#### Stereo



16-385 Computer Vision Spring 2024, Lecture 12

#### Overview of today's lecture

- Revisiting triangulation.
- Disparity.
- Stereo rectification.
- Stereo matching.
- Improving stereo matching.
- Structured light.

#### Slide credits

Some of these slides were adapted directly from:

- Kris Kitani (16-385, Spring 2017).
- Srinivasa Narasimhan (16-823, Spring 2017).

Revisiting triangulation



Left image



Right image



Left image



Right image

1. Select point in one image (how?)



Left image

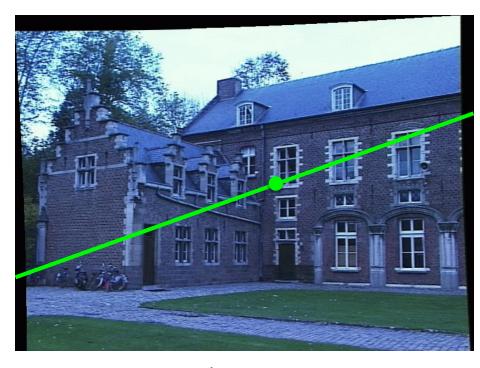


Right image

- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)



Left image



Right image

- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)
- 3. Find matching point along line (how?)



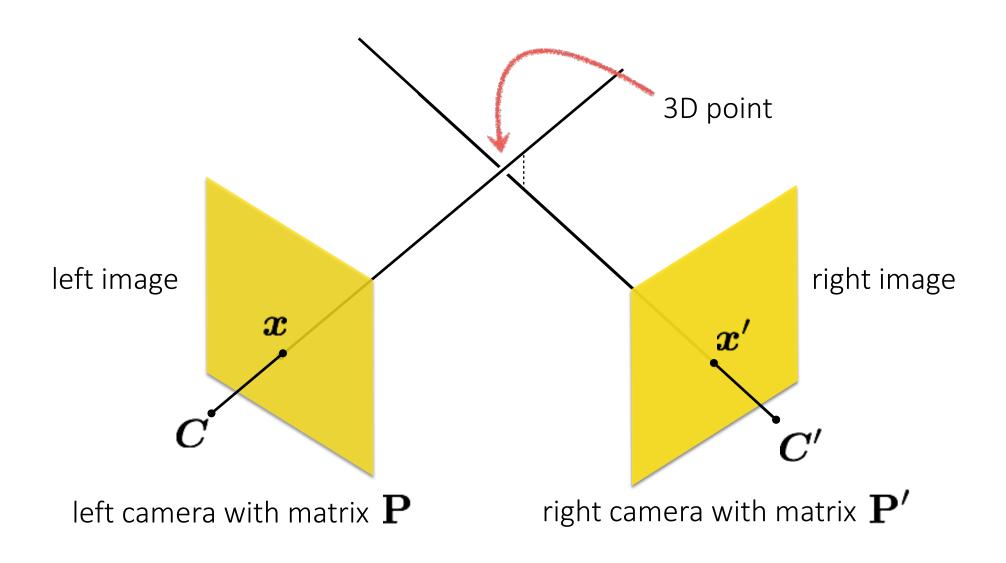




Right image

- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)
- 3. Find matching point along line (how?)
- 4. Perform triangulation (how?)

#### Triangulation









Right image

- 1. Select point in one image (how?)
- 2. Form epipolar line for that point in second image (how?)
- 3. Find matching point along line (how?)
- 4. Perform triangulation (how?)

What are the disadvantages of this procedure?

#### Stereo rectification





What's different between these two images?







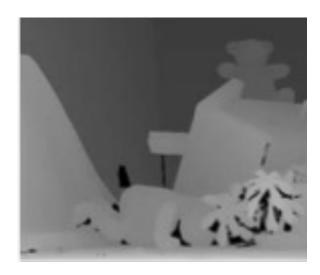


Objects that are close move more or less?

# The amount of horizontal movement is inversely proportional to ...







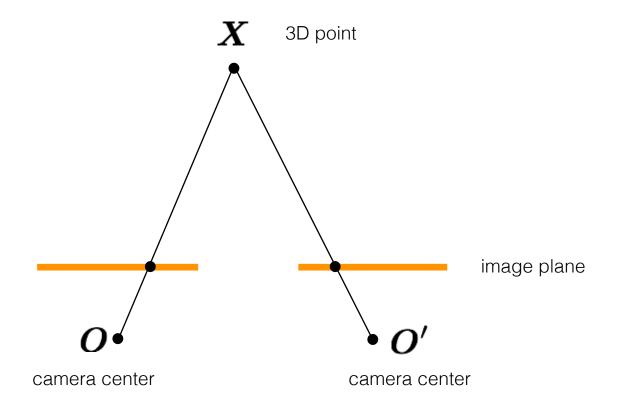
## The amount of horizontal movement is inversely proportional to ...

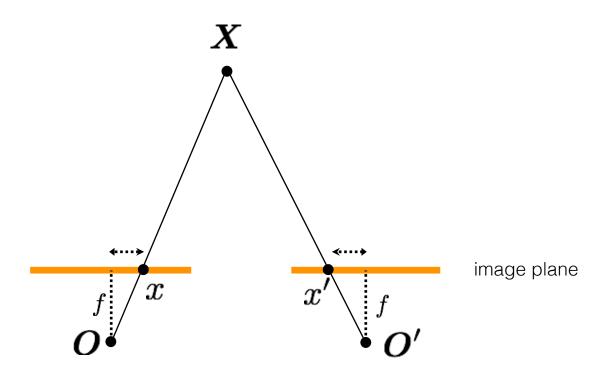


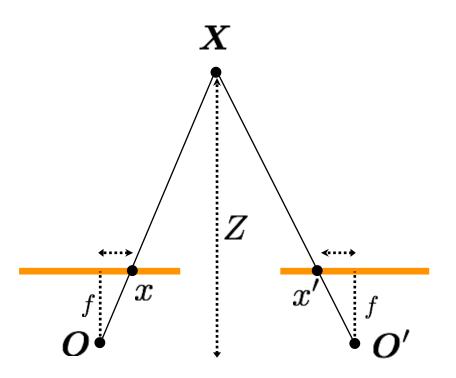


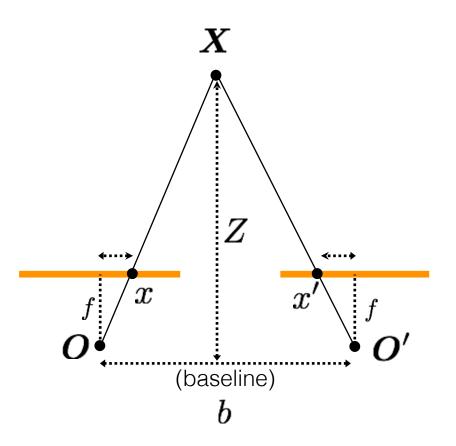


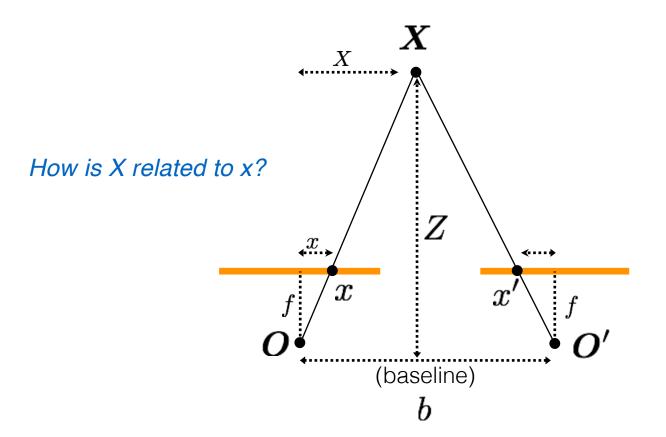
... the distance from the camera.

