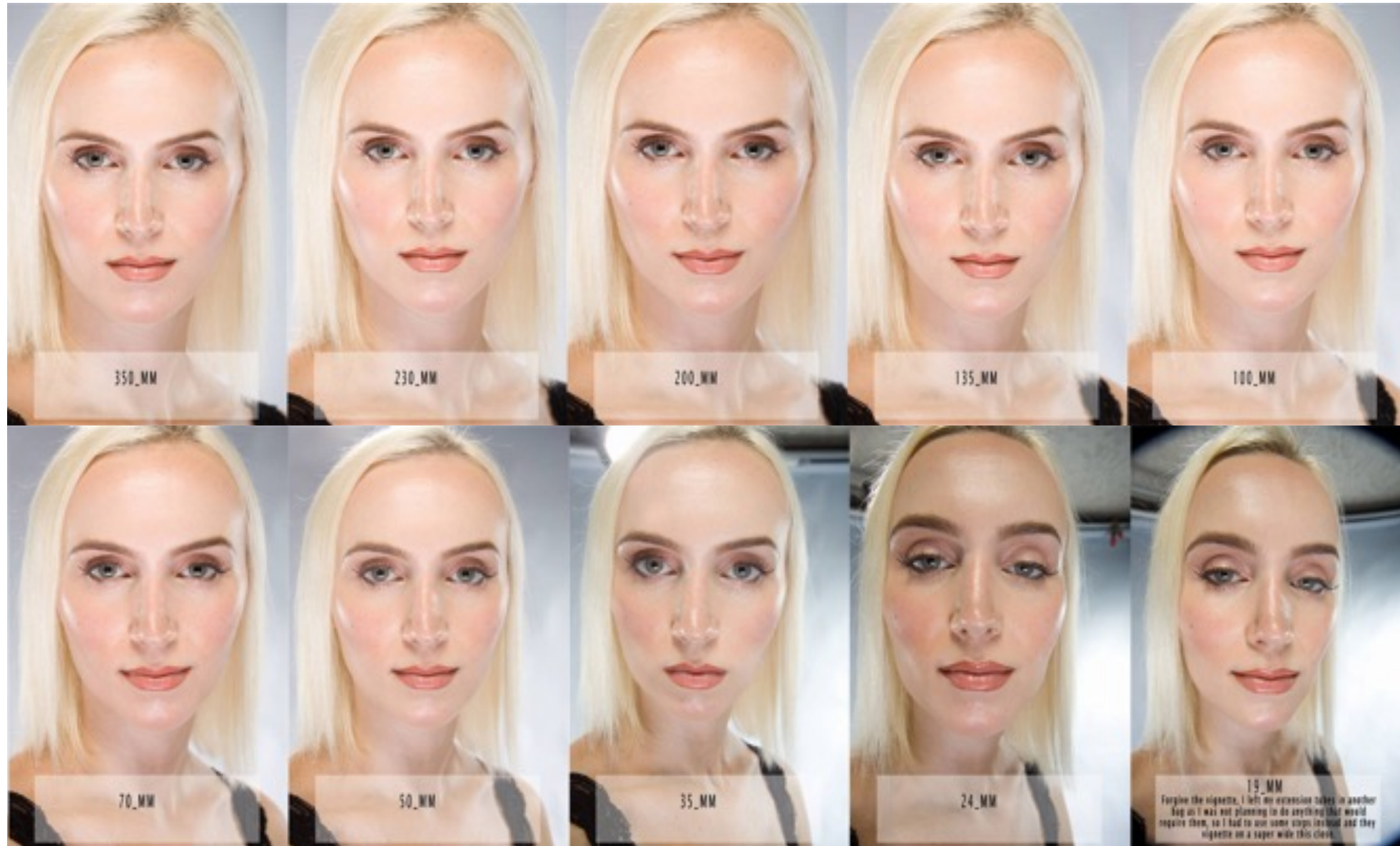


mid focal length



short focal length

Perspective distortion



Vertigo effect

Named after Alfred Hitchcock's movie

- also known as “dolly zoom”



Vertigo effect



How would you
create this effect?

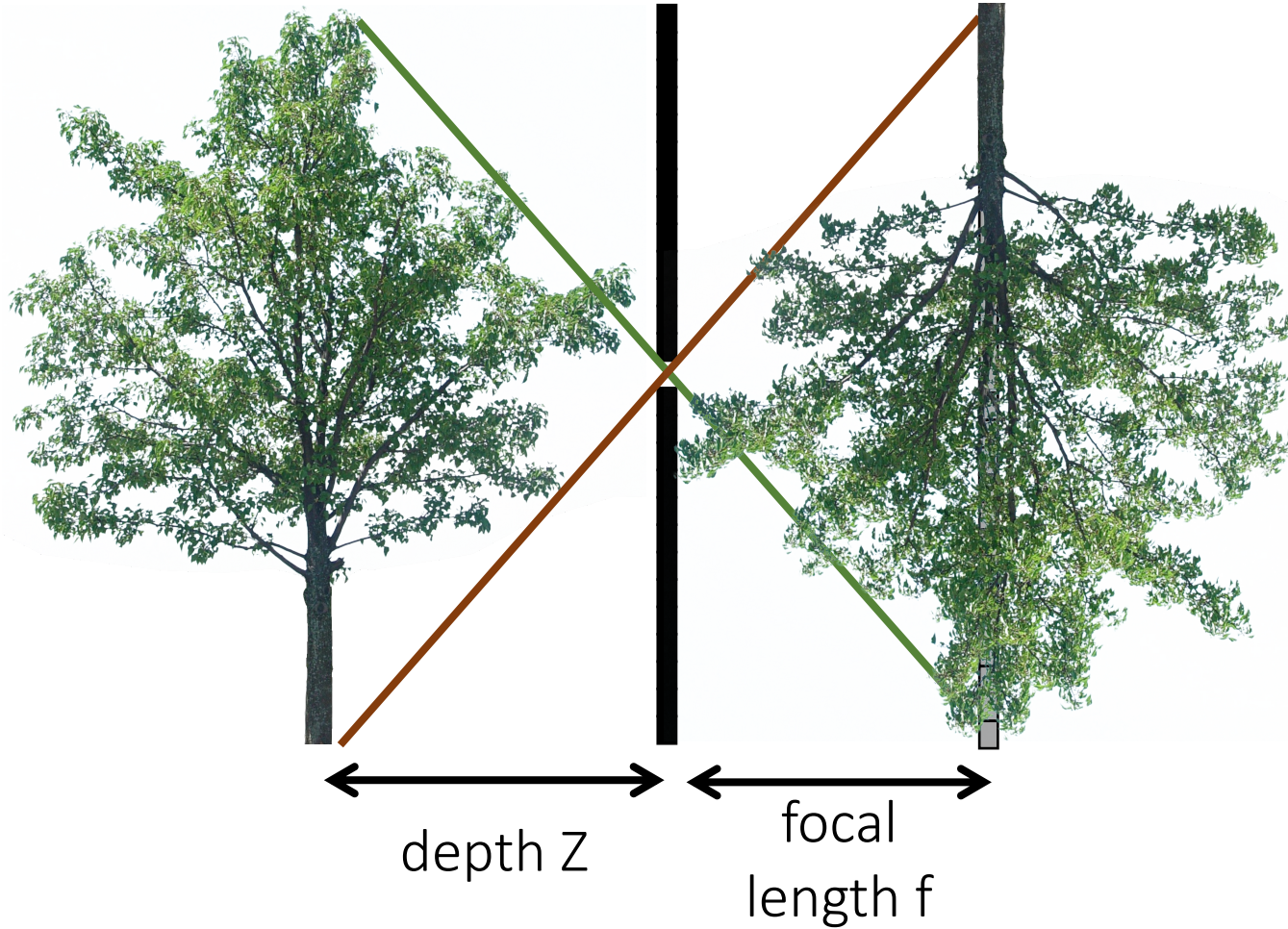
Vertigo effect



Other camera models

What if...

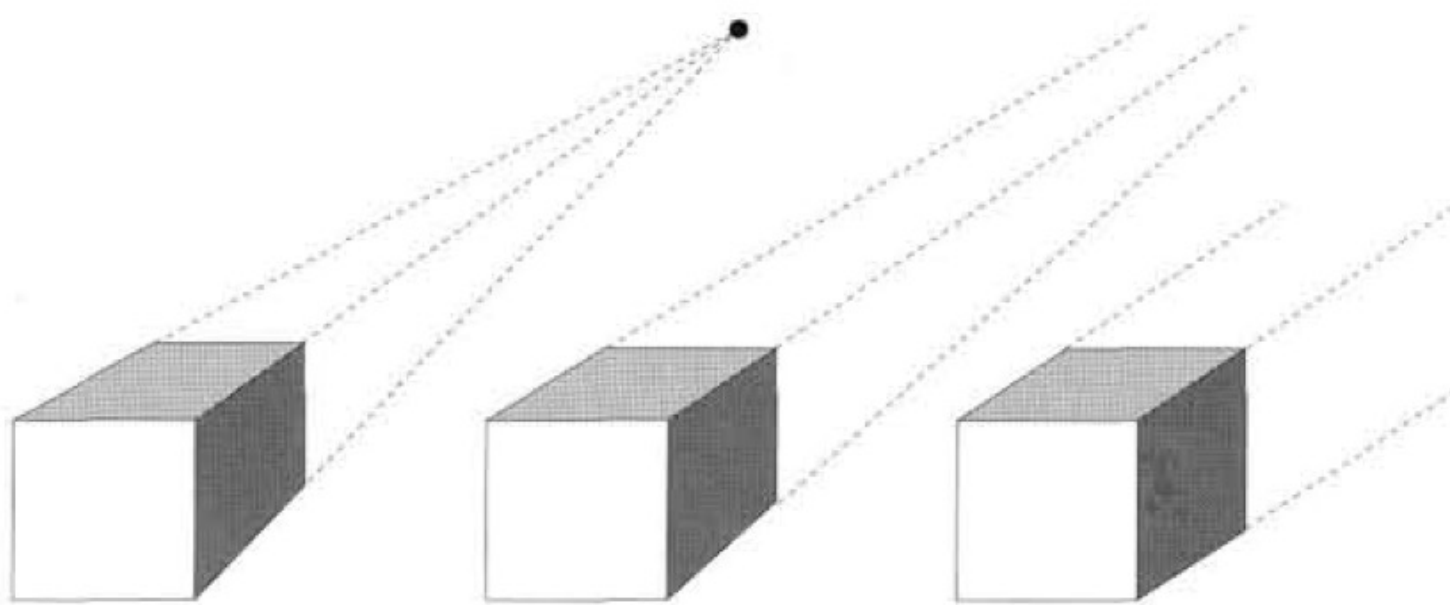
real-world
object



... we continue increasing Z
and f while maintaining
same magnification?