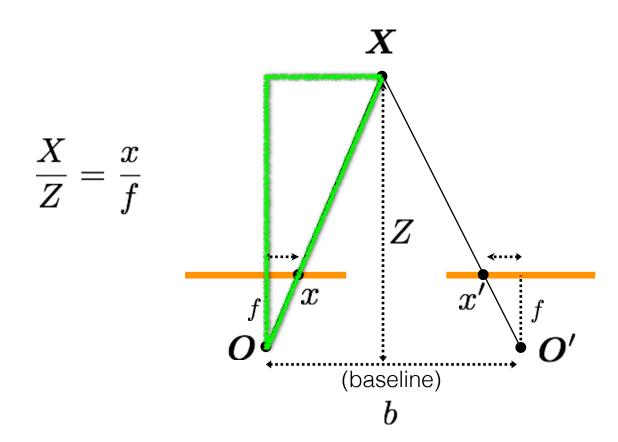


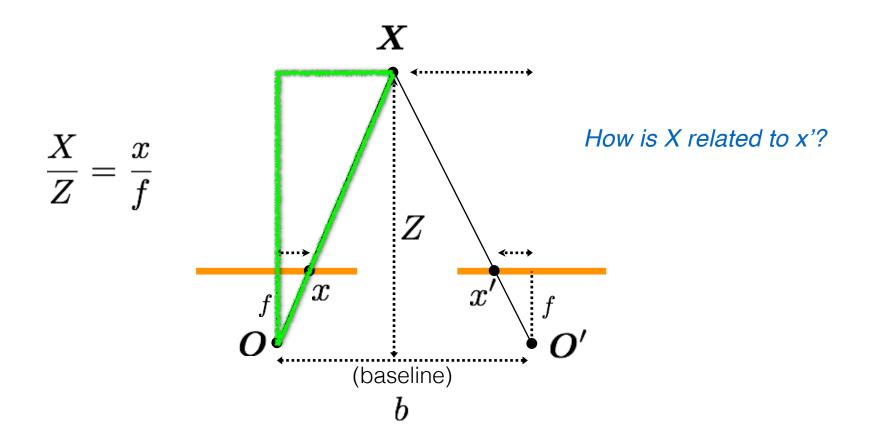


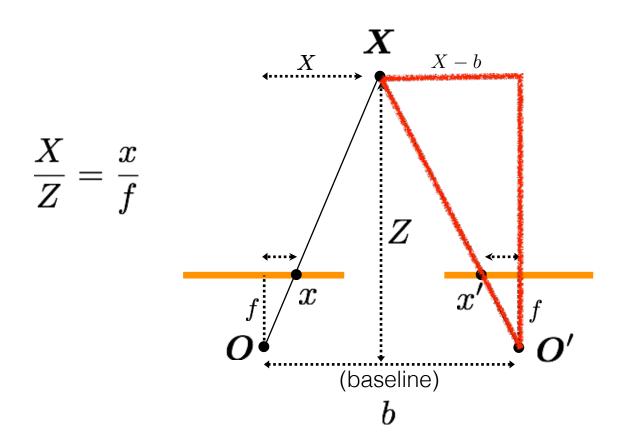




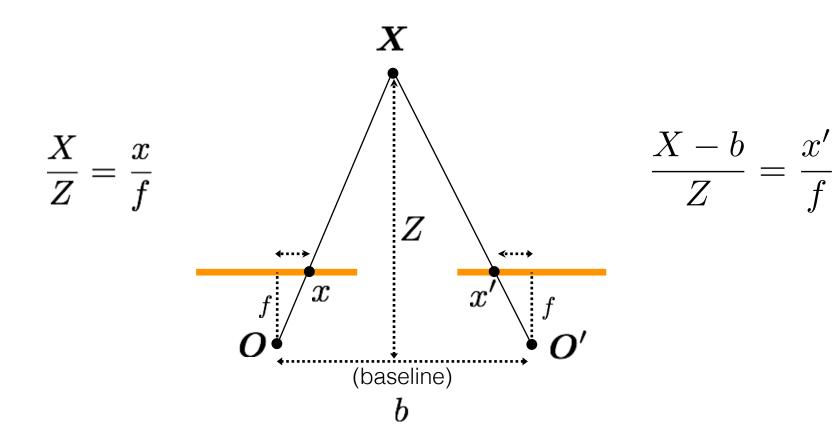






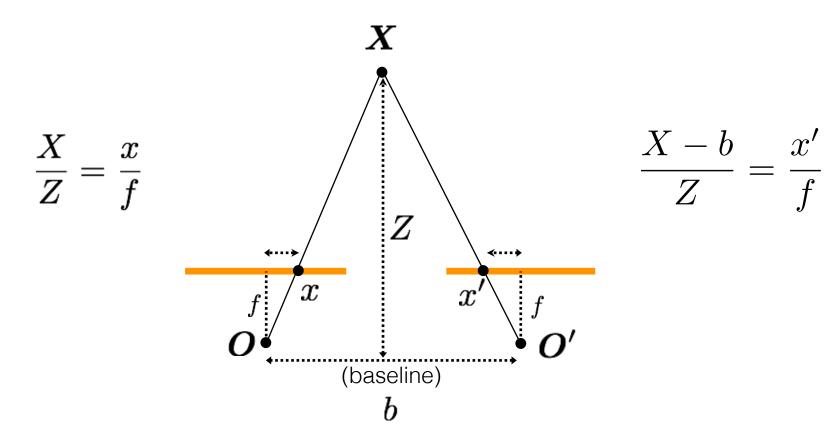


$$\frac{X-b}{Z} = \frac{x'}{f}$$



#### **Disparity**

$$d=x-x'$$
 (wrt to camera origin of image plane)  $=rac{bf}{z}$ 



#### Disparity

$$d = x - x'$$

$$= \frac{bf}{Z}$$

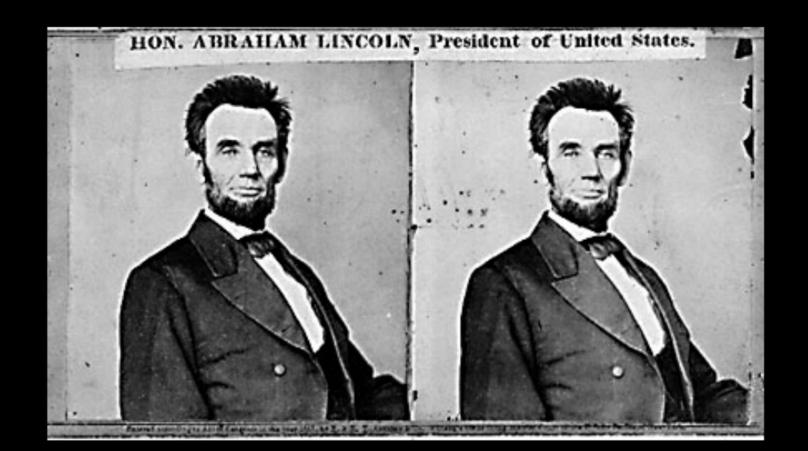
inversely proportional to depth

# What other vision system uses disparity for depth sensing?

#### Stereoscopes: A 19th Century Pastime









Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923



# Is disparity the only depth cue the human visual system uses?

# So can I compute depth from any two images of the same object?



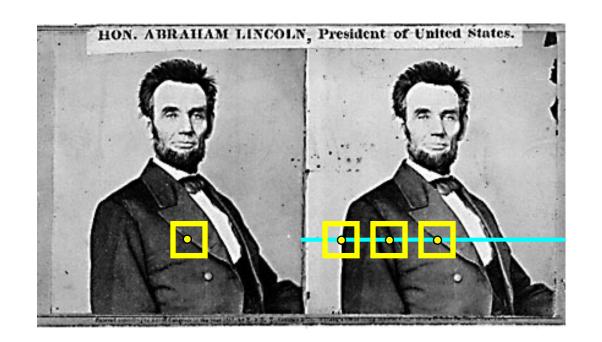


### So can I compute depth from any two images of the same object?



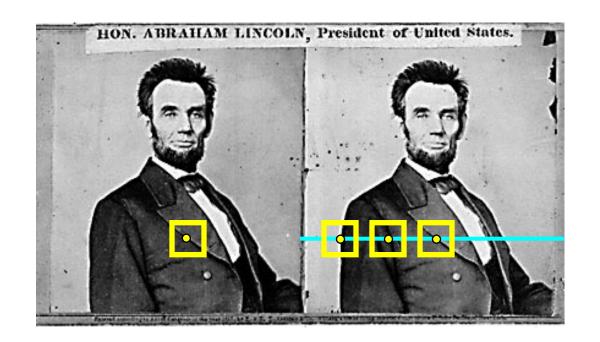


- 1. Need sufficient baseline
- 2. Images need to be 'rectified' first (make epipolar lines horizontal)

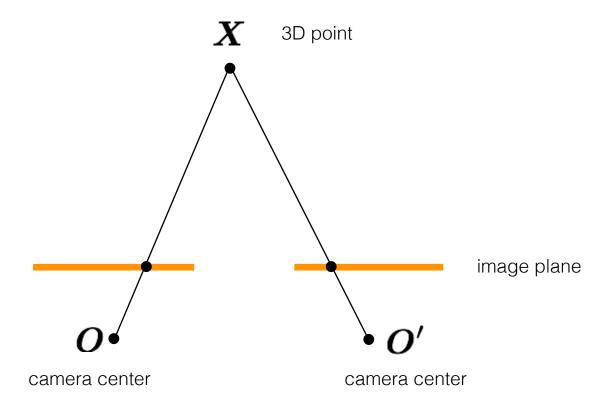


- 1. Rectify images
   (make epipolar lines horizontal)
- 2. For each pixel
  - a. Find epipolar line
  - b. Scan line for best match
  - c. Compute depth from disparity

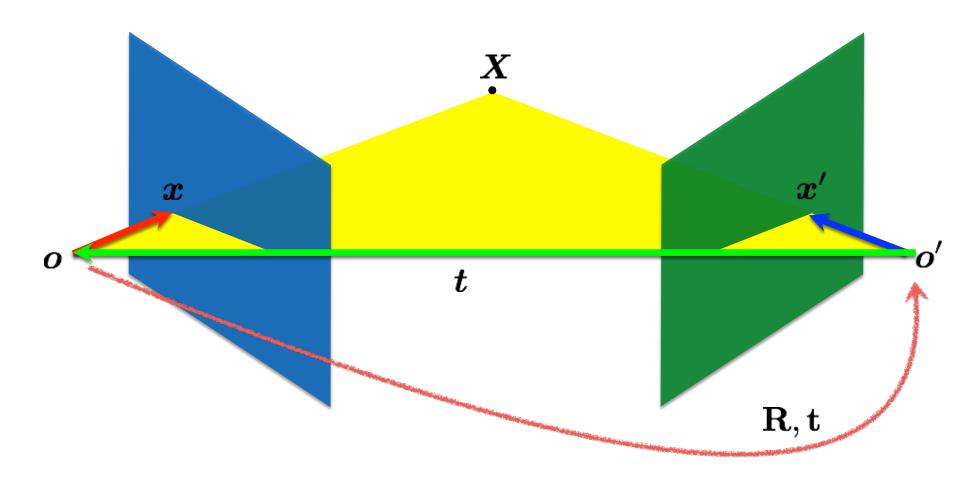
$$Z = \frac{bf}{d}$$



How can you make the epipolar lines horizontal?

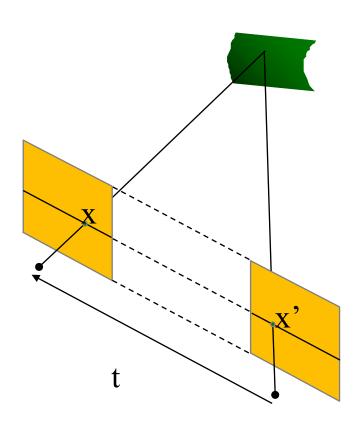


What's special about these two cameras?



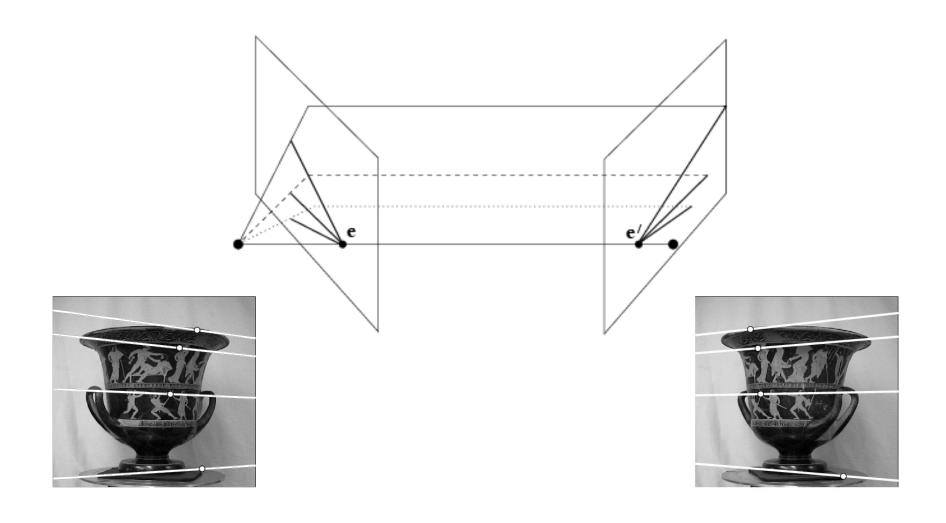
$$\boldsymbol{x}' = \mathbf{R}(\boldsymbol{x} - \boldsymbol{t})$$

#### When are epipolar lines horizontal?



#### When this relationship holds:

$$R = I \qquad \qquad t = (T, 0, 0)$$



It's hard to make the image planes exactly parallel



How can you make the epipolar lines horizontal?

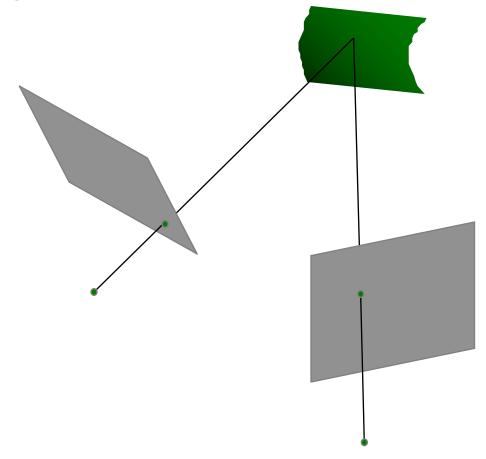




Use stereo rectification?



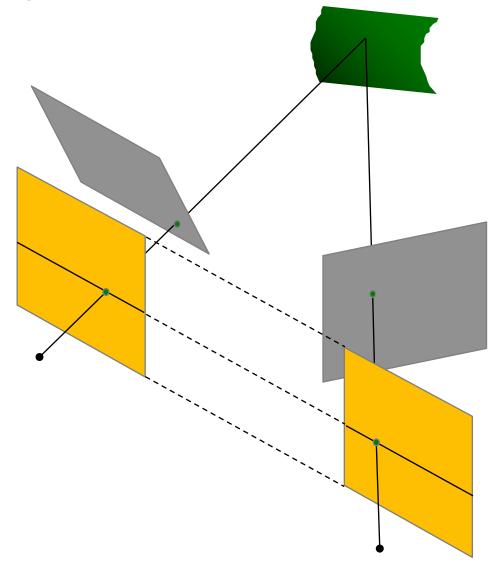
#### What is stereo rectification?