$$f \rightarrow \infty \text{ and } \frac{f}{Z} = \text{constant}$$

camera is *close*
to object and has
small focal length



perspective

weak perspective

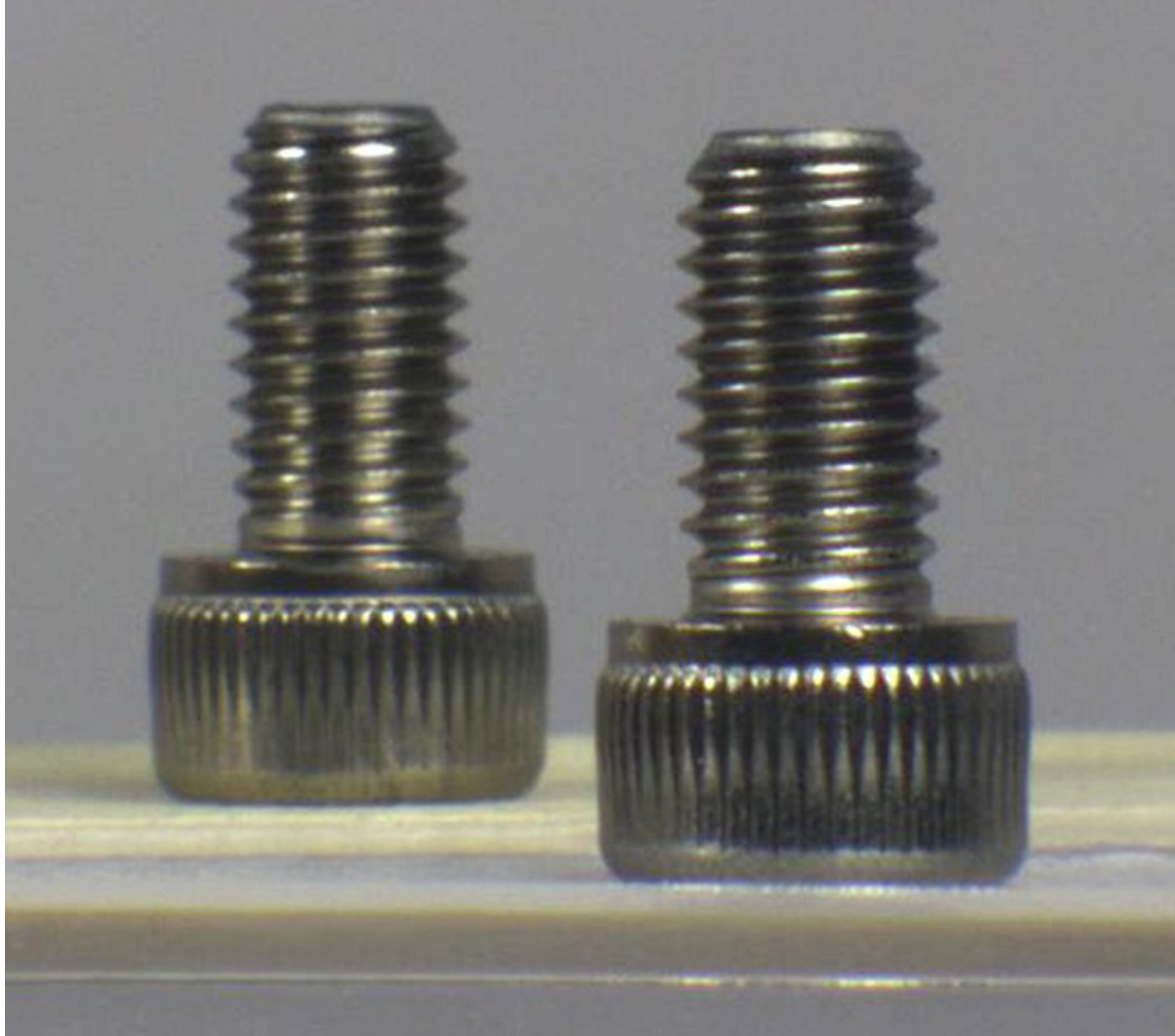
camera is *far* from
object and has
large focal length

————— increasing focal length —————>

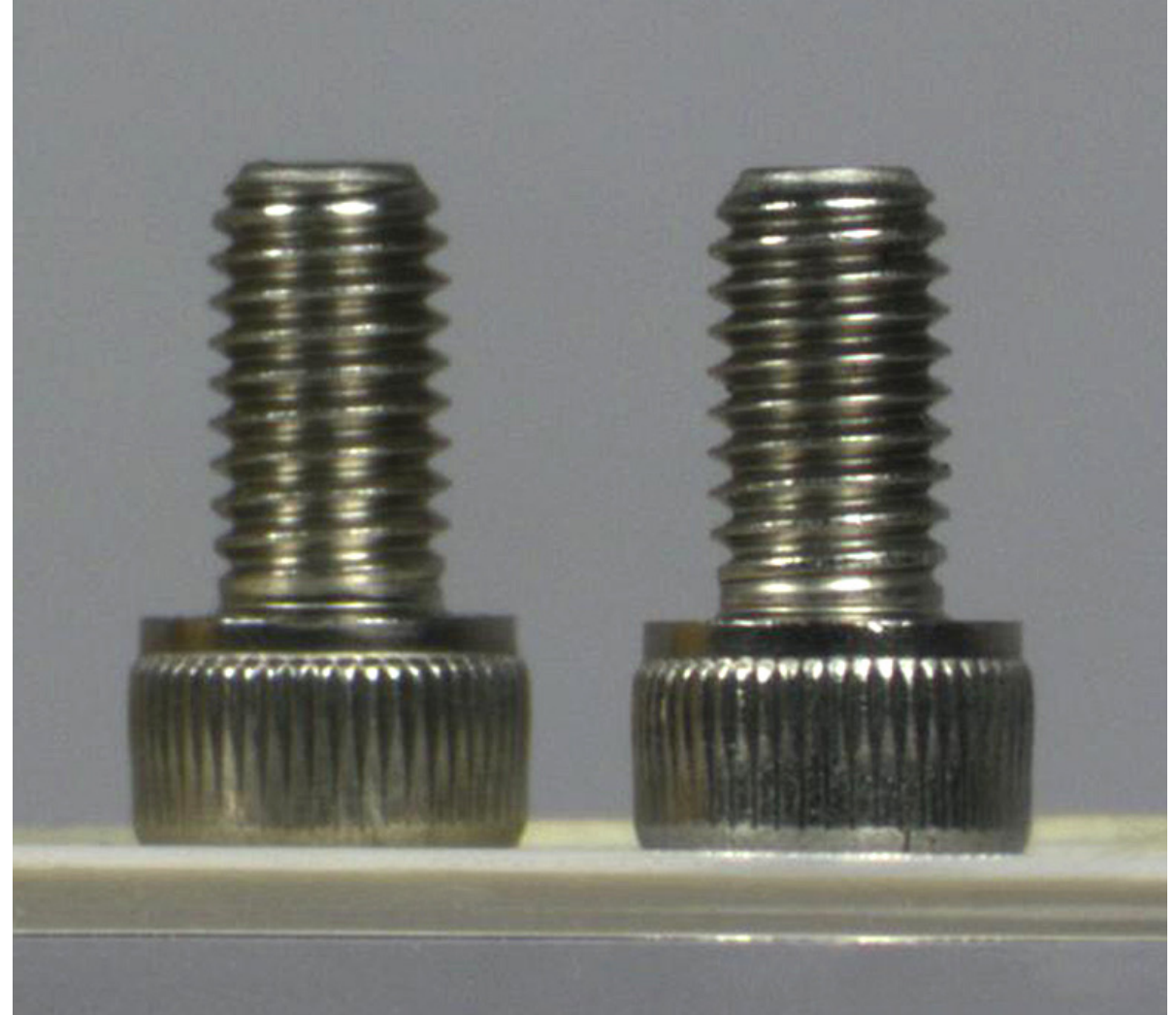
————— increasing distance from camera —————>



Different cameras

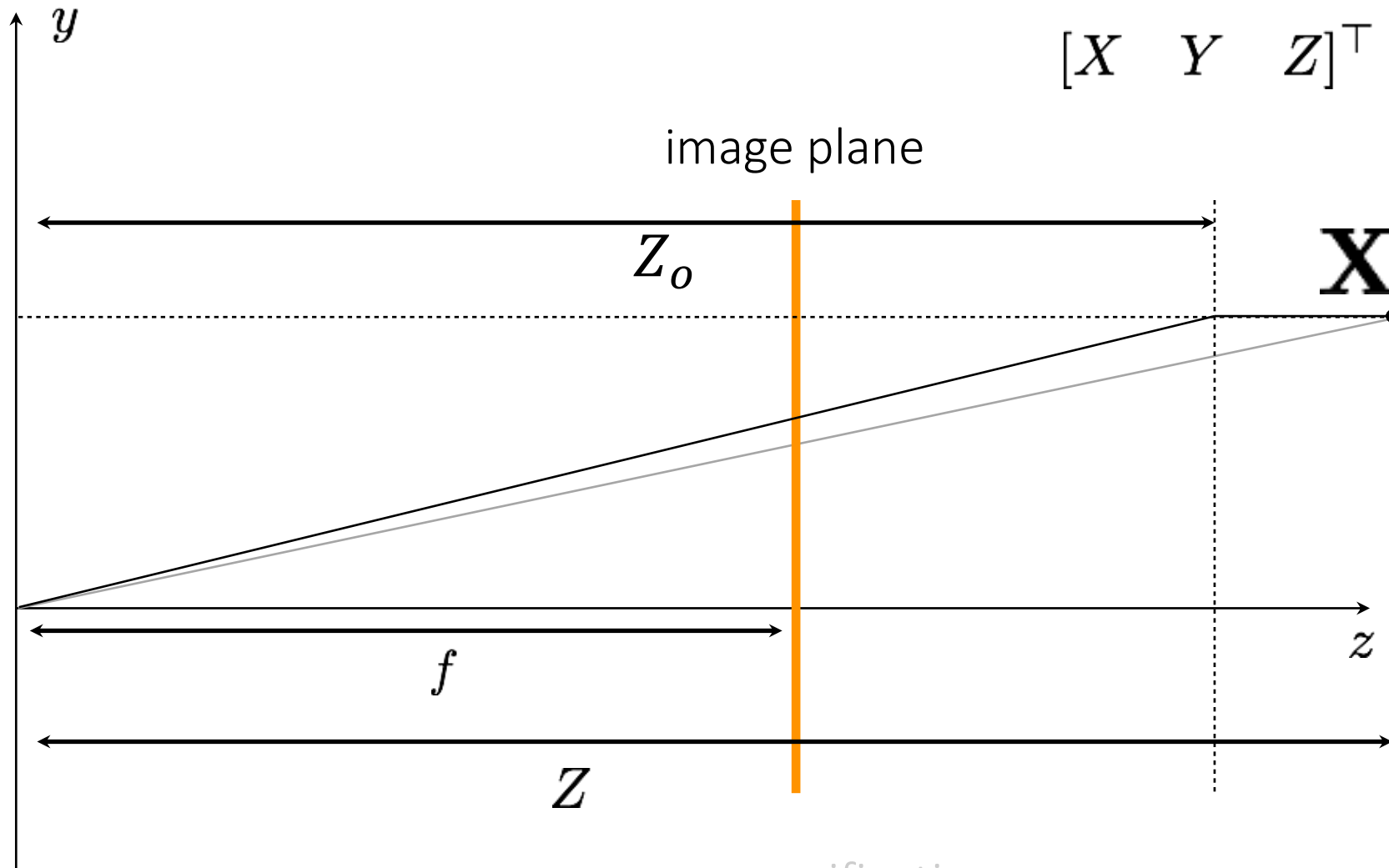


perspective camera



weak perspective camera

Weak perspective vs perspective camera



$$[X \ Y \ Z]^T \mapsto [fX/Z_0 \ fY/Z_0]^T$$

- magnification does not change with depth
- *constant* magnification depending on f and Z_0

magnification
changes with depth

$$[X \ Y \ Z]^T \mapsto [fX/Z \ fY/Z]^T$$

Comparing camera matrices

Let's assume that the world and camera coordinate systems are the same.

- The *perspective* camera matrix can be written as:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- What would the matrix of the weak perspective camera look like?

Comparing camera matrices

Let's assume that the world and camera coordinate systems are the same.

- The *perspective* camera matrix can be written as:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- The *weak perspective* camera matrix can be written as:

$$\mathbf{P} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & Z_o \end{bmatrix}$$

Comparing camera matrices

Let's assume that the world and camera coordinate systems are the same.

- The *finite projective* camera matrix can be written as:

$$\mathbf{P} = \mathbf{K} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where we now have the more general intrinsic matrix

- The *affine* camera matrix can be written as:

$$\mathbf{P} = \mathbf{K} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & Z_o \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

In both cameras, we can incorporate extrinsic parameters same as we did before.

When can we assume a weak perspective camera?

When can we assume a weak perspective camera?