#### What is stereo rectification?

Reproject image planes onto a common plane parallel to the line between camera centers

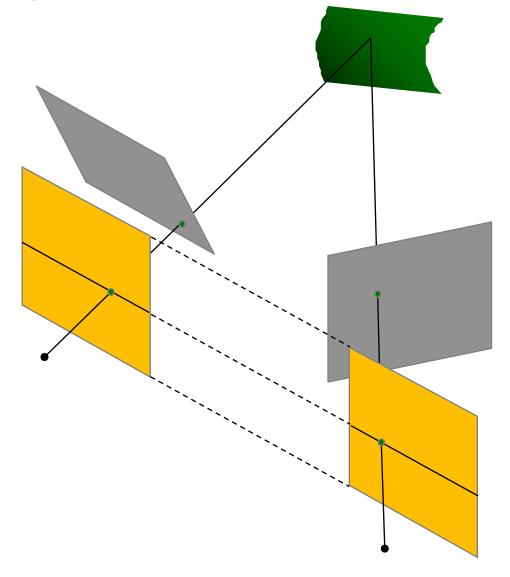
How can you do this?



#### What is stereo rectification?

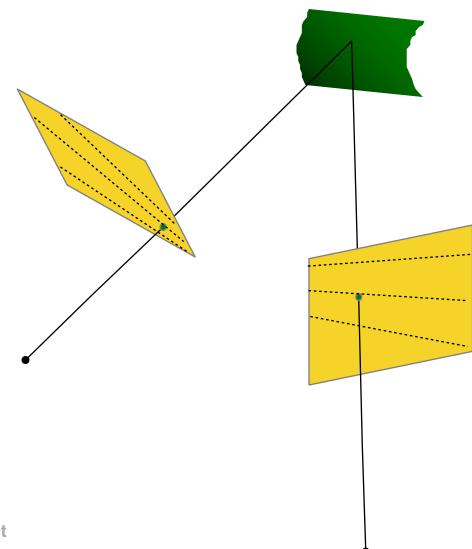
Reproject image planes onto a common plane parallel to the line between camera centers

Need two homographies (3x3 transform), one for each input image reprojection

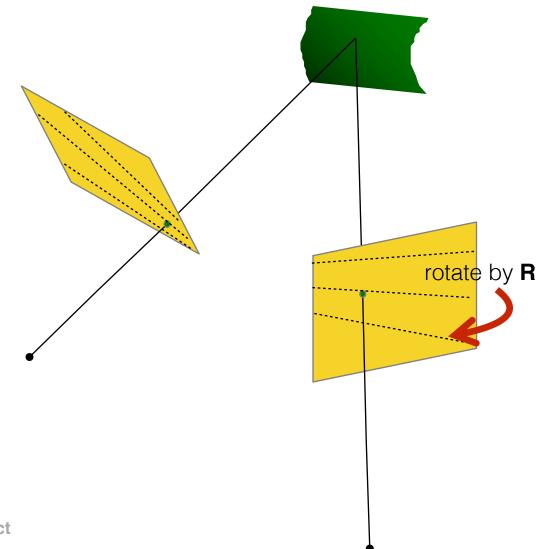


- Rotate the right camera by R

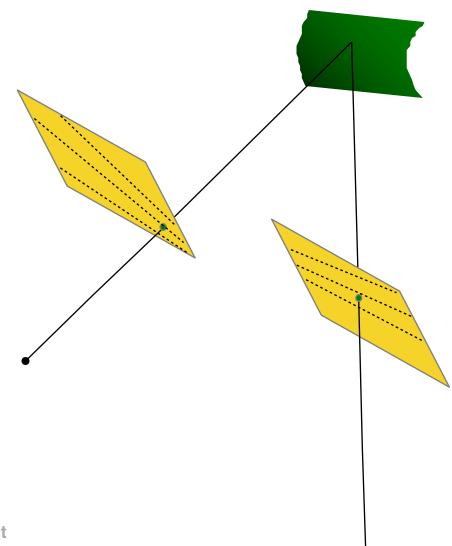
   (aligns camera coordinate system orientation only)
- 2. Rotate (**rectify**) the left camera so that the epipole is at infinity
- 3. Rotate (**rectify**) the right camera so that the epipole is at infinity
- 4. Adjust the **scale**



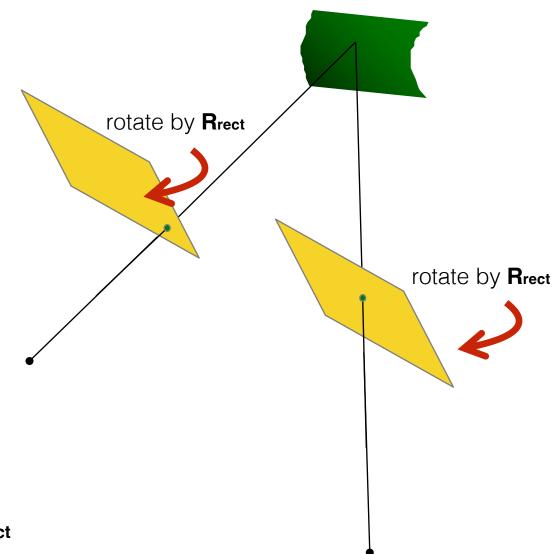
- 1. Compute **E** to get **R**
- 2. Rotate right image by R
- 3. Rotate both images by **Rrect**
- 4. Scale both images by **H**



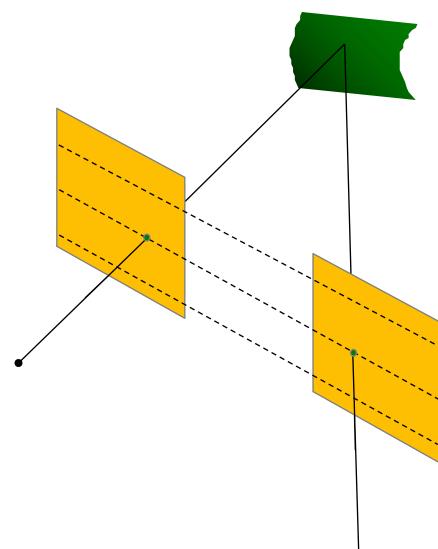
- 1. Compute **E** to get **R**
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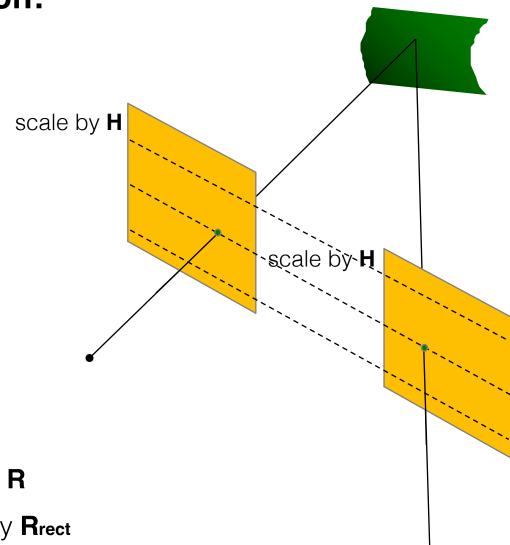
- 1. Compute **E** to get **R**
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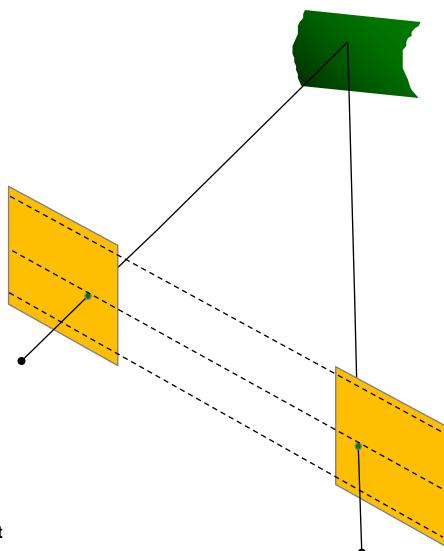
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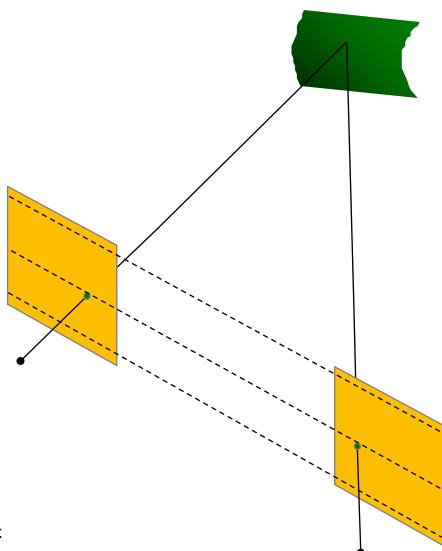
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- 1. Compute **E** to get **R**
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Suppose 
$$\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$$

## Step 1: Use E to get R

SVD: 
$$\mathbf{E} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^{\top}$$
 Let  $\mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 

We get FOUR solutions:

$$\mathbf{R}_1 = \mathbf{U} \mathbf{W} \mathbf{V}^ op \quad \mathbf{R}_2 = \mathbf{U} \mathbf{W}^ op \mathbf{V}^ op \quad egin{array}{l} \mathbf{t} \end{bmatrix}_ imes = \pm \mathbf{U} \mathbf{W} \mathbf{\Sigma} \mathbf{U}^T \ & ext{two possible translations} \end{array}$$

#### We get FOUR solutions:

$$\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathsf{T}}$$
  $\mathbf{R}_1 = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathsf{T}}$   $[\mathbf{t}]_{\times} = \mathbf{U}\mathbf{W}\mathbf{\Sigma}\mathbf{U}^T$   $[\mathbf{t}]_{\times} = -\mathbf{U}\mathbf{W}\mathbf{\Sigma}\mathbf{U}^T$ 

$$\mathbf{R}_2 = \mathbf{U}\mathbf{W}^{\mathsf{T}}\mathbf{V}^{\mathsf{T}}$$
  $\mathbf{R}_2 = \mathbf{U}\mathbf{W}^{\mathsf{T}}\mathbf{V}^{\mathsf{T}}$   $[\mathbf{t}]_{\times} = \mathbf{U}\mathbf{W}\boldsymbol{\Sigma}\mathbf{U}^T$   $[\mathbf{t}]_{\times} = -\mathbf{U}\mathbf{W}\boldsymbol{\Sigma}\mathbf{U}^T$ 

#### Which one do we choose?

Compute determinant of R, valid solution must be equal to 1

(note: det(R) = -1 means rotation and reflection)

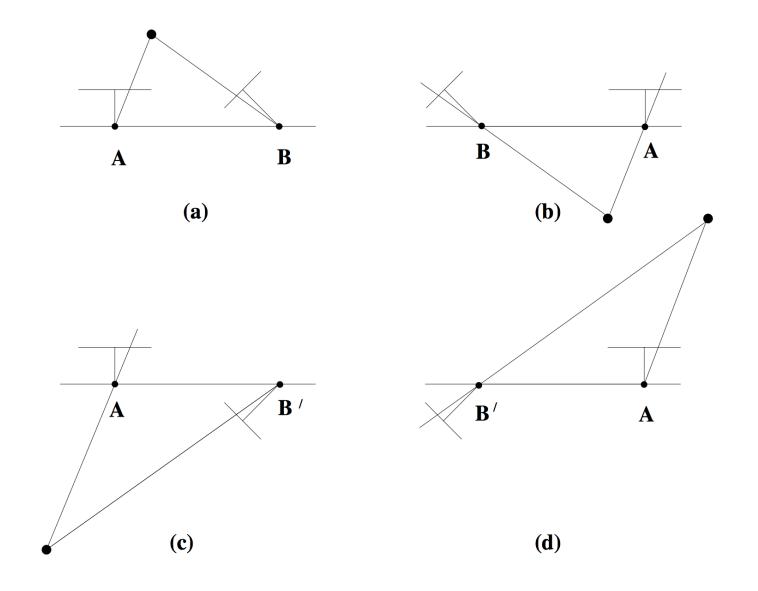
Compute 3D point using triangulation, valid solution has positive Z value (Note: negative Z means point is behind the camera)