1. When the scene (or parts of it) is very far away.

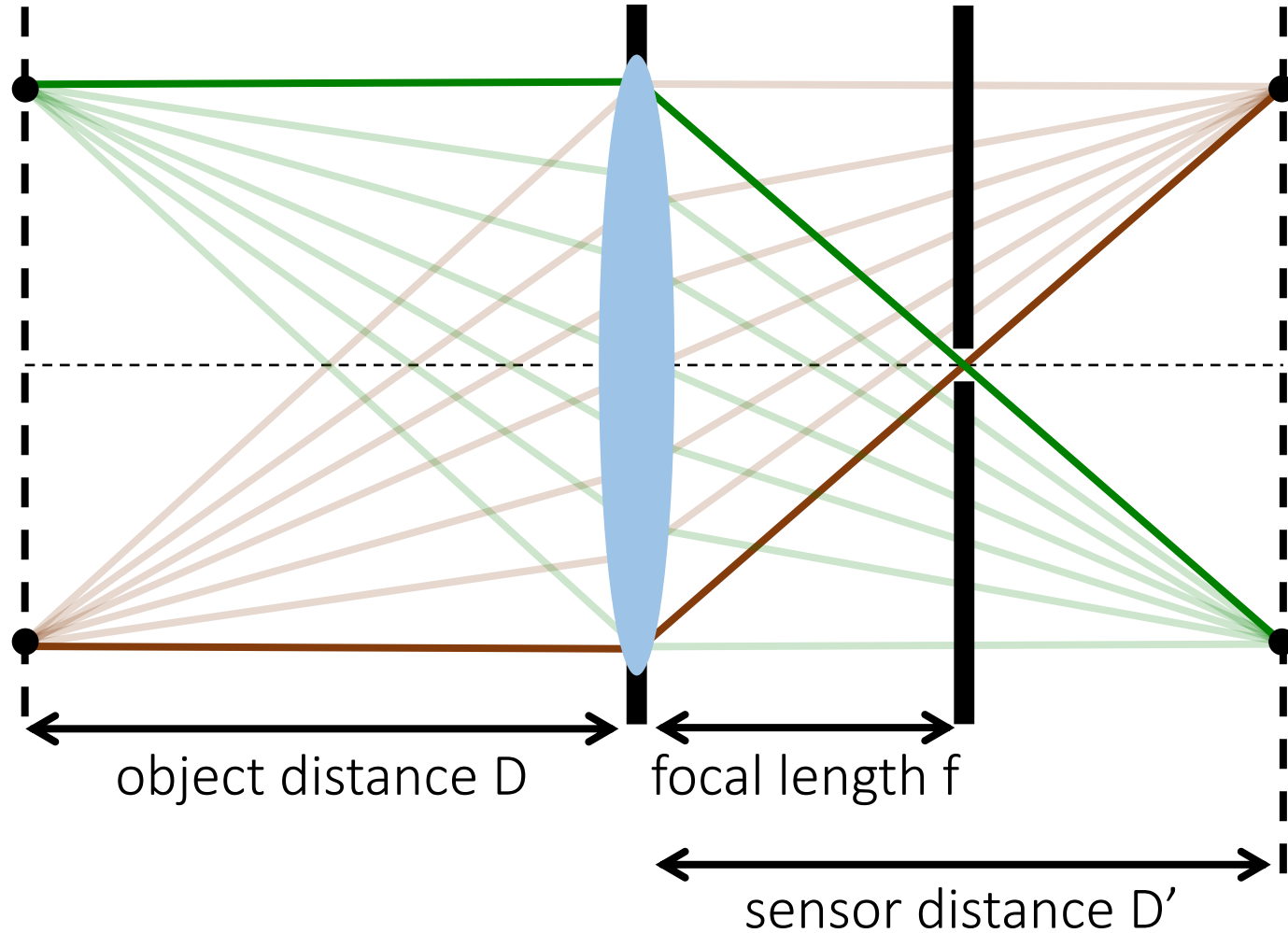


Weak perspective projection applies to the mountains.

When can we assume a weak perspective camera?

2. When we use a telecentric lens.

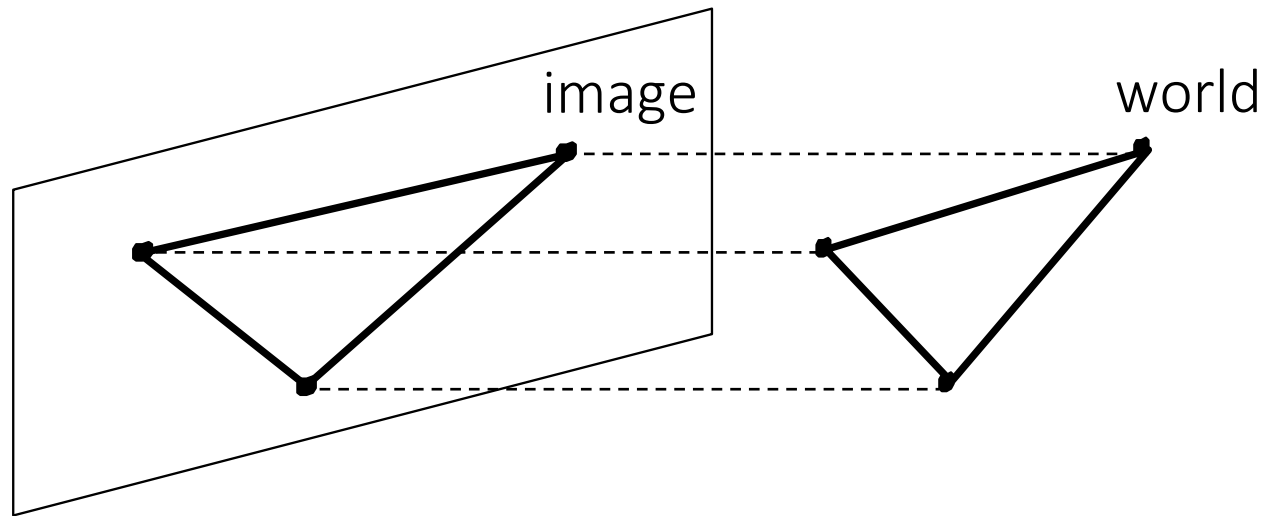
Place a pinhole at focal length, so that only rays parallel to primary ray pass through.



Orthographic camera

Special case of weak perspective camera where:

- constant magnification is equal to 1.
- there is no shift between camera and image origins.
- the world and camera coordinate systems are the same.

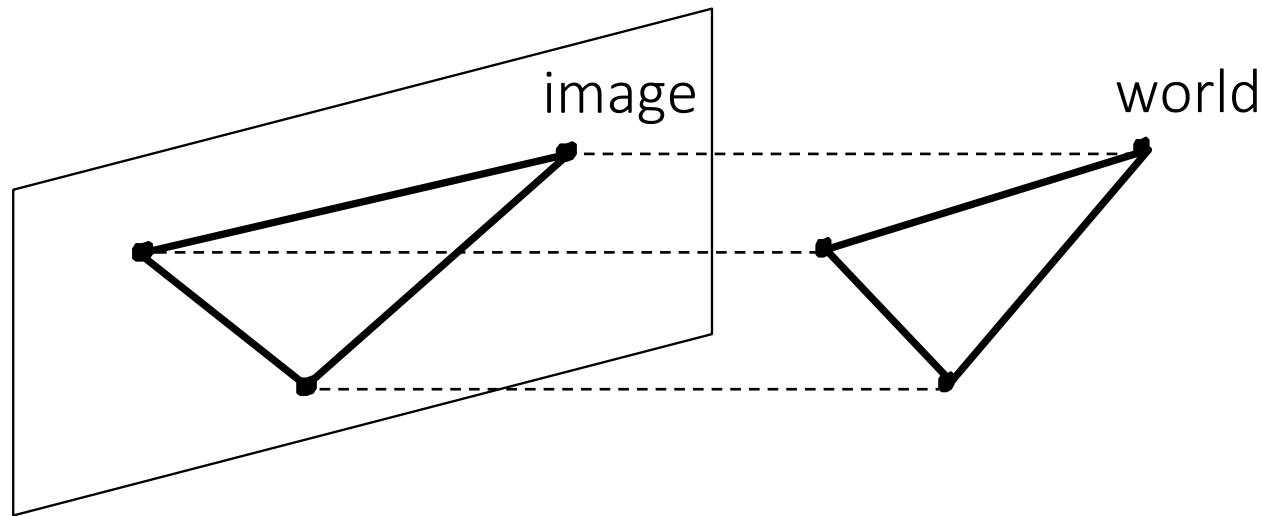


What is the camera matrix in this case?

Orthographic camera

Special case of weak perspective camera where:

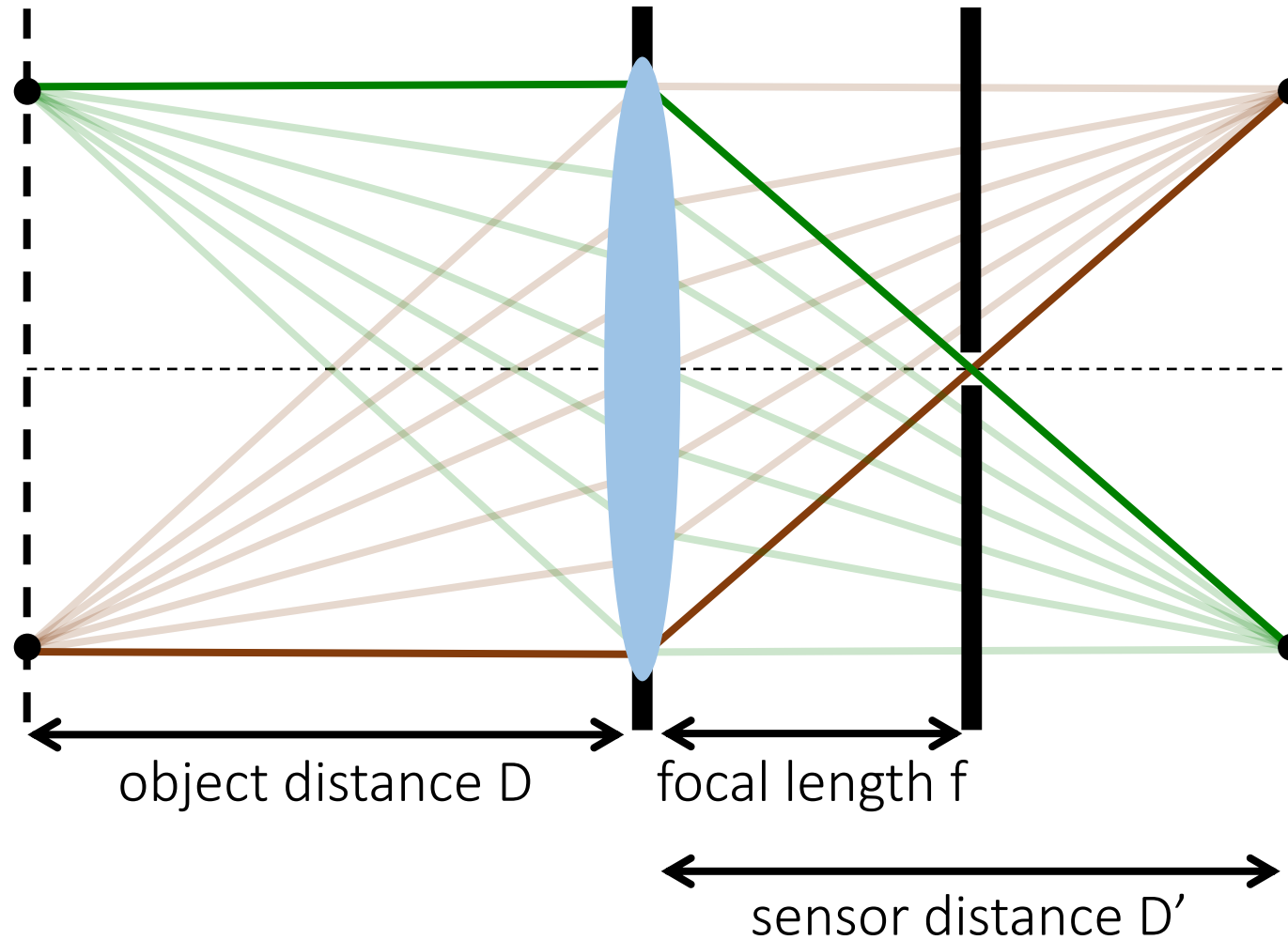
- constant magnification is equal to 1.
- there is no shift between camera and image origins.
- the world and camera coordinate systems are the same.



$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic projection using a telecentric lens

How do we make the telecentric lens act as an orthographic camera?



We set the sensor distance as:

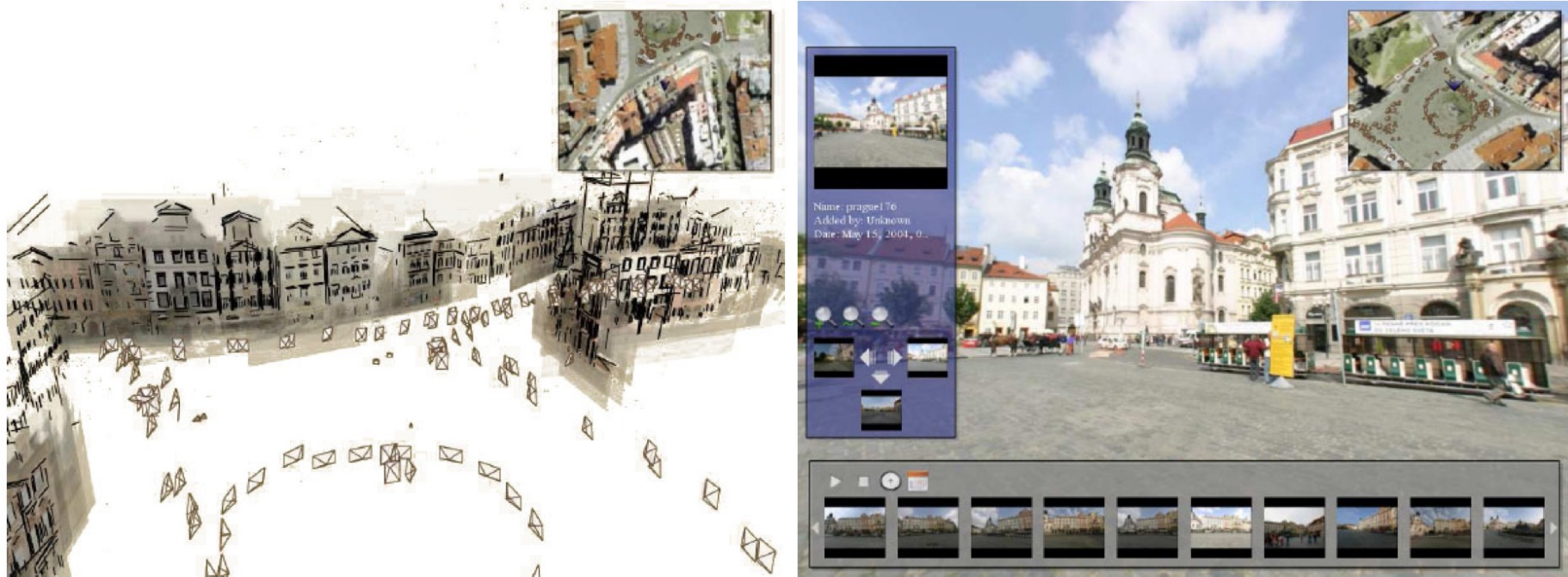
$$D' = 2f$$

in order to achieve unit magnification.

Geometric camera calibration

	Structure (scene geometry)	Motion (camera geometry)	Measurements
Camera Calibration (a.k.a. Pose Estimation)	known	estimate	3D to 2D correspondences
Triangulation	estimate	known	2D to 2D coorespondences
Reconstruction	estimate	estimate	2D to 2D coorespondences

Pose Estimation



Given a single image,
estimate the exact position of the photographer

Geometric camera calibration

Given a set of matched points

$$\{\mathbf{X}_i, \mathbf{x}_i\}$$

point in 3D
space

point in the
image

and camera model

$$\mathbf{x} = \mathbf{f}(\mathbf{X}; \mathbf{p}) = \mathbf{P}\mathbf{X}$$

projection
model

parameters

Camera
matrix

Find the (pose) estimate of

P

We'll use a **perspective** camera
model for pose estimation

Same setup as homography estimation
(slightly different derivation here)

Mapping between 3D point and image points

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What are the unknowns?

Mapping between 3D point and image points

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \text{---} & \mathbf{p}_1^\top & \text{---} \\ \text{---} & \mathbf{p}_2^\top & \text{---} \\ \text{---} & \mathbf{p}_3^\top & \text{---} \end{bmatrix} \begin{bmatrix} | \\ \mathbf{X} \\ | \end{bmatrix}$$

Heterogeneous coordinates

$$x' = \frac{\mathbf{p}_1^\top \mathbf{X}}{\mathbf{p}_3^\top \mathbf{X}} \quad y' = \frac{\mathbf{p}_2^\top \mathbf{X}}{\mathbf{p}_3^\top \mathbf{X}}$$

(non-linear relation between coordinates)

How can we make these relations linear?

How can we make these relations linear?

$$x' = \frac{\mathbf{p}_1^\top \mathbf{X}}{\mathbf{p}_3^\top \mathbf{X}} \quad y' = \frac{\mathbf{p}_2^\top \mathbf{X}}{\mathbf{p}_3^\top \mathbf{X}}$$

Make them linear with algebraic manipulation...

$$\mathbf{p}_2^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} y' = 0$$

$$\mathbf{p}_1^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} x' = 0$$

Now we can setup a system of linear equations
with multiple point correspondences

$$\mathbf{p}_2^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} y' = 0$$

$$\mathbf{p}_1^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} x' = 0$$

How do we proceed?

$$\mathbf{p}_2^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} y' = 0$$

$$\mathbf{p}_1^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} x' = 0$$

In matrix form ...

$$\begin{bmatrix} \mathbf{X}^\top & \mathbf{0} & -x' \mathbf{X}^\top \\ \mathbf{0} & \mathbf{X}^\top & -y' \mathbf{X}^\top \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = \mathbf{0}$$

How do we proceed?

$$\mathbf{p}_2^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} y' = 0$$

$$\mathbf{p}_1^\top \mathbf{X} - \mathbf{p}_3^\top \mathbf{X} x' = 0$$

In matrix form ...

$$\begin{bmatrix} \mathbf{X}^\top & \mathbf{0} & -x' \mathbf{X}^\top \\ \mathbf{0} & \mathbf{X}^\top & -y' \mathbf{X}^\top \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = \mathbf{0}$$

For N points ...

$$\begin{bmatrix} \mathbf{X}_1^\top & \mathbf{0} & -x' \mathbf{X}_1^\top \\ \mathbf{0} & \mathbf{X}_1^\top & -y' \mathbf{X}_1^\top \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^\top & \mathbf{0} & -x' \mathbf{X}_N^\top \\ \mathbf{0} & \mathbf{X}_N^\top & -y' \mathbf{X}_N^\top \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = \mathbf{0}$$

*How do we solve
this system?*

Solve for camera matrix by

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|^2 \text{ subject to } \|\mathbf{x}\|^2 = 1$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{X}_1^\top & \mathbf{0} & -x' \mathbf{X}_1^\top \\ \mathbf{0} & \mathbf{X}_1^\top & -y' \mathbf{X}_1^\top \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^\top & \mathbf{0} & -x' \mathbf{X}_N^\top \\ \mathbf{0} & \mathbf{X}_N^\top & -y' \mathbf{X}_N^\top \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

SVD!

Solve for camera matrix by

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|^2 \text{ subject to } \|\mathbf{x}\|^2 = 1$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{X}_1^\top & \mathbf{0} & -x' \mathbf{X}_1^\top \\ \mathbf{0} & \mathbf{X}_1^\top & -y' \mathbf{X}_1^\top \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^\top & \mathbf{0} & -x' \mathbf{X}_N^\top \\ \mathbf{0} & \mathbf{X}_N^\top & -y' \mathbf{X}_N^\top \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

Solution \mathbf{x} is the column of \mathbf{V}
corresponding to smallest singular
value of

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$$

Solve for camera matrix by

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{A}\mathbf{x}\|^2 \text{ subject to } \|\mathbf{x}\|^2 = 1$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{X}_1^\top & \mathbf{0} & -x' \mathbf{X}_1^\top \\ \mathbf{0} & \mathbf{X}_1^\top & -y' \mathbf{X}_1^\top \\ \vdots & \vdots & \vdots \\ \mathbf{X}_N^\top & \mathbf{0} & -x' \mathbf{X}_N^\top \\ \mathbf{0} & \mathbf{X}_N^\top & -y' \mathbf{X}_N^\top \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

Equivalently, solution \mathbf{x} is the Eigenvector corresponding to smallest Eigenvalue of

$$\mathbf{A}^\top \mathbf{A}$$

Now we have:

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix}$$

Are we done?

Almost there ...

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{bmatrix}$$

How do you get the intrinsic and extrinsic parameters from the projection matrix?

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}]$$

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} | -\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M} | -\mathbf{M}\mathbf{c}] \end{aligned}$$

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} | -\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M} | -\mathbf{M}\mathbf{c}] \end{aligned}$$

Find the camera center \mathbf{C}

What is the projection of the camera center?

Find intrinsic \mathbf{K} and rotation \mathbf{R}

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} | -\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M} | -\mathbf{M}\mathbf{c}] \end{aligned}$$

Find the camera center \mathbf{c}

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

How do we compute the camera center from this?

Find intrinsic \mathbf{K} and rotation \mathbf{R}

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} | -\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M} | -\mathbf{M}\mathbf{c}] \end{aligned}$$

Find the camera center \mathbf{c}

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of \mathbf{P} !

\mathbf{c} is the singular vector corresponding to the smallest singular value

Find intrinsic \mathbf{K} and rotation \mathbf{R}

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} | -\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M} | -\mathbf{M}\mathbf{c}] \end{aligned}$$

Find the camera center \mathbf{c}

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of \mathbf{P} !

\mathbf{c} is the singular vector corresponding to the smallest singular value

Find intrinsic \mathbf{K} and rotation \mathbf{R}

$$\mathbf{M} = \mathbf{K}\mathbf{R}$$

Any useful properties of \mathbf{K} and \mathbf{R} we can use?

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} | -\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M} | -\mathbf{M}\mathbf{c}] \end{aligned}$$

Find the camera center \mathbf{c}

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of P!

\mathbf{c} is the singular vector corresponding to the smallest singular value

Find intrinsic \mathbf{K} and rotation \mathbf{R}

$$\mathbf{M} = \mathbf{K}\mathbf{R}$$

right upper triangle orthogonal

How do we find K and R?

Decomposition of the Camera Matrix

$$\mathbf{P} = \left[\begin{array}{ccc|c} p_1 & p_2 & p_3 & p_4 \\ p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} \end{array} \right]$$

$$\begin{aligned} \mathbf{P} &= \mathbf{K}[\mathbf{R}|\mathbf{t}] \\ &= \mathbf{K}[\mathbf{R} | -\mathbf{R}\mathbf{c}] \\ &= [\mathbf{M} | -\mathbf{M}\mathbf{c}] \end{aligned}$$

Find the camera center \mathbf{c}

$$\mathbf{P}\mathbf{c} = \mathbf{0}$$

SVD of \mathbf{P} !

\mathbf{c} is the singular vector corresponding to the smallest singular value

Find intrinsic \mathbf{K} and rotation \mathbf{R}

$$\mathbf{M} = \mathbf{K}\mathbf{R}$$

QR decomposition

Geometric camera calibration

Given a set of matched points

$$\{\mathbf{X}_i, \mathbf{x}_i\}$$

point in 3D space point in the image

Where do we get these matched points from?

and camera model

$$\mathbf{x} = \mathbf{f}(\mathbf{X}; \mathbf{p}) = \mathbf{P}\mathbf{X}$$

projection model

parameters

Camera matrix

Find the (pose) estimate of

\mathbf{P}

We'll use a **perspective** camera model for pose estimation

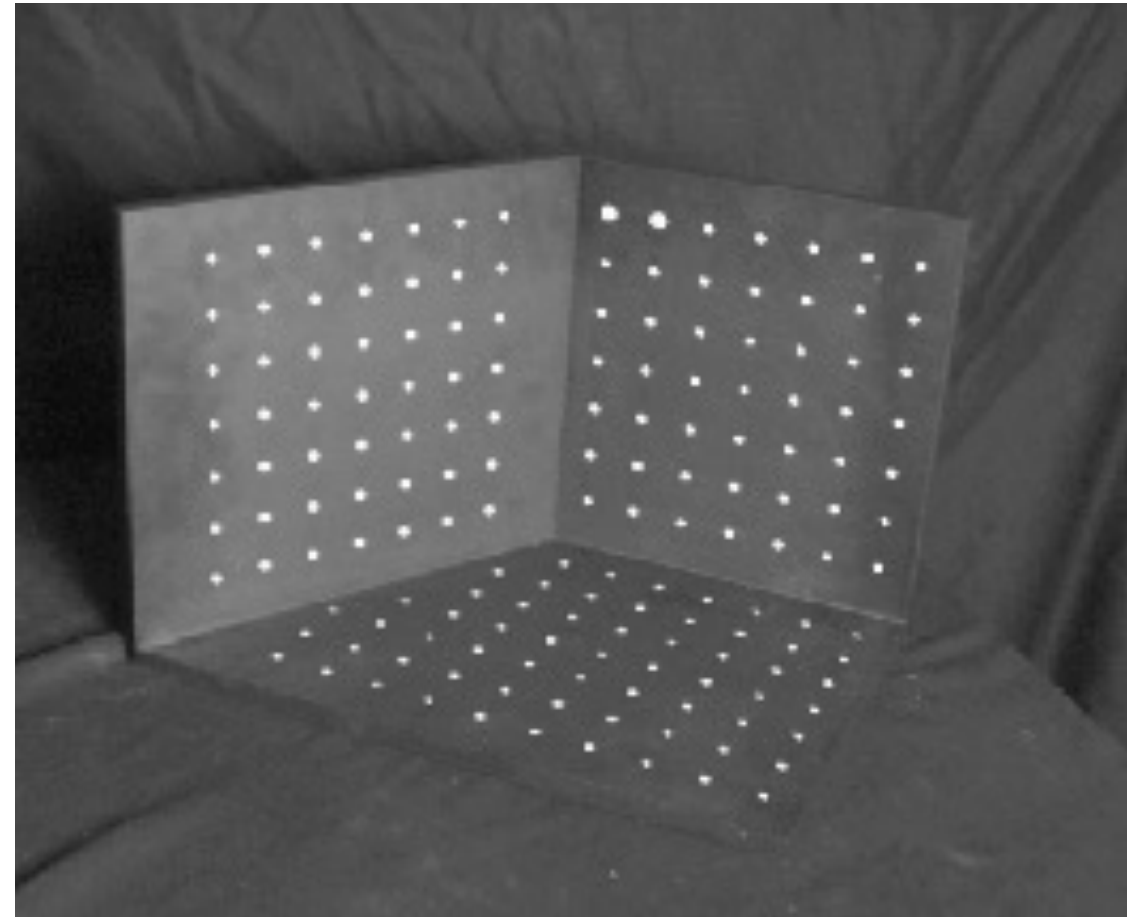
Calibration using a reference object

Place a known object in the scene:

- identify correspondences between image and scene
- compute mapping from scene to image

Issues:

- must know geometry very accurately
- must know 3D->2D correspondence



Geometric camera calibration

Advantages:

- Very simple to formulate.
- Analytical solution.

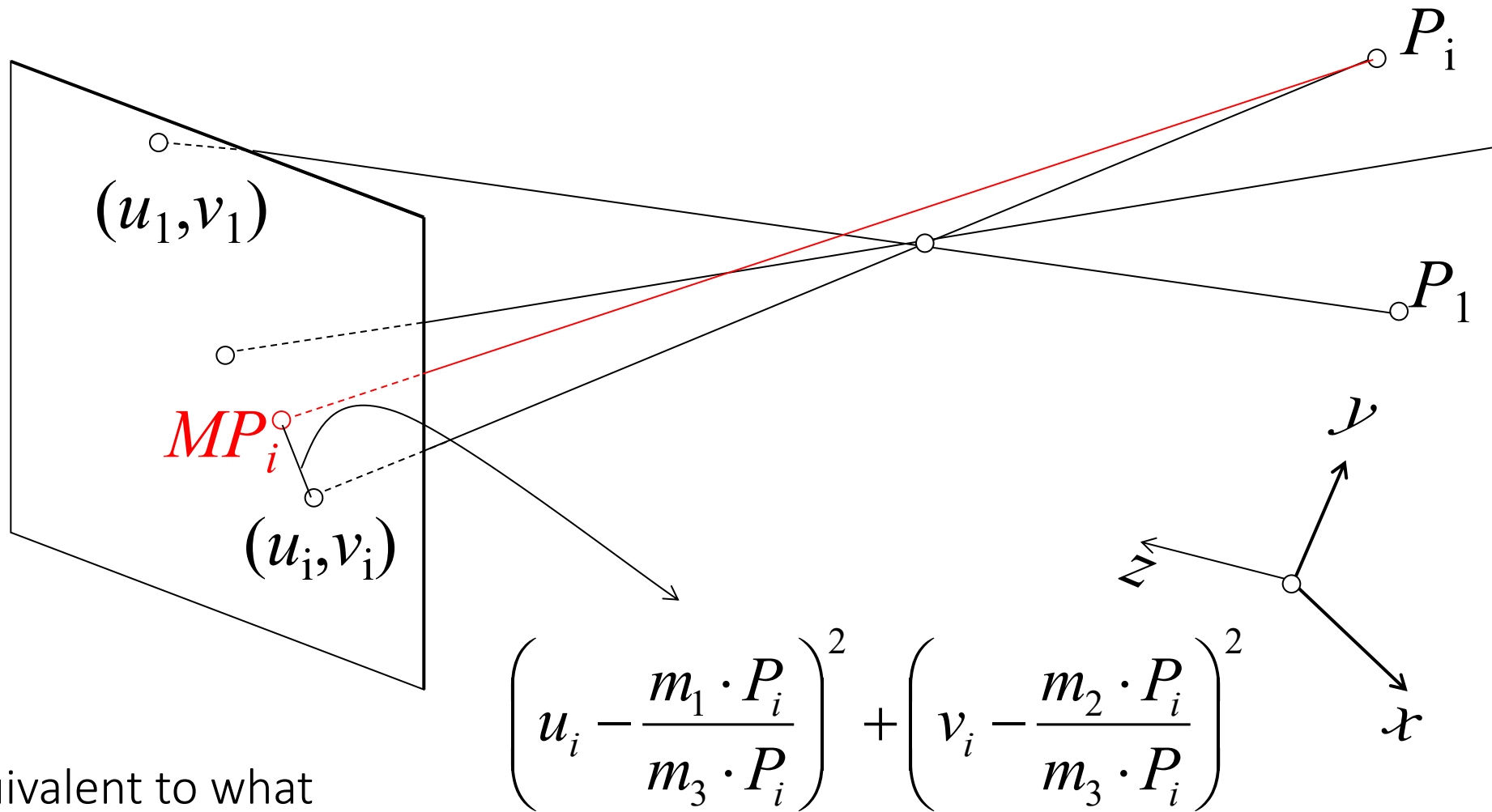
Disadvantages:

- Doesn't model radial distortion.
- Hard to impose constraints (e.g., known f).
- Doesn't minimize the correct error function.

For these reasons, *nonlinear methods* are preferred

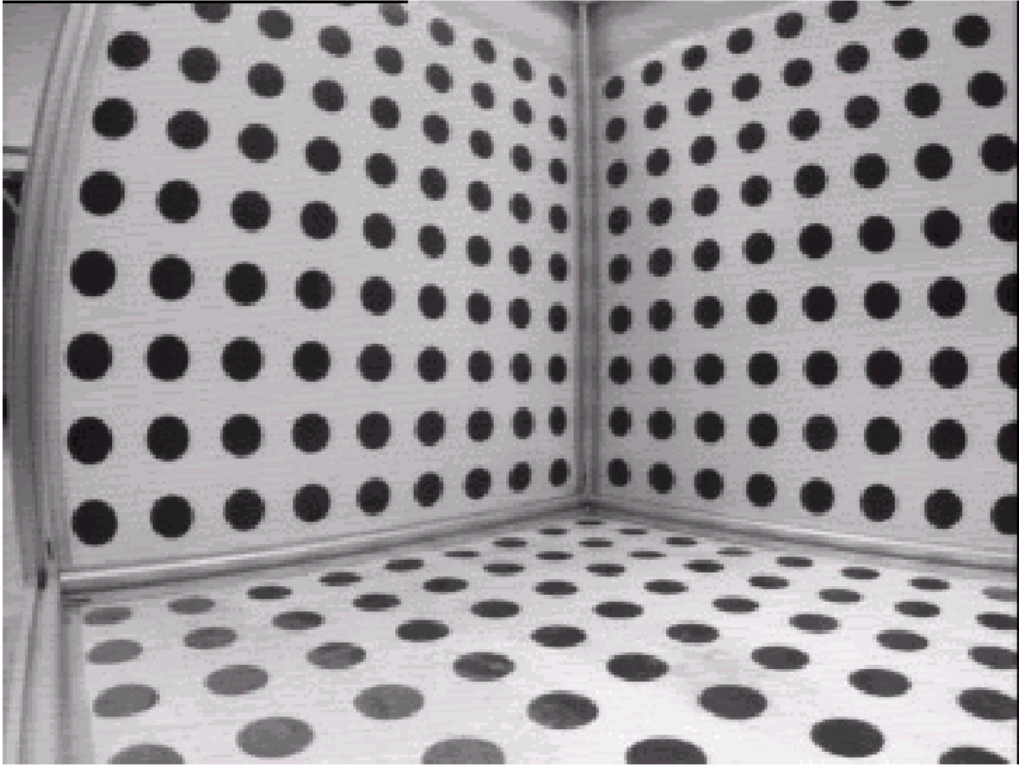
- Define error function E between projected 3D points and image positions
 - E is nonlinear function of intrinsics, extrinsics, radial distortion
- Minimize E using nonlinear optimization techniques

Minimizing reprojection error

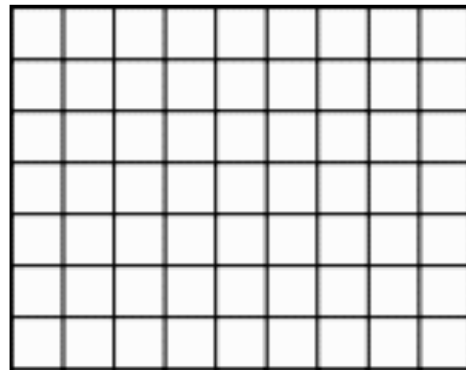


Is this equivalent to what we were doing previously?

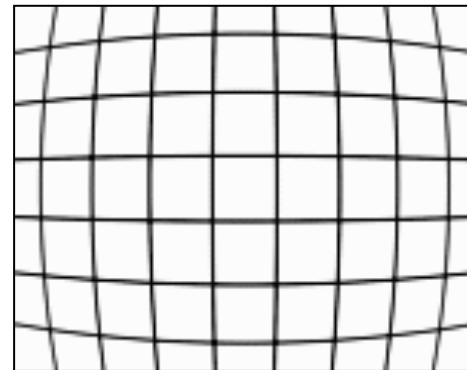
Radial distortion



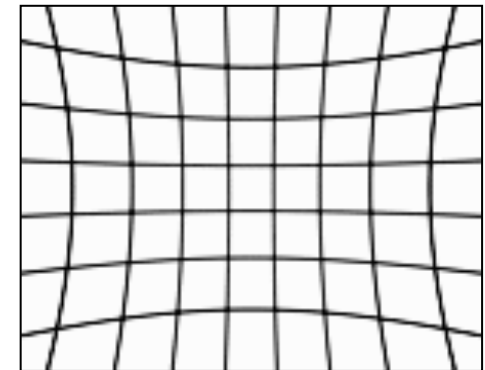
What causes this distortion?