#### Let's visualize the four configurations...

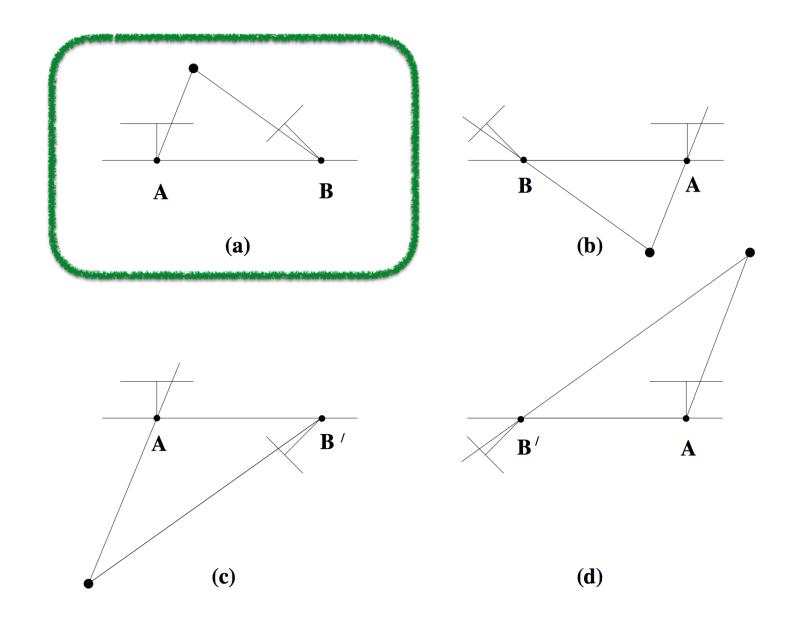


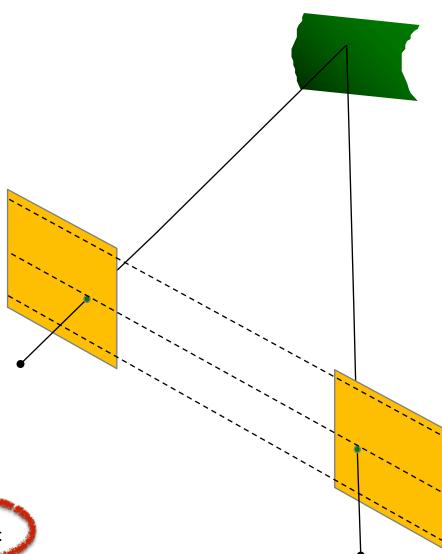
Find the configuration where the point is in front of both cameras

#### Find the configuration where the points is in front of both cameras



#### Find the configuration where the points is in front of both cameras

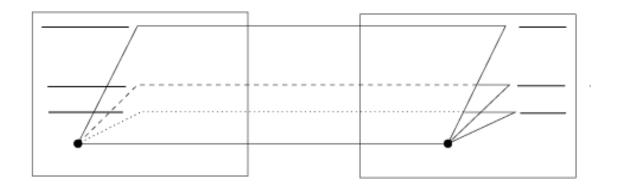


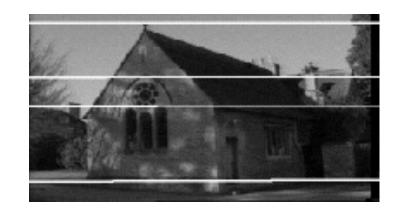


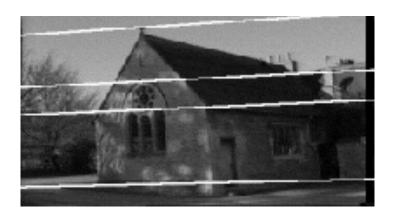
- 1. Compute **E** to get **R**
- 2. Rotate right image by R
- 3. Rotate both images by **Rrect**
- 4. Scale both images by **H**

# When do epipolar lines become horizontal?

#### Parallel cameras

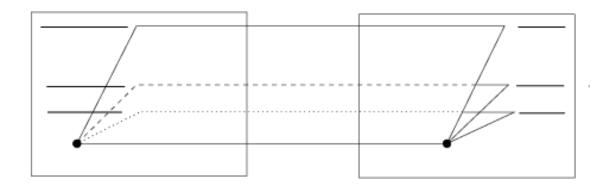


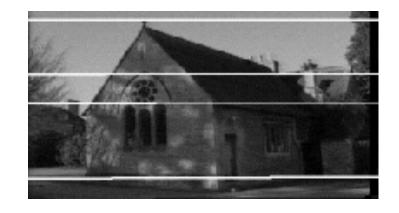


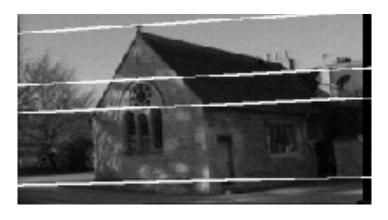


Where is the epipole?

#### Parallel cameras







epipole at infinity

# Setting the epipole to infinity

(Building **R**rect from **e**)

Let 
$$R_{
m rect}=\left[egin{array}{c} m{r}_1^{ op} \ m{r}_2^{ op} \ m{r}_3^{ op} \end{array}
ight]$$
 Given: epipole  $m{e}$  (using SVD on E) (translation from  $m{E}$ )

$$oldsymbol{r}_1 = oldsymbol{e}_1 = rac{T}{||T||}$$
 epipole coincides with translation vector

$$m{r_2} = rac{1}{\sqrt{T_x^2 + T_y^2}} \left[ egin{array}{c} -T_y & T_x & 0 \end{array} 
ight] 
ight. rac{ ext{cross product of e and}}{ ext{the optical axis}}$$

$$\boldsymbol{r}_3 = \boldsymbol{r}_1 \times \boldsymbol{r}_2$$

orthogonal vector

If 
$$m{r}_1 = m{e}_1 = rac{T}{||T||}$$
 and  $m{r}_2$   $m{r}_3$  orthogonal

then 
$$R_{ ext{rect}}oldsymbol{e}_1 = \left[egin{array}{c} oldsymbol{r}_1^ op oldsymbol{e}_1 \ oldsymbol{r}_2^ op oldsymbol{e}_1 \ oldsymbol{r}_3^ op oldsymbol{e}_1 \end{array}
ight] = \left[egin{array}{c} ? \ ? \ ? \end{array}
ight]$$

If 
$$oldsymbol{r}_1 = oldsymbol{e}_1 = rac{T}{||T||}$$
 and  $oldsymbol{r}_2$   $oldsymbol{r}_3$  orthogonal

then 
$$R_{ ext{rect}}oldsymbol{e}_1 = \left[egin{array}{c} oldsymbol{r}_1^ op oldsymbol{e}_1 \ oldsymbol{r}_2^ op oldsymbol{e}_1 \ oldsymbol{r}_3^ op oldsymbol{e}_1 \end{array}
ight] = \left[egin{array}{c} 1 \ 0 \ 0 \end{array}
ight]$$

Where is this point located on the image plane?

If 
$$oldsymbol{r}_1 = oldsymbol{e}_1 = rac{T}{||T||}$$
 and  $oldsymbol{r}_2$   $oldsymbol{r}_3$  orthogonal

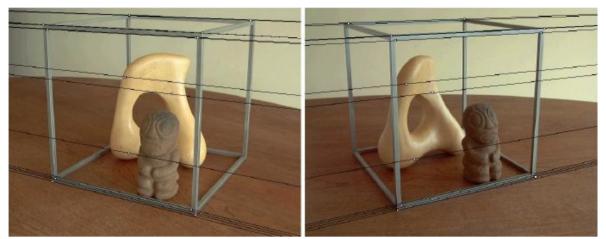
then 
$$R_{ ext{rect}}oldsymbol{e}_1 = \left[egin{array}{c} oldsymbol{r}_1^ op oldsymbol{e}_1 \ oldsymbol{r}_2^ op oldsymbol{e}_1 \ oldsymbol{r}_3^ op oldsymbol{e}_1 \end{array}
ight] = \left[egin{array}{c} 1 \ 0 \ 0 \end{array}
ight]$$

Where is this point located on the image plane?

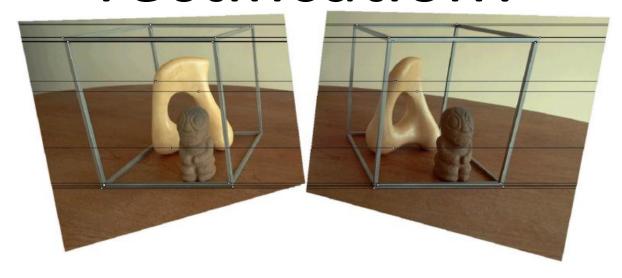
At x-infinity

#### Stereo Rectification Algorithm

- 1. Estimate E using the 8 point algorithm (SVD)
- 2. Estimate the epipole e (SVD of E)
- 3. Build Rrect from e
- 4. Decompose E into R and T
- 5. Set  $R_1=R_{rect}$  and  $R_2=R_{rect}$
- 6. Rotate each left camera point (warp image)  $[x' y' z'] = \mathbf{R}_1 [x y z]$
- 7. Rectified points as  $\mathbf{p} = f/z'[x' \ y' \ z']$
- 8. Repeat 6 and 7 for right camera points using R2



# What can we do after rectification?



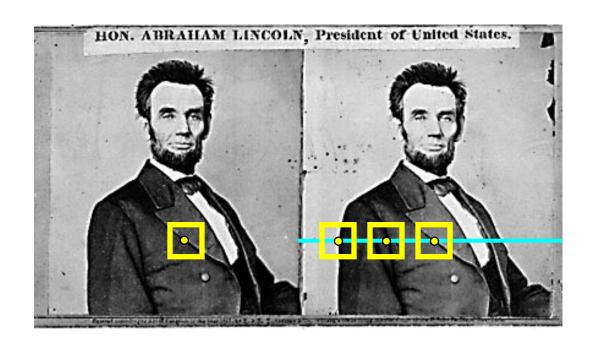
# Stereo matching





Depth Estimation via Stereo Matching





- 1. Rectify images
   (make epipolar lines horizontal)
- 2. For each pixel
  - a. Find epipolar line
  - b. Scan line for best match

c. Compute depth from disparity

$$Z = \frac{bf}{d}$$

How would you do this?

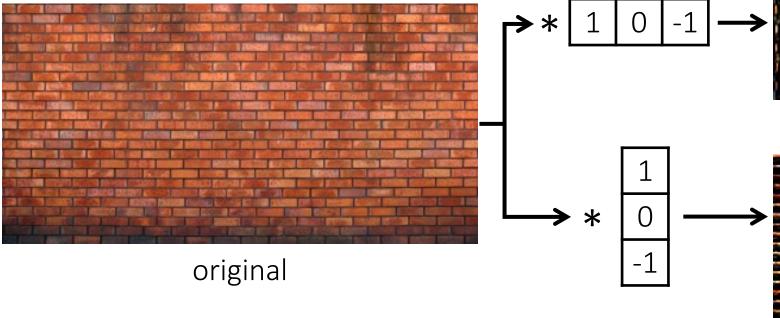
## Reminder from filtering

How do we detect an edge?