no distortion

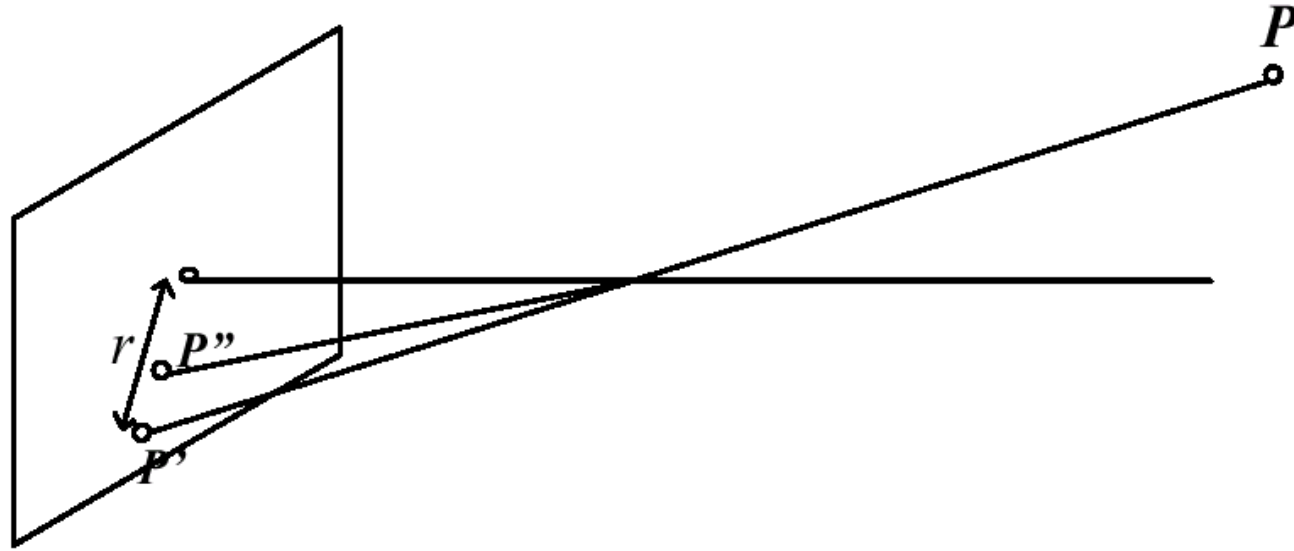


barrel distortion



pincushion distortion

Radial distortion model



Ideal:

$$x' = f \frac{x}{z}$$

$$y' = f \frac{y}{z}$$

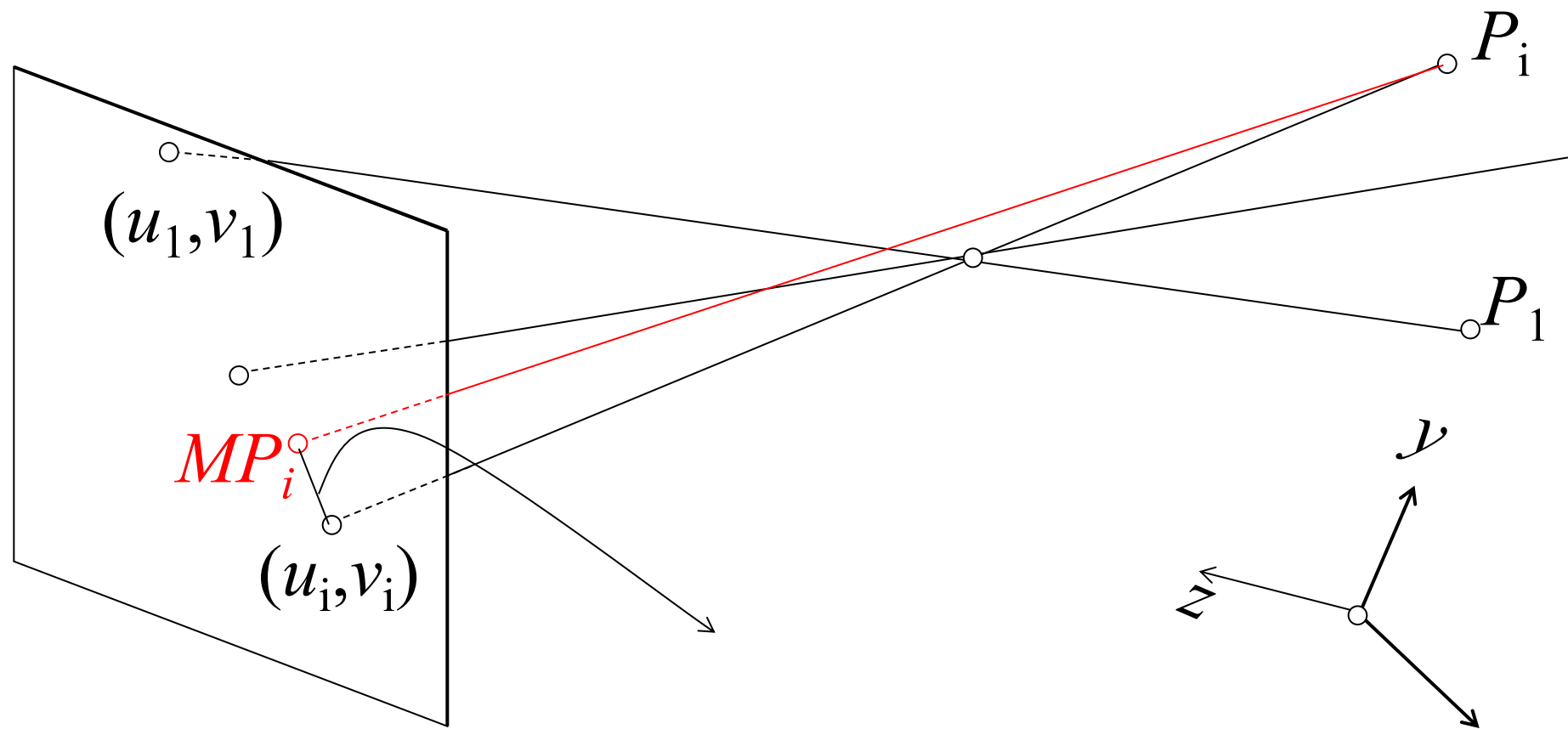
Distorted:

$$x'' = \frac{1}{\lambda} x'$$

$$y'' = \frac{1}{\lambda} y'$$

$$\lambda = 1 + k_1 r^2 + k_2 r^4 + \dots$$

Minimizing reprojection error with radial distortion



Add distortions to reprojection error:

$$\left(u_i - \frac{1}{\lambda} \frac{m_1 \cdot P_i}{m_3 \cdot P_i}\right)^2 + \left(v_i - \frac{1}{\lambda} \frac{m_2 \cdot P_i}{m_3 \cdot P_i}\right)^2$$

Correcting radial distortion

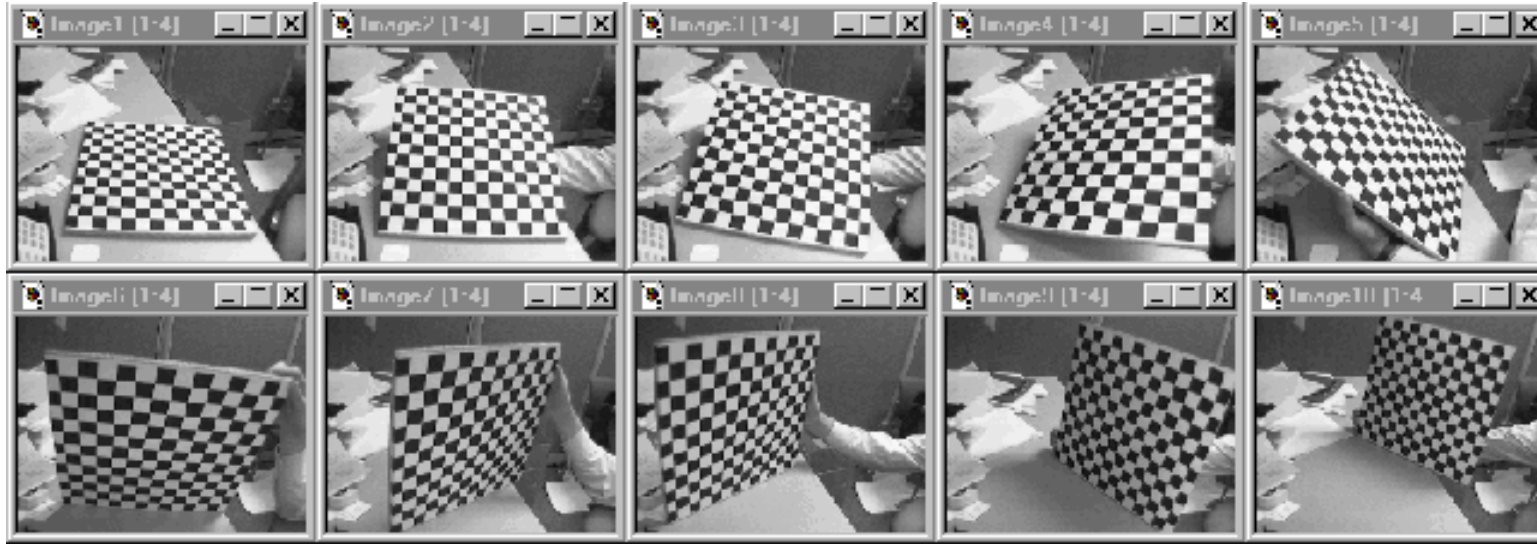


before



after

Alternative: Multi-plane calibration



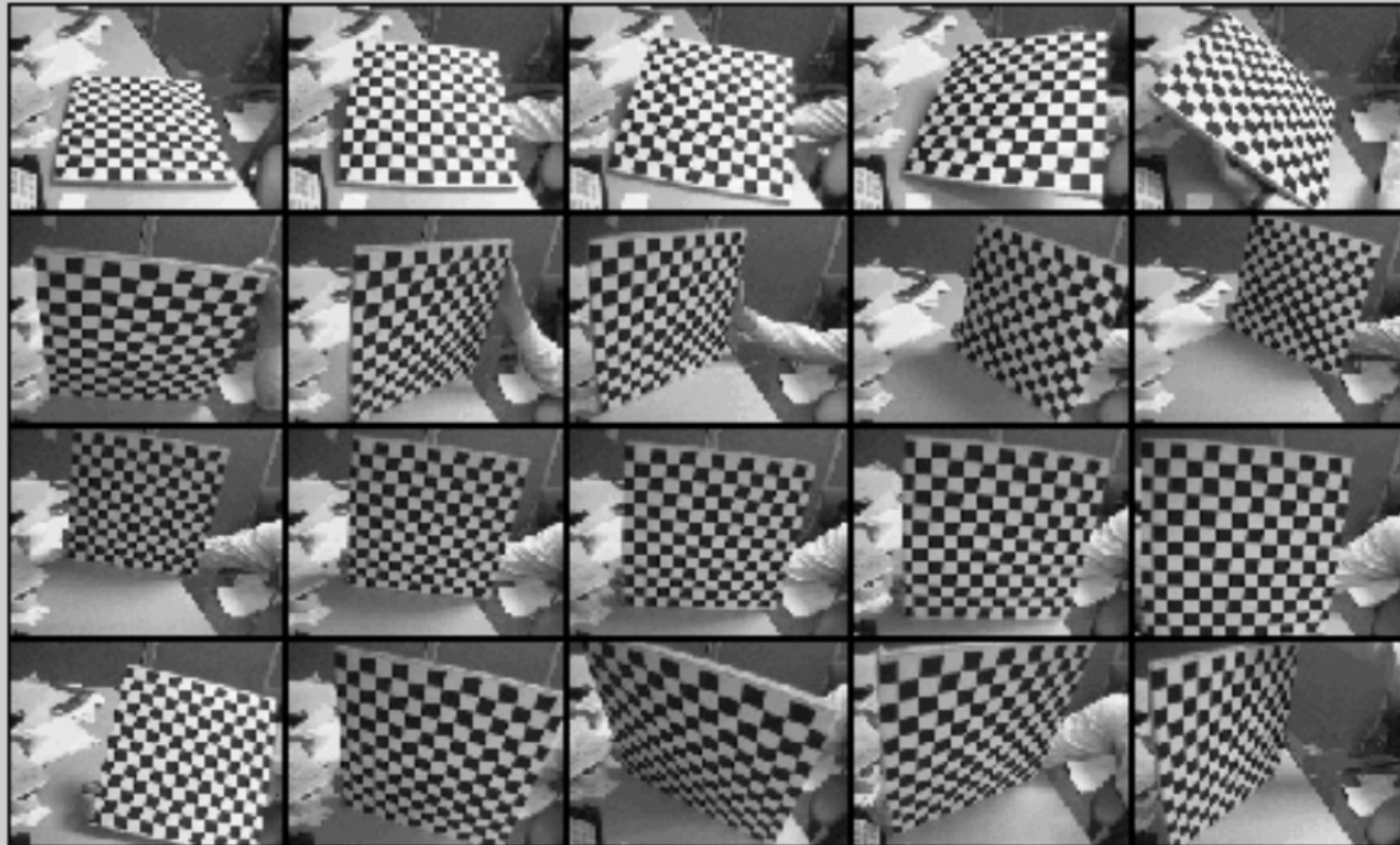
Advantages:

- Only requires a plane
- Don't have to know positions/orientations
- Great code available online!
 - Matlab version: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
 - Also available on OpenCV.