### Reminder from filtering

How do we detect an edge?

• We filter with something that looks like an edge.



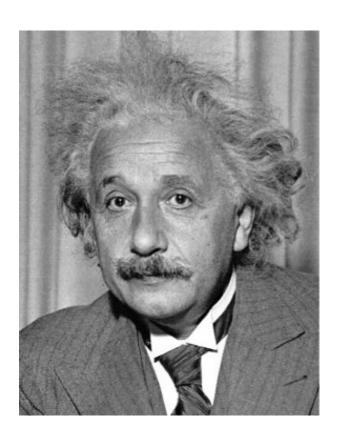
We can think of linear filtering as a way to evaluate how similar an image is *locally* to some template.



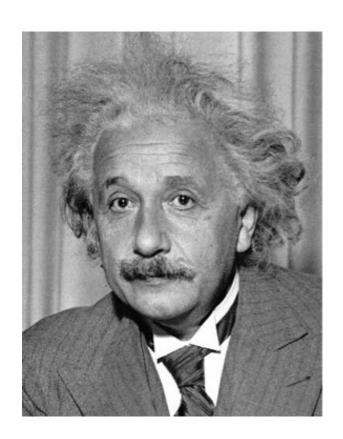
horizontal edge filter

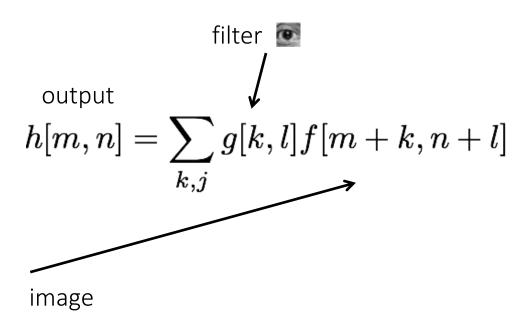


vertical edge filter



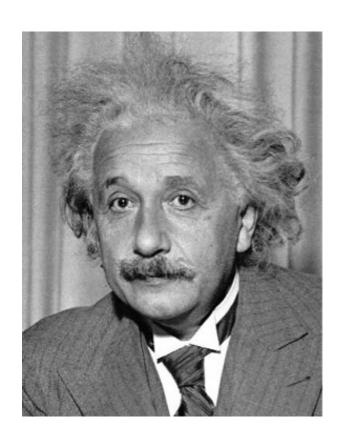
How do we detect the template **m** in he following image?

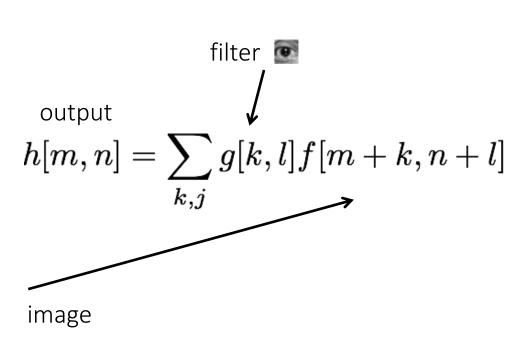


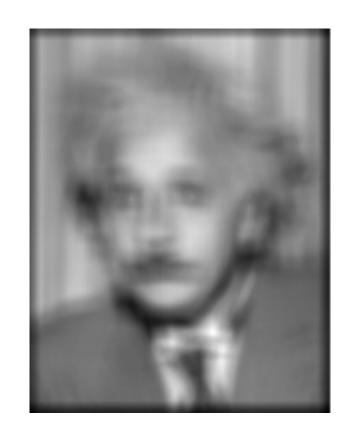


What will the output look like?

Solution 1: Filter the image using the template as filter kernel.

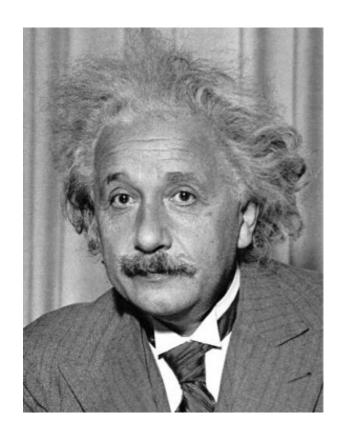


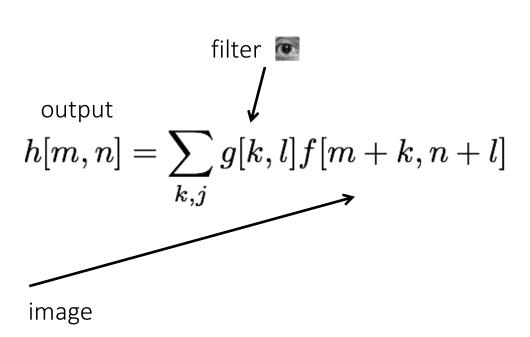




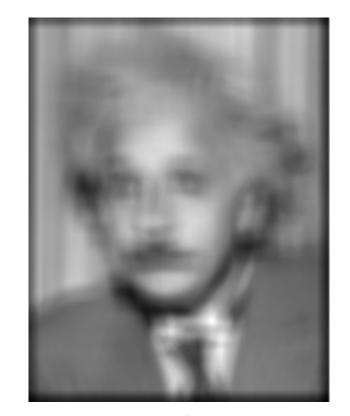
Solution 1: Filter the image using the template as filter kernel.

What went wrong?



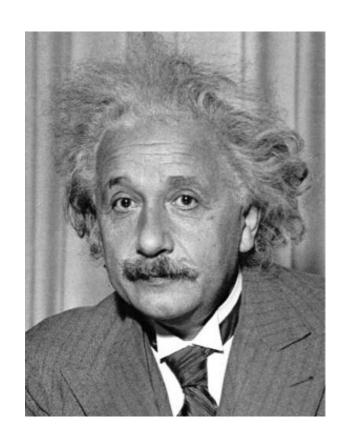


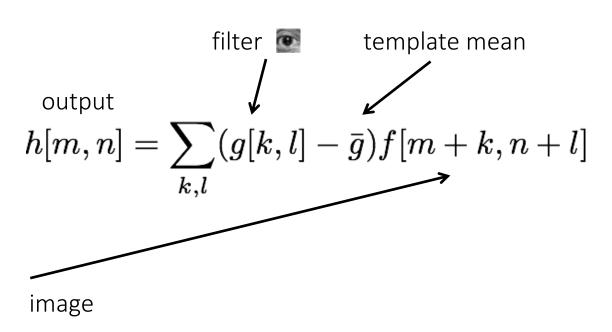
Solution 1: Filter the image using the template as filter kernel.



Increases for higher local intensities.

How do we detect the template **m** in he following image?

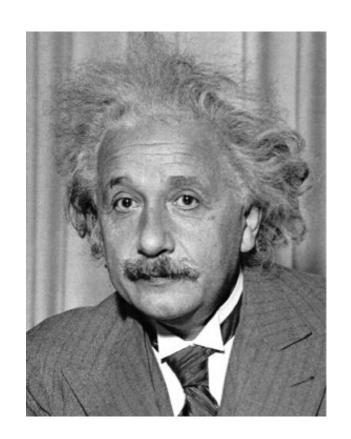




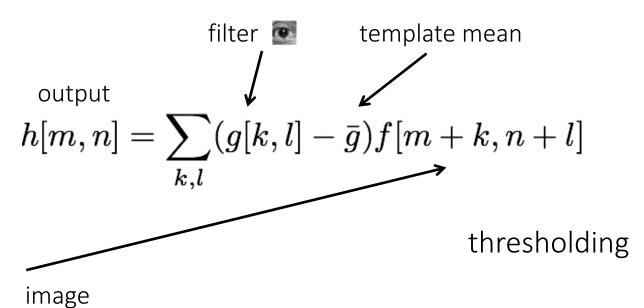
What will the output look like?

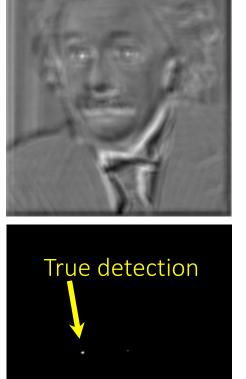
Solution 2: Filter the image using a zero-mean template.

How do we detect the template **m** in he following image?



output





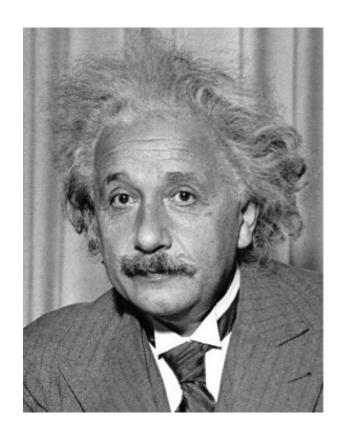
Solution 2: Filter the image using a zero-mean template.

What went wrong?

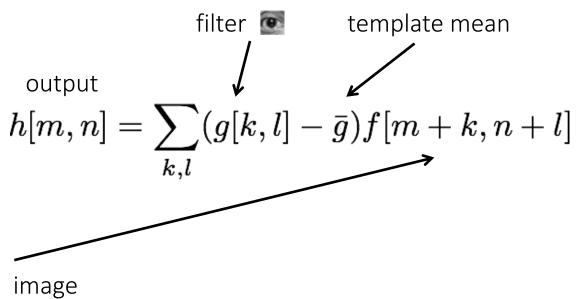
detections

False

How do we detect the template **m** in he following image?



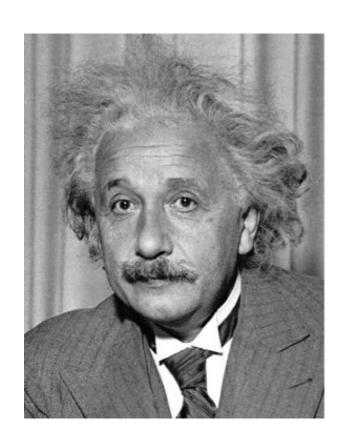
output

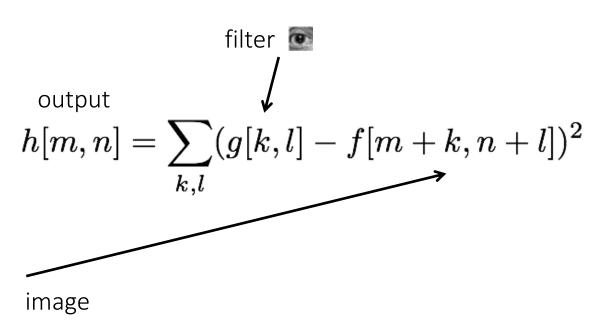


Not robust to highcontrast areas

Solution 2: Filter the image using a zero-mean template.

How do we detect the template **m** in he following image?

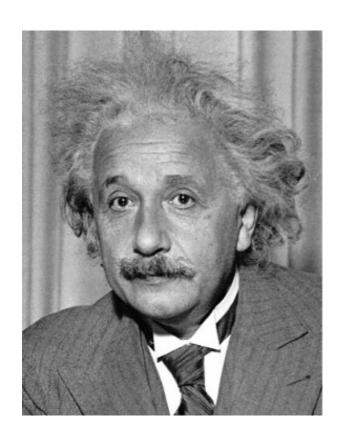




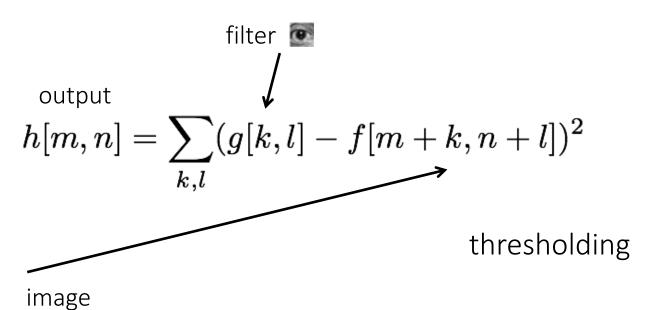
What will the output look like?