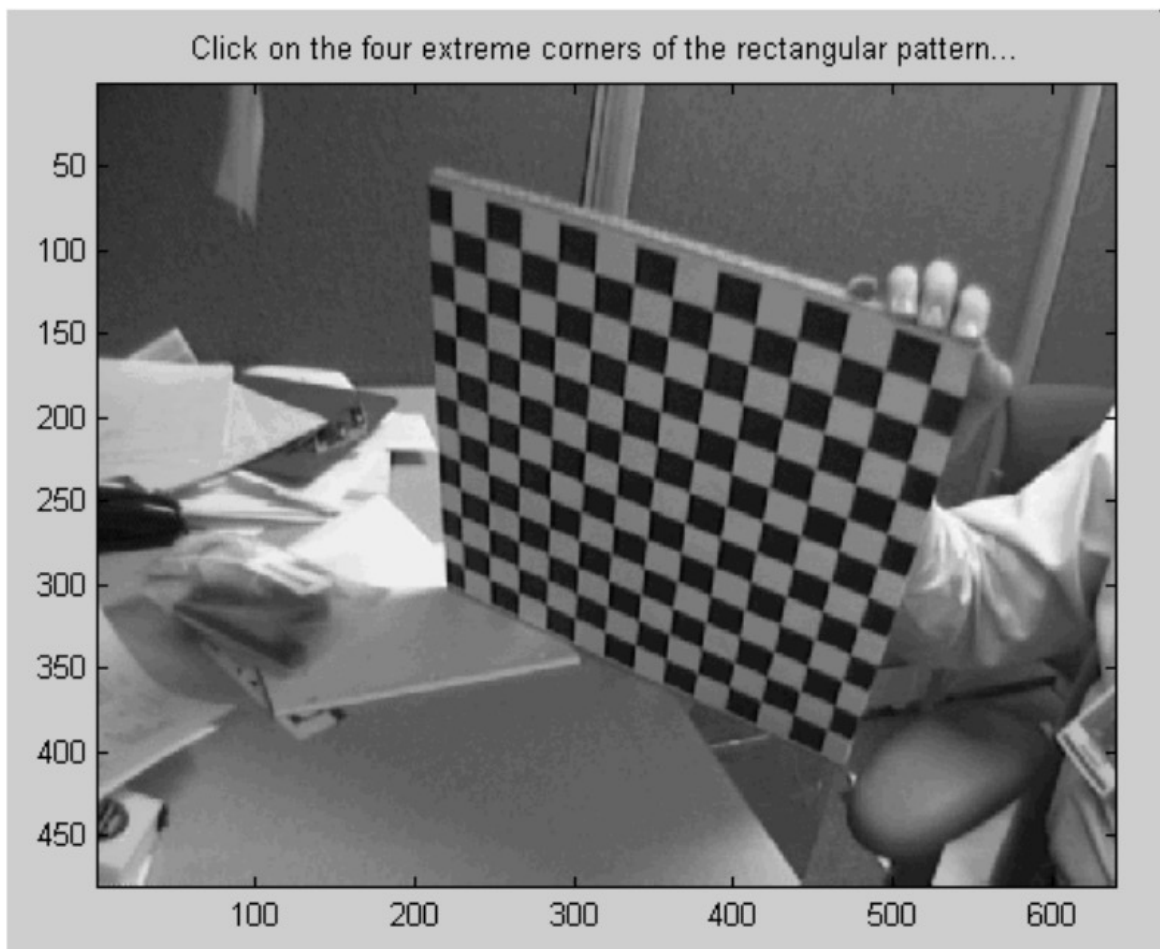
Disadvantage: Need to solve non-linear optimization problem.

Step-by-step demonstration

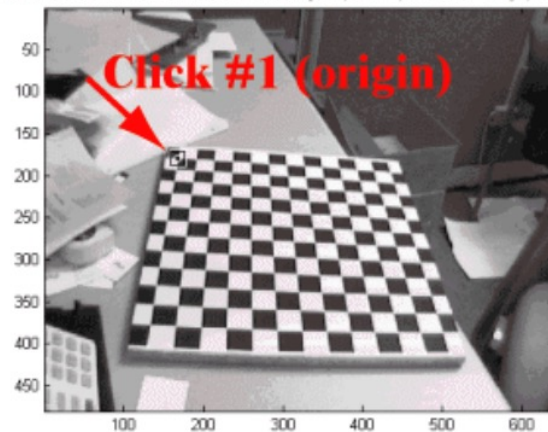
Calibration images



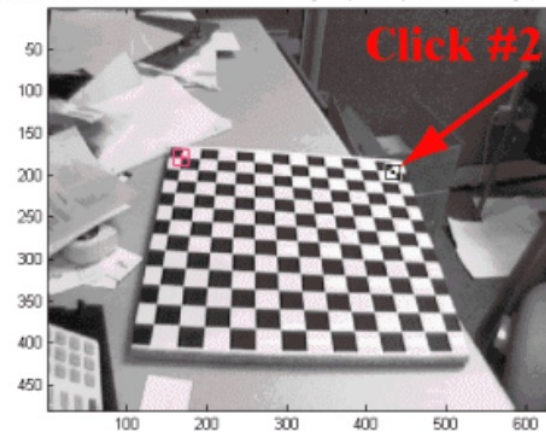
Step-by-step demonstration



Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



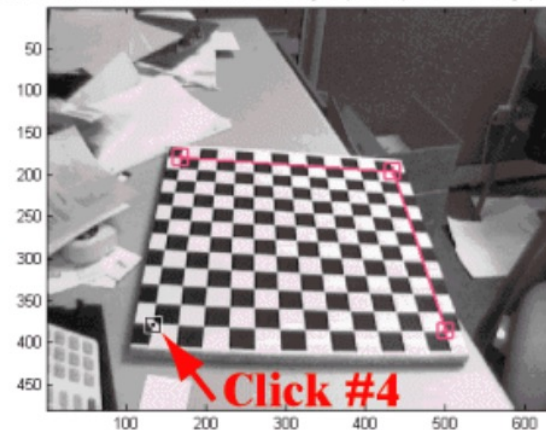
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



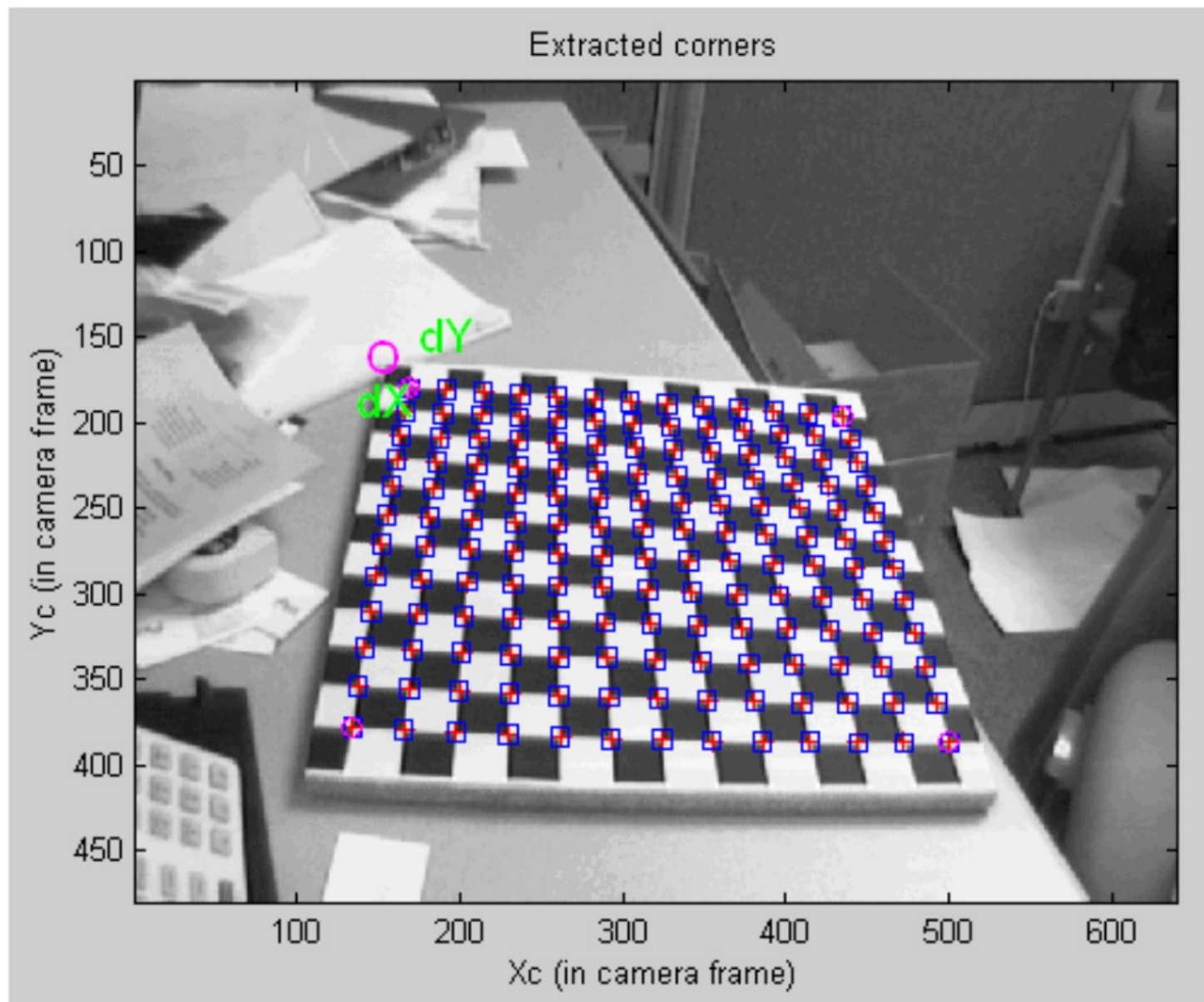
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



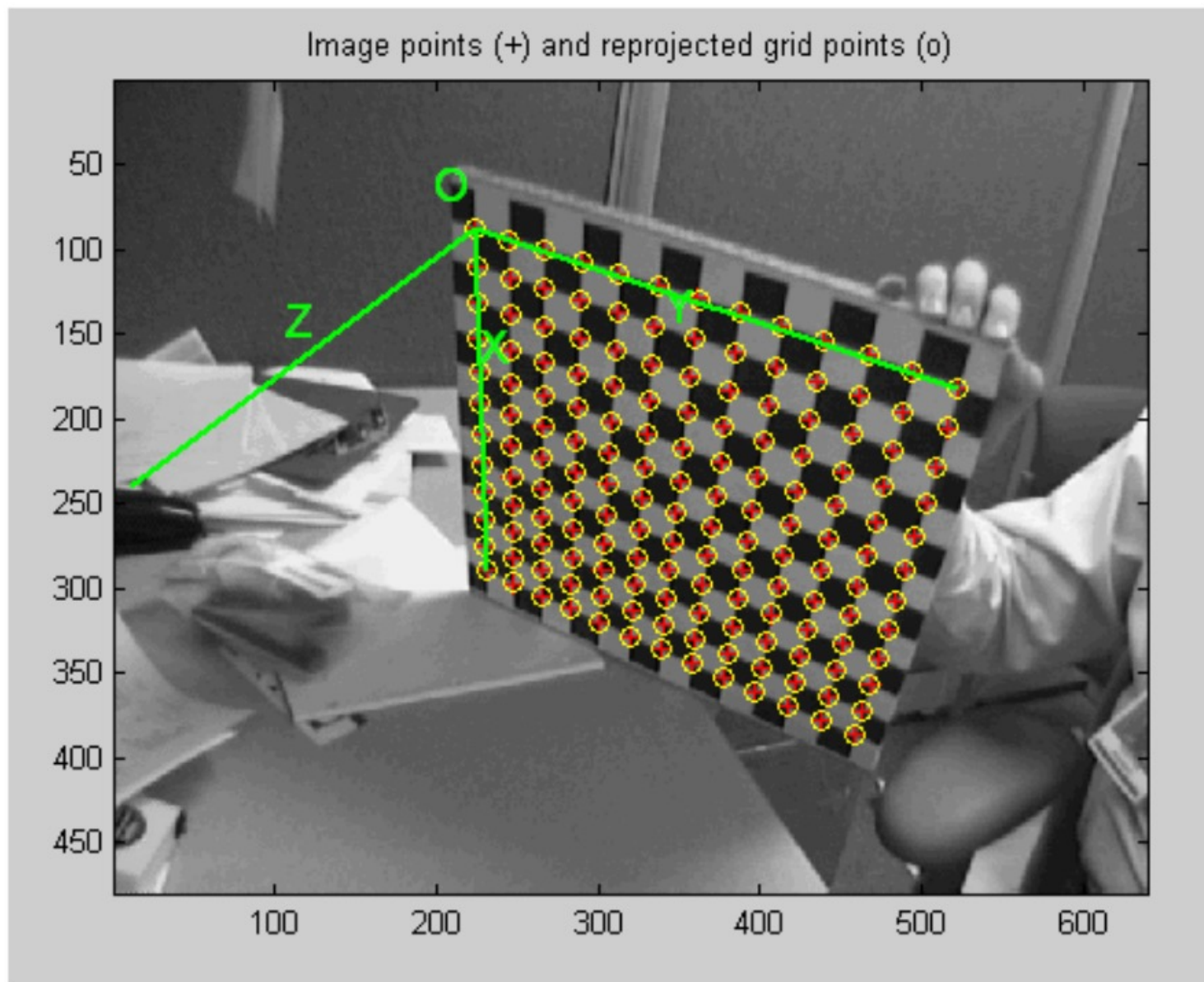
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



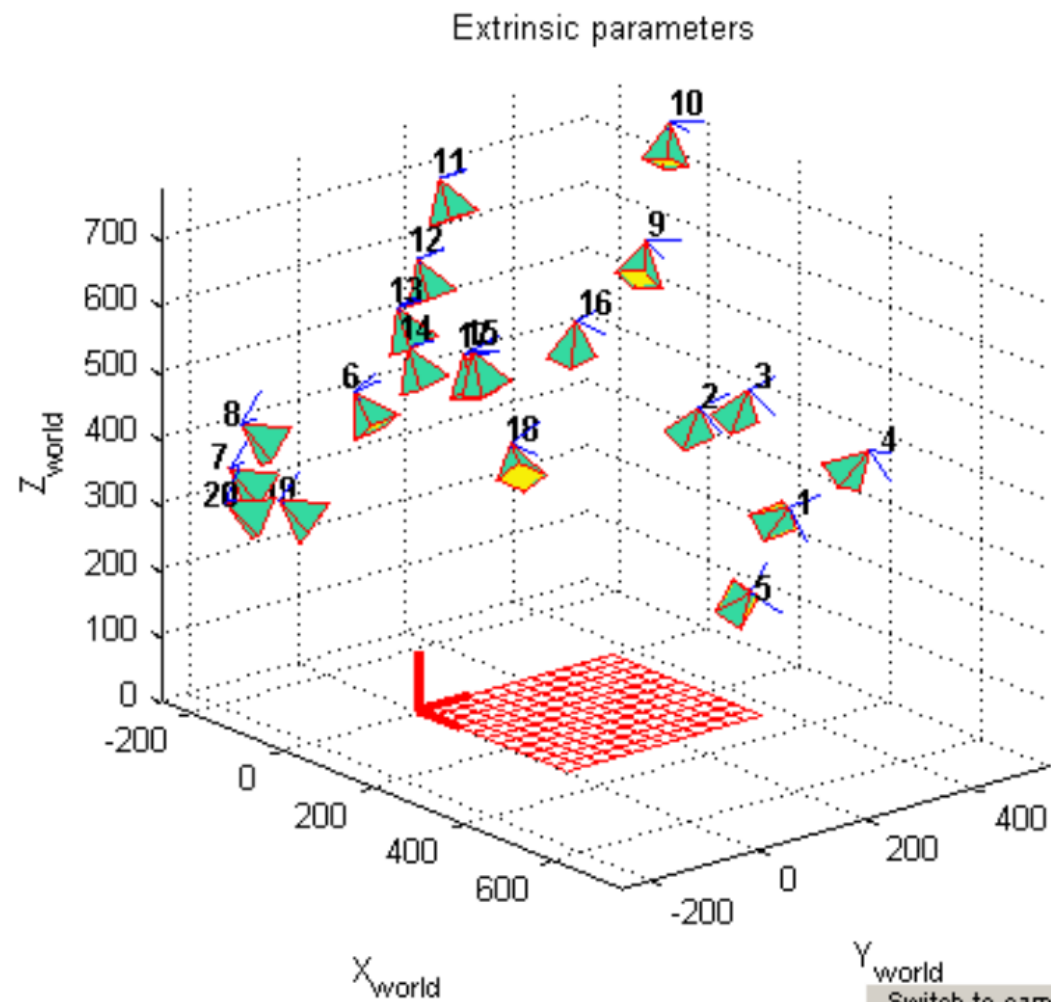
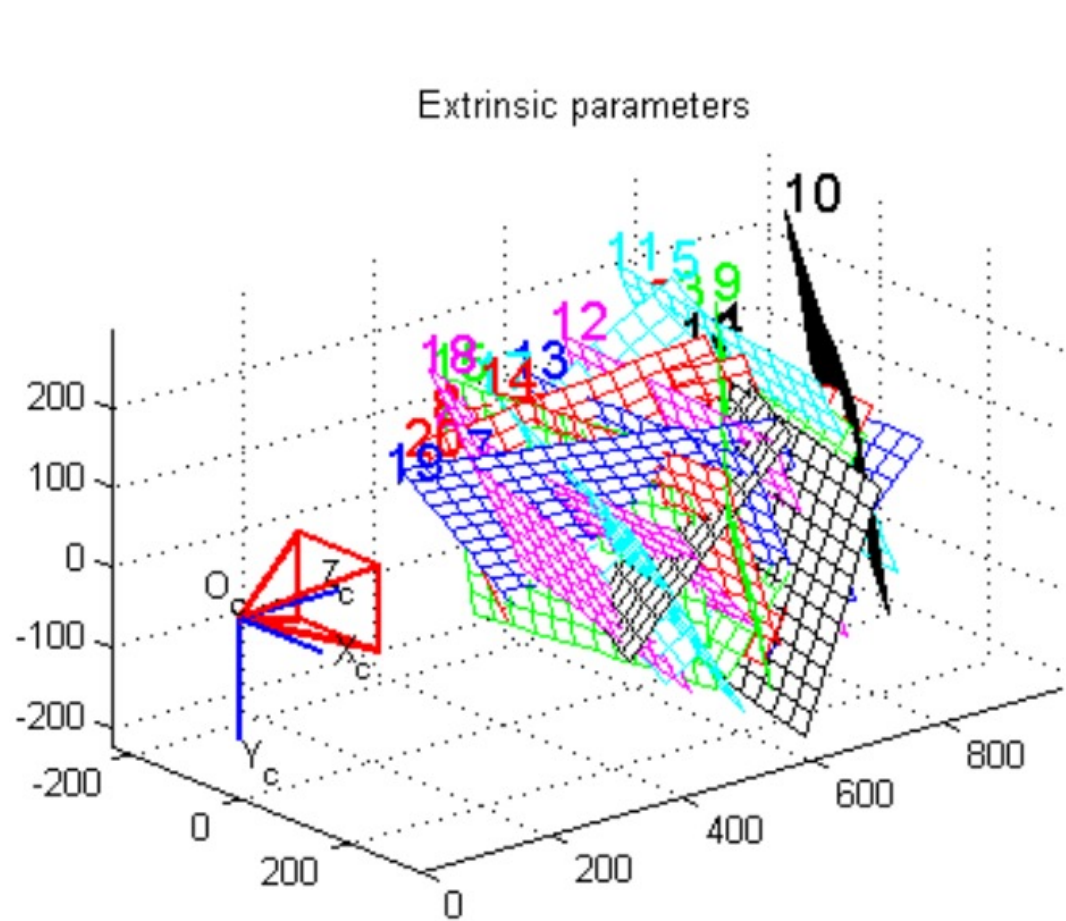
Step-by-step demonstration



Step-by-step demonstration



Step-by-step demonstration



Switch to camera-centered view

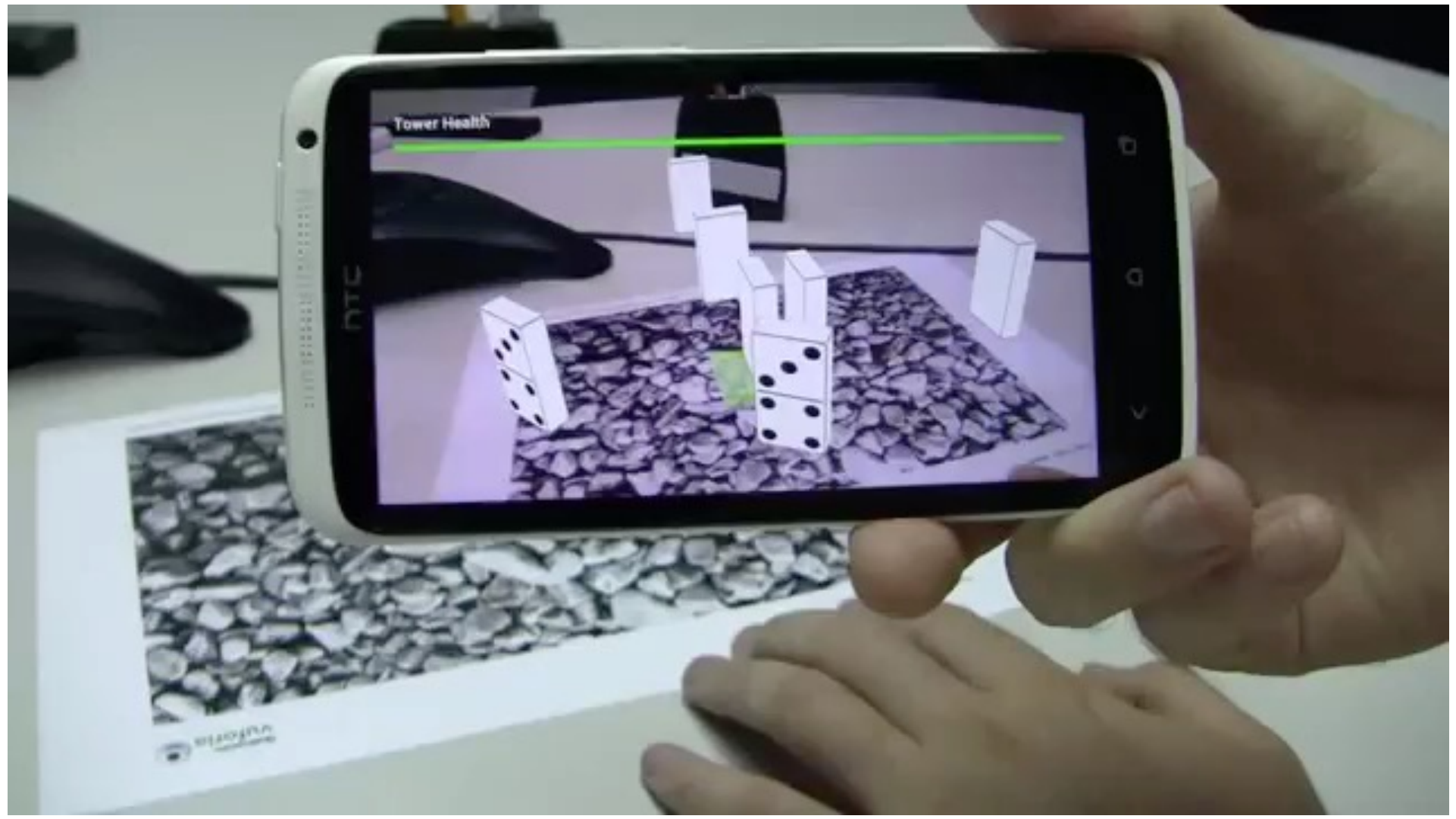
What does it mean to “calibrate a camera”?

What does it mean to “calibrate a camera”?

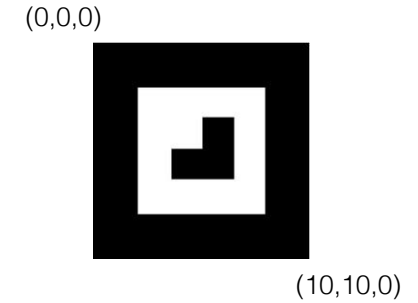
Many different ways to calibrate a camera:

- Radiometric calibration.
- Color calibration.
- Geometric calibration.
- Noise calibration.
- Lens (or aberration) calibration.

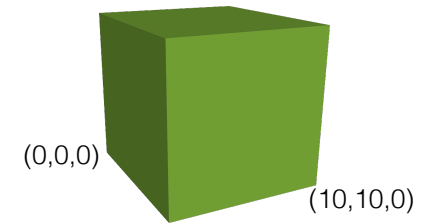
We'll briefly discuss radiometric and color calibration in later lectures. For the rest, see 15-463/663/862.



3D locations of planar marker features are known in advance



3D content prepared in advance



Simple AR program

1. Compute point correspondences (2D and AR tag)
2. Estimate the pose of the camera **P**
3. Project 3D content to image plane using **P**