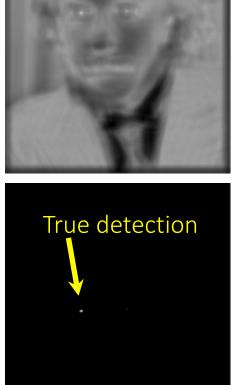
Solution 3: Use sum of squared differences (SSD).

How do we detect the template **m** in he following image?



1-output

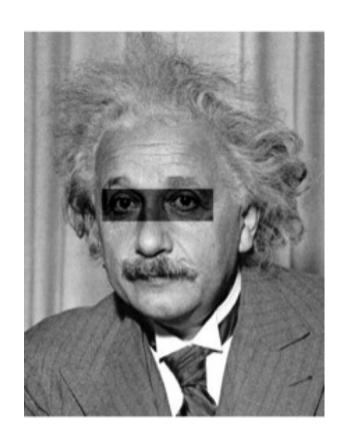




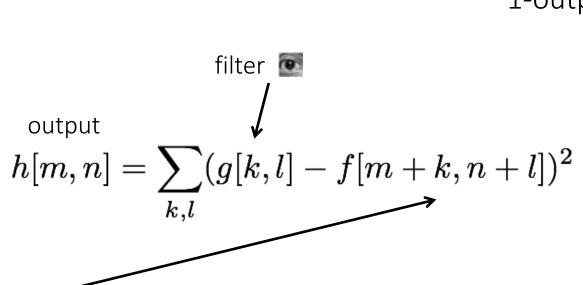
Solution 3: Use sum of squared differences (SSD).

What could go wrong?

How do we detect the template **m** in he following image?



1-output

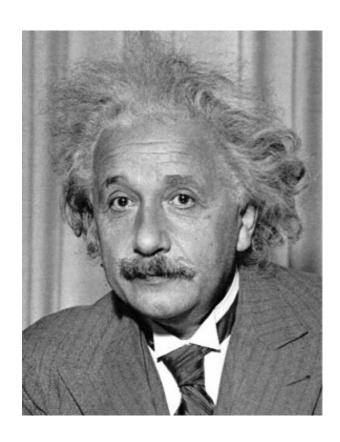


Not robust to local intensity changes

Solution 3: Use sum of squared differences (SSD).

image

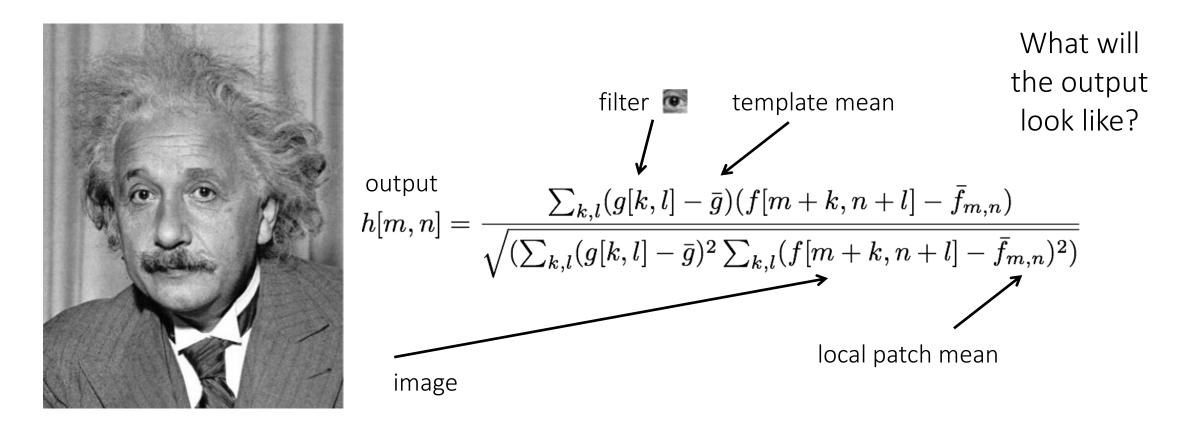
How do we detect the template **m** in he following image?



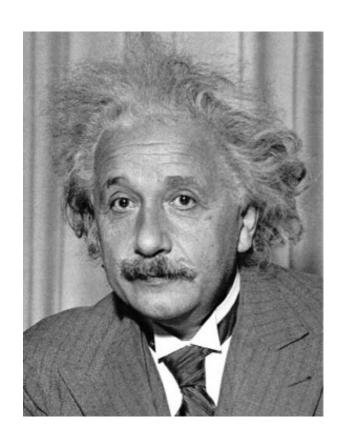
#### Observations so far:

- subtracting mean deals with brightness bias
- dividing by standard deviation removes contrast bias

Can we combine the two effects?



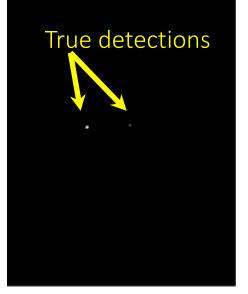
Solution 4: Normalized cross-correlation (NCC).



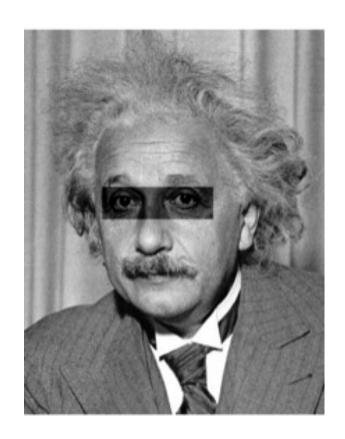
1-output



thresholding



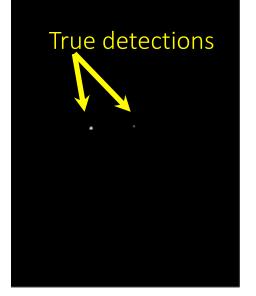
Solution 4: Normalized cross-correlation (NCC).



1-output



thresholding



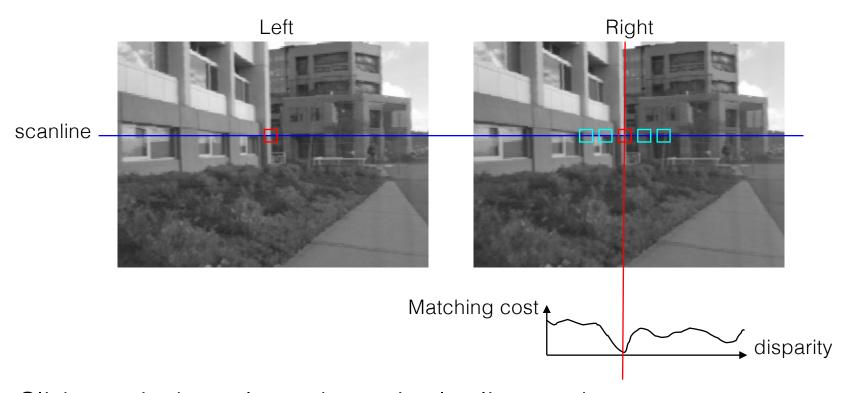
Solution 4: Normalized cross-correlation (NCC).

### What is the best method?

It depends on whether you care about speed or invariance.

- Zero-mean: Fastest, very sensitive to local intensity.
- Sum of squared differences: Medium speed, sensitive to intensity offsets.
- Normalized cross-correlation: Slowest, invariant to contrast and brightness.

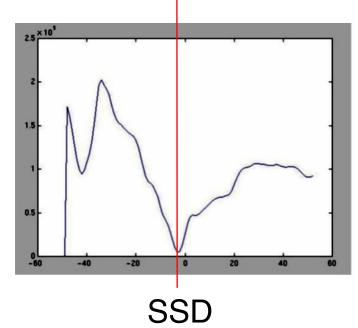
# Stereo Block Matching

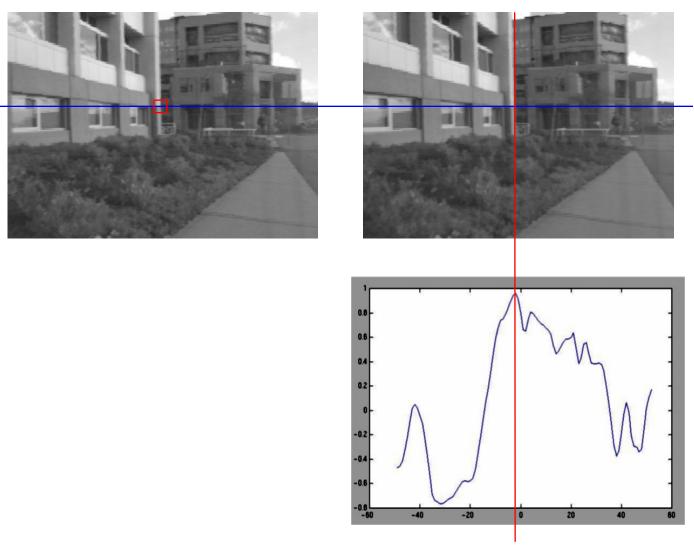


- Slide a window along the epipolar line and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation



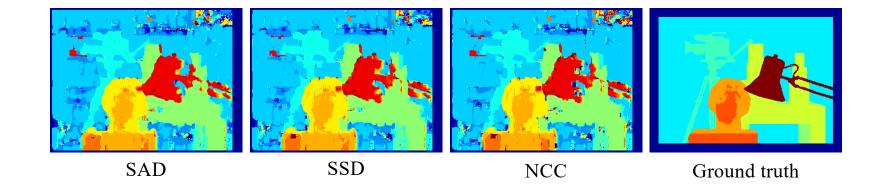






Normalized cross-correlation

Similarity Measure	Formula
Sum of Absolute Differences (SAD)	$\sum_{(i,j)\in W}  I_1(i,j) - I_2(x+i,y+j) $
Sum of Squared Differences (SSD)	$\sum_{(i,j)\in W} (I_1(i,j) - I_2(x+i,y+j))^2$
Zero-mean SAD	$\sum_{(i,j)\in W}  I_1(i,j) - \bar{I}_1(i,j) - I_2(x+i,y+j) + \bar{I}_2(x+i,y+j) $
Locally scaled SAD	$\sum_{(i,j)\in W}  I_1(i,j) - \frac{\bar{I}_1(i,j)}{\bar{I}_2(x+i,y+j)} I_2(x+i,y+j) $
Normalized Cross Correlation (NCC)	$\frac{\sum_{(i,j)\in W} I_1(i,j). I_2(x+i,y+j)}{\sqrt[2]{\sum_{(i,j)\in W} I_1^2(i,j). \sum_{(i,j)\in W} I_2^2(x+i,y+j)}}$

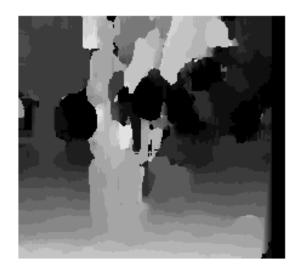


## Effect of window size









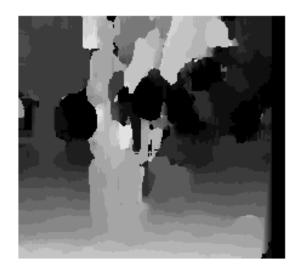
W = 20

## Effect of window size









W = 20

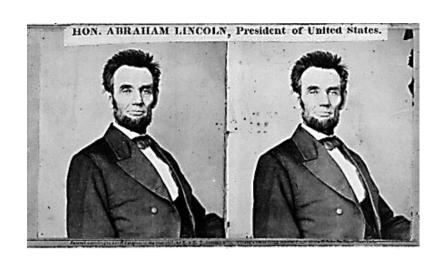
#### **Smaller window**

- + More detail
- More noise

### Larger window

- + Smoother disparity maps
- Less detail
- Fails near boundaries

### When will stereo block matching fail?

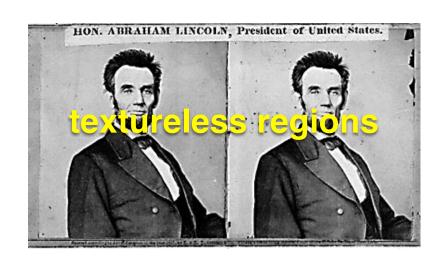








### When will stereo block matching fail?

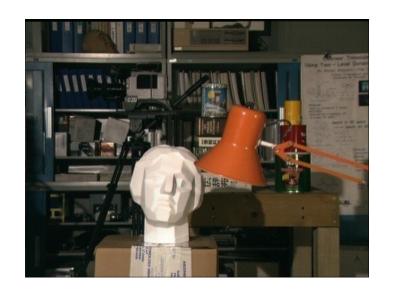




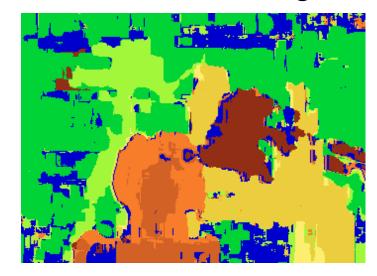




## Improving stereo matching



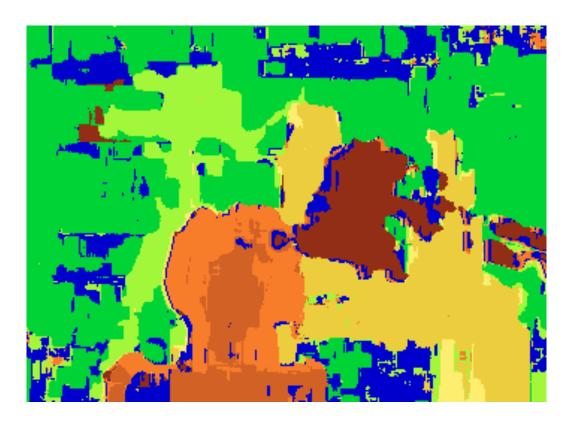
Block matching



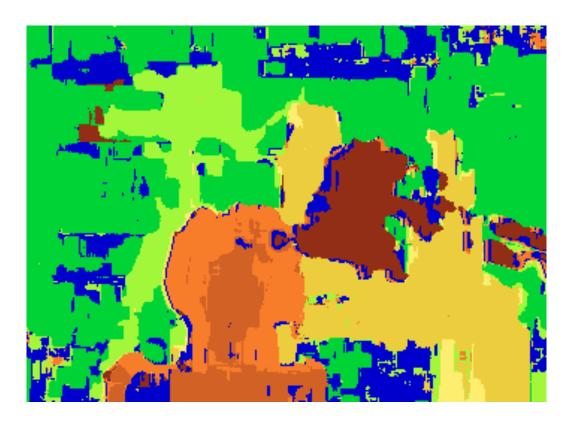
Ground truth



What are some problems with the result?



How can we improve depth estimation?



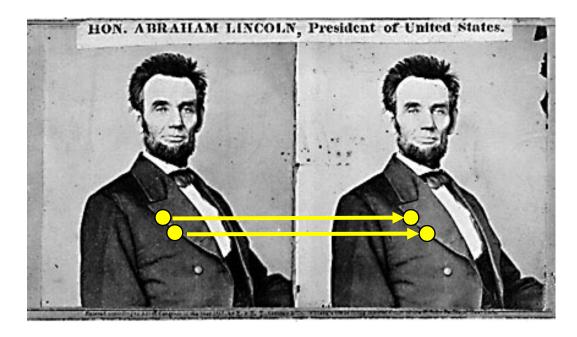
### How can we improve depth estimation?

Too many discontinuities.
We expect disparity values to change slowly.

Let's make an assumption:

depth should change smoothly

# **Energy Minimization**



What defines a good stereo correspondence?

#### 1. Match quality

Want each pixel to find a good match in the other image

#### 2. Smoothness

 If two pixels are adjacent, they should (usually) move about the same amount energy function (for one pixel)

$$E(d) = \underbrace{E_d(d)}_{\text{data term}} + \lambda E_s(d)$$

Want each pixel to find a good match in the other image (block matching result)

Adjacent pixels should (usually) move about the same amount (smoothness function)

$$E(d) = E_d(d) + \lambda E_s(d)$$

$$E_d(d) = \sum_{(x,y) \in I} C(x,y,d(x,y))$$
 data term  $(x,y) \in I$  SSD distance between windows

centered at I(x, y) and J(x+d(x,y), y)

$$E(d) = E_d(d) + \lambda E_s(d)$$

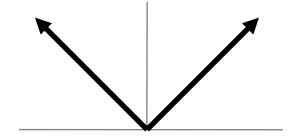
$$E_d(d) = \sum_{(x,y)\in I} C(x,y,d(x,y))$$

SSD distance between windows centered at I(x, y) and J(x+ d(x,y), y)

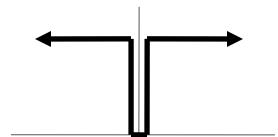
$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p,d_q)$$
 smoothness term  $(p,q) \in \mathcal{E}$  : set of neighboring pixels

$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p,d_q)$$
 smoothness term 
$$(p,q) \in \mathcal{E}$$

$$V(d_p,d_q) = |d_p - d_q|$$
 L<sub>1</sub> distance



$$V(d_p,d_q) = \begin{cases} 0 & \text{if } d_p = d_q \\ 1 & \text{if } d_p \neq d_q \end{cases}$$
"Potts model"



One possible solution...

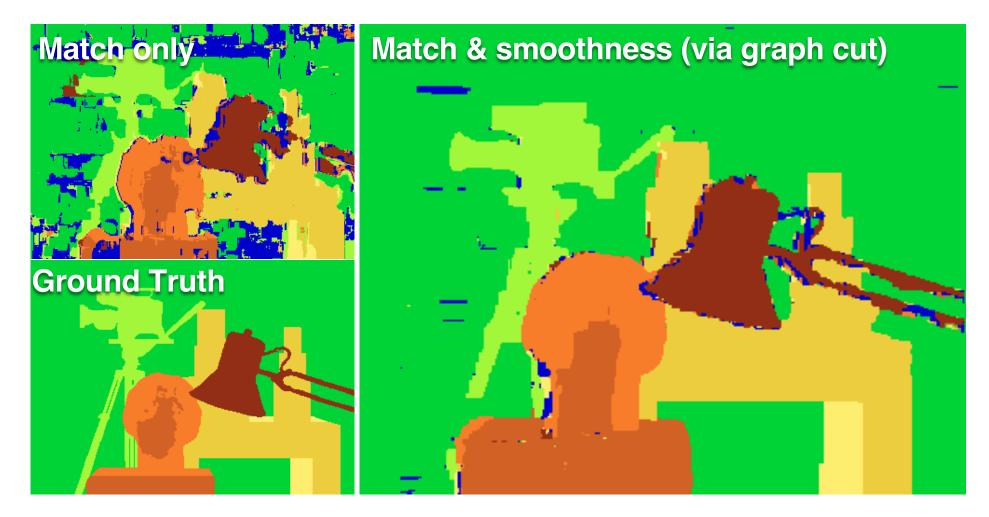
# **Dynamic Programming**

$$E(d) = E_d(d) + \lambda E_s(d)$$

Can minimize this independently per scanline using dynamic programming (DP)

D(x,y,d): minimum cost of solution such that d(x,y) = d

$$D(x, y, d) = C(x, y, d) + \min_{d'} \{D(x - 1, y, d') + \lambda |d - d'|\}$$



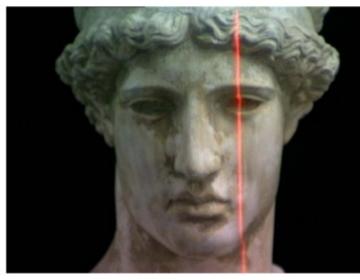
Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001

Structured light

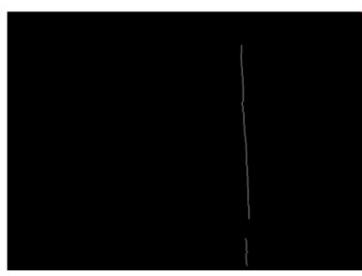
#### Use controlled ("structured") light to make correspondences easier

Disparity between laser points on the same scanline in the images determines the 3-D coordinates of the laser point on object

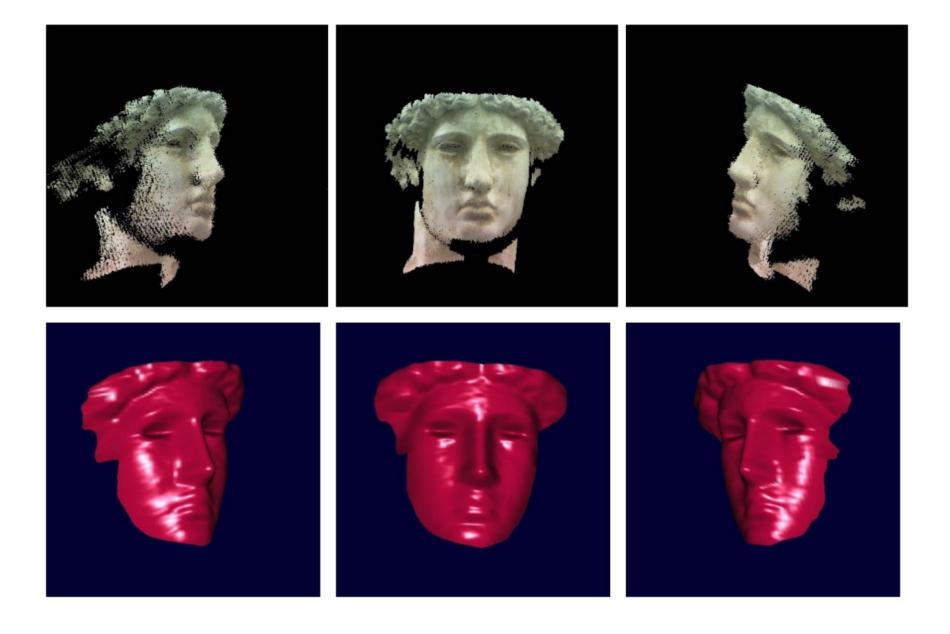




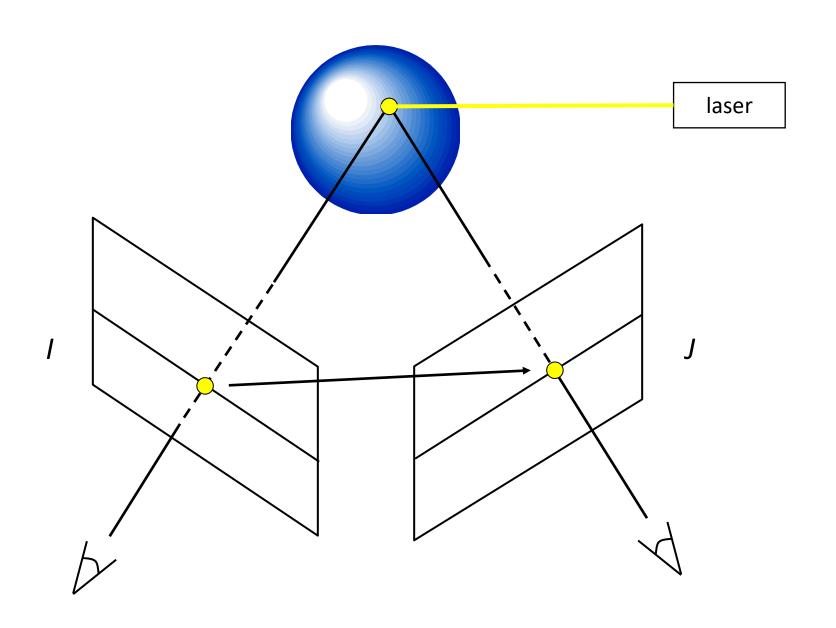




Use controlled ("structured") light to make correspondences easier

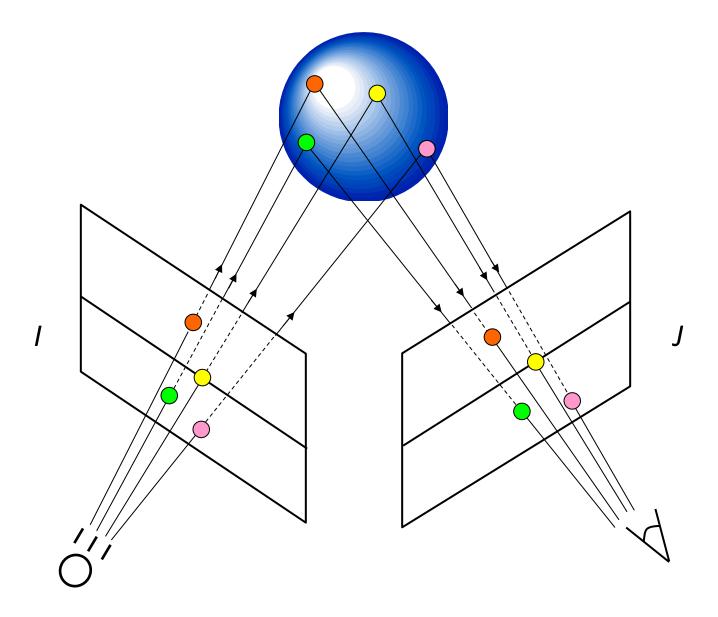


## Structured light and two cameras

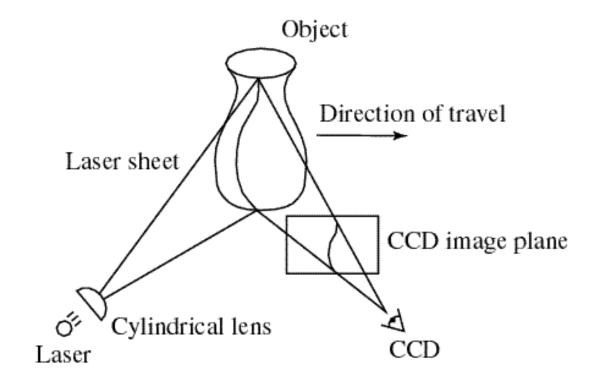


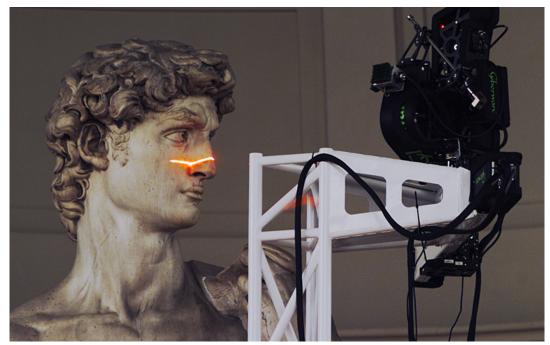
## Structured light and one camera

Projector acts like "reverse" camera



#### Example: Laser scanner





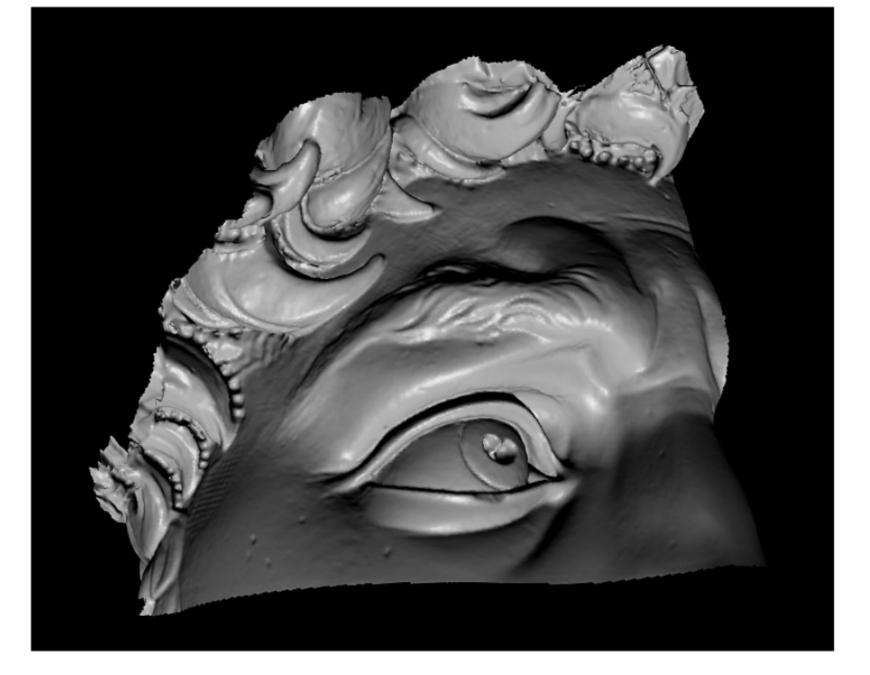
Digital Michelangelo Project <a href="http://graphics.stanford.edu/projects/mich/">http://graphics.stanford.edu/projects/mich/</a>



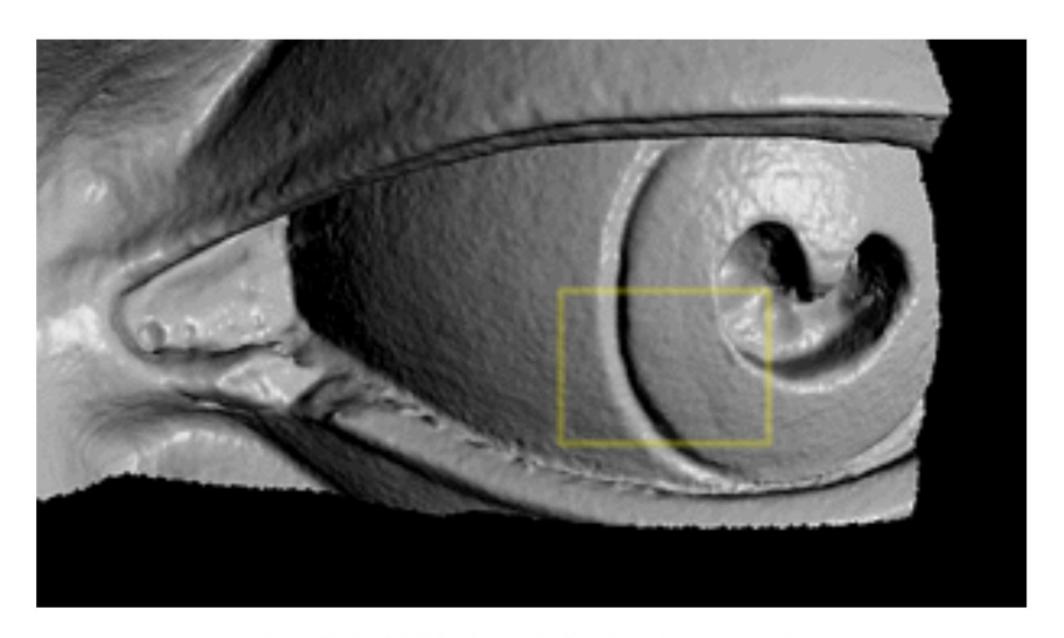
The Digital Michelangelo Project, Levoy et al.



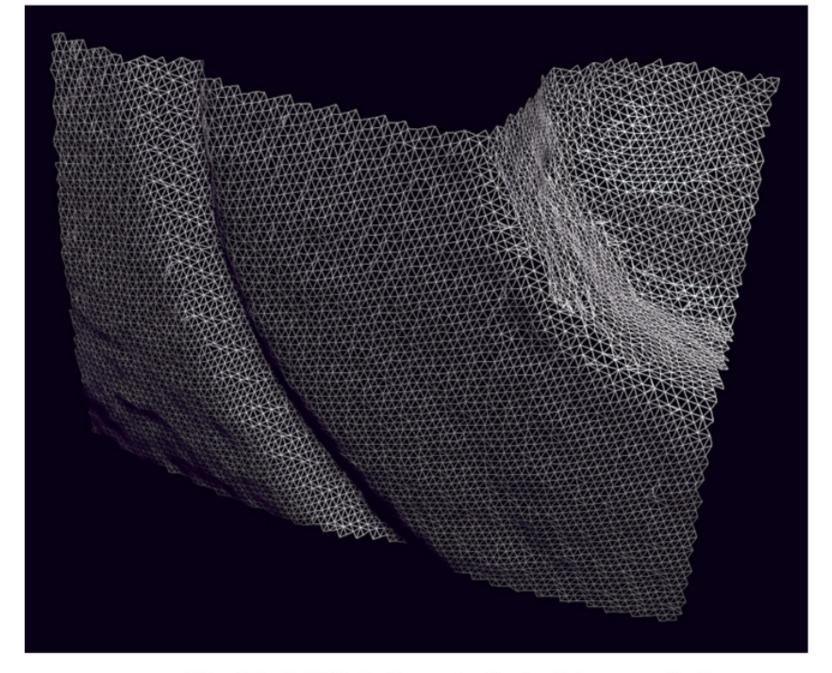
The Digital Michelangelo Project, Levoy et al.



The Digital Michelangelo Project, Levoy et al.



The Digital Michelangelo Project, Levoy et al.



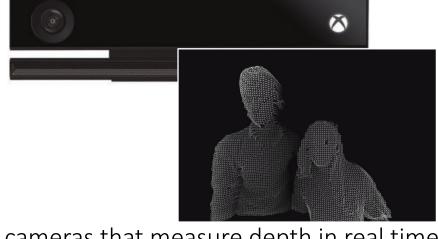
The Digital Michelangelo Project, Levoy et al.

# 15-463/15-663/15-862 Computational Photography

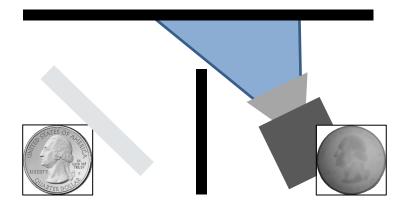
Learn about structured light and other cameras – and build some on your own!



cameras that take video at the speed of light



cameras that measure depth in real time



cameras that see around corners



cameras that capture entire focal stacks

http://graphics.cs.cmu.edu/courses/15-463/